

Extended Overstress Model with Overstress Tensor

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Summary

The overstress model is improved by incorporating the novel variable “*overstress tensor*” reaching the current stress from the subloading stress evolving in an imaginary quasi-static process of elastoplastic deformation due to the extended subloading surface model with the tangential inelastic strain rate.

Introduction

The elastoplastic stress is first defined as the stress which evolves as the actual strain rate is induced in an imaginary quasi-static process of elastoplastic deformation, while internal variables evolve with the viscoplastic strain rate calculated by the viscoplastic constitutive equation. Further, the novel variable “*overstress tensor*” reaching the current stress from the elastoplastic stress is defined. Thus, the overstress model [1] is extended so as to describe also the tangential viscoplastic strain rate induced by the overstress tensor component tangential to the yield surface, extending the *tangential inelastic strain rate* [2]. Further, the viscoplastic strain rate due to the change of stress inside the yield surface is incorporated by adopting the concept of the *subloading surface* [3].

Extended Overstress Model with Tangential Inelasticity

Quasi-static process of elastoplastic deformation

Let the strain rate \mathbf{D} (symmetric part of velocity gradient) be additively decomposed into the elastic strain rate \mathbf{D}^e and the *inelastic strain rate* \mathbf{D}^i , while the latter is further additively decomposed into the (*normal-*)*plastic strain rate* \mathbf{D}_N^p and the *tangential inelastic strain rate* \mathbf{D}_t^i induced by the stress rate components normal and tangential, respectively, to the loading surface, i.e.

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^i, \mathbf{D}^i = \mathbf{D}_N^p + \mathbf{D}_t^i \quad (1)$$

First, let \mathbf{D}^e be related to the stress rate as

$$\mathbf{D}^e = \mathbf{E}^{-1} \overset{\circ}{\boldsymbol{\sigma}} \quad (2)$$

The fourth-order tensor \mathbf{E} is the elastic modulus and $\boldsymbol{\sigma}$ is the Cauchy stress, ($\overset{\circ}{}$) denoting the proper corotational rate with the objectivity.

Now, assume the following yield surface.

$$f(\hat{\boldsymbol{\sigma}}, \mathbf{H}) = F(H), \quad \hat{\boldsymbol{\sigma}} \equiv \boldsymbol{\sigma} - \boldsymbol{\alpha} \quad (3)$$

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where H is the isotropic hardening variable, α is the kinematic hardening variable and \mathbf{H} is the anisotropic hardening variable. f is assumed to be homogeneous function of stress σ in degree-one.

The time-differentiation of Eq. (3) with the substitution of Eq. (2) leads to the consistency condition

$$\text{tr} \left\{ \frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \sigma} \mathbf{E} (\mathbf{D} - \mathbf{D}_N^p) \right\} - \text{tr} \left(\frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \sigma} \dot{\alpha} \right) + \text{tr} \left(\frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) = F' \dot{H} \quad (4)$$

where $(\dot{\cdot})$ denotes the material-time derivative and $F' \equiv dF/dH$.

Hereafter let it be assumed that the tangential-inelastic strain rate is normal to the yield surface and thus it fulfills the following equation:

$$\text{tr} \left(\frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \sigma} \mathbf{D}_t^i \right) = 0 \quad (5)$$

Assume the associated plastic-flow rule

$$\mathbf{D}_N^p = \Lambda \mathbf{N} (\Lambda > 0), \quad \mathbf{N} \equiv \frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \sigma} / \left\| \frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \sigma} \right\| \quad (\|\mathbf{N}\| = 1) \quad (6)$$

where Λ is a positive proportionality factor, $\|\cdot\|$ denoting the magnitude, which is derived by substituting Eq. (6) into Eq. (4) as follows:

$$\Lambda = \frac{\text{tr} \left[\mathbf{N} \left\{ \mathbf{E} \mathbf{D} - \frac{F'}{F} \dot{H} \sigma - \dot{\alpha} + \frac{1}{F} \text{tr} \left(\frac{\partial f(\hat{\sigma}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) \sigma \right\} \right]}{\text{tr}(\mathbf{N} \mathbf{E} \mathbf{N})} \quad (7)$$

using $\partial f(\hat{\sigma}, \mathbf{H}) / \partial \sigma = \{F / \text{tr}(\mathbf{N} \sigma)\} \mathbf{N}$ due to the Euler's theorem.

