



Optimization of Cognitive Radio System Using Enhanced Firefly Algorithm

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Abstract: The optimization of cognitive radio (CR) system using an enhanced firefly algorithm (EFA) is presented in this work. The Firefly algorithm (FA) is a nature-inspired algorithm based on the unique light-flashing behavior of fireflies. It has already proved its competence in various optimization problems, but it suffers from slow convergence issues. To improve the convergence performance of FA, a new variant named EFA is proposed. The effectiveness of EFA as a good optimizer is demonstrated by optimizing benchmark functions, and simulation results show its superior performance compared to biogeography-based optimization (BBO), bat algorithm, artificial bee colony, and FA. As an application of this algorithm to real-world problems, EFA is also applied to optimize the CR system. CR is a revolutionary technique that uses a dynamic spectrum allocation strategy to solve the spectrum scarcity problem. However, it requires optimization to meet specific performance objectives. The results obtained by EFA in CR system optimization are compared with results in the literature of BBO, simulated annealing, and genetic algorithm. Statistical results further prove that the proposed algorithm is highly efficient and provides superior results.

Keywords: Firefly algorithm; cognitive radio; bit error rate; genetic algorithm; simulated annealing; biogeography-based optimization

1 Introduction

All the radio transmissions use a part of electromagnetic spectrum. The management and regulation of the spectrum is done by Federal Communications Commission (FCC) which has



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categorized the spectrum into licensed and unlicensed spectrum [1,2]. Due to the fast growth of wireless applications, the unlicensed spectrum is becoming overcrowded, which in turn leads to spectrum scarcity. It is also observed that the spectrum utilization is not effectively carried out by the licensed users, resulting in a situation where a few channels remain unoccupied in the wireless spectrum. Cognitive radio (CR) has been proposed as an efficient way to tackle with this problem and to permit the secondary users for the usage of licensed spectrum bands when primary users are not using it [3]. A CR system is generally characterized by two important parameters: cognitive capability and re-configurability. In the earlier one, the information about frequency, bandwidth, power and modulation type of signal is collected from the surrounding environment. Re-configurability is the ability of a radio system to rapidly configure its operational parameters in accordance with the sensed information for achieving the optimal performance [4]. By utilizing the spectrum in an opportunistic manner, CR system permits secondary users to sense and select the best unoccupied channel, share spectrum access information with others and vacate the occupied channel when primary users demand it back [5]. CR is widely used in many fields, e.g., software radios, mobile broadband, public security, and in medical applications [6–8].

CR is essential for providing time-varying Quality of Service (QoS) due to the dynamic nature of spectrum availability and the characteristics of radio channels. In addition to efficient spectrum utilization, CR aims to achieve objectives such as maximizing data throughput, minimizing bit-error-rate (BER), reducing power consumption, and minimizing interference [9]. To address these goals and meet the QoS requirements of users, CR needs to regularly sense the environment and adjust transmission parameters accordingly [10]. This adaptive behavior requires a cognitive engine that is aware of the environment, user demands, transmission links, and regulatory constraints, and is capable of balancing multiple objectives. The cognitive engine makes CR intelligent by dynamically adjusting itself to changing conditions [9–11].

Optimization algorithms, particularly evolutionary algorithms, have been successfully applied to various engineering and real-world applications [12–16]. These algorithms are well-suited for solving multi-objective CR optimization problems. Optimizing the CR system using evolutionary algorithms enables decision-making, learning, and awareness processing in the cognitive functionality [17]. As a result, researchers have focused on optimizing CR systems using various optimization algorithms in the past. These studies have employed various approaches to enhance the performance and efficiency of CR systems [9–11,17–26].

The first CR engine was developed by Virginia Tech institute using genetic algorithm (GA) [9]. The result outcomes showed that the implementation of GA altered the transmission parameters in accordance with a set of objectives. GA has been also used to find the optimal transmission parameters for single-carrier as well as multi-carrier fitness functions [18]. Zhang et al. have employed the Shuffled Frog Leap Algorithm to optimize power in the CR system [23]. Biogeography based optimization (BBO) has been utilized to obtain the optimum set of CR parameters [24]. Zhao et al. have optimized the CR system for three objectives with the ant colony optimization (ACO) technique [26]. However, all these methods either converge prematurely or take too much time to attain the optimal solution. For example, SA and GA exhibit slow convergence speed and require a significant amount of time to converge to the final solution.

The Firefly Algorithm (FA) is a swarm intelligence-based metaheuristic inspired by the behavior of fireflies. It has been successfully applied to a wide range of optimization problems in various domains. However, FA may face challenges in complex problems where it tends to oscillate around the global optimum due to random walks. To address these challenges and improve performance, it is necessary

to explore alternative solutions. The No Free Lunch Theorem [27] highlights the need for developing new algorithms specifically tailored to different problem dimensions. This approach allows researchers to propose algorithm variants that are more suitable for specific areas, leading to advancements in solving complex optimization problems. In this paper, an enhanced variant of FA called enhanced FA (EFA) is introduced. EFA incorporates Mantegna's algorithm to enhance the convergence speed of FA. The proposed algorithm is applied to benchmark functions as well as the optimization of a cognitive radio (CR) system to demonstrate its capabilities and effectiveness. By continuously developing and applying new algorithm variants like EFA, researchers can make progress in addressing the challenges of complex optimization problems and further improve the performance of metaheuristic algorithms.

