# Modal Analysis Technique for Anisotropic Composite Laminates

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## Summary

Nowadays, the experimental modal analysis in composite materials is an important tool in the structural analysis of new designs in aircraft structures. It supplies data on the behavior of these materials and, when associated with numerical methods, it can also be used to identify elastic properties. However, lightweight composite materials demand the use of appropriate techniques and devices. This paper describes an experimental modal analysis technique where the response is measured without physical contact in a large number of points using a Laser Doopler Vibrometer (LDV), and the excitation is carried out on a single point by an electromechanical shaker.

### Introduction

In recent years there has been an increasing interest in composite materials, due to application in many engineering fields. These materials have particular properties and characteristics that other commonly used engineering materials lack. As there are numerous possible component combinations and arrangements, designers and engineers can meet the specific needs of a particular design.

However, the anisotropic behavior of composite materials makes their computational modeling more complex [1]. The anisotropy increases the number of independent elastic properties. As a consequent, it is more difficult to identify and determine the values of these properties in experimental tests.

The elastic properties of composite materials can be characterized through dynamic or static tests.

Static tests are more simple, and they were the most known and most used for many years. However, despite the simplicity of static tests, they have disadvantages, e.g. the fact that they are destructive and require a number of samples with fiber orientations specified by standards that are seldom coincident with directions demanded in the design [2]. Furthermore, some variables can hardly be controlled during tests and can contribute to the spread in experimental results, for example the clamped boundary conditions may introduce non-uniform stress fields that mask the test results. These aspects turn static tests less attractive for composites.

An alternative approach for evaluating elastic constants is a combination of experimental modal analysis and numerical methods. This type of approach allows the identification of elastics constants using only one sample or even the real com-

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posite structure. These elastic constants represent global values an therefore, offer the advantage of that are not influenced by local small imperfections.

In this type of tests, samples are usually thin plates that follow Kirchhoff's hypotheses, cylindrical shells, or beams. The modal analysis is required to supply the input data for the numerical method that evaluate the elastic constants. Generally, the input data of numerical methods are natural frequencies and modal shapes. Due to numerical reasons, first modes associated to lower frequencies are preferred. Many authors have proposed to evaluate elastic constants by iterative procedures using the Rayleigh-Ritz method [3-5] and finite elements [6-8].

Grediac and Paris [8] proposed an alternative approach that directly identifies the elastic parameters without using an iterative procedure. In this case, natural frequencies and mode shapes are input data to solve a linear system based on the equation that governs the transversal vibrations of the anisotropic thin plates under free response. However, because this method requires the computation of second order spatial derivatives of modes shape, these results are strongly dependent of the noise level of the experimental mode shapes. Hence, it is very difficult to use this method with experimental data obtained from standard modal analysis procedures. The experimental noise level must be reduced in order to overcome difficulties in the numerical computation of spatial derivatives of mode shape.

As composite laminates are light structures the use of accelerometers are not indicated to obtain the shape modes, provided that their mass can be confused with the mass of the plate. Furthermore, if a large number of measuring points are required, this procedure will not be feasible.

This paper presents an experimental procedure to perform modal analysis combined with numerical methods in thin lightweight composite plates, using a LDV, measuring dynamic responses without physical contact and obtaining accurate mode shapes and natural frequencies.

### **Experimental Modal Analysis**

The governing equation of transversal vibration in thin anisotropic plates is given by [8]:

$$D_{11}\frac{\partial^4 w}{\partial x^4} + D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2\left(D_{12} + D_{66}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2}$$

where  $D_{ij}$  are Kirchhoff's plate bending stiffness constants;  $\rho$  is the mass density of material; *h* is the plate thickness; *x* and *y* are coordinates of the plate; t is time; and w(x, y, t) is the deflection that represents the transversal displacement of a point the plate at an instant *t*.

In order to obtain the anisotropic plate  $D_{ij}$  stiffness constants, the required modes and natural frequencies are evaluated experimentally and are used as input

data of numerical methods to solve Equation (1), see Batista et al.[6]. The numerical methods that were used in these calculations were a finite element commercial software (ANSYS 7.0) and a boundary element code using the radial integration method, see Albuquerque et al.[9].

In the experimental modal analysis an orthotropic carbon/epoxy laminated plate whose dimensions were 444 x 346 x 3 mm and density 1.5448e3 Kg/m<sup>3</sup> was used as a sample. The laminate was eight layers of symmetric [0/90] carbon/epoxy, and was made by autoclave process.

Nylon strings are used to suspend the plate and simulated free-edge boundary conditions. Other equipments used in this analysis is a Laser Dopler Vibrometer, Polytec OFV – 303.8; a force transducer, PCP 208A 02; a eletromechanical shaker that was connected to the plate in a fixed position; signal generator; signal conditioner, PCB 484A 05; signal amplifier LDS, PA25E and a HP35650 modal analysis software, see Figure 1.



Figure 1: Experimental setup.

#### **Results and Discussion**

To make the modal analysis, 176 different points were uniformly distributed over the plate surface, as it is shown in Figure 2. The Frequency Response Functions (FRFs) [10] were processed trough HP35650 software to obtain the vibration modes and its respective vibration frequencies. Natural vibration frequencies can be obtained measuring at least one point to generate the FRFs. The experiment was performed in a frequency range of 0-200 Hz and results showed five different natural frequencies for this interval as shown in Figure 3.



Figure 2: Experimental devices.

To assess the accuracy of experiment, a finite element software (ANSYS 7.0) and a boundary element code [9], were used to model the laminated plate. The numerical results, show good agreement when compared to the experimental results as illustrated in Table 1.

Mode shapes	Experimental frequencies F <sub>EX</sub> (Hz)	Numerical frequencies F <sub>BEM</sub> (Hz)	$\frac{\left  F_{EX} - F_{BEM} \right }{F_{EX}} * 100$	Numerical frequencies F <sub>FEM</sub> (Hz)	$\frac{\left \frac{F_{_{EX}}-F_{_{FEM}}}{F_{_{EX}}}\right *100$						
						1	41.32	41.95	1.50	41.04	0.69
						2	99.59	97.04	2.62	98.39	1.20
3	126.53	125.19	1.07	128.91	1.88						
4	166.76	178.99	6.83	181.28	8.71						
5	183.97	185.90	1.15	198.70	8.00						

Table 1: Comparison among five experimental and numerical frequencies.

### Conclusions

This paper presented an experimental approach to obtain mode shapes and natural frequencies of thin light composite laminates using a Laser Doopler Vibrometer. The advantage of the presented technique is that there is no contact between



Figure 3: Experimental modes of carbon/epoxy plate.

the measuring equipment and the plate. This is ideal for light structure as composite laminates. The experimental results were compared with numerical results and showed good agreement.

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