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A Frictionless Contact Algorithm for Meshless Methods

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Summary

An approach to the treatment of contact problems involving frictionless sliding and separation under large deformations in meshless methods is proposed. The method is specially suited for non-structured spatial discretisation. The contact conditions are imposed using a contact potential for particles in contact. Interpenetration is checked as a part of the neighbourhood search. In the case of conventional SPH contact conditions are enforced on the boundary layer $2h$ thick while in the case of the normalized SPH contact conditions are enforced for the particles lying on the contact surface. The implementation of the penalty based contact algorithm for the central difference time integration scheme is described and the performance of the new contact algorithm is illustrated with an example.

Introduction

Contact from a mathematical point of view is a constraint on the solution. The mathematical expressions for the constraints on the solution by the contact condition are commonly known as the Kuhn-Tucker conditions. In order to express these conditions mathematically we introduce a gap function, g, defined using closest point projection to determine pairs of points in contact that belong to body Ω_A and Ω_B and lie on their respective contact surfaces Γ_{CA} and Γ_{CB} (see Figure 1).

$$g = \hat{\mathbf{n}}_{\Gamma_{CA}} \cdot (\mathbf{x}_B - \mathbf{x}_A), \tag{1}$$

where $\mathbf{x}_A \in \Gamma_{CA}$ and

$$\mathbf{x}_B = \arg\ \min \|\mathbf{x}_A - \mathbf{x}\| \quad \forall \mathbf{x} \in \Gamma_{CB}. \tag{2}$$

The contact boundary condition has been largely ignored in the conventional Smooth Particle Method (SPH), with contact between two bodies simply handled

by the conservation equations. Monaghan proposed a modification to the conventional SPH method to prevent the interpenetration, which he called XSPH [7], where the velocity is single valued. While Monaghan's modification does prevent interpenetration it does nothing to solve two other problems with treating contact through the conservation equations:

- Generation of non-physical tensile forces, resisting separation of two bodies.
- Generation of shear stresses preventing friction-less or low friction sliding.

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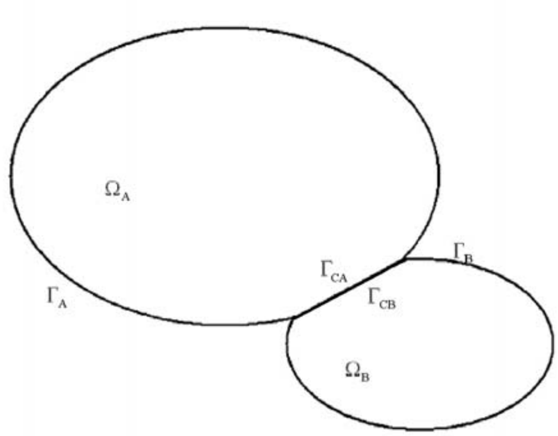

Figure 1: Schematic representation of two bodies in contact.

Several contact algorithms have been proposed for treatment of contact particle to particle and particle to FE mesh contacts [1,2,3,4,6,11,12]. The proposed methods have limitations and most of them represent extension of methods developed for FE to meshless methods.

The paper describes a new contact algorithm inspired by Monaghan's repulsive stress [10]. The new contact algorithm simplifies the calculation of direction of contact forces and avoids the problems related to non-uniqueness of the surface normal at particles. This is achieved by assigning a contact potential to all boundary particles and defining the contact force for particles in contact as the gradient of this field.

New Contact Algorithm

The main problem in treating contact between bodies discretised with particles is defining the location of the boundary of the bodies. In an SPH approximation the particle position can be regarded as the centre of a sub-domain with radius $2h$, where h is known as the smoothing length. Consequently the boundary is diffused over the distance of $2h$, where density varies smoothly from the material density, ρ, at the particle position to zero at the limit of the kernel support. In the case of corrected normalised (CN-SPH) [2] the boundary coincides with the boundary particle locations.

Consider two particles A and B belonging to two different bodies, Ω_A and Ω_B respectively. Particle B is the closest member of Ω_B to particle A. In this case the gap function can be defined as:

$$g = \left\| \mathbf{x}^{(A)} - \mathbf{x}^{(B)} \right\| - (h^A + h^B). \tag{3}$$

Where $\mathbf{x}^{(A)}$ and $\mathbf{x}^{(B)}$ are position vectors for particles A and B respectively and $\| \cdot \|$

is a norm representing the distance between the two particles. In the new contact algorithm, instead of using the projection onto the direction of the normal which is typically used in definition of a gap function a simplification is made and the actual distance between the particles is used to define the gap function,

The contact between two bodies occurs when the penetration g becomes > 0, i.e. the distance between the particles becomes smaller than two times the smoothing length. This is consistent with the SPH method.

Following the idea of body forces being defined as the gradient of a potential field [13], a similar approach was used to define contact forces. The contact potential function had the following properties: it is zero in the interior of the domain, it is always be positive, it becomes larger as the distance between to points decreases.

Defining the contact forces in this manner removes the need to determine and define the boundary surfaces of the two or more bodies and is more consistent with the SPH method. This is a critical consideration as determining boundary particles has proved to be a tough problem, with no approach developed that works efficiently and reliably in 3D to track the evolving boundary in problems such as impact [2,8,11].