The main contributions of this work are as follows:

- The proposed work addresses the challenges of local optima stagnation, poor exploration, and unbalanced exploitation and exploration operations in FA. The research introduces an enhanced version of FA called EFA, which incorporates improvements to enhance its performance and overcome the identified issues.
- EFA incorporates ideas from Mantegna's algorithm and utilizes Lévy stable distribution to improve the exploration and exploitation operations. This enhances the algorithm's ability to explore extensively and exploit effectively.
- EFA is evaluated on different benchmark problems and CR system optimization as a real-world application.

After this brief introduction, this paper is arranged as follows: [Section 2](#) discusses FA, [Section 3](#) introduces the concept of EFA, and [Section 4](#) explains the fitness functions required for CR system optimization. Results of benchmark functions and CR system optimization using EFA are presented in [Section 5](#). Finally, conclusions are given in [Section 6](#).

2 Firefly Algorithm

The attractiveness of a firefly in the Firefly Algorithm (FA) is determined by its brightness, which is related to the fitness function being optimized [28–31]. FA is inspired by the flashing behavior of fireflies, where their flashing light helps them find potential mating partners and defend against predators [32]. In FA, the following idealized rules are applied [33]: (i) All fireflies in FA are unisexual, meaning they are attracted to each other regardless of their sex. (ii) Fireflies are differentiated based on their light intensity. Less bright fireflies are attracted to brighter ones, simulating the attraction behavior observed in fireflies. (iii) The brightness of a firefly in FA is related to the fitness function that needs to be optimized. Fireflies with higher fitness (better solutions) are represented as brighter, while those with lower fitness are dimmer.

By applying these rules, FA mimics the behavior of fireflies to guide the optimization process. It utilizes the attractiveness between fireflies to search for optimal solutions in the search space. The algorithm has shown effectiveness in various optimization applications across different fields. For the optimization problem, the brightness I of any firefly i at a particular position $x = (x_1, x_2, x_3, \dots, x_d)$ is associated to the fitness value of the objective function. For a simple case, the brightness I for a certain location x is equivalent to $I(x) \propto f(x)$. On the other hand, the attractiveness coefficient β is relative and it varies with distance r_{ij} between fireflies i and j . The light intensity $I(r)$ at assumed distance r from any of the light source follows the inverse square law as given in (1). Light is also absorbed by the

media when the distance between the source and light intensity reduces, so the attractiveness varies with the degree of absorption γ . The light intensity $I(r)$ in its simplest form is given by [33].

$$I(r) = \frac{I_s}{r^2} \quad (1)$$

where I_s is the light intensity at source. For a given medium having fixed light absorption coefficient, the light intensity I varies with the distance r_{ij} [33] in the following form:

$$I(r_{ij}) = I_0 \exp(-\gamma r_{ij}^2) \quad (2)$$

where I_0 is the original light intensity.

As the attractiveness of a firefly is proportional to the light intensity seen by neighbouring fireflies, so attractiveness β of a firefly [33] is

$$\beta(r_{ij}) = \beta_0 \exp(-\gamma r_{ij}^2) \quad (3)$$

where β_0 is the attractiveness at $r_{ij} = 0$ and r_{ij} is the distance between two fireflies i and j [33] which is defined as

$$r_{ij} = ||x_i - x_j|| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (4)$$

The equation used for the attractiveness of i^{th} firefly toward j^{th} [33] is given by

$$x_{i+1} = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \epsilon_i \quad (5)$$

where the second term is due to attraction, the third term is due to randomization, ϵ_i and α are the random number vectors generated using a uniform or Gaussian distribution, and the randomization parameter is in the range of 0 and 1. We can define different set of values for β_0 and α . If $\beta_0 = 0$, it represents a simple random walk. For practical implementations, β_0 can be set to 1, 2, or a Lévy flight can be used. The value of γ in the algorithm determines the speed of convergence with the variation of attractiveness. Typically, γ is taken to be 1 for the system to be optimized. This explanation covers the entirety of the firefly algorithm.

FA is an efficient algorithm and has served as a global problem solver but with the increase in problem complexity, the algorithm takes longer time to give the appropriate results. When given enough computational time, FA is able to provide good results. However, due to the random walk mechanism in FA, the search process can be time-consuming, which can reduce the effectiveness of the algorithm.

