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Ankle Joint Model with Applications in Extreme Situations of Ligament Traumas

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Introduction

From biomechanics point of view, the ankle joint represents a structural and functional unity through witch the propulsion forces during gait, run and jump are transmitted to the foot and, in the same time, takes over part of the shock produced by the impact with ground and transmits them to the muscular structures to be absorbed.

Currently, the literature approaches the human locomotion apparatus mainly from four perspectives, as follows:

- from the resestive structure of the component tissues;
- from cinematic point of view, focussing on the movement characteristics, in the absence of the dynamic demands which initiate and maintain the movement;
- from dynamic perspective, analysing the movement characteristics (movements, speeds, accelerations) and also the dynamic characteristics (forces, moments, powers, muscular loads, etc.) appeared during the movement of the human body;
- from experimental point of view, with focus either on the experimental techniques or on the measured values and analyse them regarding the numeric simulations.

The theoretical or experimental models are analysed either for the normal locomotion apparatus, without deficiencies, or for the one with deficiencies (fractures, sprain, wrench, ligament traumas, etc.), in post-traumatic cases.

The biomechanics models are created, in many situations, as a response to same demands, as follows:

- medical ones, as joint or bones deficiencies, having pathologic causes or as a consequence of accidents requiring surgical or kinesio-therapeutical treatments;
- sportive ones, as the muscle-bones biomechanics parameters, which could be modified in order to improve, mainly, the motion performances;

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- technical ones, aiming to realize equipments used for medical or sportive purposes - for training, and also to develop robots as clouse as possible to the human body from the functional point of view.

The aim of the paper is to present a simplified static model of the ankle joint, corrected and validated using electromiographical experimental measurements, to analyse the limit situations leading to traumas of the lateral ligaments, calcaneo–fibular and deltoidian or Achilles tendon (not so frequently).

The considered structural model has three liberty degrees, as the spherical joint type, but with the hypothesis that the demands in the limiting situations mentioned above are mainly a consequence of the eversion- inversion motion and plantar-dorsal flexion motion.

Mechanical Model of the Ankle Joint

Anatomically, at the foot there are two big articulations: an upper articulation, formed by the inferior extremities of the shank bones (tibia and fibula) and the upper face of the talus, and a inferior joint, formed by talus and calcaneus. First of these joints allows the motion of flexion-extension of the foot (also called dorsal flexion-plantar flexion), and the second allows the motion of pronation-supination (a motion composed by flexion–extension and abduction–adduction). The relative position between the bones of ankle joint is maintained by ligaments, mainly by the calcaneo–fibular, talo–fibular and deltoidian ligaments (three fascicles, between tibia and talus, calcaneus and navicular), partially shown in figure 1. Realization of plantar and dorsal flexion motion, necessary to propel the body, including when jumping from run, is done, mainly, with the help of triceps sural muscle (formed by the muscles gastrocnemius and soleus), the most voluminous muscle of the shank, inferior inserted on calcaneus through the Achilles tendon. A simplified physical model of the ankle joint, where are represented the ligaments and the Achilles tendon, is shown in figure 2.

