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Evaluation of T-Stress for an Interface Crack Lying Between Dissimilar Anisotropic Solids Using BEM

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Summary

The path-independent mutual- or M-integral for the computation of the T-stress for interface cracks lying between dissimilar anisotropic, linear elastic solids is developed in this paper. For the numerical stress analysis, the Boundary Element Method (BEM) is employed and special crack-tip elements with the proper oscillatory traction singularity are used. The successful implementation of the scheme for evaluating the T-stress of an interface crack between anisotropic bi-materials with BEM is demonstrated by numerical examples.

Introduction

The study of cracks along the interface between dissimilar anisotropic materials is important in understanding the structural integrity of many modern engineered materials such as composites, thin-film coatings and bi-crystals. At the tip of such a crack, the stresses have an oscillatory singularity; this and the associated stress intensity factor K has been a subject of extensive study over the years. Recent studies have established that in addition to K, the elastic T-stress is also important in fracture mechanics analysis and fracture assesments. The T-stress, T, is the leading non-singular term of the William's eigenfunction series for the stress solution near a crack tip. It represents the stress acting parallel to the crack plane and quantifies the degree of constraint at the crack tip. Over the years, several numerical approaches have been developed to extract this quantity in homogenous isotropic bodies. For cracks in non-isotropic bodies, similar studies are more recent and are quite limited.

Works on the determination of T-stress in bimaterial interface crack are also very scarce indeed. For isotropic bodies, these studies include the M-integral approach in conjunction with BEM developed by Sladek and Sladek [1]; Moon and Earmme [2] studied such cracks between semi-infinite strips using an analytical approach with the M-integral; while Fett and Rizzi [3] employed the weight function approach. Kim, Moon and Earmme [4] extended their analytical work mentioned above to an interface crack between dissimilar anisotropic bodies of infinite and semi-infinite extents using M-integral. In a recent contribution, Song [5] has also presented a relatively new approach based on scaled boundary finite element method (SBFEM) to obtain T-stress values for cracks in isotropic and non-isotropic bimaterial interfaces. However, no numerical solutions for a generally anisotropic bimaterial interface crack were presented in above-mentioned papers. There is indeed paucity of numerical T-stress solutions for the interface crack problem in such

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materials and to authors' knowledge, the use of the BEM to this end has hitherto also not been reported in the open literature until the study presented by the present authors very recently [7]. This paper reviews the key developments reported in that study and the reader is referred to that reference for additional details. In essence, the M-integral approach implemented by Sladek and Sladek [1] to obtain T-stress in isotropic bimaterial interface cracks in conjunction with BEM is extended to the generally anisotropic case. Unlike several other schemes, the M-integral approach does not rely on the field solutions near the crack tip, thus minimizing the effect of singularity in its vicinity. A relatively simpler approach to obtain M-integral will be described here and its successful implementation in conjunction with BEM stress analysis is illustrated by some numerical examples.

M-integral for T-stress Evaluation

The stress field σ_{ij} near a crack tip along an interface between dissimilar anisotropic materials can be generally written as

$$\sigma_{ij}^{(m)} = f(K, r, \gamma, \theta) + C^{(m)}\delta_{i1j1}T + O(r^{\alpha}) \tag{1}$$

where r, θ are polar coordinates with origins at crack tip, γ is the bimaterial constant, m is 1 for material (1) or 2 for material (2) and $\alpha > 0$. The material specific coefficient $C^{(m)}$ are $C^{(2)} = 1$ and $C^{(1)} = a_{11}^{'(2)}/a_{11}^{'(1)}$ where $a_{11}^{'(m)}$ is the first element of the compliance matrix of the constitutive equations in the direction parallel to the crack plane in material m.

Consider two independent equilibrium states, A and aux, of a bi-material solid with an interface crack. The mutual integral, also commonly referred to as the M-integral, about the contour Γ_o, as given in Fig. 1a, is expressed in terms of the path independent J-integral as,

$$M = J^{(A+aux)} - J^{(A)} - J^{(aux)} = \int_{\Gamma_0} \left(\sigma_{ij}^{A}\varepsilon_{ij}^{aux}n_1 - \sigma_{ij}^{A}n_j u_{i,1}^{aux} - \sigma_{ij}^{aux}n_j u_{i,1}^{A}\right) d\Gamma \tag{2}$$