In FA, it is assumed that the fireflies are randomly distributed in the search space at the beginning of the optimization process. During the initial iterations, fireflies are indeed separated by large distances which leads to smaller value of $\beta(r_{ij})$. As the value of the attractiveness parameter is very small, the fireflies move towards each other slowly. This effect leads to the poor convergence performance of FA during the initial iterations. As the algorithm proceeds towards its final stage, the fireflies come closer to the optimal insect resulting in a higher value of $\beta(r_{ij})$. Because of the random walk operation, the solution undergoes large unwanted variation. This results in the oscillatory behaviour around the global optimum and causes the slow convergence in the final generations of FA [34]. Therefore, it is evident that there is room for improvement in FA to further enhance its performance. The Pseudocode of FA is shown in Algorithm 1.

Algorithm 1: Pseudo-code of FA

```

Begin
  1. Initialize:  $\alpha, \beta_0, \gamma$ , maximum iterations
  2. Define Population, objective function  $f(x)$ 
  3. Determine  $I$  at  $(x)$ :  $f(x_i)$ 
  4. While ( $t < \text{maximum iterations}$ )
      For  $i = 1$  to  $n$ 
          For  $j = 1$  to  $n$ 
              if ( $I_j > I_i$ )
                  update solution using Eq. (5)
              End if
          Evaluate new solutions and update  $I$ 
          End for  $j$ 
      End for  $i$ 
  5. Rank fireflies and update current best.
  6. End while
  7. Find final best
End

```

3 Enhanced Firefly Algorithm

To overcome the shortcomings of FA, a modified version of FA is proposed and is termed as EFA. The quality of solutions is improved by reducing the randomness in EFA. In general, there are three ways to carry out randomization: uniform randomization, random walk and heavy-tailed walks. Uniform randomization keeps the new solution between upper and lower bounds. For global and local randomization, random walks provide the solution depending upon the step size used. Heavy-tailed are the most suitable forms of randomization on global scale and a Lévy flight is one of its type [35].

Generating a random number via Lévy flights consist of two steps: (i) choice of a random direction drawn from a uniform distribution and (ii) generation of steps obeying a Lévy distribution which is a tricky affair [35]. In the present work, steps for a symmetric Lévy stable distribution are generated using the Mantegna algorithm [36]. The major advantage of using the Mantegna algorithm is its better efficiency and simplicity. The step length d in Mantegna’s algorithm [36] is calculated by

$$d = \frac{u}{|v|^{\frac{1}{\alpha}}} \tag{6}$$

where u and v are two normally distributed stochastic random variables used to generate distribution for d that exhibits similar behavior to a Lévy distribution.

$$u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2) \tag{7}$$

$$\text{where } \sigma_u = \left\{ \frac{\Gamma(1 + \alpha) \sin(\frac{\pi\alpha}{2})}{\Gamma\left[\frac{(1 + \alpha)}{2}\right] \alpha 2^{\frac{(\alpha - 1)}{2}}} \right\}^{\frac{1}{\alpha}}, \text{ and } \sigma_v = 1.$$

For $|d| \geq |d_0|$, where d_0 is the smallest step, this distribution obeys a Lévy distribution. In the Mantegna algorithm, the transition from the current location to the next location is achieved through two main steps: (a) Entry-wise multiplication of random integers, (b) Distance-based transition probability. By combining these two steps, the Mantegna algorithm creates a Markov chain-like process, where the current solution is modified by the random multiplication and the transition probability determines the likelihood of moving to the next location. This approach promotes the exploration by allowing its movement towards potentially better solutions while also considering the distance to the best solution. The required random variable is defined as follows:

$$k = C^{\frac{1}{\alpha}} k_{cn} \quad (8)$$

where $k_{cn} = \frac{1}{n^{1/\alpha}} \sum_1^n w_g$ converges to a Lévy stable distribution and its convergence is assured by central limit theorem and d is same as in the Mantegna's algorithm. The value of w is calculated using equation given in [37]. The FA algorithm gets enhanced by the addition of random variable from the Mantegna algorithm and generates a solution based on the attractiveness of fireflies and replace with a newly generated vector using

$$S = 0.01 \times d \times (x_i - g^*) \quad (9)$$

$$x_{i+1} = x_i + S \times k \quad (10)$$

where x_i is the old solution, g^* is the current optimal solution, x_{i+1} is the new solution. The Pseudocode of EFA is given in Algorithm 2.

Algorithm 2: Pseudo-code of EFA

Begin

1. Initialize: α, β_0, γ , maximum iterations
2. Define Population, objective function $f(x)$
3. Determine I at (x): $f(x_i)$
4. While ($t < \text{maximum iterations}$)
 - For $i = 1$ to n
 - For $j = 1$ to n
 - if ($I_j > I_i$)
 - update solution using Eqs. (5) and (10)
 - End if
 - Evaluate new solutions and update I
 - End for j
 - End for i
5. Rank fireflies and update current best.
6. End while
7. Find final best

End

4 CR System Optimization

In a CR system, two types of operating parameters are present: transmission parameters and environmental parameters. The transmission parameters of a CR system behave like decision variables

[38–44]. These are tunable parameters of the system and the radio adjusts its transmission knobs to matching values from the optimal set of parameters and are shown in Table 1.

