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Article

Computational Analysis for Computer Network Model with Fuzziness

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Abstract: A susceptible, exposed, infectious, quarantined and recovered (SEIQR) model with fuzzy parameters is studied in this work. Fuzziness in the model arises due to the different degrees of susceptibility, exposure, infectivity, quarantine and recovery among the computers under consideration due to the different sizes, models, spare parts, the surrounding environments of these PCs and many other factors like the resistance capacity of the individual PC against the virus, etc. Each individual PC has a different degree of infectivity and resistance against infection. In this scenario, the fuzzy model has richer dynamics than its classical counterpart in epidemiology. The reproduction number of the developed model is studied and the equilibrium analysis is performed. Two different techniques are employed to solve the model numerically. Numerical simulations are performed and the obtained results are compared. Positivity and convergence are maintained by the suggested technique which are the main features of the epidemic models.

Keywords: NSFD method; computer virus; fuzzy parameters; convergence; stability

1 Introduction

A computer virus is a type of malicious software (malware) that is designed to replicate itself and spread to other computers, often with the intent of causing damage to the infected system or stealing sensitive information. Computer viruses can be spread through email attachments, infected software downloads, malicious websites, or by exploiting vulnerabilities in the operating system or other software on a computer. Once a virus infects a computer, it can perform various harmful activities, such as deleting or corrupting files, stealing personal information, or using the computer as part of a botnet to launch attacks on other computers. To protect against computer viruses, it's important to use anti-virus software and keep it up to date, avoid downloading files or clicking links from untrusted sources, and keep your operating system and other software up to date with the latest security patches. It's also important to be cautious when opening email attachments or clicking on links, and to always verify the authenticity of messages or requests from unfamiliar sources. Morri's worm was one of the first computer worms to gain widespread attention. It exploited vulnerabilities in UNIX systems to replicate itself and caused significant disruption to the early internet. Melissa was one of the first email-based viruses to spread rapidly across the internet. It was a macro virus that infected Microsoft Word documents and then spread to other documents and email contacts. I LOVE YOU virus was a computer worm that spread through email and internet messaging systems. It was a simple virus that tricked users into opening an attachment that contained the virus code. Code Red was a worm that targeted Microsoft IIS web servers. It was able to replicate itself and caused significant disruption to internet traffic. Nimda was a worm that spread through email and network shares. It was able to infect a computer by exploiting multiple vulnerabilities in web servers and email systems. Sasser was a worm that spread through network shares and caused significant disruption to computer systems. It was able to infect a computer without any user interaction. Conficker was a worm that spread through network shares and USB drives. It was able to infect millions of computers worldwide and was difficult to detect and remove [1].

Mathematical modeling of computer viruses involves creating mathematical models that can simulate the behavior and spread of a virus within a computer network or system. Such models can help researchers understand the dynamics of virus spread and predict the potential impact of different virus control strategies. There are several mathematical models used to study the spread of computer viruses. Yang et al. developed the SLBS computer model for studying virus propagation [2], while Ahmed et al. proposed a Spatio-temporal computer virus model [3]. Ali et al. investigated virus propagation using padé approximation [4], and Ebenezer et al. developed a fractional model that accounted for the interaction between computers and removable devices [5]. Lanz et al. presented a virus model that included a quarantine strategy [6], and Xu et al. proposed a new model with a limited antivirus capacity [7]. Parsaei et al. developed a novel mathematical model of computer viruses [8], while Deng et al. presented a Susceptible-Infected-Recovered-Dead (SIRD) model and examined the virus transmission mechanisms [9]. Finally, Tuwairqi et al. suggested two computer virus-propagation isolation strategies [10]. Mathematical models were developed by numerous writers in the past to study the transmission of computer viruses [11–19].

The concepts of susceptible, infectious, and recovered are uncertain in the sense that different individuals in the population have varying degrees of susceptibility, infectivity, and recovery. Such differences can occur when different population groups have different habits and customs, and different age groups have varying degrees of resistance, etc. More realistic models that take into account the individuals' varying degrees of susceptibility, infectivity, and recovery are required. Such uncertainty can be handled by fuzzy theory. Fuzzy theory is effective at solving complex problems characterized by environmental uncertainty and information fuzziness. It enables the handling of uncertain and imprecise knowledge and provides a powerful reasoning framework. Zadeh first proposed the fuzzy theory in 1965 [20]. Many researchers have applied fuzzy theory to epidemiology. The fuzzy theory has been applied in various ways to develop and enhance epidemic models. Epidemic models with fuzzy transmission coefficients were studied by Mondal et al. [21]. Mishra et al. proposed a Susceptible Infectious–Recovered–Susceptible (SIRS) model for the fuzzy transmission of worms in a computer network. The three cases of epidemic control strategies of worms in the computer network– low, medium, and, high–are analyzed. Numerical methods are employed to solve and simulate the system of equations developed [22]. Padmapriya et al. investigated a model for COVID-19 prediction using a Caputo fractional derivative in a fuzzy sense. The model's numerical results for COVID-19 in the United States, India, and Italy are presented. Future outbreaks, the effectiveness of preventive measures, and potential infection control strategies are also estimated [23]. Korenevskiy et al. studied kidney injuries on the basis of fuzzy models [24]. Adak et al. proposed a mathematical model with arbitrary disease transmission and treatment control functions. The disease transmission and treatment functions are considered as fuzzy numbers [25]. Sambas et al. examined the chaotic behavior and designed a type-2 fuzzy controller for the Permanent Magnet Synchronous Generator (PMSG) in a wind turbine system [26]. Sambas et al. developed a fractional-order model financial risk dynamical system and studied its periodic and chaotic behaviors [27]. Fuzzy control is proposed and numerical simulations are performed.

