 ICCES, vol.10, no.2, pp.71-76

An algorithm for contact problem with large deformation of plane frame structures

T. Tsutsui[1], H. Obiya[1], K. Ijima[1]

Summary

This study shows an algorithm, which solves the contact phenomenon without friction with large deformation for plane frame structures. Particularly, the study mentions about a technique for the case when contact node slide to the next-door element from edge of the contact element. The technique is to use the re-division and uniting of the element, in order to avoid the computational unstable territory around the edge of the contact element. Furthermore the authors consider about unstable territory of the contact point by some numerical examples.

Introduction

There already exist some investigations[1.2)] around the numerical method of contact problem. The authors, however, don't know the simple and rational calculation process that can simulate node-element contact phenomenon with large deformation. It is necessary for the problem to consider the geometrical nonlinearity and the nonlinearity of the boundary condition at the same time. Further, one difficulty of the contact analysis is how to express the slide of the contact node from one element to next-door element.

In this study, a simple contact element with 3-nodes is proposed, and this element realizes steady convergence even if in case of extremely large deformation. This is why the tangent stiffness method[3)], which is our unique geometrical nonlinear theory, has very strong robustness. However, when a contact node slides close to an edge of an element, the element stiffness matrix becomes singular and the unbalanced forces don't converge. In such a case, if only the element is united with next-door element, stable computation becomes possible. After the contact node has passed through the 'computational unstable territory', the united element should be divided again. Furthermore, this study verify the influence of the element edge force equation to the 'computational unstable territory'. We can define two types of the element edge force equation. One is described in the simple supported coordinate and the other is in the cantilever coordinate. By a result of numerical example, the cantilever coordinate can make the 'computational unstable territory' narrower than the simple supported coordinate.

The tangent stiffness method

Let the vector of the element edge forces independent of each other be indicated by S, and let matrix of equilibrium which relates S to the general coordinate system

[1]Department of Civil Engineering, Saga University, Saga, Saga, 8408502, Japan.

be J. Then the nodal forces D expressed in general coordinate follow the equation:

$$D = JS \tag{1}$$

The tangent stiffness equation is expressed as the deferential of Eq.(1), as follows.

$$\delta D = J\delta N + \delta JN = (K_0 + K_G)\delta d \tag{2}$$

In which, K_0 is the element stiffness which provide the element behavior in element (local) coordinate, and K_G is the tangent geometrical stiffness caused by the rigid body displacement of each element.

Equilibrium condition of contact element with 3-nodes without friction

Fig.1 shows the nodal forces and element edge forces of a contact element in case that the simple supported coordinate is adopted. In this case, the rotation of the contact node is disregarded, and the contact node has two degrees of freedom. The vector of element edge forces independent each other is defined as follows.

$$S = \begin{bmatrix} N & M_i & M_j & Y \end{bmatrix}^T \tag{3}$$

Furthermore, the vector of nodal forces displayed in a general coordinate system to act on both edges of element (i, j)is defined as follows.

$$D = \begin{bmatrix} U_i & V_i & Z_i & U_j & V_j & Z_j & U_c & V_c \end{bmatrix}^T \tag{4}$$

The tangent geometrical stiffness in a contact phenomenon can be acquired by differentiating the equilibrium equation between S and D.

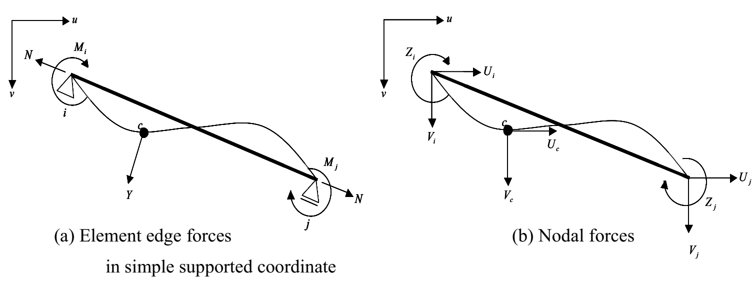
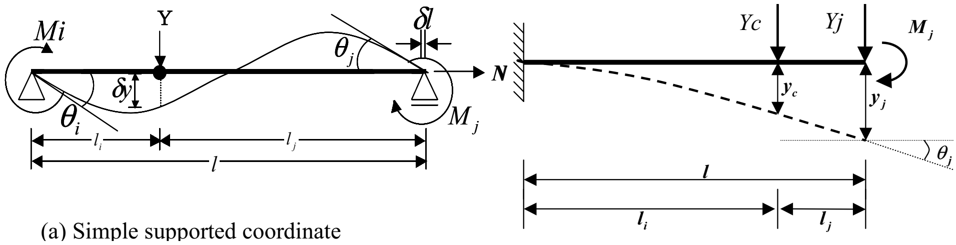


