

Physiological Cost Optimization for Bipedal Modeling with Optimal Controller Design

A. M. Mughal¹ and K. Iqbal²

Summary

Human voluntary movements are complex physical phenomenon and there are several physiological factors that control the movement and transient response, steady state position, speed of motion and other characteristics. Many experimentalists described variety of variables important for human balance and movement such as center of mass, center of pressure, ground reaction forces etc. In this study, we discuss a bipedal model for biomechanical sit to stand movement with optimal controller design. The cost optimization for gain scheduling is based upon physiological variables of center of mass, head position, and ground reaction forces. Our simulation results shows that movement profiles improve with this techniques and it provides better gain scheduling for different joint angles.

Introduction

Researchers have analyzed human movement coordination in a variety of ways using different optimization variables and functions. Many experimentalists of kinesiology and physiology have studied the human balance with variety of techniques, such as measurement of center of pressure, ground reaction forces, with no vision, slip experiments etc. These all factors are important in voluntary movement and actually human movement is combination of many physiological parameters. Ref [1] showed that the centre of mass (CoM) or position of head during the motion put constraints in movement coordination of joints, which was solved by nervous system to complete the required sit-to-stand (STS) task. Ref [2-3] presented an analytical biomechanical bipedal model for sit-to-stand transfer. This model has three holonomic constraints which are decoupled from the unconstrained variables for controller design. This scheme uses a linear quadratic regulator (LQR) for constrained system and H^∞ suboptimal controller for unconstrained system (with no holonomic constraints). This controller is totally based upon the state optimization and doesn't consider any physiological variables for gain scheduling of joint angles. In our previous work [4], we presented 2D sagittal plane model with CoM and ground reaction forces (GRF) based gain scheduling for optimal controller design and these results motivate us to adopt similar gain scheduling based upon physiological variables.

In this study, we are presenting an analytical bipedal model with physiological cost optimization based upon CoM, head position (HP), GRF and a hybrid design

¹Dept. of Applied Science, University of Arkansas at Little Rock, USA

²Dept. of Systems Engineering, University of Arkansas at Little Rock, USA

which include all the physiological parameters. First we discuss the bipedal model scheme, followed by mathematical background for cost optimization with a table of eigenvalues from different cost optimization.

Bipedal Modeling

The general structure of a bipedal model includes 2 foot segments, 2 lower limbs, 2 upper limbs connected with pelvic through a 2-dof joint. Pelvic segment is connected with head, arm, and trunk (HAT) segment as shown in Fig 1.

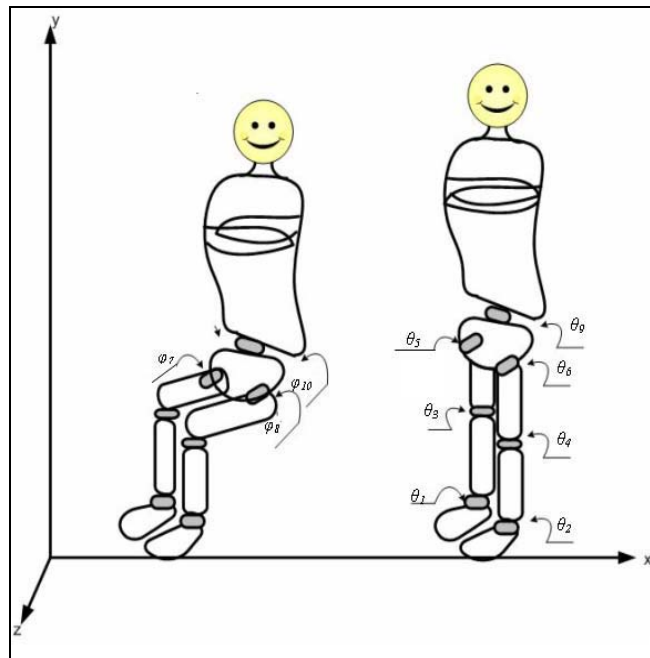


Figure 1: Bipedal Rigid Body Model, with sagittal plane angles $\theta_1 - \theta_6$ and θ_9 , frontal plane angles $\phi_7 - \phi_8, \phi_{10}$