The mathematical and numerical modeling of diseases makes substantial use of the NSFD theory, which Mickens proposed [28]. Allehiany et al. studied a Covid-19 model with fuzziness using the NSFD scheme for its numerical solution [29]. Alhebshi et al. investigated a computer virus model with fuzzy criteria [30]. Equilibrium and reproduction analysis is performed for the studied model. Forward Euler and NSFD schemes were used for the numerical solution of the model. The NSFD scheme was found to be preserving stability, convergence and positivity while the forward Euler method failed to preserve these important features of the epidemic models. Many other researchers also used the NSFD method and fuzzy theory in their studies [31–33], just to mention a few. The current work extends the computer virus propagation model by including fuzzy parameters, which enables a more thorough explanation of how infections spread within computers. The first-order explicit scheme's creation, implementation, and mathematical analysis in fuzzy environments with NSFD settings, specifically with fuzzy parameters, are novel aspects of the created technique.

2 Formulation of the Model and Mathematical Analysis

Consider the following system of 5^{th} first order ordinary differential equations representing the SEIQRS model.

$$\frac{dS}{dt} = A - \beta SI - bS + \eta R. \tag{1}$$

$$\frac{dE}{dt} = \beta SI - (b + \mu) E. \tag{2}$$

$$\frac{dI}{dt} = \mu E - (b + \alpha + \gamma + \delta) I. \tag{3}$$

$$\frac{dQ}{dt} = \delta I - (b + \alpha + \epsilon) Q. \tag{4}$$

$$\frac{dR}{dt} = \gamma I + \epsilon Q - (b + \eta) R. \tag{5}$$

The corresponding model with fuzzy parameters can be written as

$$\frac{dS}{dt} = A - \beta(\rho) SI - bS + \eta R. \tag{6}$$

$$\frac{dE}{dt} = \beta(\rho) SI - (b + \mu) E. \tag{7}$$

$$\frac{dI}{dt} = \mu E - (b + \alpha(\rho) + \gamma(\rho) + \delta) I. \tag{8}$$

$$\frac{dQ}{dt} = \delta I - (b + \alpha(\rho) + \epsilon) Q. \tag{9}$$

$$\frac{dR}{dt} = \gamma I + \epsilon Q - (b + \eta) R. \tag{10}$$

The average effective interactions with other nodes per unit of time, crashing of nodes due to the attack of malicious objects and recovery from infection are considered fuzzy numbers due to their uncertain natures. These parameters are denoted by $\beta(\rho)$, $\alpha(\rho)$ and $\gamma(\rho)$, respectively, and are defined below. Fig. 1 shows the flowchart of the model. Detail of the other variables and parameters are given in Tables 1 and 2 respectively.

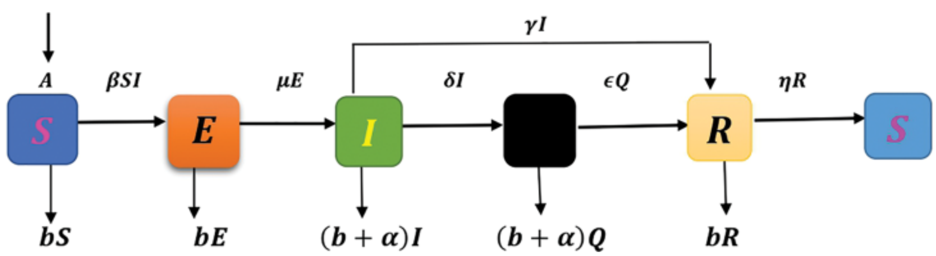

Figure 1: Flowchart of the model

Table 1: The description of the variables

Variable	Description
S	Susceptible humans
E	Endemic humans
I	Infectious humans
Q	Quarantined
R	Recovered humans
N	The total population

Table 2: The description of the parameters

Parameter	Description
A	The rate at which susceptible nodes are added
b	The rate of natural mortality
μ	The rate of nodes leaving E
δ	The rate of nodes leaving I for the Q class
ϵ	Recovery rate
η	The rate of loss of immunity

$$\beta(\rho) = \begin{cases} 0, & \rho \le \rho_{min} \\ \dfrac{\rho - \rho_{min}}{\rho_M - \rho_{min}}, & \rho_{min} < \rho \le \rho_M \\ 1, & \rho_M < \rho, \end{cases} \tag{11}$$

$$\gamma(\rho) = \frac{\gamma_0 - 1}{\rho_M}\rho + 1, \qquad 0 \le \rho \le \rho_{min}, \tag{12}$$

and

$$\alpha(\rho) = \begin{cases} \dfrac{(1-\rho) - \epsilon_0}{\rho_{min}}\rho + \epsilon_0, & 0 \le \rho \le \rho_{min} \\ 1 - \rho, & \rho_{min} < \rho. \end{cases} . \tag{13}$$

