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A meshless method based on Daubechies wavelet for 2-D elastoplaticity problems

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Summary

In this paper, a Daubechies(DB) wavelet-based meshless method is proposed to analyze 2-D elastoplasticity problems. Using DB wavelet scaling functions and wavelet functions as basis functions to approximate the unknown field functions, there is no need to construct the shape functions costly as done in FEM and conventional meshless methods. Incremental formulations are established for solution of 2-D elastoplasticity problems. In addition, the property of DB wavelet is used to make the method concise in formulations, flexible in applications and easy to realize. Due to the lack of Kroneker delta properties in scaling functions and wavelet functions, the penalty method is used to impose the essential boundary condition in this work. Numerical examples of two dimensional elastoplasticity problems illustrate that this method is very efficient and stable.

Introduction

Many numerical methods have been developed and used to solve problems of computational mechanics. Recently, one of the hottest topics in computational mechanics is the meshless or meshfree method. some meshless methods have been proposed and achieved remarkable progress, such as smooth particle hydrodynamics (SPH) [1], the diffuse element method (DEM) [2], the element-free Galerkin (EFG) method [3,4], the meshless local Petrov–Galerkin(MLPG) method [5–7], and so on. In addition, techniques for coupling meshfree methods with other established numerical methods have also been proposed, such as the MLPG/FEM/BEM [8].

In above-mentioned meshless methods, it is key and necessary to construct the so-called shape function, which is complicated, time-consuming and even hard to realize in some special cases. Furthermore, the complexity of shape function will increase the computational cost in total solution process. It is desirable to find a new method, which is simple and reasonable to construct shape functions in meshless methods. However, it seems to be a difficult task. So we should resort to some other mathematics tools.

Wavelet is a powerful mathematics tool in solving many problems in science and engineering. In recent years, there has been an increasing interest in the wavelet-based methods in several applications. Some of recent investigations on the wavelet methods include papers by Amaratunga and Williams [9], Christon and Roach [10], Kim and Jang [11], Xuefeng Chen[12].

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This paper is aimed at analyzing 2-D elastoplasticity problems using DB wavelet-based meshless method. In the approximation, scaling functions and wavelet functions in wavelet analysis are directly employed as basis functions to approximate the unknown field function. Because of its special property, DB wavelet can describe details of complicated problems accurately and conveniently. We build incremental formulations for solution of 2-D elastoplasticity problems using DB wavelet-based meshless method. Due to the lack of Kroneker delta properties in scaling functions and wavelet functions, the penalty method is used to impose the essential boundary condition in this work. The numerical examples illustrate that the present method is effective for solving 2-D elastoplasticity problems.

Numerical theory

From translation and dilation of basic scaling function $h(x)$, we can obtain the expression

$$h_{i,j}(x) = h(2^i x - j) \tag{1}$$

where j and i denote the scale and the place index, respectively, in wavelet space. According to the DB wavelet theory, the support of the function $h^k_{i,j}(x)$ is

$$\text{supp} h^k_{i,j} = [2^{-i} j, 2^{-i}(j + 2k - 1)] \tag{2}$$

In 2-D problems, the approximation of 2-D field function is relatively complicated. It is usually used to approximate 2-D function through tensor-product method. Toward a 2-D function $f(x, y)$ defined in a finite elastic body, it can be approximated through using scaling functions of 1-D wavelet. For example, In a rectangular domain $[l_1, l_2] \times [l_3, l_4]$, we have

$$u(x,y) \approx u_{i,j}(x,y) = \sum_{m=-2k+1+\lceil l_1/2^{-i} \rceil}^{\lceil l_2/2^{-i} \rceil - 1} \sum_{n=-2k+1+\lceil l_3/2^{-j} \rceil}^{\lceil l_4/2^{-j} \rceil - 1} a_{m,n} h^k_{i,m}(x) h^k_{j,n}(y) \tag{3}$$

the symbol $\lceil l/2^i \rceil$ denotes the smallest integer great than $l/2^{-i}$. The two dimensional domain that an elastic body occupies is usually irregular. In this case, we should consider the least rectangular domain which surrounds the original domain in the total computational process. Because of the limit in length of the article, it will not be discussed here.