Table 1: Transmission parameters [38]

Name of parameter	Description
Transmitted power	Raw transmission power
Modulation type	Type of modulation (QAM)
Modulation index (M)	Symbols count in given modulation scheme
Symbol rate (Rs)	Symbols count per second
Time division duplexing (TDD)	Percentage of transmit time
Bandwidth (B)	Bandwidth of the transmission signal

The environmental parameters are necessary to get the information about the surrounding environmental characteristics and provide this information to the CR system, which helps the CR control system to make the accurate decisions. The environmental variables used are given in Table 2.

Table 2: Environmental parameters

Parameter name	Description
BER	Bit error rate of particular modulation type
SNR	Ratio representing the signal to the noise power
Noise power	Provide information to the system about the approximate noise power

A CR system may have to meet a number of objectives in the wireless communication environment. Here five objectives or scenarios have been taken which are same as formulated in [23,24,39]. These objectives are given in Table 3 with their fitness functions and description about different variables.

Table 3: Objectives for CR system [44]

Parameter name	Fitness function	Description
Minimize power consumption	$f_{\min_power} = \frac{P}{P_{max}}$	Reduce the amount of power consumed by the system. P_{max} is maximum accessible transmit power and P is average transmit power
Minimize BER	$f_{\min_BER} = \frac{\log_{10}(0.5)}{\log_{10}(P_{be})}$	To decrease the BER. P_{be} is the bit error rate of the modulation type being used

(Continued)

Table 3 (continued)

Parameter name	Fitness function	Description
Maximize throughput	$f_{\min_throughput} = \frac{\log_2(M)}{\log_2(M_{max})}$	Increase the data throughput transferred by the CR system. M is modulation index of a single carrier and M_{max} is maximum modulation index
Minimize interference	$f_{\min_interference} = \frac{\{(P + B + TDD) - (P_{min} + B_{min} + 1)\}}{(P_{max} + B_{max} + 100)}$	Reduce the interference in CR system. P is average transmitted power, B is bandwidth demanded for a single carrier, B_{min} and B_{max} is minimum and maximum available bandwidths, respectively. TDD is the time used for transmission
Maximize spectral efficiency	$f_{\max_spectral_eff} = \frac{1 - (M \times B_{min} \times R_s)}{(B \times M_{max} \times R_{smax})}$	To use frequency spectrum efficiently. R_s is symbol rate and R_{smax} is maximum symbol rate.

It is not possible to achieve the best values of all the objectives simultaneously because of the fact that these are conflicting e.g., minimizing BER increases the power consumption. Hence, rather than targeting these objectives independently, a multi-objective function [39] is taken by linearly combining these factors as follows:

$$f_{five_objective} = w_1(f_{\min_power}) + w_2(f_{\min_BER}) + w_3(f_{\min_throughput}) + w_4(f_{\min_interference}) + w_5(f_{\max_spectraleff}) \quad (11)$$

The weighting factors w_1, w_2, w_3, w_4 and w_5 decide direction of search for the optimizing algorithm and shows the primacy of this objective in the CR decision making. Table 4 shows every weight vector for five objectives which are used in the algorithm. By incorporating weights with fitness functions, specific objectives can be evolved and optimized during the optimization process.

Table 4: Weighting factors used in different scenarios [44]

Outlines	Weight vectors for five modes
	w_1, w_2, w_3, w_4, w_5
Lowest power mode	[0.45 0.10 0.20 0.15 0.10]
Lowest BER mode	[0.10 0.50 0.10 0.10 0.20]
Highest throughput mode	[0.10 0.15 0.50 0.15 0.10]
Lowest interference mode	[0.10 0.10 0.20 0.50 0.10]
Highest spectral efficiency mode	[0.10 0.15 0.15 0.10 0.50]

5 Results and Discussion

5.1 Benchmark Results

The performance of the EFA algorithm is evaluated in this section by using benchmark functions. Eight unconstrained real objective benchmark functions [38,39] are employed for the optimization using EFA. The set of functions along with variable range for determining optimality is shown in Table 5. Artificial bee colony (ABC), BBO, bat algorithm (BA), and FA are used for the purpose of comparison. The associated set of initial conditions for the competitive algorithms is illustrated in Table 6. In order to ensure that the algorithm finds optimal solution consistently, each algorithm runs over 20 times. In each scenario, the maximum number of function evaluations (NFEs) has been set at $500 \times 20 = 10,000$.