2.1 The Fuzzy Basic Reproductive Number (BRN) R_0^f

The BRN R_0 is given by

$$R_0 = \frac{\beta(\rho)\left(\frac{A}{b}\right)}{\mu + \alpha(\rho) + \delta + \gamma(\rho) + b}. \tag{14}$$

Since R_0 is a direct function of a computer virus ρ can be analyzed as follows:

Case 1: If $\rho < \rho_{min}$, then we have $\beta(\rho) = 0$ and we obtain,

$R_h(\rho) = 0.$

Case 2: If $\rho_{min} < \rho \le \rho_M$, then we have $\beta(\rho) = \dfrac{\rho - \rho_{min}}{\rho_M - \rho_{min}}$ and we obtain,

$$R_0 = \frac{\beta(\rho)\left(\frac{A}{b}\right)}{\mu + \alpha + \delta + \gamma + b}.$$

Case 3: If $\rho_M < \rho < \rho_{max}$, then we have $\beta(\rho) = 1$ and we obtain,

$$R_0 = \frac{\left(\frac{A}{b}\right)}{\mu + \alpha + \delta + \gamma + b}.$$

$R_0(\rho)$ can be expressed as

$$R_0(\rho) = \left(0, \frac{\beta(\rho)\left(\frac{A}{b}\right)}{\mu + \alpha(\rho) + \delta + \gamma(\rho) + b}, \frac{\frac{A}{b}}{\mu + \alpha(\rho) + \delta + \gamma(\rho) + b}\right).$$

The fuzzy reproduction number can be found as follows:

$$R_0^f = E\left[R_h\left(\rho\right)\right], = \frac{\frac{A}{b}\left(2\beta\left(\rho\right)+1\right)}{\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b}.$$

2.2 Equilibrium Analysis

Case 1: If $\rho < \rho_{min}$, we obtain:

$$E^0\left(S^0, E^0, I^0, Q^0, R^0\right) = \left(\frac{A}{b}, 0, 0, 0, 0\right).$$

Case 2: If $\rho_{min} < \rho \leq \rho_M$, then we have $\beta\left(\rho\right) = \frac{\rho - \rho_{min}}{\rho_M - \rho_{min}}$ and we obtain $E^*\left(S^*, E^*, I^*, Q^*, R^*\right)$, where

$$S^* = \frac{\frac{A}{b}}{R_0}, E^* = \frac{b(R_0-1)}{\beta\left(\rho\right)}, I^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\frac{\mu b}{b+\alpha\left(\rho\right)}\right), Q^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\frac{\delta b}{\epsilon+b+\alpha\left(\rho\right)}\right),$$

$$R^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\gamma+\frac{\epsilon\delta\eta}{\eta+\epsilon+b+\alpha\left(\rho\right)}\right).$$

Case 3: If $\rho_M < \rho < \rho_{max}$, then we have $\beta\left(\rho\right) = 1$ and we obtain $E^{**}\left(S^{**}, E^{**}, I^{**}, Q^{**}, R^{**}\right)$, where

$$S^* = \frac{\frac{A}{b}}{R_0}, E^* = \frac{b(R_0-1)}{\beta\left(\rho\right)}, I^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\frac{\mu b}{b+\alpha\left(\rho\right)}\right), Q^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\frac{\delta b}{\epsilon+b+\alpha\left(\rho\right)}\right),$$

$$R^* = \frac{R_0-1}{\beta\left(\rho\right)}\left(\gamma+\frac{\epsilon\delta\eta}{\eta+\epsilon+b+\alpha\left(\rho\right)}\right).$$

2.3 Sensitivity Analysis

For β we have

$$\xi\left(\beta\right) = \frac{\beta\left(\rho\right)}{R_0}.\frac{dR_0}{d\beta\left(\rho\right)} = \frac{\beta\left(\rho\right)}{\frac{\beta\left(\rho\right)\left(\frac{A}{b}\right)}{\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b}}.\frac{d\left(\frac{\beta\left(\rho\right)\left(\frac{A}{b}\right)}{\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b}\right)}{d\beta\left(\rho\right)} = 1.$$