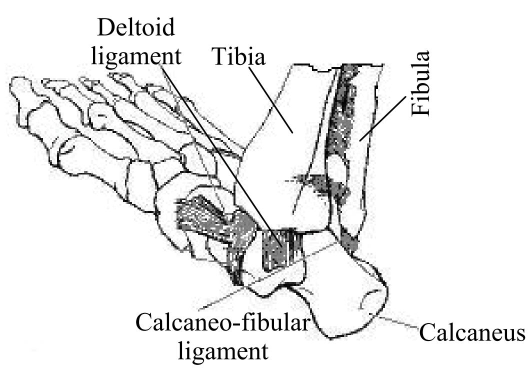


Figure 1: Ligaments of the ankle joint

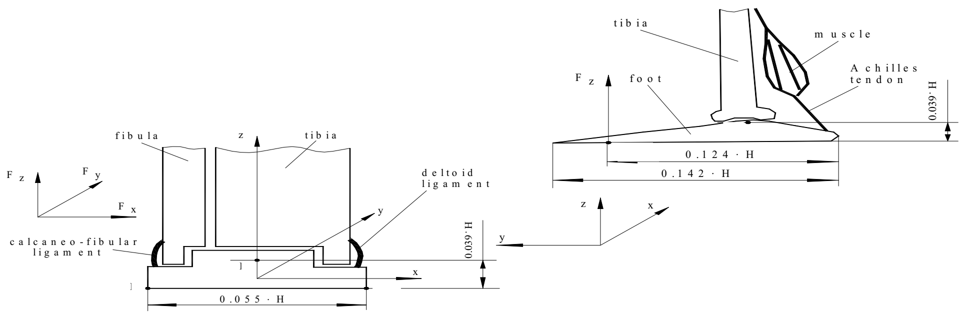


Figure 2: Physical model of the ankle joint

The mechanic model of the foot is presented in figure 3. The body mass was noted with M and g is the gravitational acceleration. In case of a “wrong step”, the foot reaches the position presented in figure 4, when all the support is in point A, Achilles tendon, BC and calcaneo–fibular ligament, DE being elongated. In order to write the equilibrium equations, one has to determine the coordinates of the points A, B, C, D, E, P and H. For this purpose, a reference having the origin in the spherical articulation (figure 4) has to be considered. To determine the coordinates of the points A, B, D and P an auxiliary reference point $Ox'_1y'_1z'_1$ has to be firstly considered, rotated around the Oy axes with an α angle, and after that a new rotation take place, around the Ox'_1 axes, with a β angle.

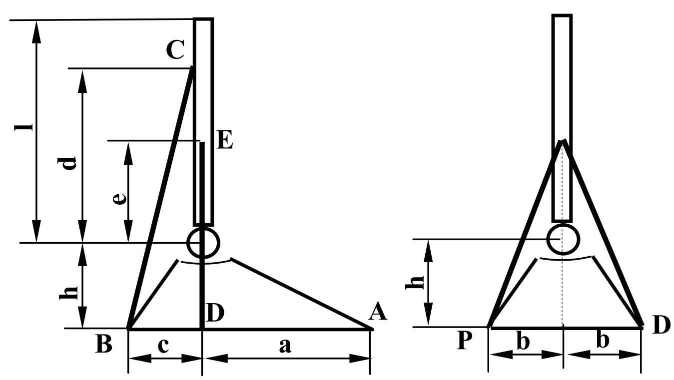


Figure 3: The mechanic model of the ankle joint

To determine the coordinates of the points A, B, D and P, a second auxiliary reference has to be considered$Ox'_2y'_2z'_2$. After performing all the mathematical calculations, the following position vectors are found:

$$\vec{r}_A = (a\cdot\cos\alpha - b\cdot\sin\alpha\cdot\sin\beta - h\cdot\sin\alpha\cdot\cos\beta)\vec{i} + (-b\cdot\cos\beta + h\cdot\sin\beta)\vec{j} + (-a\cdot\sin\alpha - b\cdot\cos\alpha\cdot\sin\beta - h\cdot\cos\alpha\cdot\cos\beta)\vec{k} \quad (1)$$

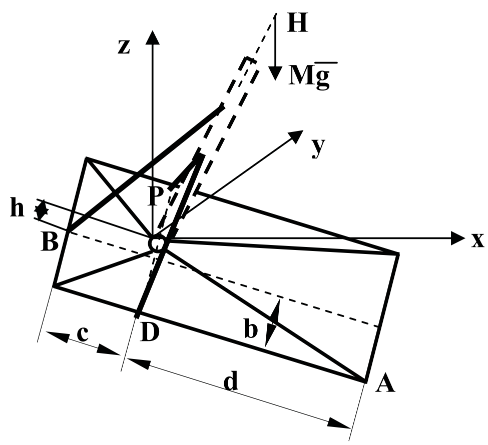


Figure 4: Position in the “wrong step” case

$$\vec{r}_B = (-c\cdot\cos\alpha - h\cdot\sin\alpha\cdot\cos\beta)\vec{i} + (h\cdot\sin\beta)\vec{j} + (c\cdot\sin\alpha - h\cdot\cos\alpha\cdot\cos\beta)\vec{k} \tag{2}$$