The variation of internal structure of material is induced by the inelastic deformation and is described by the evolution of internal variables. The inelastic part of the strain rate is induced as the viscoplastic strain rate in the overstress model. Then, the rates of internal variables in the consistency condition (4) have to be calculated by the overstress model formulated later and thus the plastic-flow rule (6) is not substituted to them.

While the inelastic strain rate is induced also by the tangential stress rate, it depends only on the deviatoric component of tangential stress rate [4]. Then, let the tangential-inelastic strain rate be given by the following equation [2].

$$\mathbf{D}_t^i = T(\sigma, H, \alpha, \mathbf{H}) \hat{\sigma}_t^* \quad (8)$$

where $\hat{\sigma}_t^*$ is the *deviatoric-tangential stress rate* given as follows:

$$\hat{\sigma}^* = \hat{\sigma}_n^* + \hat{\sigma}_t^*, \quad \hat{\sigma}_n^* = \text{tr}(\mathbf{n}^* \hat{\sigma}^*) \mathbf{n}^*, \quad \hat{\sigma}_t^* = \hat{\sigma}^* - \hat{\sigma}_n^*, \quad \mathbf{n}^* \equiv \mathbf{N}^* / \|\mathbf{N}^*\| \quad (\|\mathbf{n}^*\| = 1) \quad (9)$$

Hereafter, let the elastic modulus be given in Hooke's type of rate form, i.e.

$$E_{ijkl} = \{K - (2/3)G\}\delta_{ij}\delta_{kl} + G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \quad (10)$$

where K and G are the elastic bulk and shear moduli, respectively, and δ_{ij} is the Kroneker's delta.

It holds from Eqs. (1), (2), (6) and (10) that

$$\mathbf{D}^* = \mathring{\boldsymbol{\sigma}}^* / (2G) + \Lambda \mathbf{N}^* + T \mathring{\boldsymbol{\sigma}}_t^* \quad (11)$$

from which, noting $\mathbf{N}^* - \text{tr}(\mathbf{n}^* \mathbf{N}^*) \mathbf{n}^* = 0$, we have $\mathbf{D}_t^* \equiv \mathbf{D}^* - \text{tr}(\mathbf{n}^* \mathbf{D}^*) \mathbf{n}^* = \{(1/2G) + T\} \mathring{\boldsymbol{\sigma}}_t^*$. Substituting it into Eq. (8), the tangential-inelastic strain rate is described by the stress rate as follows:

$$\mathbf{D}_t^i = [1 + 1/\{2GT(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H})\}] \mathring{\boldsymbol{\sigma}}_t^* \quad (12)$$

The stress rate is given from Eqs. (1), (2), (6), (7) and (12) by the following equation, noting the positiveness of the proportionality factor Λ .

$$\mathring{\boldsymbol{\sigma}} = \begin{cases} \mathbf{ED} - \langle \Lambda \rangle \mathbf{EN} - \frac{2G}{(1/2G) + T(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H})} \mathbf{D}_t^* & \text{for } f(\hat{\boldsymbol{\sigma}}, H) - F(H) = 0 \\ \mathbf{ED} & \text{for others} \end{cases} \quad (13)$$

where $\langle \ \rangle$ is the McCauley's bracket.

Overstress model with overstress tensor

Let \mathbf{D} be additively decomposed into the elastic strain rate \mathbf{D}^e and the *viscoplastic strain rate* \mathbf{D}^{vp} which is further additively decomposed into the *normal-viscoplastic strain rate* \mathbf{D}_N^{vp} and *tangential-viscoplastic strain rate* \mathbf{D}_t^{vp} , i.e.

