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HYBRID a powerful Boundary Element-Finite Element Method(BEM/FEM) software for analysis of seismic response of multiphase porous media

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Summary

This document summarizes the basic concepts and steps of establishment of the set of equations of wave propoagation in far field and of the dynamic behaviour of porous media in the near field. A breif description of HYBRID software as a powerful tool for evaluation of local seismic site effect is presented. The Combination of the FEM and BEM and improvement of numerical algorithm for the time truncation are described.

Introduction

As it is stated, the response of a site to a seismic solicitation depends on topography and geotechnical local characteristics. A considerable amount of theoretical work has been reported in the literature of geotechnics and seismology, in order to model, quantify and predict the effects of the basin topography. Concerning the stragraphical effects, based on 1D models of soil columns, many softwares are developped (SHAKE family softwares) which do not take into account many significant feature of porous media to estimate this kind of site effects. In order to model site effects in more realistic circumstances (for P-SV waves and for a arbitrary shape of topographical feature), numerical methods have to be used. The finite difference method, the finite element method, the discrete wavenumber method, and the boundary element method are the most frequently used. Domain-based methods such as the Finite Element Method (FEM) represent excellent tools in analyzing heterogeneity and non-linearity in the soil. However, the size of the problem can easily exceed computing capacities and time because of the difficulty of modeling wave propagation in unbounded domains. In recent years, the boundary element methods (BEM), based on the discretization of integral equations, have gained importance in the resolution of wave propagation problems. These techniques can avoid the introduction of fictitious boundaries and reduce the dimensionality of the problem. In order to benefit from the advantages of both domain- and boundary-based methods, the BEM was coupled with the FEM and the finite difference method. In this paper, The basic theoretical development and numerical implementation and optimization of HYBRID software is briefly given. Analysis of two-dimensional wave scattering due to the presence of topographical irregularities and sediment filling is studied with the aid of a hybrid numerical technique, combining finite elements in the near field and boundary elements in the far field.

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The HYBRID program is developed by Gatmiri and his coworkers [2, 3, 4]. The integration process is approximated in the domain by time truncation [5]. Hence calculations are performed faster, with a good accuracy compared to traditional boundary integration methods.

Formulation of problems combining BEM and FEM

The finite-element method (FEM) is particularly adapted to work with anelastic or non linear soils. The boundary-element method (BEM) reduces the problem by one dimension and is relevant for half-plane problems. The study of site effects requires the resolution of mechanical wave radiation equations in irregular configurations, defined by specific topographical and geotechnical conditions. That is why hybrid models combing both methods are often used. In our study, sediments are modeled by finite elements. Substratum is represented by boundary elements, which is adapted to the study in the far field.. All materials are supposed to be elastic.

For the near field which is finite element domain, dynamic behaviour of saturated porous media is formulated based on the theory of Biot. The u-p form of porodynamic equations is used [1,2,3]. The details of the equations are given in the mentionned references. The final matrix form of the FEM équation are given in this paper. In addition of the linear and nonlinear elastic behaviour, the mixed hardening multi-yield surface model (Prevost) is implemented in HYBRID for simulation of elastoplastic behaviour of saturated soils. The "cutting plane" algorithme (Ortiz and Simo) is employed to integrate the elasto-plastic constitutive equations.

In the finite-element domain, application of the modified Newton-Raphson iterative method leads to:

$$M \cdot \ddot{U}^{t+\Delta t(k)} + K^t \cdot (U^{t+\Delta t(k)} - U^{t+\Delta t(k-1)}) = R^{t+\Delta t} - F^{t(k)} \tag{1}$$

where M is the mass matrix, and K^t is the rigidity matrix at instant t. $U^{t+\Delta t(k)}$ is the displacement vector for the k^{th} iteration done to reach the load increment $R^{t+\Delta t}$ imposed at $t + \frown t$. $F^{t(k)}$ is the force calculated by the behavior law of the material at the k^{th} iteration.

Using the Newmark method in which:

$$U^{t+\Delta t} = U^t + \frac{\Delta t}{2} \cdot (\dot{U}^t + \dot{U}^{t+\Delta t}) \tag{2}$$

equation (1) becomes:

$$\bar{K}^t \cdot \Delta U^{(k)} = R^{t+\Delta t} - F^{t(k)} - \left(\frac{4}{\Delta t^2} M\right) \cdot (U^{t+\Delta t(k-1)} - U^t) + \left(\frac{4}{\Delta t} M\right) \cdot \dot{U}^t + M \cdot \ddot{U}^t \tag{3}$$

where:

$$\bar{K}^t = K^t + \frac{4}{\Delta t^2} M \tag{4}$$

Adding $\bar{K}^t \cdot U^{t+\Delta t(k-1)}$ at both sides of equation (3) and assuming that zone 1 is modeled with finite elements:

$$\bar{K}_1^t \cdot U_1^{t+\Delta t(k)} = R_1^{t+\Delta t} - Z_1^{t+\Delta t(k)} \tag{5}$$

where:

$$Z_1^{t+\Delta t(k)} = F_1^{t(k)} - K_1^t \cdot U_1^{t+\Delta t(k-1)} - \left(\frac{4}{\Delta t^2} \cdot U_1^t + \frac{4}{\Delta t} \cdot \dot{U}_1^t + \ddot{U}_1^t \right) \cdot M_1 \tag{6}$$