Table 5: Benchmark functions used in simulation

Test problems	Objective function	Search range	Optimum value	D
Hartmann function 3	$f_1(x) = -\sum_{i=1}^4 \alpha_i \exp\left[-\sum_{j=1}^3 A_{ij}(x_j - P_{ij})^2\right]$	[0, 1]	-3.86278	3, M
Hartmann function 6	$f_2(x) = -\sum_{i=1}^4 \alpha_i \exp\left[-\sum_{j=1}^6 A_{ij}(x_j - P_{ij})^2\right]$	[0, 1]	-3.32237	6, M
Shekel function 5	$f_3(x) = -\sum_{j=1}^5 \left[\sum_{i=1}^4 \left((x_i - C_{ij})^2 + \beta_j\right)^{-1}\right]$	[0, 10]	-10.1532	4, M
Shekel function 7	$f_4(x) = -\sum_{j=1}^7 \left[\sum_{i=1}^4 \left((x_i - C_{ij})^2 + \beta_j\right)^{-1}\right]$	[0, 10]	-10.4029	4, M
Shekel function 10	$f_5(x) = -\sum_{j=1}^{10} \left[\sum_{i=1}^4 \left((x_i - C_{ij})^2 + \beta_j\right)^{-1}\right]$	[0, 10]	-10.5364	4, M
Rastrigin function	$f_6(x) = 10D + \sum_{i=1}^D [x_i^2 - 10 \cos(2\pi x_i)]$	[-5.12, 5.12]	0	30, M, NC
Six Hump Camel function	$f_7(x) = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$	[-5, 5]	-1.0316	2, M
Goldstein & price function	$f_8(x) = \left(1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)\right) (30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))$	[-2, 2]	3	2, M

Table 6: EFA and FA parameter settings

Algorithm	Parameters	Values
FA	Number of fireflies	20
	Alpha (α)	0.25
	Beta (β)	0.20
	Gamma (γ)	1
	Maximum number of iterations	500
	Stopping Criteria	Max Iteration.

(Continued)

Table 6 (continued)

Algorithm	Parameters	Values
BA	Population size	20
	Loudness	0.5
	Pulse rate	0.5
	[Qmin, Qmax]	[0, 1]
	Maximum number iterations	1000
	Stopping Criteria	Max Iteration.
ABC	Colony size (SN)	20
	Number of food sources	SN/2
	Limit	100
	Maximum number iterations	500
	Stopping Criteria	Max Iteration.
BBO	Population Size	20
	Mutation probability	0.25
	Habitat modification probability	1
	Maximum number of iterations	500
	Stopping Criteria	Max Iteration.
EFA	Number of fireflies	20
	Alpha (α)	0.25
	Beta (β)	0.20
	Gamma (γ)	1
	Maximum number of iterations	500
	Stopping Criteria	Max Iteration.

In Table 7, best values are shown in the bold text. For functions, f_2, f_5, f_6, f_7 and f_8 , the standard deviation of EFA is much better except for f_1 in which FA is better, f_3 where ABC is better and f_4 where BA is better. The mean value attained by proposed algorithm is better for seven function except for only f_2 and f_3 where FA is better. As far as, the best value is concerned, EFA gives best for most of the test function except for f_6 where BA is better. The results of the experiments demonstrate that the proposed EFA outperforms other algorithms such as ABC, BBO, BA, and FA across most of the test functions. EFA exhibits better mean and standard deviation values compared to the competing algorithms, indicating its superior performance.

Table 7: Simulation results for benchmark functions

Objective function	Algorithm	Best	Worst	Mean	Standard deviation
$f_1(x)$	ABC	-3.7754	-2.4110	-3.2397	4.15E-01
	BBO	-3.2234	-0.0024	-0.9673	9.55E-01
	BA	-3.8628	-3.0898	-3.7855	0.2379

(Continued)

Table 7 (continued)

Objective function	Algorithm	Best	Worst	Mean	Standard deviation
$f_2(x)$	FA	-3.8628	-3.8628	-3.8628	3.3120e-007
	EFA	-3.8628	-3.7951	-3.8548	1.68E-02
	ABC	-2.1963	-0.7290	-1.3816	4.64E-01
	BBO	-3.1452	-1.9059	-2.7501	3.02E-01
	BA	-3.3224	-3.2031	-3.2627	0.0612
	FA	-3.3224	-3.1915	-3.2672	0.0626
	EFA	-3.3224	-3.0941	-3.2514	8.67E-03
$f_3(x)$	ABC	-10.1073	-2.5928	-6.6114	3.0956
	BBO	-10.1532	-2.6304	-6.1444	3.4791
	BA	-10.1525	-2.6305	-5.0186	3.1879
	FA	-10.1528	-2.6304	-6.7884	3.8155
	EFA	-10.1532	-2.6305	-7.9095	3.5164
$f_4(x)$	ABC	-10.5054	-1.6680	-5.9055	3.2257
	BBO	-10.4028	-2.7659	-7.6097	3.5463
	BA	-10.4029	-1.8376	-4.0264	2.4908
	FA	-10.4028	-2.7519	-9.2542	2.8025
	EFA	-10.4029	-2.7519	-9.2567	2.7995
$f_5(x)$	ABC	-10.4642	-1.8508	-5.3552	3.4295
	BBO	-10.5363	-2.8066	-7.3243	3.6585
	BA	-10.5364	-1.6766	-4.1937	3.3005
	FA	-10.5362	-10.5347	-10.5355	4.8553e-004
	EFA	-10.5364	-10.5364	-10.5364	7.29E-06
$f_6(x)$	ABC	4.21E+01	9.27E+01	6.66E+01	1.37E+01
	BBO	9.6741	2.03E+01	1.65E+01	3.1049
	BA	8.1068e-009	12.9344	4.0793	3.1940
	FA	3.6393e-006	1.1007e-004	4.0660e-005	3.2287e-005
	EFA	1.3160e-008	2.4979e-007	1.0142e-007	8.5511e-008
$f_7(x)$	ABC	-1.0316	-1.0261	-1.0305	1.50E-03
	BBO	-1.0234	-0.0479	-0.7314	3.45E-01
	BA	-1.0316	-0.2155	-0.7868	0.3837
	FA	-1.0316	-1.0316	-1.0316	1.2261e-006
	EFA	-1.0316	-1.0316	-1.0316	4.58E-09
$f_8(x)$	ABC	3.0003	3.0904	3.0190	2.52E-02
	BBO	3.0000	3.0000	3.0000	0
	BA	3.0000	84.0000	16.5000	25.5394
	FA	3.0000	3.0000	3.0000	1.4827e-005
	EFA	3.0000	3.0000	3.0000	2.44E-08