In most cases, the field function in 2-D problems we want to find is complicated. So there will not be enough accuracy only in single scale. In this case, we should use multiscale method. In this method, the field function $u$can be approximated below

$$u_{i,j}(x,y)=\sum_m\sum_n a_{m,n}h_{i0,m}(x)h_{j0,n}(y)+\sum_{m1}\sum_{n1}\sum_{k=i0}^{i} b^1_{m1.n1}h_{i0,m1}(x)g_{k,n1}(y)+ \\ \sum_{m2}\sum_{n2}\sum_{l=i0}^{j} b^2_{m2,n2}h_{i0,m2}(y)g_{l,n2}(x)+\sum_{m3}\sum_{n3}\sum_{k=i0}^{i}\sum_{l=j0}^{j} b^3_{m3,n3}g_{k,m3}(x)g_{l,n3}(y) \quad (4)$$

where the choice of the last three wavelet basis functions is related to the requirements in practical computations.

Numerical implementation

We use incremental method to established formulations for 2-D elastoplasticity problems. The incremental virtual work statement for a system undergoing elastoplasticity is written as: if the stress $\sigma^t_{ij}+\Delta\sigma_{ij}$, volume load $\bar{F}^t_i+\Delta\bar{F}_i$ and boundary load $\bar{T}^t_i+\Delta\bar{T}_i$ in time $t+\Delta t$ satisfied the condition of equilibrium, we have

$$\int_V(\sigma^t_{ij}+\Delta\sigma_{ij})\delta(\Delta\varepsilon_{ij})dV-\int_V(\bar{F}^t_i+\Delta\bar{F}_i)\delta(\Delta u_i)dV \\ -\int_{S_\sigma}(\bar{T}^t_i+\Delta\bar{T}_i)\delta(\Delta u_i)dS=0 \quad (5)$$

where $\delta(\Delta u_i)$ and $\delta(\Delta\varepsilon_{ij})$ are respectively the incremental virtual displacement and strain. Introducing the stress-strain relation into the above equation, the equation can be eventually expressed as the matrix form

$$\int_V\delta(\Delta\varepsilon)^{T\,t}D^{ep}\Delta\varepsilon dV-\int_V\delta(\Delta u)^T\Delta\bar{F}dV-\int_{S_\sigma}\delta(\Delta u)^T\Delta\bar{T}dS \\ =-\int_V\delta(\Delta\varepsilon)^T\sigma^t dV+\int_V\delta(\Delta u)^T\bar{F}^t dV+\int_{S_\sigma}\delta(\Delta u)^T\bar{T}^t dS \quad (6)$$

We can introduce the new meshless method into the incremental virtual work principle. The displacement field functions can be approximated as

$$\Delta u=N\Delta a \quad (7)$$

At last, we can obtain equations as follows

$$^tK_{ep}\Delta a=\Delta Q \quad (8)$$

where$^tK_{ep},\Delta a,\Delta Q$ are the elastoplastic stiffness matrix, incremental displacement vector and lopsided force vector, respectively. Due to the lack of Kroneker delta properties in scaling functions and wavelet functions, the penalty method is used to impose the essential boundary condition in this work.

Numerical examples

Figure 1 depicts a quarter of panel with a central circular hole of $r=0.25$. The uniform tension $p=150$N/m in the horizontal direction is applied on the left and right edges of the panel. As a plasticity model, the von Mises flow rule is

