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Identification of Dimensions and Position of Tumor Region on the Basis of Skin Surface Temperature Using the Gradient Method Coupled with the Multiple Reciprocity BEM

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Formulation of the Problem

The body surface temperature is controlled by the blood perfusion, local metabolism and the boundary conditions determining the heat exchange between skin and environment. It is well known that the apparition of tumor region leads to the increase of a local blood perfusion and a capacity of metabolic heat source [1, 2]. In this case the skin surface temperature is not uniform due to a difference of internal heat sources capacities in sub-domains of tumor and healthy tissue. So, the abnormal temperature at the skin surface can be used in order to predict the location and dimensions of tumor region.

From the mathematical point of view the heat transfer processes in the domain of biological tissue are described by the Pennes equation [1, 2, 6]. If we consider the steady-state problem concerning the healthy tissue and the tumor region then we obtain the following system of equations

$$x \in \Omega_e : \quad \lambda_e \nabla^2 T_e(x) + k_e [T_B - T_e(x)] + Q_{me} = 0 \tag{1}$$

where e =1, 2 identifies the sub-domains of healthy tissue and tumor (Figure 1), λ_e is the thermal conductivity, $k_e = G_e c_B$ is the perfusion coefficient (G_e is the blood perfusion rate, c_B is the volumetric specific heat of blood), T_B is the blood temperature, Q_{me} is the metabolic heat source.

On the surface between tissue and tumor the ideal thermal contact is assumed

$$x \in \Gamma_c : \begin{cases} q_1(x) = -q_2(x) = q(x) \\ T_1(x) = T_2(x) = T(x) \end{cases} \tag{2}$$

where $q_e(x) = -\lambda_e \partial T(x)/\partial n_e$ is the heat flux, $\partial T(x)/\partial n_e$ denotes the directional derivative at the boundary point considered, while $n_e = [\cos\alpha_{1e}, \cos\alpha_{2e}]$ is the external unit normal vector.

On the remaining parts of the boundary the following boundary conditions (Figure 1) can be accepted

$$\begin{array}{l} x \in \Gamma_1 : \; q_1(x) = 0 \\ x \in \Gamma_2 : \; T_1(x) = T_b \end{array} \tag{3}$$

where T_b is the known boundary temperature.

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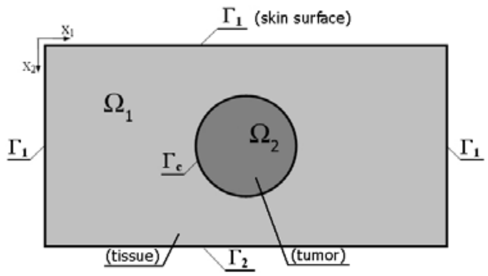

Figure 1: Tissue with a tumor

The inverse problem considered here reduces to the assumption that the geometrical parameters of the tumor region, this means the position of tumor center (x_s, $y_s) = (x_1^s, x_2^s)$ and its radius R_s are regarded as unknown, while the other quantities appearing in the equations (1) – (3) are assumed to be known.

Additionally, it is assumed, that the values of temperatures T_{di}, i=1, 2, ..., M at the points x^i located on the skin surface resulting from adequate measurements are known.

Gradient Method

In order to solve the inverse problem formulated, the gradient method has been used [5, 6]. The unknown geometrical parameters of tumor region can be estimated on the basis of minimization of the least squares criterion [5, 6]

$$S = \sum_{i=1}^{M} (T_i - T_{di})^2 \tag{4}$$

where T_{di}, T_i are the measured and calculated temperatures at the points x^i located on the skin surface. The calculated temperatures are obtained from the solution of direct problem given by equations (1), (2), (3), by using the current available estimate for the vector of unknown parameters z^T=[x_s, y_s, R_s]. Function $T_i = T(x^i)$ is expanded in a Taylor series about known values of parameters z_j^k

$$T_i = T_i^k + \sum_{i=1}^{3} \left.\frac{\partial T_i}{\partial z_j}\right|_{z_j = z_j^k} \left(z_j^{k+1} - z_j^k\right) = T_i^k + \sum_{i=1}^{3} U_{j,i}^k \left(z_j^{k+1} - z_j^k\right) \tag{5}$$