To validate the significant improvement offered by EFA, two statistical tests Wilcoxon's rank-sum test and Friedman rank (f-rank) test were conducted. The f-rank test assigns rank to each algorithm based on their performance. From the first row of [Table 8](#), it is evident that EFA significantly

outperforms the other algorithms and secures the first rank in the benchmark suite. The rank-sum test is performed for each individual function to determine whether EFA is significantly better or not. The performance of EFA is expressed as $win(w)/loss(l)/tie(t)$ in the second row of Table 8. The situation $win(w)$ arises when the algorithm being tested performs better than EFA and is denoted by a '+' sign. Conversely, the situation $loss(l)$ occurs when the performance of the test algorithm is worse than EFA and is denoted by a '-' sign. The last situation, $tie(t)$, indicates that there is no statistical difference between the algorithms under test and is denoted by an '=' sign. From the $w/l/t$ row in Table 8, it can be observed that EFA is significantly better than the other algorithms in most of the cases. This further reinforces the superior performance of the proposed EFA algorithm compared to the competing algorithms. In addition to the benchmark functions, EFA is further applied to real-life application of CR system.

Table 8: Statistical results for benchmark functions

Objective function		Algorithm				
		ABC	BBO	BA	FA	EFA
$f_1(x)$	p-rank	-	-	-	+	N/A
	f-rank	4	5	3	1	2
$f_2(x)$	p-rank	-	-	-	-	N/A
	f-rank	5	4	3	2	1
$f_3(x)$	p-rank	-	-	-	-	N/A
	f-rank	5	2	4	3	1
$f_4(x)$	p-rank	-	-	+	-	N/A
	f-rank	5	4	1	3	2
$f_5(x)$	p-rank	-	-	-	-	N/A
	f-rank	4	5	3	2	1
$f_6(x)$	p-rank	-	-	-	-	N/A
	f-rank	5	4	3	2	1
$f_7(x)$	p-rank	-	-	-	-	N/A
	f-rank	3	4	5	2	1
$f_8(x)$	p-rank	-	+	-	-	N/A
	f-rank	4	1	5	3	2
w/l/t		0/8/0	1/7/0	1/7/0	1/7/0	
Overall f-rank		35	29	27	18	10
Average f-rank		5	4	3	2	1

5.2 Simulation Results for CR Optimization

The optimization of CR systems is considered a complex problem due to the conflicting nature of the parameters that need to be optimized. As mentioned in previous literature [23–24], five scenarios have been selected for optimization in this study. These scenarios are determined by five transmission variables, namely transmitted power (P), bandwidth (B), modulation index (M), time-division duplex (TDD), and symbol rate (R_s). The min. and max. values for each of these variables are provided in Table 9. Each firefly in the EFA algorithm is represented by a set of values for these transmission variables [P, B, M, TDD, R_s].

Table 9: Range for transmission variables

Transmission variable	Range
Power transmitted (P) in mW	[0.158–251]
Bandwidth (B) in MHz	[2–32]
Number of symbols (M)	[2–256]
TDD in %	[25–100]
Symbol rate (R_s) in ksps	[125–1000]

Values of different parameters taken in EFA as well as in FA are given as follows:

- Population size = 20
- Generations = 200
- Randomization parameter, $\alpha = 0.25$
- Attractiveness coefficient $\beta_o = 0.2$
- Absorption coefficient, $\gamma = 1$

The fitness values found by EFA are compared with that of SA [23], GA [23], and BBO [24]. The FA is also applied to the CR system in order to contrast the performance of EFA with FA. The best values of fitness attained in each case are highlighted in the last column of Table 10.

