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On micromechanical formulation to accommodate the second-order perturbation due to interactions of microcracks and inclusions in brittle composites

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Summary

This paper summarizes the results of an analytical study (Lee and Ju, 2006) conducted to develop an approximate micromechanical analytical formulation to accommodate all the possible second-order, ensemble-volume averaged perturbations due to the interactions of randomly located microcracks and inclusions in brittle composites. To account for the three-dimensional effects of interactions among constituents, an approximate solution of a micromechanical framework considering the pairwise microcrack interactions, the pairwise inclusion interactions and the interactions between microcracks and inclusions have been systematically presented. The proposed pairwise interacting micromechanical damage models are compared to illustrate the influence of constituent interactions on effective elastic-damage moduli of composites

Introduction

A key issue concerning the development of micromechanical material-damage models for brittle composites is how to take into account the effects of randomly distributed heterogeneities and their interactions upon the overall elastic-damage behavior of composites. As the density of microcracks increases, strong local microcrack interactions become more significant and must be incorporated into the effective material models for accurate predictions and simulations of the performance and degradation of composites.

Bazant (1987) showed that micromechanics-based damage analysis in heterogeneous media cannot ignore the interactions between microcracks and inclusions or voids. Ju and Chen (1994a, 1994b) and Ju and Tseng (1995) proposed a novel two-dimensional statistical evolutionary micromechanical damage framework for brittle solids with many randomly located, interacting microcracks.

In this paper, a 3-D statistical micromechanical framework for deriving a stationary damage formulation and corresponding homogenized constitutive equations to model multi-phase brittle composites containing spatially randomly located, locally interacting microcracks and interacting inclusions is proposed. In particular, to account for effects of interactions among randomly located constituents, an approximate analytical micromechanical formulation is established to compute the pairwise microcrack interactions, the pairwise inclusion interactions, and the pairwise microcrack-inclusion interactions.

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Overall compliance for a composite with locally interacting microcracks and inclusions

Let us consider a brittle composite consisting of an elastic matrix, randomly dispersed spherical inclusions of radius a, and randomly distributed penny-shaped microcracks of radius c. Following Ju and Tseng (1992, 1995) and Ju and Chen (1994a, 1994b), the local ensemble-averaged perturbed strain tensor at a material point $\mathbf{x}$ within the RVE in the composite can be expressed as:

$$< \varepsilon' > (\mathbf{x}) \ = < \mathbf{S}^{*} > (\mathbf{x}) \ : \ \boldsymbol{\sigma}^{o} \tag{1}$$

where $< \mathbf{S}^{*} > (\mathbf{x})$ is the ensemble-averaged perturbed compliance tensor, and $\boldsymbol{\sigma}^{o}$ denotes the far-field stress tensor. We systematically consider the following six components of the ensemble-volume averaged perturbed effective compliance in order to obtain all possible first- and second-order ensemble-volume averaged solutions for $< \bar{\mathbf{S}}^{*} > (\mathbf{x})$ within the context of the existence and interactions of inclusions and microcracks:

$$< \bar{\mathbf{S}}^{*} > = \ (< \bar{\mathbf{S}}^{*1} > + < \bar{\mathbf{S}}^{*2} >) + (< \bar{\mathbf{S}}^{*3} > + < \bar{\mathbf{S}}^{*4} > + < \bar{\mathbf{S}}^{*5} > + < \bar{\mathbf{S}}^{*6} >) \tag{2}$$

where the first two terms represent the first-order contributions while the latter four terms signify the second-order contributions. The details of the definition of the six components in Eq. (2) can be found in Lee and Ju (2006).

The second-order formulation: Interacting solution for brittle composites with microcracks and inclusions

The overall compliance due to the pairwise inclusion interaction

The ensemble-volume averaged perturbed compliance tensor defining the second-order pairwise inclusion interaction effects takes the form

$$< \bar{\mathbf{S}}^{*3} > (\mathbf{x}) \ = -\phi \, \mathbf{E} \cdot (\boldsymbol{\Gamma} - \mathbf{I}) \cdot [(\mathbf{A} + \mathbf{E})^{-1} \cdot \mathbf{C}_0^{-1}] \equiv P \tag{3}$$

where the components of the isotropic fourth-rank tensor P are given by

$$P_{ijkl} = P_1 \ \delta_{ij} \delta_{kl} + P_2 \ (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \tag{4}$$

in which the parameters P_1 and P_2 and the details of the fourth-rank tensors in Eq. (3) can be found in Lee and Ju (2006).

The overall compliance due to the pairwise microcrack interaction

The ensemble-volume averaged perturbed compliance due to the local pairwise microcrack interaction can be expressed as

$$< \bar{\mathbf{S}}^{*4} >= \frac{16(1-\nu_0^2)}{3E(2-\nu_0)}\omega^2 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (2-\nu_0)\hat{k}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\hat{k}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\hat{k}_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

where the numerical estimation of the coefficients $\hat{k}_1$ and $\hat{k}_2$ as well as the details of the overall compliance due to the pairwise microcrack interaction can be found in Ju and Tseng (1992) and Lee and Ju (2006).

The overall compliance due to microcracks interacting pairwisely with surrounding inclusions

The ensemble-volume averaged perturbed compliance due to microcracks which interact pairwisely with surrounding inclusions within the RVE takes the form

$$< \bar{\mathbf{S}}^{*5} > = \frac{4\,(1-\nu_0^2)}{\pi E\,(2-\nu_0)}\,\mathbf{g}\cdot\, < \hat{\mathbf{M}} > \omega\,\phi \tag{6}$$

where the (volume-averaged) microcrack concentration ω, the volume fraction of inclusions ϕ and the details of the fourth-rank tensors in Eq. (6) can be found in Lee and Ju (2006).