Similarly,

$$\xi\left(d\right) = \frac{b}{R_0}.\frac{dR_0}{db} = \frac{-(\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+2d)d^2}{\beta\left(\rho\right)A(\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b)},$$

$$\xi\left(\eta\right) = \frac{\eta}{R_0}.\frac{dR_0}{d\eta} = 0,$$

$$\xi\left(\mu\right) = \frac{\mu}{R_0}.\frac{dR_0}{d\mu} = \frac{-\mu}{(\mu+\alpha+\delta+\gamma+b)},$$

$$\xi\left(\alpha\right) = \frac{\alpha}{R_0}.\frac{dR_0}{d\alpha} = \frac{-\alpha\left(\rho\right)}{(\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b)},$$

$$\xi\left(\gamma\right) = \frac{\gamma}{R_0}.\frac{dR_0}{d\gamma} = \frac{-\gamma}{(\mu+\alpha\left(\rho\right)+\delta+\gamma\left(\rho\right)+b)},$$

$$\xi(\delta) = \frac{\delta}{R_0}.\frac{dR_0}{d\delta} = \frac{-\delta}{(\mu + \alpha(\rho) + \delta + \gamma(\rho) + b)},$$

$$\xi(\epsilon) = \frac{\epsilon}{R_0}.\frac{dR_0}{d\epsilon} = \frac{-\epsilon}{(\mu + \alpha(\rho) + \delta(\rho) + \gamma + b)},$$

$$\xi(A) = \frac{A}{R_0}.\frac{dR_0}{dA} = \frac{\beta}{b(\mu + \alpha(\rho) + \delta + \gamma(\rho) + b)}.$$

3 Numerical Modelling

3.1 Forward Euler Method

$$S^{n+1} = S^n + h(A - \beta(\rho) S^n I^n - bS^n + \eta R^n), \tag{15}$$

$$E^{n+1} = E^n + h(\beta(\rho) S^n I^n - (b + \mu) E^n), \tag{16}$$

$$I^{n+1} = I^n + h(\mu E^n - (b + \alpha(\rho) + \gamma(\rho) + \delta) I^n), \tag{17}$$

$$Q^{n+1} = Q^n + h(\delta I^n - (b + \alpha(\rho) + \epsilon) Q^n), \tag{18}$$

$$R^{n+1} = R^n + h(\gamma(\rho) I^n + \epsilon Q^n - (b + \eta) R^n). \tag{19}$$

3.2 NSFD Scheme

$$S^{n+1} = \frac{S^n + h(A + \eta R^n)}{1 + h(\beta(\rho) i^n + hb)} \tag{20}$$

$$E^{n+1} = \frac{S^n + h\beta(\rho) s^n I^n}{1 + h(b + \mu)}, \tag{21}$$

$$I^{n+1} = \frac{I^n + h\mu E^n}{1 + h(b + \alpha(\rho) + \gamma(\rho) + \delta)}, \tag{22}$$

$$Q^{n+1} = \frac{Q^n + h\delta I^n}{1 + h(b + \alpha(\rho) + \epsilon)}, \tag{23}$$

$$R^{n+1} = \frac{R^n + (\gamma(\rho) I^n + \epsilon Q^n) h}{1 + h(b + \mu)}. \tag{24}$$

3.2.1 Convergence Analysis

In this section convergence analysis of the NSFD scheme of the SEIQR model will be done at disease free equilibrium (DFE) point.

$$A_1 = \frac{S + h(A + \eta R)}{1 + h(\beta(\rho) I + hb)},$$

$$A_2 = \frac{S + h\beta(\rho) SI}{1 + h(b + \mu)},$$

$$A_3 = \frac{I + h\mu E}{1 + h(b + \alpha\,(\rho) + \gamma\,(\rho) + \delta)},$$

$$A_4 = \frac{Q + h\delta I}{1 + h(b + \alpha\,(\rho) + \epsilon)},$$

$$A_5 = \frac{R + (\gamma\,(\rho)\,I + \epsilon Q)\,h}{1 + h(b + \mu)}.$$

$$J = \begin{bmatrix} \frac{dA_1}{ds} & \frac{dA_1}{de} & \frac{dA_1}{di} & \frac{dA_1}{dr} & \frac{dA_1}{dv} \\ \frac{dA_2}{ds} & \frac{dA_2}{de} & \frac{dA_2}{di} & \frac{dA_2}{dr} & \frac{dA_2}{dv} \\ \frac{dA_3}{ds} & \frac{dA_3}{de} & \frac{dA_3}{di} & \frac{dA_3}{dr} & \frac{dA_3}{dv} \\ \frac{dA_4}{ds} & \frac{dA_4}{de} & \frac{dA_4}{di} & \frac{dA_4}{dr} & \frac{dA_4}{dv} \\ \frac{dA_5}{ds} & \frac{dA_4}{de} & \frac{dA_4}{di} & \frac{dA_4}{dr} & \frac{dA_4}{dv} \end{bmatrix},$$