Table 10: Comparison of fitness values obtained by EFA with SA [23], GA [23] and BBO [24], and FA for different scenarios

Scenario	Algorithm	Optimized parameters					Fitness value
		Transmitted power (mw)	Modulation index (M)	Bandwidth (MHz)	TDD (%)	Symbol rate (ksps)	
Lowest power mode	EFA	4.500	256	2.000	61.134	950.0639	0.0229
	FA	4.400	256	2.000	63.8	1000	0.0229
	BBO [24]	3.960	256	2.153	25	1000	0.0308
	SA [23]	3.64	256	21.860	56.24	915.72	0.03661
	GA [23]	4.39	256	2.000	31.40	698.01	0.05478
Lowest BER mode	EFA	36.2	256	2.000	37.2	1000	0.0313
	FA	36.2	256	2.000	37.2	1000	0.0313
	BBO [24]	36.25	256	2.114	25	998.90	0.0425
	SA [23]	25.83	256	2.00	85.60	901.68	0.07004
	GA [23]	16.83	256	2.01	65.60	839.98	0.08674

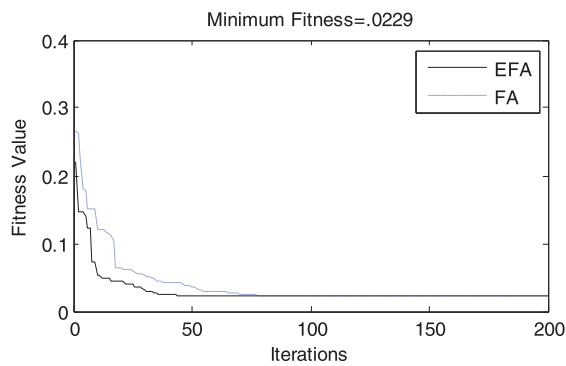
(Continued)

Table 10 (continued)

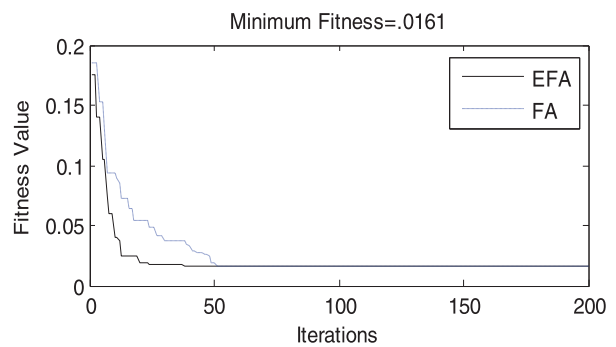
Scenario	Algorithm	Optimized parameters					
		Transmitted power (mw)	Modulation index (M)	Bandwidth (MHz)	TDD (%)	Symbol rate (ksps)	Fitness value
Highest throughput mode	EFA	17.5	256	2.00	42.1	1000	0.0161
	FA	17.5	256	2.00	42.1	1000	0.0161
	BBO [24]	17.739	256	2.114	25	998.98	0.0425
	SA [23]	15.27	256	2.00	33.80	923.852	0.02380
	GA [23]	12.684	256	2.031	63.2	540.970	0.0635
Minimum interference mode	EFA	13.5	256	2.00	65.7	1000	0.0128
	FA	12.4913	256	2.00	57.48	980.49	0.0148
	BBO [24]	13.369	256	2.032	25	998.79	0.015
	SA [23]	15.23	256	2.00	1000	047.30	0.04924
	GA [23]	7.659	256	2.00	65.8	353.484	0.0786
Highest efficiency mode	EFA	24.6	256	2.000	78.6	1000	0.0161
	FA	24.55	256	2.000	78.6	1000	0.0168
	BBO [24]	16.539	256	2.036	25	1000	0.0251
	SA [23]	34.57	256	2.00	59.60	099.18	0.0194
	GA [23]	12.366	256	2.010	29.4	962.498	0.0380

From Table 10, it can be seen that in each of the five scenarios, EFA was consistently able to secure better value of fitness as compared to SA [23], GA [23], and BBO [24]. In comparison to FA, EFA performed significantly better in the minimum interference and maximum efficiency scenarios, while still achieving comparable results in the other three scenarios. Both FA and EFA were able to score the same fitness values except in the maximum efficiency mode where EFA performed better than FA. This demonstrates the capability of EFA to effectively identify the global optimum in various scenarios.

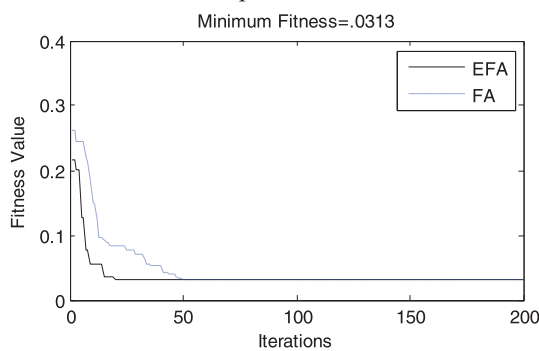
The convergence performance of an optimization algorithm is indeed a crucial factor in determining its practicality. If an algorithm takes excessively long to reach a near-optimal solution, it may not be suitable for real-world applications. The convergence curves for all the five objectives of optimization are shown in Fig. 1. The convergence performance of EFA is compared with basic FA. Fig. 1a shows the convergence curve obtained by EFA and FA for the minimum power consumption mode. FA took nearly 80 generations or iterations to converge whereas EFA converged in 40 iterations. EFA also outperformed BBO which required 90 iterations to attain its final value for the same scenario [24]. In the maximum throughput mode, EFA needed 40 iterations to converge to minimum value whereas FA found its minimum value in about 150 generations. In this case, EFA managed to show a big improvement over FA. Figs. 1d and 1e give the convergence properties of EFA and FA for the minimum interference mode and maximum efficiency mode respectively. Here, EFA not only achieved faster convergence but also delivered better value of the fitness as compared to FA. This may be attributed to changes made in the basic firefly algorithm to prevent oscillation of the algorithm around the final solution which results in finding the optimum more efficiently.