where ε_{ij} and u_i are the strains and displacements, respectively, and n_i is the outward normal at the contour Γ_o. The first state A corresponds to the boundary value problem being analysed. The second state aux, also called the auxiliary field, is chosen to be the solution of a semi-infinite crack loaded by a point (line) force f applied at the crack tip in the direction parallel to the crack plane as shown in Fig. 1b; it can be derived from the solution of an anisotropic composite wedge subjected to a point force at its apex [6]. The M-integral is path independent and can be expressed in terms of an arbitrary circular contour with radius ε shrunk to zero as follows,

$$M = \lim_{\varepsilon \to 0} \int_{\Gamma_e} (\sigma_{ij}^{A}\varepsilon_{ij}^{aux}n_1 - \sigma_{ij}^{A}n_j u_{i,1}^{aux} - \sigma_{ij}^{aux}n_j u_{i,1}^{A})\, d\Gamma \tag{3}$$

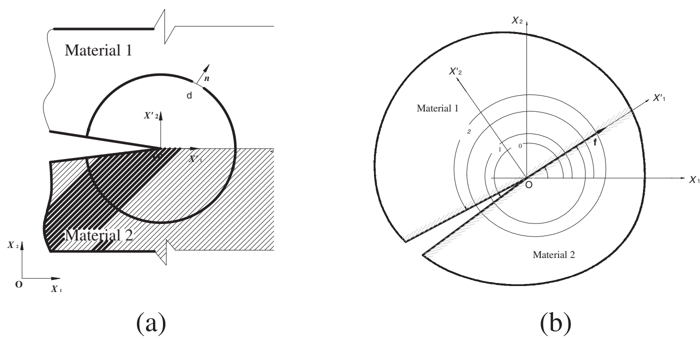


Figure 1: (a) Contour Γ_0 around the crack tip of bimaterial interface crack, (b) a composite wedge and a point force f acting on the apex.

Equation (3), after integration along the contour Γ_ε yields the relationship between T-stress (along material 2) and M-integral in local coordinates as,

$$T = M/(a'^{(2)}_{11} f) \quad (4)$$

A detailed derivation of the auxiliary fields and T-stress and M-integral relations is given in [7].

Numerical Results

The M-integral formulation to obtain the T-stress for a crack at an interface between dissimilar anisotropic materials outlined above has been implemented into a BEM code based on the quadratic isoparametric element formulation with special crack-tip elements representing the proper oscillatory stress singularity for the bi-material interface crack. Three examples are presented here to demonstrate the veracity and capability of the developed formulations.

For the purpose of verification, the first problem investigated (Example 1) was a bi-material rectangular plate comprising of dissimilar isotropic materials 1 (E_1, ν_1) and 2 (E_2, ν_2) with a central interface crack and subjected to a uniform tensile stress σ_o. It is as shown in Figure 2. The cases considered were a/W = 0.5 and H/W = 2; E_1/E_2 was varied as 1, 2, 5 and 10 with $\nu_1 = \nu_2$= 0.3. Plane strain conditions were assumed. The normalized results of T/σ_o obtained in the present work are listed in Table 1 and compared with those obtained by Sladek and Sladek [1] and Song [5], where it can be seen that there is excellent agreement.

In the second example, Example 2, the T-stress for an interface crack between dissimilar orthotropic materials was obtained for the same CCP specimen under remote tension, σ_o as in the previous example, but for relative crack lengths as a/W

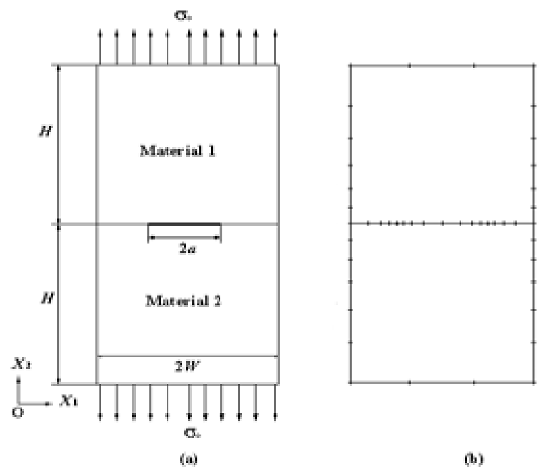


Figure 2: (a) A centre cracked plate (CCP) under remote load σ_o.(b) BEM mesh

Table 1: Normalised T-stress (T/σ_o) for Example 1.