$$J = \left[\begin{matrix} \frac{1}{1 + h(\beta\,(\rho)\,I + b)} & 0 & -\frac{h\beta\,(\rho)\,(S + h(A + \eta R)}{(1 + h(\beta\,(\rho)\,I + b))^2} \\ \frac{\beta\,(\rho)\,I}{1 + h(b + \mu)} & \frac{1}{1 + h(b + \mu)} & \frac{\beta\,(\rho)\,S}{1 + h(b + \mu)} \\ 0 & \frac{h\mu}{1 + h(b + \alpha\,(\rho) + \gamma\,(\rho) + \delta)} & \frac{1}{1 + h(b + \alpha\,(\rho) + \gamma\,(\rho) + \delta)} \\ 0 & 0 & \frac{h\delta}{1 + h(b + \alpha\,(\rho) + \epsilon)} \\ 0 & 0 & \frac{h\gamma\,(\rho)}{1 + h(b + \mu)} \end{matrix} \right.$$

$$\times \left. \begin{matrix} 0 & \frac{h\eta}{1 + h(\beta\,(\rho)\,I + b)} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{1 + h(b + \alpha\,(\rho) + \epsilon)} & 0 \\ \frac{h\epsilon}{1 + h(b + \mu)} & \frac{1}{1 + h(b + \mu)} \end{matrix} \right]$$

Jacobian at the DFE is

$$J(p_0) = \begin{bmatrix} \frac{1}{1+hb} & 0 & 0 \\ 0 & \frac{1}{1+h(b+\mu)} & 0 \\ 0 & \frac{h\mu}{1+h(b+\alpha(\rho)+\gamma(\rho)+\delta)} & \frac{1}{1+h(b+\alpha(\rho)+\gamma(\rho)+\delta)} \\ 0 & 0 & \frac{h\delta}{1+h(b+\alpha(\rho)+\epsilon)} \\ 0 & 0 & \frac{h\gamma(\rho)}{1+h(b+\mu)} \end{bmatrix}$$

$$\times \begin{bmatrix} 0 & \frac{1}{1+hb} \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{1+h(b+\alpha(\rho)+\epsilon)} & 0 \\ \frac{h\epsilon}{1+h(b+\mu)} & \frac{1}{1+h(b+\mu)} \end{bmatrix}$$

Here

$$\lambda_1 = \lambda_5 = \frac{1}{1+hb}, \lambda_2 = \frac{1}{1+\mathrm{h}(\mathrm{b}+\mu)}, \lambda_3 = \frac{1}{1+h(b+\alpha(\rho)+\gamma(\rho)+\delta)}, \lambda_4 = \frac{1}{1+h(b+\alpha(\rho)+\epsilon)}.$$

3.2.2 Consistency Analysis

In this section, the consistency analysis of the NSFD integration scheme is performed by using Taylor's series expansion.

From Eq. (20), we have

$$S^{n+1}(1+h(\beta(\rho)i^n+hb)) = S^n + h(A+\eta R^n). \tag{25}$$

Taylor's series expansions of S^{n+1} is given below:

$$S^{n+1} = S^n + h\frac{ds}{dt} + \frac{h^2}{2!}\frac{d^2s}{dt^2} + \frac{h^3}{3!}\frac{d^3s}{dt^3} + \ldots,$$

Apply Taylor's series expansions of S^{n+1} to the Eq. (25), we have

$$S^n + S^n h(\beta i^n + hb) + h\frac{ds}{dt} + h\frac{ds}{dt}(h(\beta i^n + hb)) + \frac{h^2}{2!}\frac{d^2s}{dt^2} + \frac{h^2}{2!}\frac{d^2s}{dt^2}(h(\beta i^n + hb))$$
$$+ \frac{h^3}{3!}\frac{d^3s}{dt^3}(1+h(\beta i^n + hb)) + \ldots = S^n + h(A+\eta R^n).$$

By applying $h \longrightarrow 0$, we obtain as follows:

$$S^n \beta i^n + \frac{ds}{dt} = (A + \eta R^n),$$

$$\frac{ds}{dt} = (A + \eta R^n) - S^n \beta i^n,$$

Or

$$\frac{dS}{dt} = (A + \eta R) - S\beta(\rho) I.$$

Similarly, from Eq. (21), we have

$$E^{n+1}(1 + h(b + \mu)) = E^n + h\beta(\rho) s^n I^n. \tag{26}$$

Taylor's series expansions of E^{n+1} is given below:

$$E^{n+1} = E^n + h\frac{dE}{dt} + \frac{h^2}{2!}\frac{d^2E}{dt^2} + \frac{h^3}{3!}\frac{d^3E}{dt^3} + \ldots.$$

From Eq. (26), we have

$$(E^n + h\frac{dE}{dt} + \frac{h^2}{2!}\frac{d^2E}{dt^2} + \frac{h^3}{3!}\frac{d^3E}{dt^3} + \ldots)(1 + h(b + \mu)) = E^n + h\beta(\rho) s^n I^n,$$

By applying $h \longrightarrow 0$, we obtain as follows:

$$E^n(b + \mu) + \frac{dE}{dt} = \beta(\rho) s^n I^n,$$

$$\frac{dE}{dt} = \beta(\rho) s^n I^n - E^n(b + \mu),$$

$$\frac{dE}{dt} = \beta(\rho) SI - E(b + \mu).$$

From Eq. (22), we have

$$I^{n+1}(1 + h(b + \alpha + \gamma + \delta)) = I^n + h\mu E^n, \tag{27}$$

Applying Taylor's series expansions of I^{n+1} is given by

$$I^{n+1} = I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \ldots,$$

Eq. (27) becomes

$$(I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \ldots)(1 + h(b + \alpha + \gamma + \delta)) = I^n + h\mu E^n,$$