The boundary integral equation of elastodynamics in time-domain for a homogeneous isotropic elastic medium, occupying a volume Ω, bounded by a surface Γ, and subjected to an incident plane wave is:

$$c_{ij}(\xi) u_j(\xi) = \int_\Gamma [G_{ij}(\xi,x,t) * t_j(x,t) - F_{ij}(\xi,x,t) * u_j(x,t)] d\Gamma + u_i^{eq}(\xi,t) \tag{7}$$

if the contributions of initial conditions and body forces are neglected. ξ is the source point, x is the field point; u_i and t_i are the amplitudes of the i-th component of displacement and traction vectors respectively, at the boundary; u_i^{eq} represents the incident wave; symbol $*$ indicates a Riemann convolution integral; c_{ij} is the discontinuity term depending on the local geometry of the boundary at ξ and on the Poisson's ratio; G_{ij} and F_{ij} are the fundamental solutions representing the displacement and traction at x in direction i due to a unit point force applied at ξ in the j-direction. Two-dimentional elstodynamic kernels used in HYBRID are given by Gatmiri and Kamalian (2002a); Gatmiri and Nguyen (2005).

The numerical implementation of equation (7) requires a discretization in both time and space. For this purpose, the boundary Γ is discretized into a defined number of elements, and time axis is divided into N equal intervals so that $t = N. \frown t$. Both constant and linear temporal variations can be used for each field variable. Space discretization gives the following matricial expression at instant $t = N. \frown t$:

$$F^1 \cdot U^N = G^1 \cdot T^N + \sum_{n=1}^{N-1} (G^{N+1-n} \cdot T^n - F^{N+1-n} \cdot U^n) \tag{8}$$

The equations obtained from the FEM are expressed in force and displacement whereas in the BEM, stresses replace forces. Therefore, equations need to be adapted. In the last term of (8), stresses are transformed into forces as follows:

$$Z'^{t(k)} = N \cdot (G^1) \cdot \sum_{n=1}^{N-1} (F^{N+1-n} \cdot U^n - G^{N+1-n} \cdot T^n) \tag{9}$$

Consider that zone 2 modeled by boundary elements and has a common frontier with zone 1, which is modeled by finite elements (See 5, 6). The governing matricial equation of zone 2 can be written in the same way as (5):

$$\bar{K}_2^t \cdot U_2^{t+\Delta t(k)} = R_2^{t+\Delta t(k)} - Z_2'^{t+\Delta t(k)} \tag{10}$$

General formulation of the improved integration method

Assuming that the number of time steps chosen for the integration approximation sums to m, the N^{th} equation is expressed as:

$$c.u_N = \int_\Gamma \int_{t_{N-m}}^{t_N} G_{ij} * t_j(x,t).d\Gamma - \int_\Gamma \int_{t_{N-m}}^{t_N} F_{ij} * u_j(x,t).d\Gamma + \int_\Omega G_{ij}^m.F^{N-m}.d\Omega \tag{11}$$

The domain integral can only be approximated as:

$$\int_\Omega G_{ij}^m.F^{N-m}.d\Omega \approx \psi_m \cdot \int_\Omega G_{ij}^{m-1}.F^{N-m}.d\Omega \tag{12}$$

where:

$$\psi_m = \left[\int_\Omega G_{ij}^m.d\Omega \right]^{-1} \cdot \int_\Omega G_{ij}^m.d\Omega \tag{13}$$

Writing equation (11) at the instant $t = (N-1). \frown t$ for m and m-1 yields:

$$\begin{aligned} \int_\Omega G_{ij}^m.F^{N-m}.d\Omega = & \int_\Gamma \int_{t_{N-m-1}}^{T_{N-m}} G_{ij} * t_j(x,t).d\Gamma - \int_\Gamma \int_{t_{N-m-1}}^{T_{N-m}} F_{ij} * u_j(x,t).d\Gamma \\ & + \int_\Omega G_{ij}^m.F^{N-m-1}.d\Omega \end{aligned} \tag{14}$$