$$\begin{aligned}\vec{r}_D = &(-b\cdot\sin\alpha\cdot\sin\beta - h\cdot\sin\alpha\cdot\cos\beta)\vec{i} + (-b\cdot\cos\beta + h\cdot\sin\beta)\vec{j}\\ &+(-b\cdot\cos\alpha\cdot\sin\beta - h\cdot\cos\alpha\cdot\cos\beta)\vec{k}\end{aligned} \tag{3}$$

$$\begin{aligned}\vec{r}_P = &(b\cdot\sin\alpha\cdot\sin\beta - h\cdot\sin\alpha\cdot\cos\beta)\vec{i} + (b\cdot\cos\beta + h\cdot\sin\beta)\vec{j}\\ &+(b\cdot\cos\alpha\cdot\sin\beta - h\cdot\cos\alpha\cdot\cos\beta)\vec{k}\end{aligned} \tag{4}$$

Two successive rotation movements of the reference $Oxyz$are to be considered in order to determine the position vectors of the E, C and H. Firstly, reference $Ox'_3y'_3z'_3$ will be considered, obtained by rotating the reference $Oxyz$ around axes 0y with a θ angle, and then the reference is rotated around Ox'_3axes with a φangle, meaning overlapping the reference $Ox'_4y'_4z'_4$ where Ox'_3 and Ox'_4 are identical. After performing all the calculations, one will determine:

$$\vec{r}_C = (d\cdot\cos\varphi\cdot\sin\theta)\bar{i} + (d\cdot\sin\varphi)\bar{j} + (d\cdot\cos\varphi\cdot\cos\theta)\bar{k} \tag{5}$$

$$\vec{r}_H = (l\cdot\cos\varphi\cdot\sin\theta)\bar{i} + (l\cdot\sin\varphi)\bar{j} + (l\cdot\cos\varphi\cdot\cos\theta)\bar{k} \tag{6}$$

The static equilibrium

Assuming that the foot reached the equilibrium position with known α, β, θ, φ angles, our aim to determine the forces appeared in the joint, in the Achilles tendon and in the elongated calcaneo–fibular side ligament. Fe_1 and Fe_2 are the two forces, N is the normal reaction from the contact point and F_f is the friction force. The isolation of the corps method will be used, considering separately the foot, and the tibia and peroneus bones (figure 5 and figure 6).

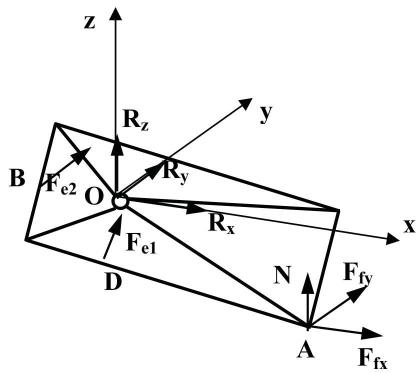


Figure 5: The forces of the foot segment

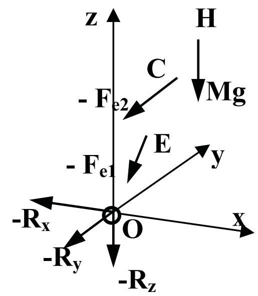


Figure 6: Forces of the shank segment

The scalar equations for the equilibrium forces for the two bodies are:

$$\begin{aligned} F_{e1x}+F_{e2x}+R_x+F_{fx} &= 0 \\ F_{e1y}+F_{e2y}+R_y+F_{fy} &= 0 \\ F_{e1z}+F_{e2z}+R_z+N &= 0 \\ -F_{e1x}-F_{e2x}-R_x &= 0 \\ -F_{e1y}-F_{e2y}-R_y &= 0 \\ -F_{e1z}-F_{e2z}-R_z-M\cdot g &= 0 \end{aligned} \tag{7}$$