The starting point for the derivation of the contact force was the strong form, i.e. the partial differential equations with the boundary and initial conditions representing the motion of two deformable bodies A and B in contact. Galerkin method vas used to derive the week form for the contact problem given by equation (4).

$$\int_{\Gamma_t} w\sigma \cdot n \, d\Gamma - \int_{\Omega} \nabla w \cdot \sigma \, dV = \int_{\Omega} w\rho(\mathbf{a}-\mathbf{b}) \, dV - \int_{\Gamma_c} w\bar{\mathbf{t}} d\Gamma. \tag{4}$$

Where: is w by test/weighting function, σ is Cauchy stress and ρ is current density.

Using $w = \mathbf{N}d$, $u = \mathbf{N}d$ where $N_{ij} = \frac{m_j}{\rho_j} \frac{W_j(x_i)}{\sum_{j=1}^{np} W_j(x_i)}$ and the contact potential defined as $\phi(x_i) = \sum_{j}^{NCONT} \frac{m_j}{\rho_j} K \left(\frac{W(r_{ij})}{W(\Delta p_{avg})} \right)^n$ this equation can now be discretised in space yielding the SPH form for the contact force (for a detailed derivation see [14]):

$$\mathbf{f_c}(x_i) = \sum_{j}^{NCONT} \frac{m_j}{\rho_j} \frac{m_i}{\rho_i} Kn \frac{W(r_{ij})^{n-1}}{W(\Delta p_{avg})^n} \nabla_{\mathbf{x_i}} W(r_{ij}). \tag{5}$$

I = particle at which shape function is evaluated, J = particle around which the shape function is centred, np = number of neighbours for the i particle, d = nodal displacement vector, W = SPH kernel function ($W(x_A - x_B) = 0$if x_A and x_B belong to the same body), K is a user defined scalar contact stiffness penalty parameter, NCONT defines the list of neighbour particles that belong to a different body to particle i.

It is important to notice that direction of the contact force is determined by the SPH approximation of the gradient of the contact potential. For the conventional SPH method it is acting along the line connecting the pair of particles in contact and in the case of CN-SPH it has the direction of normal to the contact surface. The contact force is applied to boundary particles as soon as they get within $2h$ from each other, where h is the smoothing distance. The approach is in keeping with the meshless techniques in general and its implementation in 3D is not complex. The contact algorithm allows surfaces to come together and separate in a physically correct manner. Furthermore this approach does not require the identification of boundary particles (hard to do robustly in 3D). The algorithm is numerically efficient because of its consistency with the general SPH approach and the fact that it utilizes the already generated existing neighbour lists.

With respect to the variational consistency the contact force itself is derived using Galerkin method approach. No assumptions were made about the contact potential function except that it depends on the distance between the two bodies. The contact constraint is imposed directly through the shape functions of the particles in contact, which control the penalty stiffness at the same time. Within a time step contact forces are evaluated at the time of integration of constitutive equation i.e. $t = t_{n+\frac{1}{2}}$.

Numerical Example

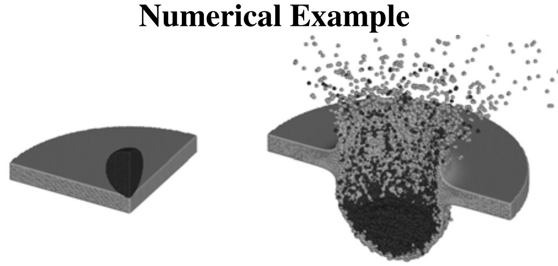

Figure 2: Isometric View of 3D Hypervelocity Impact at 0.0 and 0.2μs

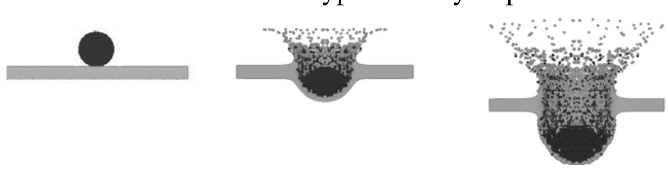

Figure 3: Cross Section Plots of 3D Hypervelocity Impact at 0.0, 0.1 and 0.2μs.

In order to demonstrate that this contact algorithm performs correctly in 3D a hypervelocity impact of a sphere on a plate was simulated. This type of problem is a severe test for the robustness of the contact algorithm as both objects are subjected to extremely large deformations. The radius of the sphere is 0.3mm, the plate thickness and radius is 0.2mm and 0.15mm. The material of both objects is aluminium and an elasticplastic hydrodynamic material model with Gruneisen equation of state was used. The sphere has an initial velocity of 7km/s. Due to the symmetry of the problem a quarter model was created (see figure 2). Cross section plots after 0.0, 0.1 and 0.2μs are shown in figure 2 and figure 3. From these plots it can be seen that the algorithm does not cause any mixing of particles, even at very large deformations. No experimental data, such as hole diameter or debris cloud angle, was available to perform a more qualitative validation of the simulation results.

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