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^{vp}, \quad \mathbf{D}^{vp} = \mathbf{D}_N^{vp} + \mathbf{D}_t^{vp} \quad (14)$$

The overstress has been evaluated merely by the scalar quantity describing the expansion of the dynamic-loading surface from the yield surface. Here, we introduce the imaginary stress defined as the stress on the yield surface, which evolves as the actual strain rate is induced in an imaginary quasi-static process of elastoplastic deformation, and let it be called the *elastoplastic stress*, denoting it by the notation $\boldsymbol{\sigma}^{ep}$. The viscoplastic strain rate could be formulated more precisely by introducing the tensor of stress reaching the current stress from the elastoplastic stress. The elastoplastic stress rate is given by the following equation with the replacement of the stress $\boldsymbol{\sigma}$ to $\boldsymbol{\sigma}^{ep}$ in Eq. (13).

$$\mathring{\boldsymbol{\sigma}}^{ep} = \begin{cases} \mathbf{ED} - \Lambda^{ep} \frac{\mathbf{EN}^{ep}}{\text{tr}(\mathbf{N}^{ep} \mathbf{EN}^{ep})} - \frac{2G}{1 + 1/\{2GT(\boldsymbol{\sigma}^{ep}, H, \boldsymbol{\alpha}, \mathbf{H})\}} \mathbf{D}_t^{ep*} & \text{for } (\hat{\boldsymbol{\sigma}}, \mathbf{H}) - F(H) = 0 \\ \mathbf{ED} & \text{for others} \end{cases} \quad (15)$$

where

$$\Lambda^{ep} \equiv \frac{\text{tr} \left[\mathbf{N}^{ep} \left\{ \mathbf{E}\mathbf{D} - \frac{F'}{F} \dot{\mathbf{H}} \boldsymbol{\sigma}^{ep} - \dot{\boldsymbol{\alpha}} + \frac{1}{F} \text{tr} \left(\frac{\partial f(\boldsymbol{\sigma}^{ep}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) \boldsymbol{\sigma}^{ep} \right\} \right]}{\text{tr}(\mathbf{N}^{ep} \mathbf{E} \mathbf{N}^{ep})} \quad (16)$$

$$\dot{\mathbf{H}} = \text{tr} \{ \mathbf{f}_H(\boldsymbol{\sigma}^{ep}, H, \boldsymbol{\alpha}, \mathbf{H}) \mathbf{D}_N^{vp} \}, \quad \dot{\boldsymbol{\alpha}} = f_\alpha(\boldsymbol{\sigma}^{ep}, H, \boldsymbol{\alpha}, \mathbf{H}) \|\mathbf{D}_N^{vp}\|, \quad \dot{\mathbf{H}} = f_H(\boldsymbol{\sigma}^{ep}, H, \boldsymbol{\alpha}, \mathbf{H}) \|\mathbf{D}_N^{vp}\| \quad (17)$$

$$\mathbf{N}^{ep} \equiv \frac{\partial f(\boldsymbol{\sigma}^{ep})}{\partial \boldsymbol{\sigma}^{ep}} / \left\| \frac{\partial f(\boldsymbol{\sigma}^{ep})}{\partial \boldsymbol{\sigma}^{ep}} \right\| \quad (\|\mathbf{N}^{ep}\| = 1), \quad \mathbf{n}^{ep*} \equiv \frac{\mathbf{N}^{ep*}}{\|\mathbf{N}^{ep*}\|} \quad (\|\mathbf{n}^{ep*}\| = 1) \quad (18)$$

$$\mathbf{D}_t^{ep*} \equiv \mathbf{D}^* - \text{tr}(\mathbf{n}^{ep*} \mathbf{D}^*) \mathbf{n}^{ep*} \quad (19)$$

The second-order tensor \mathbf{f}_H and the scalars f_α and f_H are functions of $\boldsymbol{\sigma}^{ep}, H, \boldsymbol{\alpha}, \mathbf{H}$.

Define the *overstress tensor* to be the tensor of stress reaching the current stress from the elastoplastic stress and denote it by $\overleftarrow{\boldsymbol{\sigma}}$, i.e.

$$\overleftarrow{\boldsymbol{\sigma}} \equiv \boldsymbol{\sigma} - \boldsymbol{\sigma}^{ep} \quad (20)$$