where T_i^k are the calculated temperatures under the assumption that $z_j = z_j^k$, j=1, 2, 3, while k is the number of iteration. It should be pointed out that z_j^0 are arbitrary assumed values of parameters z_j (for $k>0$ they result from the previous iteration). In equation (5) ($U_{j,i}^k$) are the sensitivity coefficients. Putting (5) into (4) and using the necessary condition of minimum one obtains the following system of equations

$$\sum_{i=1}^{M} \sum_{j=1}^{3} U_{j,i}^k U_{l,i}^k \left(z_j^{k+1} - z_j^k\right) = \sum_{i=1}^{M} \left(T_{di} - T_i^k\right) U_{l,i}^k \tag{6}$$

where l =1, 2, 3. The system of equations (6) can be written in the matrix form

$$\left(\mathbf{U}^T\right)^k \mathbf{U}^k \mathbf{z}^{k+1} = \left(\mathbf{U}^T\right)^k \mathbf{U}^k \mathbf{z}^k + \left(\mathbf{U}^T\right)^k \left(\mathbf{T}_d - \mathbf{T}^k\right) \tag{7}$$

where

$$\mathbf{U}^k = \begin{bmatrix} U_{1,1}^k & U_{1,2}^k & U_{1,3}^k \\ U_{2,1}^k & U_{2,2}^k & U_{2,3}^k \\ \cdots & \cdots & \cdots \\ U_{M,1}^k & U_{M,2}^k & U_{M,3}^k \end{bmatrix} \tag{8}$$

Solution of the system of equations (6), this means the values z_j^{k+1} constitutes the input data for the next iteration. The iteration process is stopped when $k = K$, where K is the assumed number of iterations or when the predefined value of criterion (4) is achieved.

Multiple Reciprocity BEM

For each iteration the problem (1)-(3) with the parameters $(\mathbf{z}^T)^k = [x_s^k, \ y_s^k, \ R_s^k]$ has been solved using the multiple reciprocity boundary element method [3, 4, 6].

The multiple reciprocity boundary element method leads to the following integral equations corresponding to equations (1)

$$\begin{gathered} B_e(\xi)\, T_e(\xi) + \sum_{l=0}^{\infty} \left(\frac{k_e}{\lambda_e}\right)^l \int_\Gamma q_e(x)\, V_{le}^*(\xi, x)\, d\Gamma = \\ \sum_{l=0}^{\infty} \left(\frac{k_e}{\lambda_e}\right)^l \int_\Gamma T_e(x)\, Z_{le}^*(\xi, x)\, d\Gamma - \frac{Q_e}{\lambda_e} \sum_{l=1}^{\infty} \left(\frac{k_e}{\lambda_e}\right)^{l-1} \int_\Gamma Z_{le}^*(\xi, x)\, d\Gamma \end{gathered} \tag{9}$$

where ξ is the observation point, $B_e(\xi) \in (0,\ 1]$, $q_e(x) = -\lambda_e \partial T(x)/\partial n_e$ is the boundary heat flux, $Q_e = k_e T_B + Q_{me}$, for healthy tissue: $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_c$, for tumor region: $\Gamma = \Gamma_c$ – c.f. Figure 1. In equation (9) the functions $V_{le}^*(\xi,\ x)$ for 2D problem are defined as follows [3, 4, 6]

$$V_{le}^*(\xi, x) = \frac{1}{2\pi\lambda_e} r^{2l} \left(A_l \ln \frac{1}{r} + B_l\right), \ l = 0, 1, 2, \ldots \tag{10}$$

where

$$\begin{array}{lll} A_0 = 1, & A_l = \frac{A_{l-1}}{4l^2}, & l = 1, 2, 3, \ldots \\ B_0 = 0, & B_l = \frac{1}{4l^2}\left(\frac{A_{l-1}}{l} + B_{l-1}\right), & l = 1, 2, 3, \ldots \end{array} \tag{11}$$

and r is the distance between the points ξ and x

$$r = \sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2} \tag{12}$$

The heat fluxes resulting from the fundamental solutions (10)

$$Z_{le}^*(\xi, x) = -\lambda_e \frac{\partial V_{le}^*(\xi, x)}{\partial n}, \ l = 1, 2, \ldots \tag{13}$$

can be calculated analytically and

$$Z^*_{le}(\xi,x) = \frac{d}{2\pi} r^{2l-2} \left[A_l - 2l \left(A_l \ln\frac{1}{r} + B_l \right) \right] \tag{14}$$

where

$$d = (x_1 - \xi_1)\cos\alpha_1 + (x_2 - \xi_2)\cos\alpha_2 \tag{15}$$

while $\cos\alpha_1$, $\cos\alpha_2$ are the directional cosines of the normal outward vector n.