The overall compliance due to inclusions interacting pairwisely with surrounding microcracks

The ensemble-volume averaged perturbed compliance due to inclusions which interact pairwisely with surrounding microcracks within the RVE is given by

$$< \bar{\mathbf{S}}^{*6} > = -\phi\;\omega\,\mathbf{E}\cdot\, < \hat{\mathbf{N}} > \,\cdot\,[(\mathbf{A}+\mathbf{E})^{-1}\cdot\,\mathbf{C}_0^{-1}] \equiv \mathbf{Q} \tag{7}$$

where the details of the fourth-rank tensors in Eq. (7) can be found in Lee and Ju (2006).

Numerical examples

To illustrate the effective elastic-damage behavior of the proposed micromechanical framework for brittle composites, the proposed approximate analytical scheme is implemented to compute the overall elastic-damage compliances of brittle composites. The material properties involved in this simulation are adopted according to Smith (1976) as follows: E_0=3.0 GPa, ν_0=0.4, E_1=76 GPa, and ν_1=0.23 for the particulate-filled glassy polymer composites (Smith, 1976). The estimation of coefficients (e.g., $\hat{k}_1$ and $\hat{k}_2$; $\hat{m}_1$ and $\hat{m}_2$; $\hat{z}_1$ and $\hat{z}_2$) in the present micromechanical model for this simulation can be found in Lee and Ju (2006).

Figures 1 and 2 show the predicted longitudinal normal and shear compliances $<\bar{S}_{33}>$ and $<\bar{S}_{55}>$, respectively, of the particulate-filled glassy polymer composites, illustrating the effects of the first- and second-order contributions on the behavior of the composites. The effects of interactions among constituents on the overall effective moduli of brittle composites are clearly shown from the figures.

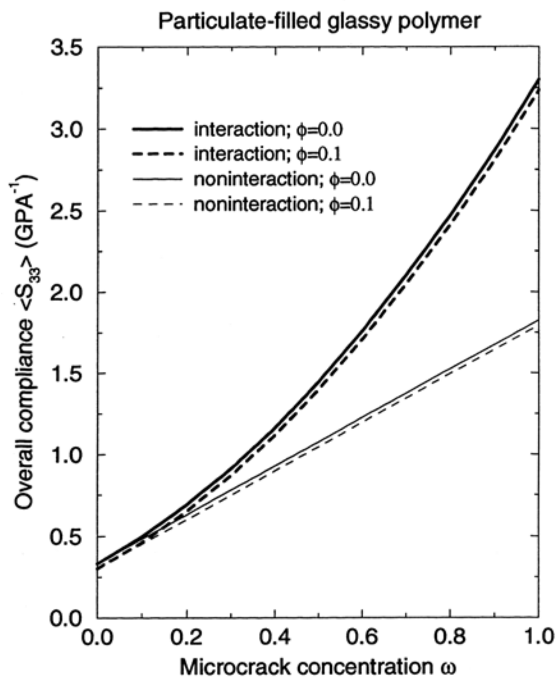

Figure 1: The comparison of overall longitudinal normal compliance $<\bar{S}_{33}>$ of particulate-filled glassy polymer composites between the interacting and first-order models at the volume fractions of inclusions of 0 and 0.1.

Concluding remarks

Based on the concept of the ensemble-volume average and the pairwise interactions among constituents, a three-dimensional, statistical micromechanical formulation has been proposed for brittle composites containing many randomly located, interacting penny-shaped microcracks and randomly dispersed, interacting spherical inclusions. Specifically, approximate explicit (analytical) solutions for the three-dimensional interactions between microcracks and inclusions are derived in detail.

Some numerical examples are presented to illustrate the effective elastic behavior of the proposed methodology. The model predictions clearly exhibit the effects of interactions among constituents on the overall effective moduli of brittle com-

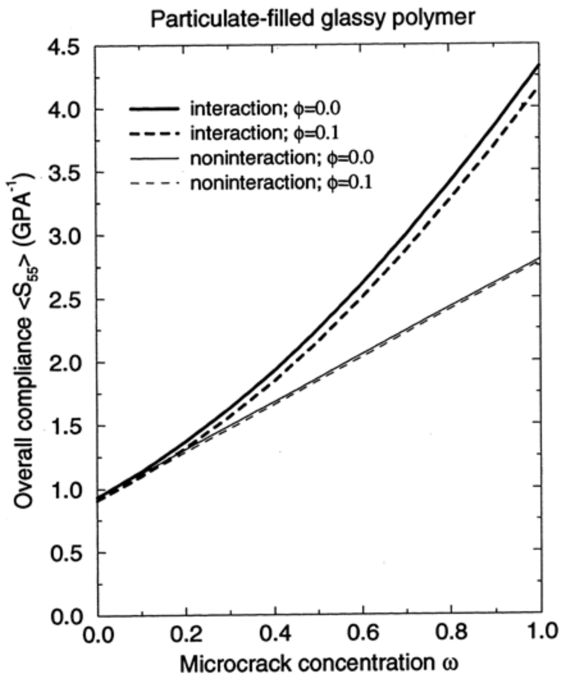

Figure 2: The comparison of overall longitudinal shear compliance $< \bar{S}_{55} >$ of particulate-filled glassy polymer composites between the interacting and first-order models at the volume fractions of inclusions of 0 and 0.1.

posites. Applications may be made to the aligned matrix cracks, the aligned fiber breaks, and the aligned delaminations of brittle composites.

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