Then, the deviatoric part $\overleftarrow{\boldsymbol{\sigma}}^*$ is decomposed into the normal-deviatoric component $\overleftarrow{\boldsymbol{\sigma}}_n^*$ and the tangential-deviatoric component $\overleftarrow{\boldsymbol{\sigma}}_t^*$ as follows:

$$\overleftarrow{\boldsymbol{\sigma}}^* \equiv \overleftarrow{\mathbf{I}}^* \overleftarrow{\boldsymbol{\sigma}} = \overleftarrow{\boldsymbol{\sigma}}_n^* + \overleftarrow{\boldsymbol{\sigma}}_t^*, \quad \overleftarrow{\boldsymbol{\sigma}}_n^* = \text{tr}(\mathbf{n}^* \overleftarrow{\boldsymbol{\sigma}}) \mathbf{n}^*, \quad \overleftarrow{\boldsymbol{\sigma}}_t^* = \overleftarrow{\boldsymbol{\sigma}}^* - \overleftarrow{\boldsymbol{\sigma}}_n^* \quad (21)$$

Then, let \mathbf{D}_N^{vp} and \mathbf{D}_t^{vp} be given by

$$\mathbf{D}_N^{vp} = C_N(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \left\langle \frac{f(\hat{\boldsymbol{\sigma}}, \mathbf{H})}{F(H)} - 1 \right\rangle^N \mathbf{N}, \quad (22)$$

$$\mathbf{D}_t^{vp} = \begin{cases} C_t(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \frac{\overleftarrow{\boldsymbol{\sigma}}_t^*}{F(H)} & \text{for } f(\hat{\boldsymbol{\sigma}}, \mathbf{H}) - F(H) \geq 0 \\ 0 & \text{for others} \end{cases} \quad (23)$$

where N is the material constant and T is the absolute temperature. Then, the strain rate and the stress rate are given from Eqs. (2), (14), (22) and (23) by

$$\mathbf{D} = \begin{cases} \mathbf{E}^{-1} \dot{\boldsymbol{\sigma}} + C_N(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \left\langle \frac{f(\hat{\boldsymbol{\sigma}}, \mathbf{H})}{F(H)} - 1 \right\rangle^N \mathbf{N} \\ \quad + C_t(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \frac{\overleftarrow{\boldsymbol{\sigma}}_t^*}{F(H)} & \text{if } f(\hat{\boldsymbol{\sigma}}, \mathbf{H}) - F(H) \geq 0 \\ \mathbf{E}^{-1} \dot{\boldsymbol{\sigma}} & \text{for others} \end{cases} \quad (24)$$

$$\dot{\boldsymbol{\sigma}} = \begin{cases} \mathbf{E}\mathbf{D} - C_N(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \left\langle \frac{f(\hat{\boldsymbol{\sigma}}, \mathbf{H})}{F(H)} - 1 \right\rangle^N \mathbf{E}\mathbf{N} \\ \quad - 2GC_t(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \frac{\overleftarrow{\boldsymbol{\sigma}}_t^*}{F(H)} & \text{if } f(\hat{\boldsymbol{\sigma}}, \mathbf{H}) - F(H) \geq 0 \\ \mathbf{E}\mathbf{D} & \text{for others} \end{cases} \quad (25)$$

Subloading-overstress model

The *dynamic-loading surface* on which the current stress lies for the normal-yield surface of Eq. (3) is described as

$$f(\bar{\boldsymbol{\sigma}}, \mathbf{H}) = \bar{R}F(H), \quad \bar{\boldsymbol{\sigma}} \equiv \boldsymbol{\sigma} - \bar{\boldsymbol{\alpha}}, \quad \bar{\boldsymbol{\alpha}} \equiv \bar{R}\boldsymbol{\alpha} + (1 - \bar{R})\mathbf{s} \quad (\boldsymbol{\alpha} - \mathbf{s} = \bar{R}(\boldsymbol{\alpha} - \mathbf{s})) \quad (26)$$

where \bar{R} is the ratio of the size of dynamic-loading surface to that of normal-yield surface, called the *dynamic-loading ratio*. $\bar{\boldsymbol{\alpha}}$ is the conjugate point in the subloading surface for the back stress $\boldsymbol{\alpha}$ in the normal-yield surface. \mathbf{s} is the *similarity-center* of the normal-yield and the dynamic-loading surfaces. On the other hand, the elastoplastic stress lies on the subloading surface, fulfilling

$$f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H}) = \bar{R}^{ep}F(H), \quad \bar{\boldsymbol{\sigma}}^{ep} \equiv \boldsymbol{\sigma}^{ep} - \bar{\boldsymbol{\alpha}}^{ep}, \quad \bar{\boldsymbol{\alpha}}^{ep} = \mathbf{s} - \bar{R}^{ep}\hat{\mathbf{s}} \quad (27)$$