By applying $h \longrightarrow 0$, we obtain as follows:

$$\frac{dI}{dt} = \mu E - I(b + \alpha + \gamma + \delta).$$

Similarly, taking the last two equations and applying Taylor's series expansions of Q^{n+1} and R^{n+1}, we obtain as follows

$$\frac{dQ}{dt} = \delta I - Q(b + \alpha + \epsilon),$$

$$\frac{dR}{dt} = (\gamma I + \epsilon Q) - R(b + \mu).$$

Hence, our discretized implicit numerical integration scheme is consistent with the ODES above system.

4 Numerical Simulations

The graphical results of the two schemes developed above are shown and their behavior is discussed in this section.

Dynamics of the subpopulations are shown in Fig. 2 for DFE and EE points at different step sizes using Euler's method. All compartments of the studied model are clearly converging to their steady states in all three cases at small step sizes. The method starts nonphysical oscillations and produces non positive values when the step size is increased slightly in all cases. Positivity is one of the very important features of the epidemic models as negative values in these models are meaningless. Euler's method does not keep the positivity with an increase in the value of the step size. It can be concluded that the increase in the value of the time step sizes affects the convergence of the scheme. This shows that the method is not a good tool to study the long-term behavior of the model. From this behavior, we also concluded that the method is not suitable to study disease dynamics epidemic models. Fig. 3 shows the graphical behavior of the NSFD scheme for DFE and EE points at different step sizes. This time, the method remains positive and convergent for all values of the step sizes. The graphs are positively converging to their steady states in all cases. The behavior of the method is not affected with an increase in the values of the step size. It can be concluded that the NSFD theory can be used to study the long-term behavior of the model. This is an interesting feature of the developed method which many other classical methods such as Euler and many other do not keep at increasing step sizes. This behavior shows the superiority of the NSFD scheme over Euler's method in fuzzy conditions. We can conclude from this behavior that the NSFD method is capable of reflecting the dynamics of the studied model in fuzzy conditions. The typical standard schemes that exist in the literature can cause chaos and misleading variations for some passions of the discretization constraints [34,35].

Effects of quarantine on infected class are shown in Fig. 4 which displays an inverse relation. The infection decreases as we increase the quarantine and vice versa. In computer networks, quarantine can be an effective method of preventing the spread of viruses and other types of malwares. It is a critical component of network security because it allows network administrators to quickly identify and isolate potentially infected devices before they cause widespread damage. Quarantine can help limit the impact of a virus attack by containing it and preventing it from spreading to other parts of the network. By isolating infected or potentially compromised devices, network administrators can prevent them from infecting other devices or spreading the infection to other parts of the network. In a computer network, quarantine can be implemented in a variety of ways. One approach is to use endpoint security software that can detect and isolate infected devices. For example, antivirus software can be configured to automatically quarantine devices that have been infected with a known virus or malware. NAC solutions can also be used to enforce security policies and restrict network resource access based on predefined criteria. In addition to isolating infected devices, quarantine can be used to limit access to devices that violate security policies. A device that does not have up-to-date antivirus software or has not been patched with the most recent security updates, for example, may be quarantined until the problem is resolved. This can help keep the device from posing a threat to the rest of the network. It is important to note, however, that quarantine is only one component of a comprehensive network security strategy. Other measures to protect against computer viruses include regular patching, employee training, and robust backup and recovery procedures.