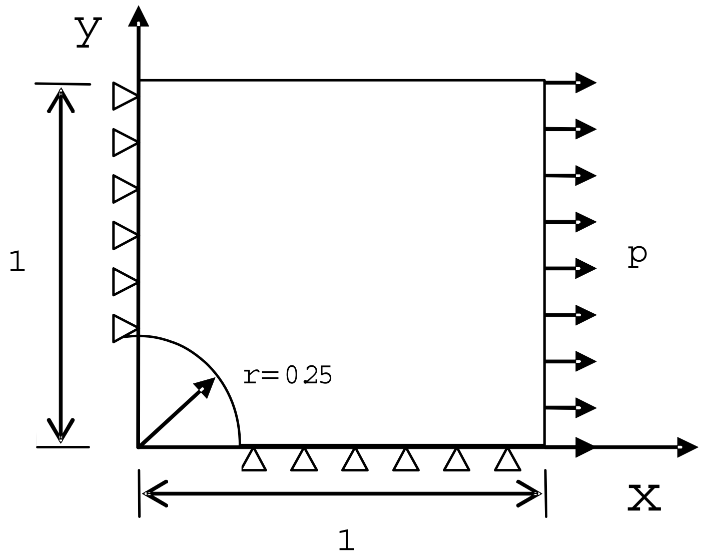

Figure 1: A quarter of panel with a central circular hole

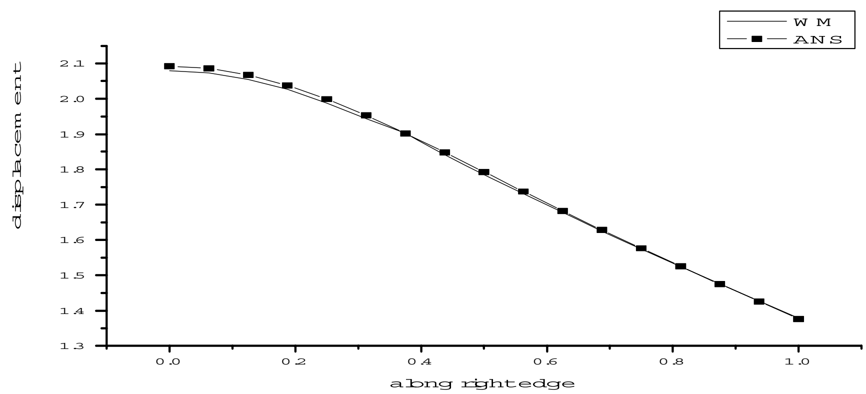

Figure 2: Displacement (u_x) distribution along the right edge

used. Young's modulus$E = 2.1 \times 10^5$MPa, plastic modulus$E_p = 1.0 \times 10^5$MPa, Poisson's ratio$v = 0.3$and the initial yield stress $\sigma_{s0} = 280$MPa. The calculated results are respectively obtained from the present method and FEM(ANSYS with 1110 Plain143 elements). Figure 2 shows the comparison of displacement (u_x) along the right edge. Figure 3 shows the comparison of stress (σ_x) along y axis. We can see that the results from the present method agree very well with those generated by the FEM(ANSYS).

Conclusions

In this paper, a Daubechies wavelet-based meshless method is presented to

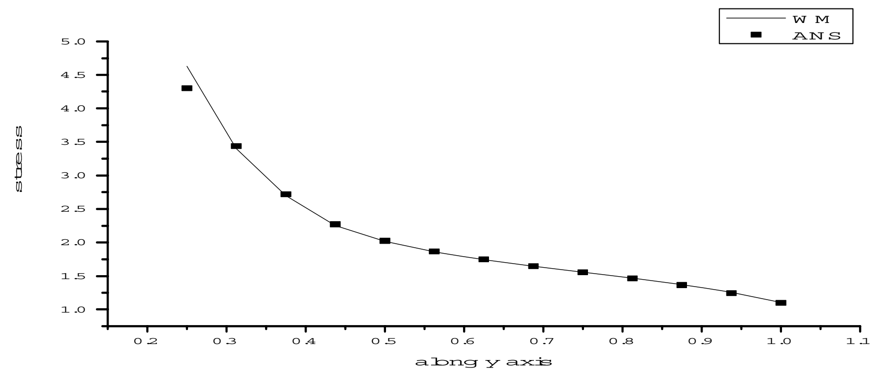

Figure 3: Stress (σ_x) distribution along y axis

analyze 2-D elastoplasticity problems. Incremental formulations are established for solution of 2-D elastoplasticity problems using this new meshless method. The property of DB wavelet is used to make the method concise in formulations, flexible in applications and easy to realize. Due to the lack of Kroneker delta properties in scaling functions and wavelet functions, the imposition of boundary condition is also discussed. Numerical examples of two dimensional elastoplasticity problems illustrate that this method is effective and stable.

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