There are 7 joints in the systems connecting in yz or sagittal plane with x as axis of rotation representing joint angles $\theta_1 - \theta_6$ and θ_9 . Hip-pelvic and pelvic-HAT joints can also move the limbs in xy or frontal plane with z -axis of rotation, and these angles will be modeled with Euler angle representations shown as ϕ_7 , ϕ_8 and ϕ_{10} in Fig 1. These joints are represented as either revolute or universal joints in DynaFlexPro environment; ref [2] discusses these in details. This model has 10 joint angles, 7 in sagittal plane and three in frontal plane, and three left foot position variables due to free foot joint at left foot; whereas, right foot is modeled as weld or 0-DOF joint which doesn't move during sit-to-stand (STS) maneuver. This model also does not move from its position for a normal gait due to this restriction of a weld joint. The system has 10 degrees of freedom and it is modeled

using 13 generalized coordinates coupled by 3 algebraic or holonomic constraints. We decoupled the constrained system of left foot position from unconstrained or joint angles system for controller design [3]. This design scheme allows us to use different linear combinations of state variables for measurement and regulation. This also provides better regulation of noise and smaller gains with respect to LQR design. The overall system including measured and regulated variables is given as:

$$S = \begin{bmatrix} A & B_w & B_u \\ C_y & D_{yw} & D_{yu} \\ C_m & D_{mw} & D_{mu} \end{bmatrix} \quad (1)$$

In the S matrix, A and B are 20×20 and 20×10 state space formulation for linearized bipedal model of unconstrained system (joint angles only). B_w is a matrix for external disturbances. C_m is a matrix of state measurements, and D_{mw} is a full row rank matrix of input disturbances. The regulated state and input variables are given by C_y and D_{yu} (full column) matrices, and, D_{mu} and D_{yw} are zero matrices in most general cases. The whole system S must also satisfy design and stability requirement given in ref [5] as well as decoupling stability conditions given in ref [3].

Physiological Cost Optimization

State weighting matrix Q and input weighting matrix R are the key elements for controller design. Weights assigned to each state leads to minimization of errors in the state. Ref [1] discussed that STS movement coordination takes CoM and HP position as a constraint to be satisfied for the task. In ref [4], we used this approach with the horizontal component of CoM for 3-link biomechanical model for STS task, and this research showed the improvements in results. Now, we use x , y and z components of CoM and HP, and GRF in 3 directions of position ' p ' and velocity ' v ' to compute combined state weighting matrix accordingly.

$$\begin{aligned} CoM_{x|y|z} &= f_{x|y|x}(\vec{x}) = f_{x|y|x} \\ HP_{x|y|x} &= g_{x|y|z}(\vec{x}) = g_{x|y|z} \\ GRF_{x|y|z} &= h_{x|y|z}(\vec{x}, \vec{u}) = h_{x|y|z} \end{aligned} \quad (2)$$

A regulated C_y matrix with state, and a physiological function cost optimization is given as

$$C_y = \begin{bmatrix} C_{ysv} & 0 \\ 0 & C_{ysp} \\ C_{yFv} & C_{yFp} \end{bmatrix} \quad (3)$$

C_{yF} is 3×20 function F based state optimization matrix, where $F = \{f|g|h\}$ can be

either CoM, HP or GRF, with each row for x , y , and z coordinates as follows

$$C_{yF} = [C_{yFv} \ C_{yFp}] = \begin{bmatrix} \frac{\partial F_x}{\partial x_j} \\ \frac{\partial F_y}{\partial x_j} \\ \frac{\partial F_z}{\partial x_j} \end{bmatrix}_{x=x_e, u=u_e} \quad \text{for } (j = 1, \dots, 20) \quad (4)$$

C_{yFv} is obtained by either differentiating Eq(2) with respect to time, for CoM and HP, then further differentiating with respect to velocity states or for simplicity we can assume $C_{yFv} = s_c \cdot C_{yFp}$ where s_c is some scalar for optimization. CoM and HP based optimization doesn't provide regulation for input and measurements so D_{yu} and D_{mw} matrices are arbitrary according to closed loop pole placement for steady state response. Nonlinear GRF functions provides optimization in 3×10 C_{yhv} matrix for velocity states as well as inputs through D_{yu} matrix as

$$D_{yu} = \begin{bmatrix} D_{us} \\ D_{uh} \end{bmatrix}, \quad D_{ur} = \begin{bmatrix} \frac{\partial h_x}{\partial u_j} \\ \frac{\partial h_y}{\partial u_j} \\ \frac{\partial h_z}{\partial u_j} \end{bmatrix} \quad (5)$$

Where D_{us} is a 10×10 matrix for input optimization and D_{uh} is 3×10 matrix from GRF function $h(\vec{x}, \vec{u})$. A hybrid cost optimization matrix generated from states, CoM, HP and GRF weighting (regulation) matrix is given as

$$C_y = \begin{bmatrix} C_{ysv} & 0 \\ 0 & C_{ySp} \\ C_{yfv} & C_{yfp} \\ C_{ygv} & C_{ygp} \\ C_{yhv} & C_{yhp} \end{bmatrix}, \quad D_{yu} = \begin{bmatrix} D_{us} \\ D_{uh} \end{bmatrix} \quad (6)$$

C_y and D_{yu} are 29×20 and 23×10 matrices respectively for states and input regulation. We provide the closed loop eigenvalues and achieved γ (gains bound for Riccati equations) with H_∞ optimal design for physiological variables based cost optimization in Table 1.