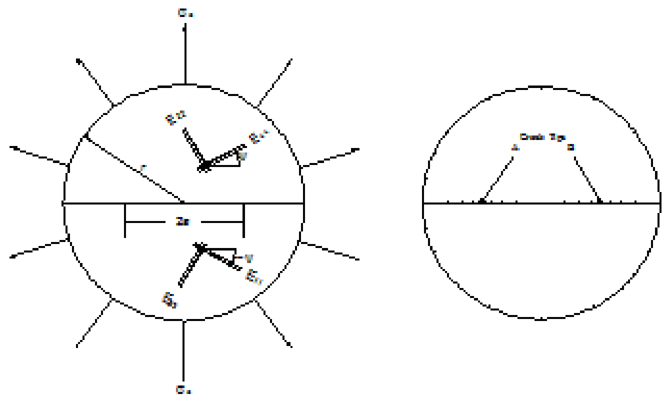
$\frac{E_1}{E_2}$	T/σ_o				
	Present	**Song [9]**	% Δ	**Sladek & Sladek [5]**	% Δ
1	-1.257	-1.260	0.2	-1.272	1.2
2	-0.846	-0.847	0.1	-0.861	1.7
5	-0.436	-0.437	0.2	-0.450	3.1
10	-0.244	-0.244	0	-0.260	6.2

= 0.1, 0.2, 0.3, 0.4 and 0.5. The material properties chosen in the analysis were E_{11}= 1000; E_{22}= 500, G_{12}=100.1, ν_{12}= 0.3 for material 1 and E_{11}= 200, E_{22}= 60, G_{12}= 15.7, ν_{12}= 0.3 for material 2. Plane stress conditions were assumed here. The numerical results of the normalized T-stress, T/σ_o, are shown in Table 2 for the range of crack lengths considered along with the values for the corresponding case of isotropic bimaterials with E_1/E_2= 500/60 and ν= 0.3. It is evident that material orthotropy has quite a significant influence on the value of the T-stress.

Table 2: The normalized T-stress, T/σ_o, for Example 2.

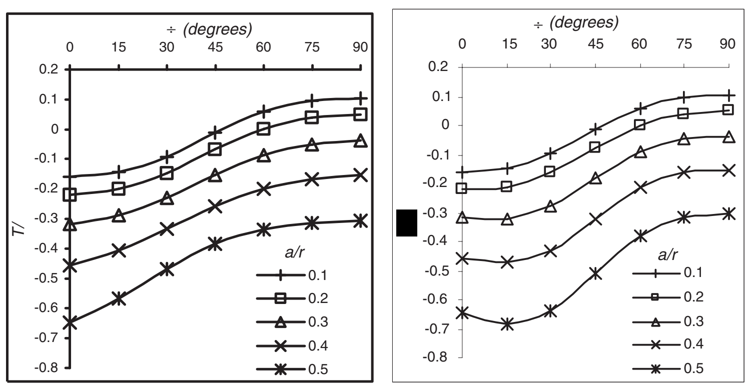
	T/σ_o				
a/W	0.1	0.2	0.3	0.4	0.5
Orthotropic	-0.522	-0.534	-0.556	-0.591	-0.651
Isotropic	-0.222	-0.227	-0.238	-0.255	-0.283

Finally, a cracked anisotropic bi-material disc subjected to uniform radial tension σ_o, as shown in Fig. 4 was investigated (Example 3). The material chosen was single crystal silicon in the [110] plane with the following properties [8]: E_{11}=



(a) (b)

Figure 3: Example 3: (a) Cracked silicon [110] disc (b) BEM mesh.



(a) (b)

Figure 4: Variation of the normalised T-stress, T/σ_o, with angle ψ in centrally cracked interface silicon [110] disc at (a) crack tip A (b) crack tip B.

169.1 GPa, E_{22}= 130.1 GPa, μ_{12}=79.6 GPa and ν_{12} = 0.362. The angles of orientation of the material principal axes with the global Cartesian axes, ψ, for material 1 and -ψ for material 2, were varied together from 0^o to 90^o. The relative crack lengths analyzed were a/r = 0.1 to 0.5 and plane stress conditions were assumed. The effect of ψ on the normalized T-stress values (T/σ_o) for both crack tips are presented in Fig. 6. The degree of anisotropy clearly has a significant effect on the value of the T-stress.

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