where \bar{R}^{ep} is the ratio of the size of subloading surface to that of normal-yield surface, called the *subloading ratio*. The material-time derivative of Eq. (26) leads to

$$\begin{aligned} & \text{tr} \left\{ \frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \mathbf{E}(\mathbf{D} - \mathbf{D}_N^p) \right\} - \bar{R}^{ep} \text{tr} \left(\frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \dot{\bar{\boldsymbol{\alpha}}} \right) + \text{tr} \left(\frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) - \bar{R}^{ep} F' \dot{H} \\ & - (1 - \bar{R}^{ep}) \text{tr} \left(\frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \dot{\hat{\mathbf{s}}} \right) - \left\{ F - \text{tr} \left(\frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \mathbf{s} \right) \right\} \dot{\bar{R}}^{ep} = 0 \end{aligned} \quad (28)$$

Substituting the associated flow rule

$$\mathbf{D}_N^p = \bar{\Lambda} \bar{\mathbf{N}}^{ep} \quad (\bar{\Lambda} > 0), \quad \bar{\mathbf{N}}^{ep} \equiv \frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} / \left\| \frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \right\| \quad (\|\bar{\mathbf{N}}^{ep}\| = 1) \quad (29)$$

into Eq. (28), the elastoplastic stress is derived as follows:

$$\dot{\boldsymbol{\sigma}}^{ep} = \begin{cases} \mathbf{E}\mathbf{D} - \bar{\Lambda}^{ep} \mathbf{E} \bar{\mathbf{N}}^{ep} - \frac{2G}{1 + \frac{2G\hat{\xi}^{ep}(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H})\bar{R}^{ep\tau}}{2G}} \bar{\mathbf{D}}_t^{ep*} & \text{for } \bar{R} > \bar{R}^{ep} \text{ or } \bar{\Lambda}^{ep} > 0 \\ \mathbf{E}\mathbf{D} - \frac{2G}{1 + \frac{2G\hat{\xi}^{ep}(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H})\bar{R}^{ep\tau}}{2G}} \bar{\mathbf{D}}_t^{ep*} & \text{for others} \end{cases} \quad (30)$$

where

$$\bar{\Lambda}^{ep} \equiv \frac{\text{tr} \left[\bar{\mathbf{N}}^{ep} \left\{ \mathbf{E}\mathbf{D} - \bar{R}^{ep} \dot{\bar{\boldsymbol{\alpha}}} - \frac{1}{\bar{R}^{ep} F} \left\{ \frac{F'}{F} \dot{H} - \text{tr} \left(\frac{\partial f(\bar{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) \right\} \bar{\boldsymbol{\sigma}}^{ep} - (1 - \bar{R}^{ep}) \dot{\hat{\mathbf{s}}} - \frac{\dot{\bar{R}}^{ep}}{\bar{R}^{ep}} \bar{\boldsymbol{\sigma}}^{ep} \right\} \right]}{\text{tr}(\bar{\mathbf{N}}^{ep} \mathbf{E} \bar{\mathbf{N}}^{ep})} \quad (31)$$

where the rate of *subloading ratio* \bar{R}^{ep} ($0 \leq \bar{R}^{ep} \leq 1$) is given by

$$\dot{\bar{R}}^{ep} = \bar{U}^{ep}(\bar{R}^{ep}) \|\mathbf{D}_N^{vp}\| \quad \text{for } \mathbf{D}_N^{vp} \neq \mathbf{0} \quad (32)$$