In numerical realization, the boundary $\Gamma_1 \cup \Gamma_2 \cup \Gamma_c$ is divided into N linear boundary elements. Now, the following denotations are introduced (c.f. Figure 1)

- $\mathbf{T}_{11}$ is the vector of function T on the boundary Γ_1 of domain Ω_1,
- $\mathbf{q}_{12}$ is the vector of function q on the boundary Γ_2 of domain Ω_1,
- $\mathbf{T}_{c1}$, $\mathbf{T}_{c2}$, $\mathbf{q}_{c1}$, $\mathbf{q}_{c2}$ are the vectors of functions T and q on the contact surface Γ_c between domains Ω_1 and Ω_2.

Using above notations and taking into account the boundary conditions (3) one obtains the following systems of equations

- for the healthy tissue domain

$$\begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{c1} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{q}_{12} \\ \mathbf{q}_{c1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{c1} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{11} \\ \mathbf{T}_b \\ \mathbf{T}_{c1} \end{bmatrix} + \mathbf{W}_1 \tag{16}$$

- for the tumor region

$$\mathbf{G}_{c2}\mathbf{q}_{c2} = \mathbf{H}_{c2}\mathbf{T}_{c2} + \mathbf{W}_2 \tag{17}$$

The condition (2) written in the form

$$x \in \Gamma_c : \begin{cases} \mathbf{T}_{c1} = \mathbf{T}_{c2} = \mathbf{T} \\ \mathbf{q}_{c1} = -\mathbf{q}_{c2} = \mathbf{q} \end{cases} \tag{18}$$

should be introduced to the equations (16), (17). Finally, one obtains the following system of equations

$$\begin{bmatrix} -\mathbf{H}_{11} & \mathbf{G}_{12} & -\mathbf{H}_{c1} & \mathbf{G}_{c1} \\ 0 & 0 & -\mathbf{H}_{c2} & -\mathbf{G}_{c2} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{11} \\ \mathbf{q}_{11} \\ \mathbf{T} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{12}\mathbf{T}_b + \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix} \tag{19}$$

from which the 'missing' boundary values of temperatures or heat fluxes can be determined. The form of matrices appearing in (19) results from the assumed approximation of the boundaries of the domains considered. The details are presented in [6].

Sensitivity Coefficients

The sensitivity coefficients (8) can be determined in different ways. Here the approach called the implicit differentiation method [6, 7] has been applied. In this case the differentiation takes place on the stage of numerical approximation of the problem considered and concerns the system of equations (19) resulting from numerical method application. So, differentiation of (19) with respect to the shape parameter z_i, i=1, 2, 3 ($z_1 = x_s$, $z_2 = y_s$, $z_3 = R_s$) leads to the following three systems of equations

$$\begin{bmatrix} -\mathbf{H}_{11} & \mathbf{G}_{12} & -\mathbf{H}_{c1} & \mathbf{G}_{c1} \\ 0 & 0 & -\mathbf{H}_{c2} & -\mathbf{G}_{c2} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{D}\mathbf{T}_{11}}{\mathrm{D}z_i} \\ \frac{\mathrm{D}\mathbf{q}_{12}}{\mathrm{D}z_i} \\ \frac{\mathrm{D}\mathbf{T}}{\mathrm{D}z_i} \\ \frac{\mathrm{D}\mathbf{q}}{\mathrm{D}z_i} \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{D}\mathbf{H}_{11}}{\mathrm{D}z_i}\mathbf{T}_{11} - \frac{\mathrm{D}\mathbf{G}_{12}}{\mathrm{D}z_i}\mathbf{q}_{12} + \frac{\mathrm{D}\mathbf{H}_{c1}}{\mathrm{D}z_i}\mathbf{T} - \frac{\mathrm{D}\mathbf{G}_{c1}}{\mathrm{D}z_i}\mathbf{q} + \frac{\mathrm{D}\mathbf{H}_{12}}{\mathrm{D}z_i}\mathbf{T}_b + \frac{\mathrm{D}\mathbf{W}_1}{\mathrm{D}z_i} \\ \frac{\mathrm{D}\mathbf{H}_{c2}}{\mathrm{D}z_i}\mathbf{T} + \frac{\mathrm{D}\mathbf{G}_{c2}}{\mathrm{D}z_i}\mathbf{q} + \frac{\mathrm{D}\mathbf{W}_2}{\mathrm{D}z_i} \end{bmatrix} \tag{20}$$

where $\mathrm{D}(\cdot)/\mathrm{D}z_i$ is the material derivative associated with the shape parameter z_i [7].