Results: F_{fx}=0, F_{fy}=0 and $N = M \cdot g$, showing that no tendency of sliding appears. To write the equations of the equilibrium of moments, one will calculate firstly the moments related with the O pole of the forces acting on the first body:

$$\vec{M}_0(N) = \bar{r}_A \times \vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ 0 & 0 & N \end{vmatrix} = y_A N\vec{i} - x_A N\vec{j}, \tag{8}$$

where x_A, y_A and z_A are the coordonates of the point A,

$$\begin{aligned} \vec{M}_0(\vec{F}e_1) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_D & y_D & z_D \\ Fe_{1x} & Fe_{1y} & Fe_{1z} \end{vmatrix} \\ &= (y_D Fe_{1z} - z_D Fe_{1y})\vec{i} + (z_D Fe_{1x} - x_D Fe_{1z})\vec{j} + (x_D Fe_{1y} - y_D Fe_{1x})\vec{k} \end{aligned} \tag{9}$$

where x_D, y_D and z_D are the coordonates of the point D,

$$\begin{aligned} \vec{M}_0(\vec{F}e_2) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_B & y_B & z_B \\ Fe_{2x} & Fe_{2y} & Fe_{2z} \end{vmatrix} \\ &= (y_B Fe_{2z} - z_B Fe_{2y})\vec{i} + (z_B Fe_{2x} - x_B Fe_{2z})\vec{j} + (x_B Fe_{2y} - y_B Fe_{2x})\vec{k} \end{aligned} \tag{10}$$

The following scalar equations for the equilibrium of moments are resulting:

$$\begin{aligned} y_A N + y_D Fe_{1z} - z_D Fe_{1y} + y_B Fe_{2z} - z_B Fe_{2y} &= 0 \\ -x_A N + z_D Fe_{1x} - x_D Fe_{1z} + z_B Fe_{2x} - x_B Fe_{2z} &= 0 \\ x_D Fe_{1y} - y_D Fe_{1x} + x_B Fe_{2y} - y_B Fe_{2x} &= 0 \end{aligned} \tag{11}$$

For the second body, the forces moments are:

$$\begin{aligned} \vec{M}_0(-Fe_1) &= \vec{r}_E x \vec{F}e_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_E & y_E & z_E \\ -Fe_{1x} & -Fe_{1y} & -Fe_{1z} \end{vmatrix} \\ &= (-y_E Fe_{1z} + z_E Fe_{1y})\vec{i} + (-z_E Fe_{1x} + x_E Fe_{1z})\vec{j} + (-x_E Fe_{1y} + y_E Fe_{1x})\vec{k} \\ \vec{M}_0(-Fe_2) &= \vec{r}_C x \vec{F}e_2 \\ &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_C & y_C & z_C \\ -Fe_{2x} & -Fe_{2y} & -Fe_{2z} \end{vmatrix} \\ &= (-y_C Fe_{2z} + z_C Fe_{2y})\vec{i} + (-z_C Fe_{2x} + x_C Fe_{2z})\vec{j} + (-x_C Fe_{2y} + y_C Fe_{2x})\vec{k} \\ \vec{M}_0(\vec{M}_g) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_H & y_H & z_H \\ 0 & 0 & -M \cdot g \end{vmatrix} \\ &= -y_H \cdot M \cdot g \cdot \vec{i} + x_H \cdot M \cdot g \cdot \vec{j} \end{aligned}$$