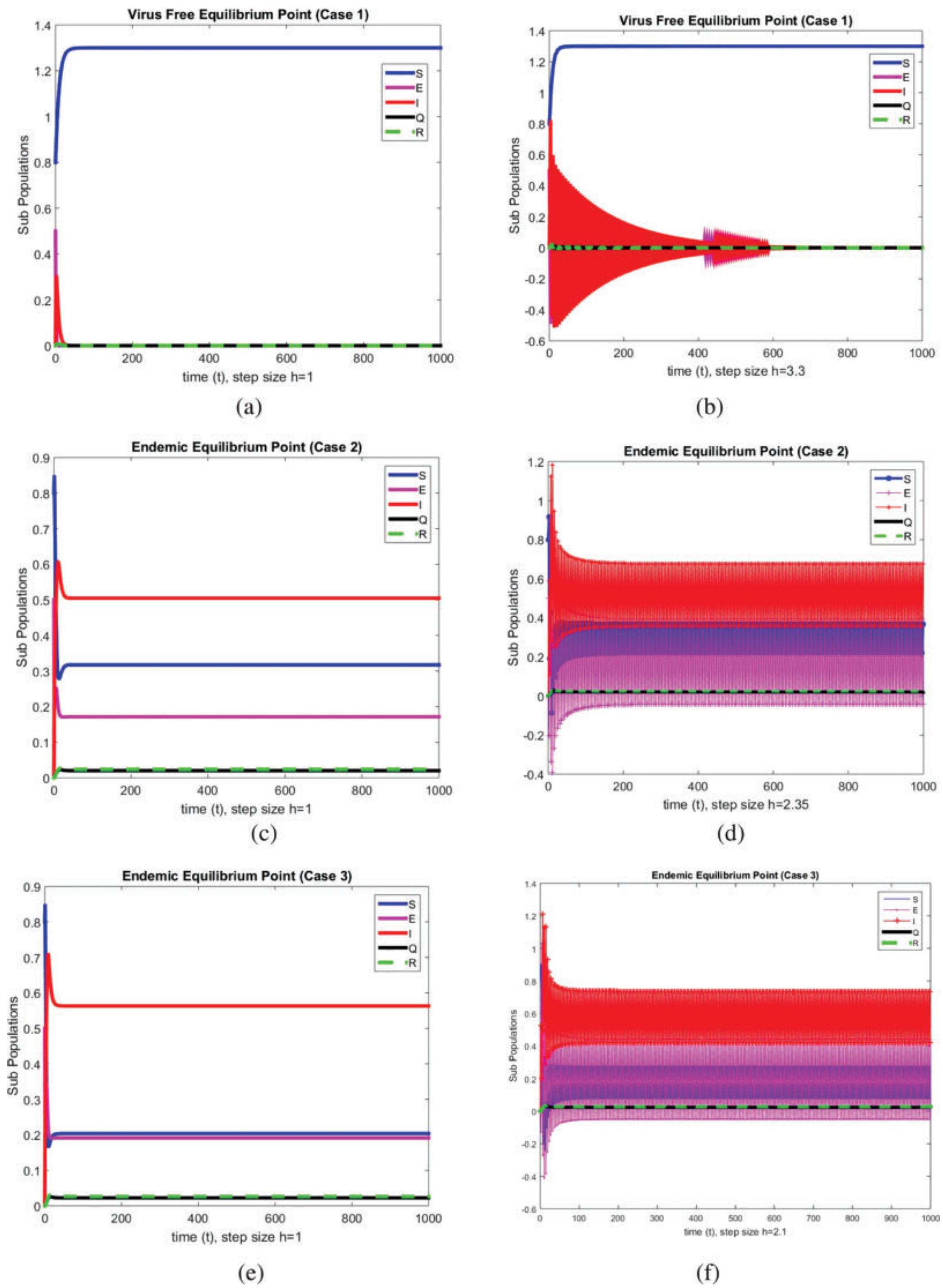

Figure 2: Dynamics of subpopulations using Euler's method

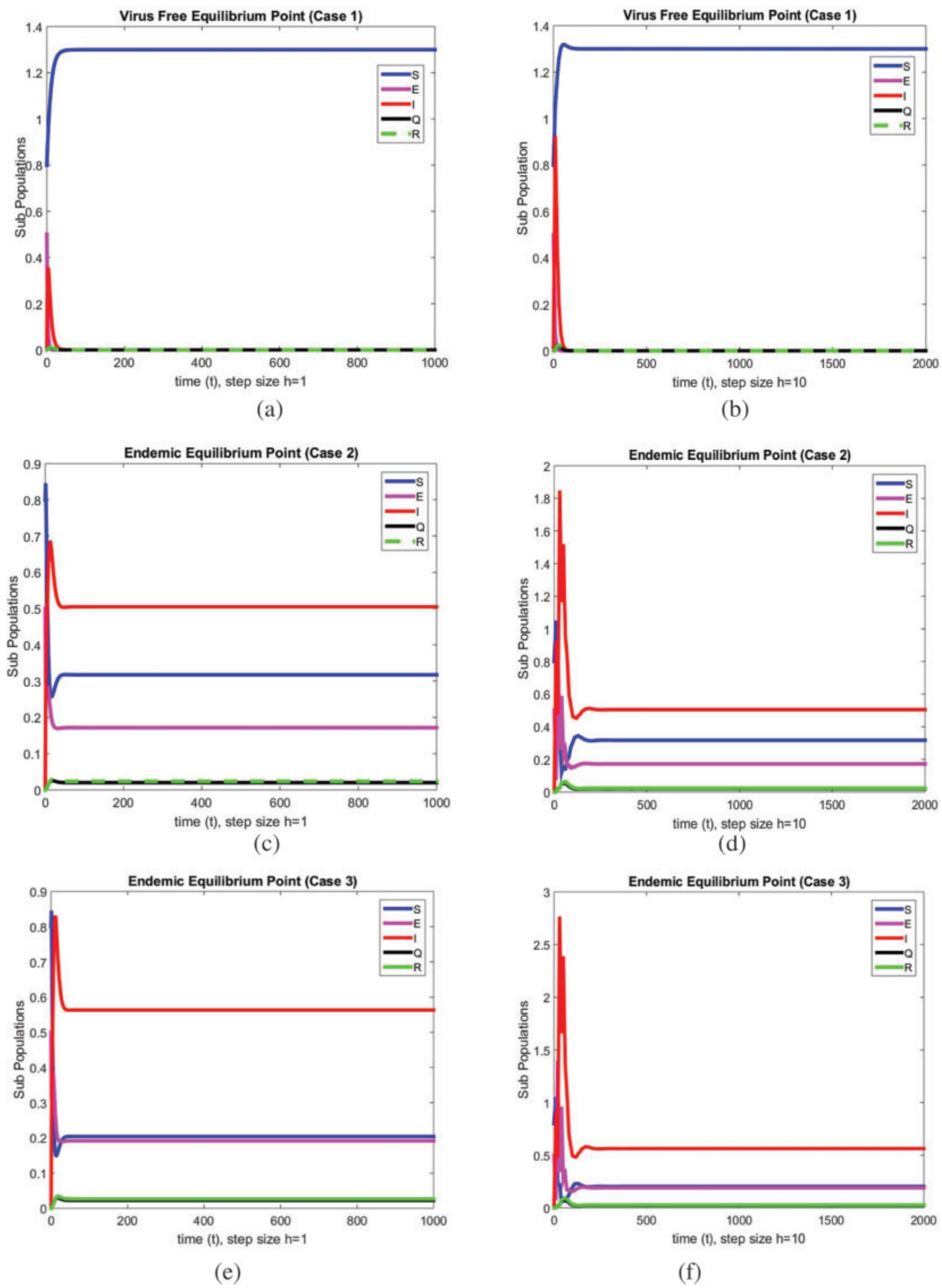

Figure 3: Dynamics of subpopulations using NSFD method

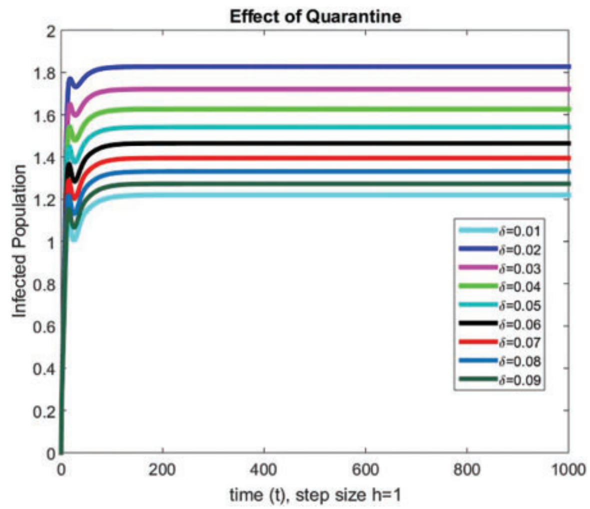

Figure 4: Effect of quarantine on infected class

5 Conclusion

Epidemic modeling is a useful tool to understand the spread and control of infectious diseases. However, in real world situations, the parameters used to model the disease may not be precisely known. In such situations, fuzzy theory can be used to incorporate uncertainty into the models. A SEIQR mathematical model for computer virus propagation with fuzzy parameters by introducing forward Euler and NSFD techniques is investigated in this work. It is assumed that the virus transmission and the recovery of the infected computers are not the same for all PC's under consideration. These are treated as fuzzy numbers depending on the amount of the virus on the single individual PC. In classical models, each parameter is assigned a fixed value independent of the virus load. In this context, the model with fuzziness is more valuable and reliable. The model is analyzed mathematically. Equilibrium analysis, reproduction analysis and stability analysis are performed for the studied model. From a mathematical perspective, the model has produced three equilibrium points that can be used to represent disease-free and endemic conditions, respectively. Two different numerical techniques are used to solve the model numerically and the simulation results are compared. The NSFD approach preserved the essential features of the disease dynamical models like convergence and positivity etc. for all values of the step sizes. The forward Euler method approach is contrasted with the suggested method. According to the simulation, the Euler method was unable to produce an accurate result, even at very small step sizes. Fuzzy theory can be a useful tool in network security to detect and prevent virus spread. It can handle uncertainty and imprecision in disease data. Traditional epidemiological models frequently assume that everyone in a population is either susceptible, infected, or recovered, with no gray area in between. Individuals may have varying degrees of susceptibility or immunity to a disease, which can affect disease spread in a population. Individuals in a population's susceptibility or immunity can be modeled using fuzzy theory. Each individual can be assigned a degree of membership based on their age, health status, vaccination status, and other relevant factors. The model can then calculate each individual's overall risk, which can help predict their likelihood of becoming infected or spreading the disease. Fuzzy theory can be used to model and analyze the spread of infectious diseases in mathematical epidemiology. The major objective of the current work is to incorporate triangular fuzzy numbers as membership functions. The membership functions of the trapezoidal, pentagonal, and other fuzzy numbers can likewise serve as potential future directions for us. Stochastic, delayed, and fractional dynamics with the fuzziness of the studied model can also be considered as a future direction. This study will open some new windows for researchers in this field. Delayed, stochastic and fractional models respectively with fuzziness and many more directions can also be considered as future directions.

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