Figure 1: Equilibrium condition on contact element

Two types of definition of the element edge force equation

When the element edge forces and the element deformation are expressed in the simple supported coordinate as Fig.2-(a), the element force equation for the contact problem becomes like eq.(5). On the other hand, When the element edge



(a) Simple supported coordinate

(b) Cantilever coordinate

Figure 2: Element edge forces and element deformations

forces and the element deformation are expressed in the cantilever coordinate as Fig.2-(b), the element force equation for the contact problem becomes like eq.(6).

$$\begin{bmatrix} N \\ M_i \\ M_j \\ Y \end{bmatrix} = \left\{ \frac{3EIl}{l_{i0}l_{j0}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \left(\frac{l_{j0}}{l_0}\right)^2 & -\frac{l_{i0}l_{j0}}{l_0^2} & -\frac{1}{l_{i0}} \\ 0 & -\frac{l_{i0}l_{j0}}{l_0^2} & \left(\frac{l_{i0}}{l_0}\right)^2 & \frac{1}{l_{j0}} \\ 0 & -\frac{1}{l_{i0}} & \frac{1}{l_{j0}} & \left(\frac{l}{l_{i0}l_{j0}}\right) \end{bmatrix} + \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 \\ 0 & \frac{4EI}{l} & \frac{2EI}{l} & 0 \\ 0 & \frac{2EI}{l} & \frac{4EI}{l} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \\ \delta y \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} N \\ M_i \\ M_j \\ Y \end{bmatrix} = \left\{ \frac{3EIl}{l_{i0}l_{j0}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \left(\frac{l_{j0}}{l_0}\right)^2 & -\frac{l_{i0}l_{j0}}{l_0^2} & -\frac{1}{l_{i0}} \\ 0 & -\frac{l_{i0}l_{j0}}{l_0^2} & \left(\frac{l_{i0}}{l_0}\right)^2 & \frac{1}{l_{j0}} \\ 0 & -\frac{1}{l_{i0}} & \frac{1}{l_{j0}} & \left(\frac{l}{l_{i0}l_{j0}}\right) \end{bmatrix} + \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 \\ 0 & \frac{4EI}{l} & \frac{2EI}{l} & 0 \\ 0 & \frac{2EI}{l} & \frac{4EI}{l} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} \delta l \\ \theta_i \\ \theta_j \\ \delta y \end{bmatrix} \quad (6)$$

The handling of slide for contact node

When the target point contacts the segment, the contact element is divided into two elements, because the curvature should be sharp around the contact node. (Fig3.-(a)) When contact node slides toward left in the figure, the element of next door is divided into two elements. (Fig3.-(b)) After that, when the contact node approaches close to an element edge, the unbalanced forces diverge, and element A is united with element B. (Fig3.-(c)) After the contact node have passed through the unstable territory, the united element should be divided into element A and B again. (Fig3.-(d)) Using this calculation process, contact points can go over from an element to another element smoothly.

