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Some Application of MLPG in Large Deformation Analysis of Hyperelasto-Plastic Material

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Summary

The Meshless Local Petrov-Galerkin (MLPG) Method is applied to solve large deformation problems of elasto-plastic materials. In order to avoid re-computation of the shape functions, the supports of MLS approximation functions cover the same sets of nodes during the deformation; fundamental variables are represented in spatial configuration, while the numerical quadrature is conducted in the material configuration; the derivation of shape function to spatial coordinate is pushed back to material coordinate by tensor transformation. For simulating both large strain and large rotation, the multiplicative hyperelasto-plastic constitutive model is adopted for path-dependent material. Numerical results indicate that the MLPG method can solve large deformation and large rotation of elasto-plastic materials accurately. Moreover, it can simulate the strain localization phenomenon induced by material instability of strain softening materials.

Introduction

The meshless methods have advantages over FEM in solving problems involving large deformation and discontinuities. Many kinds of meshless methods have been proposed, but most of those methods are based on global weak form, and the global domain quadrature needs background cells. In order to eliminate the necessity of introducing background quadrature cells, the meshless local Petrov-Galerkin (MLPG) method was developed by Atluri et al [1]. The MLPG method construct weighted residual formulation on local test subdomain, which can overlap each other. In this paper, the MLPG method is applied to large deformation problems of hyperelasto-plastic materials. A nonlinear local weak form for these problems is developed and linearized, and some numerical examples are provided to verify the efficiency and accuracy.

Nonlinear MLPG formulation for large deformations problems of elasto-plasticity

The plasticity deformation is path dependent, so usually the rate form of constitutive equation is used and fundamental variables are represented in spatial configuration. Using variables related to the material figuration, the rate

form of equilibrium equations for the solid subjected to finite deformation is

$$\dot{P}_{iJ,J} + \dot{b}_i = 0 \tag{1}$$

where $\dot{P}_{iJ}$ and $\dot{b}_i$ are the rates of first Piola-Kirchhoff stress and the body force respectively.

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According to the weak form over a referential local volume Ω_X, using the divergence theorem and neglecting the body force, the local weakform can be written as

$$\int_{\Omega_X} \dot{P}_{iJ} w_{i,J}\, \mathrm{d}\Omega - \int_{\Gamma_{st}} \dot{\bar{t}}_i w_i \mathrm{d}\Gamma - \int_{\Gamma_{su}+\Gamma_{Ls}} \dot{P}_{iJ} w_i n_J \mathrm{d}\Gamma = 0 \tag{2}$$

where w_i are the test functions, n_J are the components of a unit outward normal to the boundary of the local subdomainΩ_X, $\Gamma_{st}$and Γ_{su} are the intersection between local boundary and global boundary with natural boundary conditions and essential boundary conditions respectively. $\Gamma_{Ls}$is the part of local boundary totally inside the domain. Generally, the local boundary $\partial\Omega_X = \Gamma_{Ls} \cup \Gamma_{su} \cup \Gamma_{st}$ and $\Gamma_{Ls} \cap \Gamma_{su} \cap \Gamma_{st} = \emptyset$.

In the MLPG method, assume w to be a Heaviside function, the MLPG5 formulation is deduced,

$$\int_{\Gamma_{su}+\Gamma_{Ls}} \dot{P}_{iJ} n_J \mathrm{d}\Gamma + \int_{\Gamma_{st}} \dot{\bar{t}}_i \mathrm{d}\Gamma = 0 \tag{3}$$

Now we adopt a rate form constitutive equation as

$$L_v \tau_{mn} = c_{klmn} v_{k,l} = c_{klmn} d_{kl} \tag{4}$$

where $d_{kl} = 1/2\,(v_{k,l} + v_{l,k})$ is the spatial rate of deformation tensor, c_{klin} is the spatial elasticity tensor.

Considering

$$\dot{P}_{iJ} = \dot{F}_{iK} S_{KJ} + F_{iK} \dot{S}_{KJ} = (v_{i,m}\, \tau_{mn} + L_v \tau_{in})\, F_{nJ}^{-\mathrm{T}} \tag{5}$$

where $F_{iK}$is the deformation gradient, and$S_{KJ}$is the second Piola-Kirchhoff stress, $v_{i,m} = \partial v_i / \partial x_m$is the spatial velocity gradient tensor, $\tau_{mn}$is the Kirchhoff stress and $L_v \tau_{in}$is the Lie derivative of the Kirchhoff stress with

$$L_v \tau_{in} = \dot{\tau}_{in} - v_{i,m}\, \tau_{mn} - \tau_{im} v_{n,m} \tag{6}$$

the weak form of MLPG5 can be rewritten as

$$\int_{\Gamma_{su}+\Gamma_{Ls}} (\nabla v_{im} \tau_{mn} + c_{klin} d_{kl})\, F_{nJ}^{-T} n_J d\Gamma + \int_{\Gamma_{st}} \dot{\bar{t}}_i d\Gamma = 0 \tag{7}$$