In Table 1, it is shown that CoM and HP based optimization doesn't vary in the spectra of eigenvalues and thus speed of controller, but these does assign different eigenvalues in the middle spectra of minimum and maximum eigenvalues. GRF based cost optimization provides altogether different values for controller energy function for input torques, and finally a hybrid scheme produces a better optimization with respect to any individual optimization. Appropriate weight assignment is very critical in controller design to meet the physiologically relevant movement

Table 1: Eigenvalues and γ achieved for closed loop H^∞ controller of Decoupled unconstrained Designs for STS Movement

| With States Only | Physiological variable based designs | | | |
|--|--------------------------------------|----------|----------|----------|
| | CoM | HP | GRF | Hybrid |
| Achieved γ values for gain scheduling | | | | |
| 88.549 | 13.266 | 66.893 | 39.164 | 48.21 |
| Eigenvalues | | | | |
| -79443 | -79560 | -79453 | -376940 | -83022 |
| -29614 | -29667 | -29617 | -48431 | -44408 |
| -14144 | -16753 | -14149 | -20903 | -20903 |
| -2143.7 | -2163 | -2168.1 | -18470 | -1618.3 |
| -3385.7 | -3396.5 | -3401.3 | -1619.6 | -4045.4 |
| -424.25 | -483.73 | -448.99 | -358.35 | -357.97 |
| -37.08 | -263.29 | -54.45 | -4316.4 | -927.43 |
| -247.24 | -52.592 | -301.34 | -34.555 | -35.193 |
| -6.8393 | -9.62 | -7.0104 | -5.5773 | -5.3011 |
| -0.49797 | -7.1773 | -0.49837 | -0.55288 | -0.55726 |
| -2.5967 | -0.4958 | -1.0666 | -2.4081 | -2.3429 |
| -1.0724 | -2.4328 | -2.3555 | -2.1546 | -2.154 |
| -2.2364 | -1.0575 | -1.7706 | -1.958 | -1.9573 |

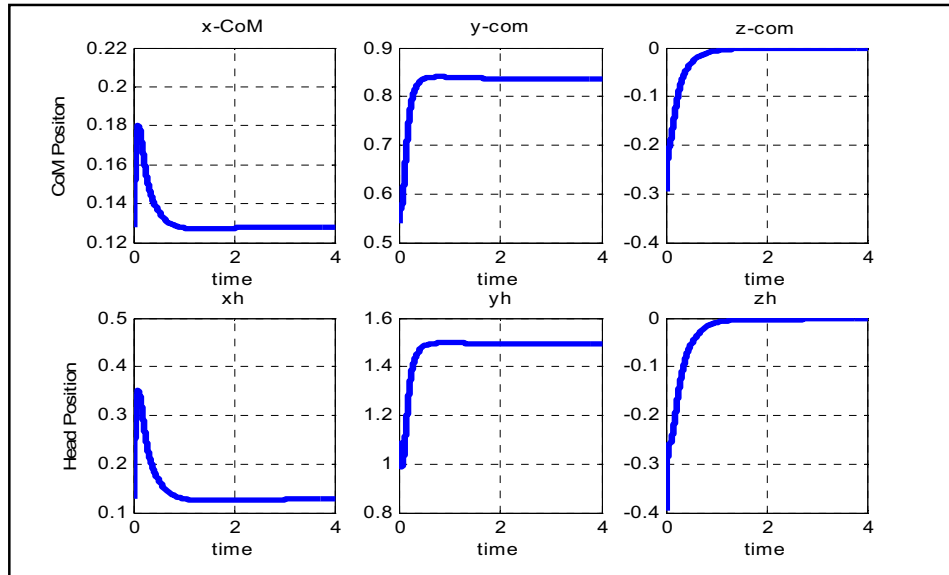


Figure 2: CoM and HP profiles with a hybrid controller design

profiles. Cost optimization based upon physiological variables allows smoothing

the physiological profiles and achieving the STS movement with lower gains. Fig 2 shows the profiles of CoM movement and head movement in x , y and z axis with hybrid design scheme. x -CoM and x -head positions settles at $1/2$ distance between the feet, y -CoM settles at length reaching midway in pelvic height and y -head settles at total body height, Both z -CoM and z -head settles at 0 at steady state, that a person in sitting or standing posture with right foot on origin and left feet on x -axis.

References

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