The same thing can be done at $t = (N-2). \frown t$ and using (11), (12) and (14) yields:

$$
\begin{aligned}
c.u_N \approx & \int_{\Gamma} \int_{t_{N-m}}^{T_N} G_{ij} * t_j(x,t).d\Gamma - \int_{\Gamma} \int_{t_{N-m}}^{T_N} F_{ij} * u_j(x,t).d\Gamma \\
& + \psi_m \cdot \left[\int_{\Gamma} \int_{t_{N-m-1}}^{T_{N-m}} G_{ij} * t_j(x,t).d\Gamma - \int_{\Gamma} \int_{t_{N-m-1}}^{T_{N-m}} F_{ij} * u_j(x,t).d\Gamma \right] \\
& + \psi_m \cdot \int_{\Omega} G_{ij}^{m}.F^{N-m-1}.d\Omega
\end{aligned} \tag{15}
$$

Approximating the domain integral in equation (14) results in:

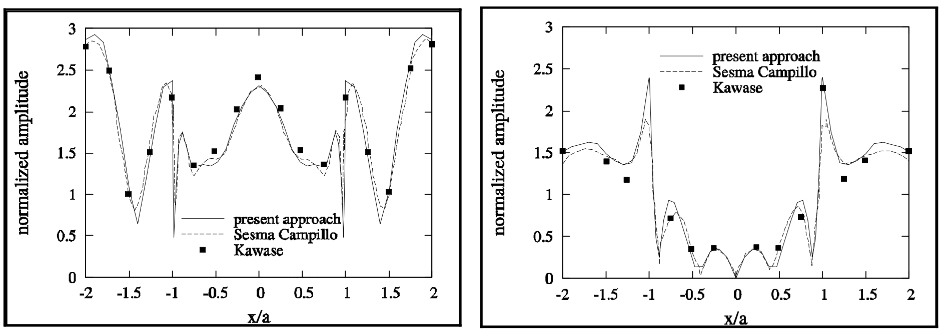
$$
\begin{aligned}
\int_{\Omega} G_{ij}^{m-1}.F^{N-m}.d\Omega \approx & \int_{\Gamma} \int_{t_{N-m-1}}^{T_{N-m}} G_{ij} * t_j(x,t).d\Gamma - \int_{\Gamma} \int_{t_{N-m-1}}^{T_{N-m}} F_{ij} * u_j(x,t).d\Gamma \\
& + \psi_m \cdot \int_{\Omega} G_{ij}^{m-1}.F^{N-m-1}.d\Omega
\end{aligned} \tag{16}
$$

Rewriting equation (15), by equations (11), (12), and (16) and repeating the process till the domain integral affects the initial forces at t=t_0, yields:

$$
\begin{aligned}
c.u_N \approx & \int_{\Gamma} \int_{t_{N-m}}^{T_N} G_{ij} * t_j(x,t).d\Gamma - \int_{\Gamma} \int_{t_{N-m}}^{T_N} F_{ij} * u_j(x,t).d\Gamma \\
& + \sum_{i=1}^{N-m} \psi_m^i \cdot \left[\int_{\Gamma} \int_{t_{N-m-i}}^{T_{N-m-i+1}} G_{ij} * t_j(x,t).d\Gamma - \int_{\Gamma} \int_{t_{N-m-i}}^{T_{N-m-i+1}} F_{ij} * u_j(x,t).d\Gamma \right] \\
& + \psi_m^{N-m} \cdot \int_{\Omega} G_{ij}^{m}.F^{0}.d\Omega
\end{aligned} \tag{17}
$$

When $\psi_m < 1$, $\lim_{i\to\infty} \psi_m^i = 0$. It is possible to ignore the time steps at a certain distance from the limit of backtracking. The integration process continues until a convergence criterion $\psi_m^q < L_m$ is satisfied for some small tolerance L_m. In other words, the time integration is truncated after $m+q$ steps ($m+q < N$) and the determination of the current state requires only the knowledge of $m+q$ former states. Therefore, computation time decreases considerably. Moreover, only the first $m+q$ pairs of coefficients (G_{ij}, F_{ij}) need to be stored instead of all N pairs, which reduces temporary storage requirement. The precision is controlled by the tolerance L_m and by the number of time steps m. In the half-plane problems treated in the following examples, m and L_m are selected to be equal 5 and 0.1 respectively, which gives a reasonable precision. The diffraction of a plane SV wave of vertical incidence by a semi-circular canyon is studied for an elastic half-plane. The Poisson ratio is ν = 1/3 and the dimensionless frequency η of the input signal equals 2. The dimensionless frequency η is the ratio of the characteristic dimension of the relief to the wavelength.

The horizontal and vertical motion amplitudes given by the time truncation method are compared with the results obtained by other authors. Amplitudes are normalized by the displacement measured on the outcrop. Fig. 1 shows an excellent agreement between the diverse modeling techniques.



a. Horizontal displacements b. Vertical displacements

Figure 1: Normalized displacement amplitudes for vertical incidence of a harmonic plane SV wave upon a semi-circular canyon.

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