The solution of the system of equations (20) allows to determine the sensitivity functions with respect to the geometrical parameters of the tumor region. It is visible, that the main matrix of the system of equations (19) for the basic problem and for the additional problems (20) is the same. The system of equations (20) is coupled with (19) because the functions $\mathbf{T}$ and $\mathbf{q}$ should be known. Additionally, the elements of matrices appearing on the right-hand side of equations (20) should be differentiated with respect to the shape parameters z_i. The details are presented in [6].

Results of Computations

The domain of biological tissue of dimensions 0.06×0.03 [m] has been considered. The following thermophysical parameters have been assumed: λ_1=0.5 [W/(mK)], k_1=1998.1 [W/(m^{3}K)], Q_{m1}=420 [W/m^3], λ_2=0.75 [W/(mK)], k_2=7992.4 [W/(m^{3}K)], Q_{m2}=4200 [W/m^3], blood temperature T_B=37^{o}C. On the arbitrary assumed internal boundary Γ_2 the temperature T_b=37^{o}C can be accepted.

The boundary $\Gamma_1 \cup \Gamma_2$ has been divided into 90 linear boundary elements, the boundary Γ_c has been divided into 16 linear boundary elements.

The information concerning the temperature distribution (Figure 2) has been obtained from the direct problem solution under the assumption that the radius of tumor region equals 0.0075 [m], the position of its center (0.03 [m], 0.015 [m]) - Figure 1.

In order to solve the inverse problem, it is assumed that unknown values of

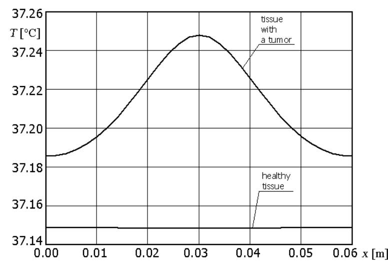

Figure 2: Skin surface temperature

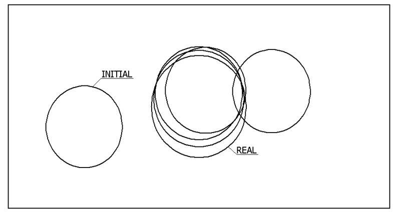

Figure 3: The result of identification for $x_S^0 = 0.012$, $y_S^0 = 0.012$, $R_S^0 = 0.006$

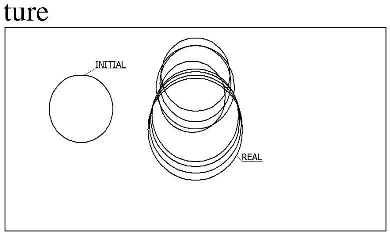

Figure 4: The result of identification for $x_S^0 = 0.012$, $y_S^0 = 0.018$, $R_S^0 = 0.005$

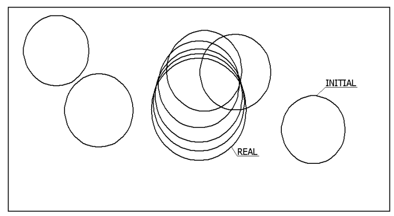

Figure 5: The result of identification for $x_S^0 = 0.048$, $y_S^0 = 0.012$, $R_S^0 = 0.005$

parameters x_s, y_s, R_s are from the intervals

$$\begin{array}{l} 0.012 \le x_S \le 0.048 \\ 0.012 \le y_S \le 0.018 \\ 0.005 \le R_S \le 0.01 \end{array} \tag{21}$$

On a stage of computations, the different initial values of identified parameters x_s, y_s, R_s have been assumed (Figures 3, 4, 5). In comparison with the direct problem solution the identified position and shape of tumor region are practically the same. It seems that the results obtained can be useful as a tool for non-invasive diagnosis while the algorithm proposed is sufficiently exact and effective.

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