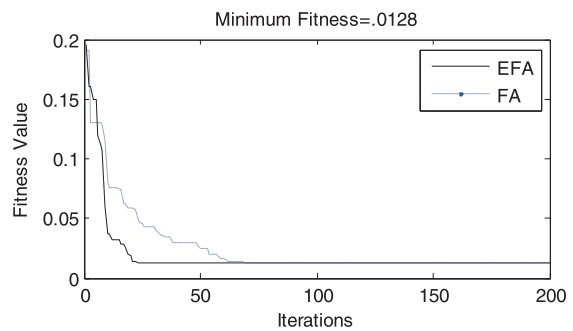
(a) Convergence curve for minimum power consumption mode



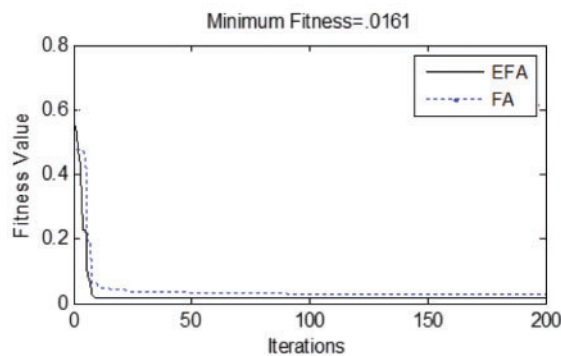
(c) Convergence curve for maximum throughput mode



(b) Convergence curve for minimum BER mode



(d) Convergence curve for minimum interference mode



(e) Convergence curve for maximum efficiency mode

Figure 1: The convergence curves for all the five objectives of optimization

6 Discussion

The EFA has demonstrated its efficiency in solving the benchmark functions and the CR system optimization problem. It has shown significant advancements compared to other state-of-the-art algorithms. The simplicity of its structure and ease of implementation make it suitable for inclusion in the expert and hybrid intelligent systems. The convergence properties of EFA further confirm its suitability as an optimization algorithm.

However, like other stochastic algorithms, EFA is susceptible to getting stuck at local minima and may not always achieve the global optima for all benchmark functions. The benchmark set used in the evaluation consists of various types of problems, including unimodal, multimodal, and composite functions. An algorithm that can solve the entire benchmark set and consistently reach the global solution can be considered as a state-of-the-art algorithm. While EFA may encounter local optima in some cases, it still possesses the potential to become a standard algorithm.

Further research and improvements are needed to address the limitations of EFA and enhance its ability to achieve global solutions. By addressing these challenges, EFA can progress towards becoming a widely accepted and recognized state-of-the-art algorithm.

7 Conclusion

This paper introduced a novel variant of FA named EFA. In the proposed technique, the unique and fascinating features of FA are retained and performance is improved by reducing the randomness in the search mechanism using Mantegna's algorithm. The proposed technique was employed to optimize benchmark functions and the CR system. EFA yielded superior results in the optimization of benchmark functions in contrast to ABC, FA, BA, and BBO. CR transmission parameters, such as transmitted power, modulation index, bandwidth, TDD and symbol rate, have been optimized by EFA in different environments to meet various objectives. The EFA scored better fitness values than BBO, SA, GA. In addition, EFA converged in almost half or fewer generations than basic FA and BBO. Both these factors make EFA an attractive choice as an optimization tool. Finally, it can be concluded that EFA is a robust optimization technique, and it is anticipated that it can be used to optimize other real-world problems like animal tracking, cancer classification, logistics, coal mine workers' tracking, gene expression modeling, feature selection, clustering problems, underwater wireless sensor networks, and various industrial applications.

In addition to these applications, exploring the implementation of another hybrid meta-heuristic algorithm can offer improved accuracy and reduced convergence time for EFA. Future work can focus on incorporating a balanced exploration and exploitation strategy to further enhance the algorithm's performance. Introducing different mutation operators and chaotic maps can also be considered to analyze their impact on the performance of the EFA algorithm. To enhance the global and local search capabilities of EFA, new exploratory and exploitative search equations can be introduced. These additions can help improve the algorithm's ability to effectively explore the solution space and exploit promising regions. By incorporating these enhancements, the algorithm's overall performance can be further improved, making it more efficient and effective in solving optimization problems.

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