\bar{U}^{ep} is the monotonically-decreasing function of \bar{R}^{ep} fulfilling the conditions $\bar{U}^{ep} \rightarrow \infty$ for $\bar{R}^{ep} = 0$ and $\bar{U}^{ep} = 0$ for $\bar{R}^{ep} = 1$. τ is the material constant. The explicit example is given by $\bar{U}^{ep} = -u \ln \bar{R}^{ep}$, u being the material constant. The rate of \mathbf{s} is given as

$$\dot{\mathbf{s}} = c \|\mathbf{D}_N^{vp}\| \frac{\tilde{\boldsymbol{\sigma}}^{ep}}{\bar{R}^{ep}} + \dot{\boldsymbol{\alpha}} + \frac{1}{F} \left\{ F' \dot{H} - \text{tr} \left(\frac{\partial f(\hat{\mathbf{s}}, \mathbf{H})}{\partial \mathbf{H}} \dot{\mathbf{H}} \right) \right\} \hat{\mathbf{s}}, \quad \tilde{\boldsymbol{\sigma}}^{ep} \equiv \boldsymbol{\sigma}^{ep} - \mathbf{s}, \quad \hat{\mathbf{s}} \equiv \mathbf{s} - \boldsymbol{\alpha} \quad (33)$$

$$\bar{\mathbf{N}}^{ep} \equiv \frac{\partial f(\tilde{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} / \left\| \frac{\partial f(\tilde{\boldsymbol{\sigma}}^{ep}, \mathbf{H})}{\partial \boldsymbol{\sigma}^{ep}} \right\| \quad (\|\bar{\mathbf{N}}^{ep}\| = 1), \quad \bar{\mathbf{n}}^{ep*} \equiv \frac{\bar{\mathbf{N}}^{ep*}}{\|\bar{\mathbf{N}}^{ep*}\|} \quad (34)$$

$$\bar{\mathbf{D}}_t^{ep*} = \mathbf{D}^* - \text{tr}(\bar{\mathbf{n}}^{ep*} \mathbf{D}^*) \bar{\mathbf{n}}^{ep*} \quad (35)$$

τ is the material constant.

The strain rate and the stress rate are given by

$$\mathbf{D} = \mathbf{E}^{-1} \dot{\boldsymbol{\sigma}} + \hat{C}_N(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \langle \bar{R} - \bar{R}^{ep} \rangle^N \bar{\mathbf{N}} + \hat{\zeta}_t(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \bar{R}^{\kappa_t} \frac{\bar{\boldsymbol{\sigma}}_t}{F} \quad (36)$$

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} \mathbf{D} - \hat{C}_N(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \langle \bar{R} - \bar{R}^{ep} \rangle^N \mathbf{E} \bar{\mathbf{N}} - 2G \hat{\zeta}_t(\boldsymbol{\sigma}, H, \boldsymbol{\alpha}, \mathbf{H}, T) \bar{R}^{\kappa_t} \frac{\bar{\boldsymbol{\sigma}}_t}{F} \quad (37)$$

where

$$\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{\sigma}}_n + \bar{\boldsymbol{\sigma}}_t, \quad \bar{\boldsymbol{\sigma}}_n = \text{tr}(\bar{\mathbf{n}}^* \bar{\boldsymbol{\sigma}}) \bar{\mathbf{n}}^*, \quad \bar{\boldsymbol{\sigma}}_t = \bar{\boldsymbol{\sigma}} - \bar{\boldsymbol{\sigma}}_n \quad (38)$$

κ_t is the material constant.

Concluding Remarks

The rational formulation of the extended subloading-overstress model with the overstress tensor which is capable of describing the smooth elastic-plastic transition and the tangential viscoplastic strain rate. On the other hand, the former formulation [4], in which the novel physical quantity “*overstress tensor*” was introduced first, involves the impertinence that the plastic flow rule was substituted into the evolutions of the subloading ratio and the similarity-center, although they have to be updated by the viscoplastic strain rate. Therefore, the elastoplastic stress rate in that formulation leads irrationally to $\text{tr}(\bar{\mathbf{N}}^{ep} \dot{\boldsymbol{\sigma}}^{ep}) \neq 0$ in the infinite rate of deformation ($\|\mathbf{D}\| \rightarrow \infty$: $\mathbf{D} = \mathbf{D}^e$, $\mathbf{D}_N^{vp} = \mathbf{0}$) although in fact $\text{tr}(\bar{\mathbf{N}}^{ep} \dot{\boldsymbol{\sigma}}^{ep}) = 0$ must hold for that rate as fulfilled in Eq. (30) on account of $\dot{H} = \dot{\bar{R}} = 0$, $\dot{\boldsymbol{\alpha}} = \dot{\mathbf{H}} = \dot{\mathbf{s}} = \mathbf{0}$.

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