A computational example of sliding pass over element edge

As shown in Fig.4, M_1 of Torque is loaded step by step on the free edge of a

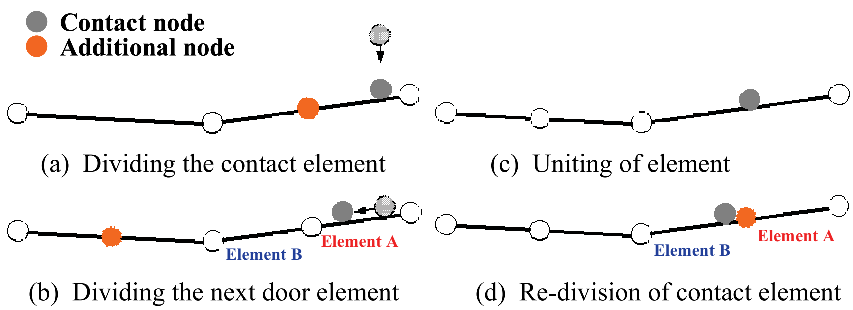


Figure 3: Process of sliding

cantilever. The simple supported coordinate is adopted for the element edge force equation. The contact node (No.20) hits at the element between node7 and node8. (Fig.5-(a)) Next, the contact element is divided and element length has become 1/2. (Fig.5-(b)) Then, the element on the left side of a contact element is divided into two elements. (Fig.5-(c)) If the contact node reaches into 'computational unstable territory' and the unbalanced forces diverge, the calculation restarts form previous converged solution. Further, as shown in Fig.5-(d), two elements are connected and the length of the element becomes twice the previous step. When the contact node slides to 1/2 way of element length, the contact element is divided again. By an example of contact phenomenon between a node and a frame structure, the authors were able to express that a contact node slides across the edge of the contact element.

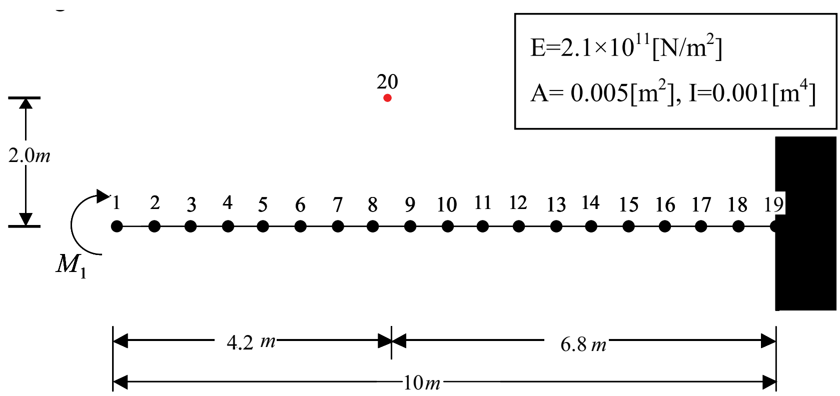


Figure 4: Initial shape of cantilever beam of computational example

Consideration about computational unstable territory

Compulsive displacement is given on a contact node(No.20) as shown in Fig.6. The example is to compare the performance around the 'computational unstable territory' of two coordinate systems. One is the simple supported coordinate, and the other is the cantilever coordinate. Pitch of the compulsive displacement is 0.02m