The derivation of the spatial configuration in this equation can be transformed to the material configuration by

$$\frac{\partial}{\partial x_i} = \frac{\partial}{\partial X_J} F_{Ji}^{-1} \tag{8}$$

Let n, νdenoted time step and iteration counters respectively, introducing essential BC using penalty factor α, the incremental equation can be written as

$$\Delta T = (R)_n - (T)_n^{\nu} \tag{9}$$

where

$$T = -\int\limits_{\Gamma_{su}+\Gamma_{Ls}} \tau_{in} F_{nJ}^{-T} n_J d\Gamma \;+\; \alpha \int\limits_{\Gamma_{su}} (u_i - \overline{u_i})\, d\Gamma, \quad R = \int\limits_{\Gamma_{st}} \dot{\bar{t}}_i d\Gamma \tag{9a}$$

$$\Delta T = -\int\limits_{\Gamma_{su}+\Gamma_{Ls}} \left(\Delta u_{i,m} \tau_{mn} F_{nJ}^{-T} n_J + c_{klin} \Delta u_{k,l} F_{nJ}^{-T} n_J\right) d\Gamma + \alpha \int\limits_{\Gamma_{su}} \Delta u_i d\Gamma \tag{9b}$$

The multiplicative hyperelasto-plasticity model

When large material deformation occurred, classical hypoelasto-plasticity model made errors more large and made convergence slowly. Simo [2] has presented a formulation of static and dynamic plasticity at finite strains based on the multiplicative decomposition which inherits all the features of the classical models of infinitesimal plasticity. In this model, the stress is deduced from strain-energy function, so the principle of frame invariability is satisfied, a drawback of hypoelastoplasticity, which require objective stress integration, is avoided, and better convergence can be achieved.

For static problem, if the yield criterion is pressure insensitive, the plastic deformation maintain the same volume, the deformation gradient is decomposed into a product form written as

$$F_{iJ} = F_{iK}^{e} F_{KJ}^{p} \tag{10}$$

where$F^e$is the elastic part and $F^p$is the plastic part. The left Cauchy-Green deformation tensor and the logarithmic strains can be written as:

$$b_{ij}^{e} = F_{iK}^{e} F_{jK}^{e} = \sum_m \left(\lambda_m^e\right)^2 n_i n_j, \quad \varepsilon_m = \log\left(\lambda_m^e\right) \tag{11}$$

where λ_m^e are principal strain components.

The Neo-Hookean energy function is used as the free energy function $\phi = \phi\left(b^e, \xi\right)$, ξ is the equivalent plastic strain. Accordingly, the hyperelastic stress-strain relation is

$$\tau = 2 \frac{\partial \phi\left(b^e, \xi\right)}{\partial b^e} : b^e \tag{12}$$

The von Mises yield criterion of isotropic hardening material, and the corresponding flow rule are:

$$f(\tau,\ \xi) = \sqrt{3/2}\left[\|\mathrm{dev}(\tau)\| - k(\xi)\right] = 0 \tag{13}$$

$$\dot{b}_e = (v\nabla)\cdot b_e + b^e\cdot(\nabla v) - 2\dot{\lambda}\frac{\partial f}{\partial \tau} : b^e \tag{14}$$

where $k(\xi)$stands for the hardening law, for linear isotropic hardening material $k(\xi) = \sigma_Y + K\xi$. The plastic deformation satisfies Kuhn-Tucker conditions:

$$\dot{\lambda} > 0,\ \ f \leq 0,\ \ \dot{\lambda} f = 0 \tag{15}$$

This form permits a consistent linearization of the algorithm resulting in optimal performance when Newton-Raphson solution scheme is used. When simulating the path-dependent material, a secant Newton method is necessary when the algorithm consistent constitutive modulus is adopted in the computation.

Numerical examples

1. Large deformation bending of a variable-cross-section beam

A variable-cross-section beam with a hole is simulated under the conditions that left end is fixed and right end with a displacement u_y. The material is considered to be linear hardening with material constants: $E = 206.9$ GPa, $\nu = 0.29006$, $K = E/10$, $\sigma_y = 0.45GPa$. The distribution of von Mises stress obtained by FEM and MLPG are shown in Fig. 1. The results show that the nonlinear MLPG method derived here can solve the problems with large strain and rotation accurately.

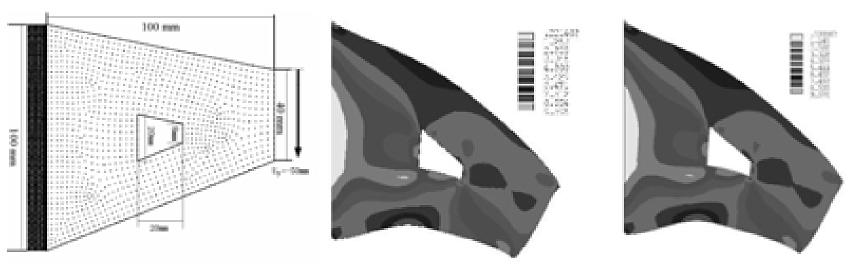

Figure 1: A variable-cross-section beam and node distribution (left), and distribution of von Mises stress by FEM (middle) and MLPG (right)