The scalar equations for the balance of forces for the second body are:

$$\begin{aligned} -y_E \cdot Fe_{1z} + z_E \cdot Fe_{1y} - y_C \cdot Fe_{2z} + z_C \cdot Fe_{2y} - y_H \cdot M \cdot g &= 0 \\ -z_E \cdot Fe_{1x} + x_E \cdot Fe_{1z} - z_C \cdot Fe_{2x} + x_C \cdot Fe_{2z} + x_H \cdot M \cdot g &= 0 \\ -x_E \cdot Fe_{1y} + y_E \cdot Fe_{1x} - x_C \cdot Fe_{2y} + y_B \cdot Fe_{2x} &= 0 \end{aligned} \tag{12}$$

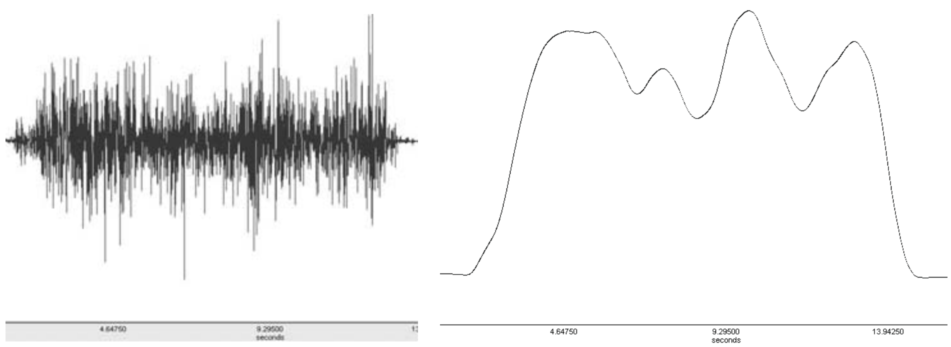
The system of scalar equilibrium equations written for the whole system contains 9 equations with the following 9 unknown parameters: Fe_{1x}, Fe_{1y}, Fe_{1z}, Fe_{2x}, Fe_{2y}, Fe_{2z}, R_x, R_y, R_z, N, F_{fx}, F_{fy}, where the three last parameters are know.

Experimental determinations

To test and correct the analytic model presented above, different experimental measurements were performed, the tension variation in the Achilles tendon being determined on electromiographyc way.

The tests were performed on eleven volunteers, the electromiographyc activity of the gastronomies muscle being recorded in “standing” position, supported on the whole foot, and supported on toes, with the foot in plantar flexion. Figure 7 presents a sequence from the electromiographyc tests, and figure 8a and 8b present one of electromiographyc records obtained and the muscle tension obtained by integrated the electric signal. The variation of the muscle tension during plantar or dorsal flexion is the same with the force variation in Achilles tendon, so the comparison between the experimental records and the graphic obtained after the numeric solution of the analytical model can be made.

Figure 7: A sequence from the electromiographyc tests



a. **b.**

Figure 8: a. Variation of the electromiographyc signal in plantar flexion; b. Variation of the gastronomies muscle in plantar flexion

To make the proper correlations in the analytical model, using, for instance, the non-linear correction coefficients, one has to know the numerical values of the force in Achilles tendon. Due to the fact that the electromiographyc tests only supplied relative and not absolute values, one can compare the interpolation function, determined on very tight temporal domains (0.01 or 0.001 seconds), for the graphics of force from Achilles tendon, namely the one resulted from the experimental measurements compared with the one resulted from the numeric solution of the model.

Conclusions

The proposed analytic model offers the possibility to perform numeric simulations for disadvantage cases of eversion and inversion motions, leading to breaking of the ligaments of the lateral ligaments. In the same time, the mechanic demands from the ankle joint, considered as a spherical joint, can be determined, when the human body is in static equilibrium, knowing that the joint tension is important characteristic in post-traumatically rehabilitation.

The analytic model of the ankle joint can be used also for simulation regarding the motion degree, the amplitude of the joint motion, respectively, having direct applications in sport. Performance in sport is often obtained by increasing the joint mobility, due to ligament elasticity determined by training.

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