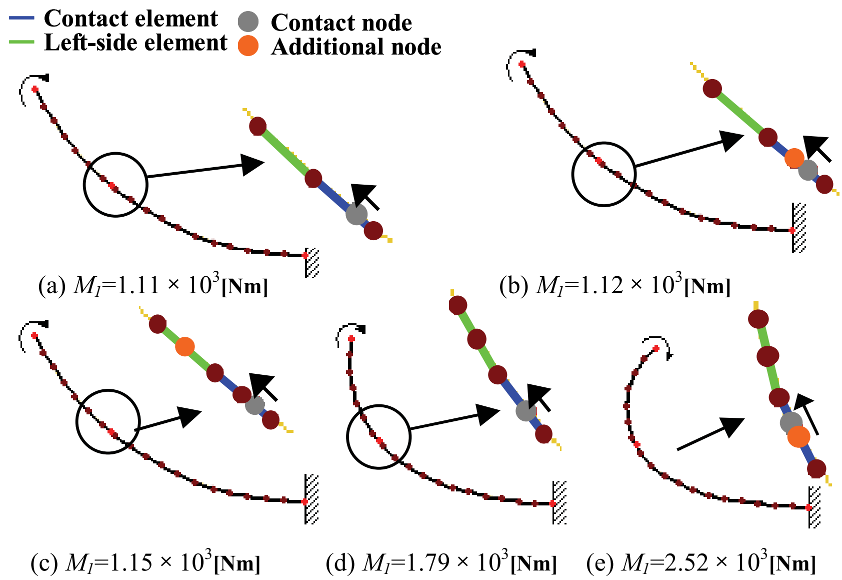


Figure 5: Appearance of transformation in the cantilever beam

toward upper direction and the contact node hits at element between node8 and node7. Calculation is continued until the unbalanced forces diverge. Fig.7 shows relation between the total magnitude of displacement after contact and the li/l. li is the distance between the contact point and the element edge, and l is the element length. (Both are shown in Fig.3,4.) The values in legend of Fig.7 show initial coordinate values of a contact node, and larger horizontal coordinate value means that the contact node is more close to element edge. The circles in the figure show the points where the unbalanced forces have diverged. Namely, the 'computational unstable territory' exists from these circles until li/l =1. Therefore, the cantilever coordinate has quite narrower 'computational unstable territory' than simple supported coordinate.

This example suggests that the cantilever coordinate may provide more rational calculation system for the contact and slide analysis.

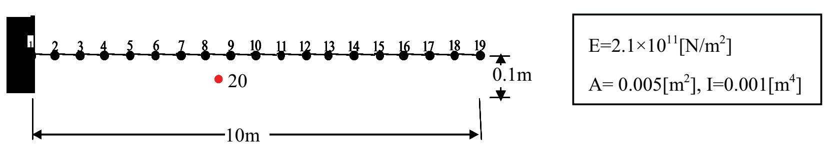


Figure 6: Initial shape of cantilever

References

1. Anders KLARBRING: A Mathematical Programming Approach to Three-

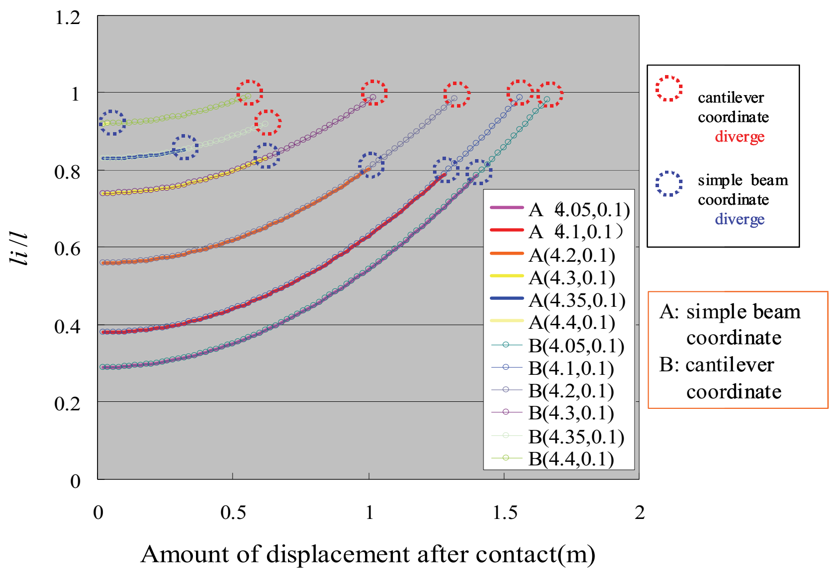


Figure 7: Relation of Li/L and the amount of displacement after contact

Dimensional Contact Problems with Friction Computer Methods in Applied Mechanics and Engineering 58, pp. 175-200 (1986).

2. Xian CHEN: Sensitivity Analysis for Frictional Contact Problems with Large Deformation The Japan Society of Mechanical Engineers 65, pp. 1859-1866 (1999)

3. Shigeo GOTO, Goro HANE and Tatsuro TANAKA: Tangent Stiffness Method for Large Deformation Analysis of Frame Structure Proc. of JSCE, No.238, pp. 31-42(1975).