2. The necking of a Circular Bar

A circular bar with a radius of 10mm and length of 100mm is subjected to prescribed tension at the ends. To simulate the necking, a geometric imperfection is introduced by a linear reduction of radius along the length, with radius at the center to be 90% of the radius at the end. In the simulation we adopted a multiplicative hyperelasto-plasticity model with the constants: $E = 206.9$ GPa, $\nu = 0.29006$,and a general isotropic nonlinear hardening law of the form

$$k(\xi) = \sigma_Y + K\xi + (\sigma_Y^\infty - \sigma_Y)[1 - \exp(-\delta\xi)],\ \ \delta \geq 0$$

where $k(\xi)$is the yield radius, ξ is the effective plastic strain and $\sigma_Y = 0.45$ GPa, $\sigma_Y^\infty = 0.715$ GPa, $K = 0.12924$ GPa, $\delta = 16.93$. The initial and deformed shapes of MLPG model with 213 nodes are shown in Fig. 2, where the necking area is refined. The analysis proceeds until the ratio of current radius to initial radius at the necking section reaches 0.133.

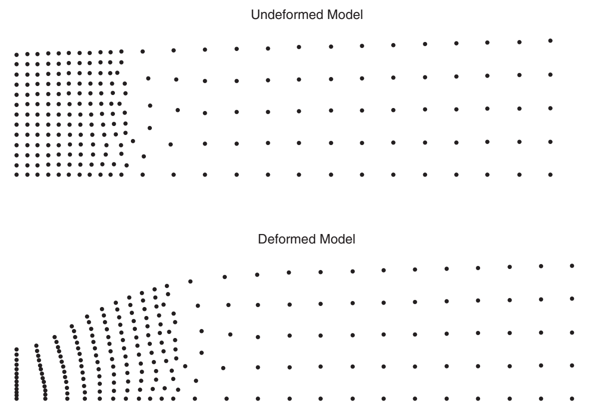

Figure 2: The initial and deformed shape of MLPG model

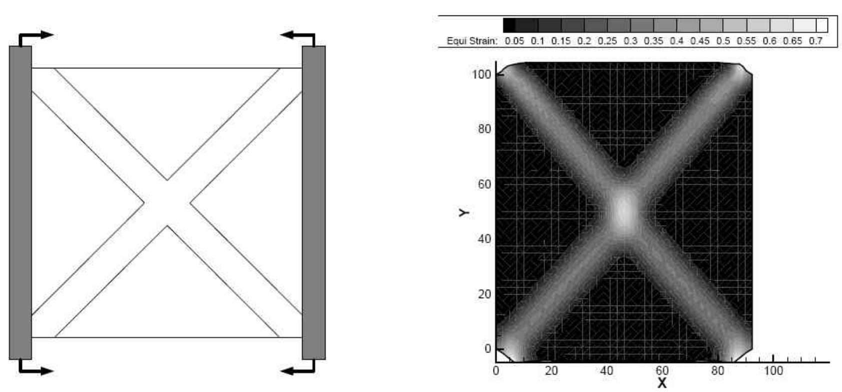

Figure 3: Illustration and formation of shear band under uniaxial compression

3. Deformation of the shear band

In strain-softening materials, uniaxial tension and compression can induce a cross shear band. We have performed MLPG simulation for a square plate of linear strain-soften material: $E = 206.9$ GPa, $\nu = 0.29006$, $K = -0.2$ GPa. The simulated uniaxial compression and tension induced shear band is shown in Fig. 3. Fig. 4 shows the distortion of fictitious "mesh" resulting from large deformation locally in the material after the formation of shear band. The difficulty of this problem is

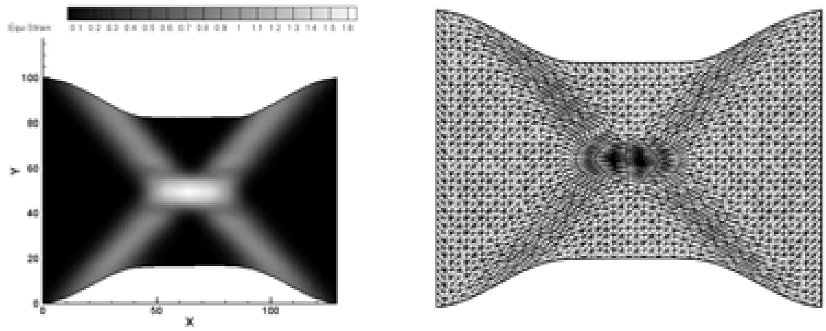

Figure 4: Formation of shear band and the deformation under uniaxial tension

the nearly incompressibility of elastoplastic material and the mesh distortion due to large deformation and the strain localization resulting from the buckling. The MLPG method can be applied to simulating such problem successfully.

Concluding remarks

The formulation of the MLPG method for path-dependent materials using stress and strain components in the space coordinate is presented, and the integration is computed in the material coordinate. The numerical results have shown that the presented MLPG scheme can relieve the difficulties resulting from the volume locking, tolerate larger mesh distortion and avoid the structural instabilities due to the formation of shear band.

References

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