Intelligent Automation & Soft Computing
DOI: 10.32604/iasc.2023.034234
Article

Approximations by Ideal Minimal Structure with Chemical Application

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Received: 11 July 2022; Accepted: 26 October 2022

Abstract: The theory of rough set represents a non-statistical methodology for analyzing ambiguity and imprecise information. It can be characterized by two crisp sets, named the upper and lower approximations that are used to determine the boundary region and accurate measure of any subset. This article endeavors to achieve the best approximation and the highest accuracy degree by using the minimal structure approximation space $MSAS$ via ideal $\mathcal{J}$. The novel approach (indicated by $\mathcal{J}MSAS$) modifies the approximation space to diminish the boundary region and enhance the measure of accuracy. The suggested method is more accurate than Pawlak's and EL-Sharkasy techniques. Via illustrated examples, several remarkable results using these notions are obtained and some of their properties are established. Several sorts of near open (resp. closed) sets based on $\mathcal{J}MSAS$ are studied. Furthermore, the connections between these assorted kinds of near-open sets in $\mathcal{J}MSAS$ are deduced. The advantages and disadvantages of the proposed approach compared to previous ones are examined. An algorithm using MATLAB and a framework for decision-making problems are verified. Finally, the chemical application for the classification of amino acids (AAs) is treated to highlight the significance of applying the suggested approximation.

Keywords: Ideal; minimal structure spaces; rough set theory; approximation spaces

1 Introduction

Topological structures and their generalizations are of crucial importance in data analysis, which have manifested in different fields, for example, in physics [1], chemistry [2], medicine [3], soft set theory [4], etc. Lashin et al. [5] used topological notions to study different issues in rough set theory in order to generalize Pawlak's concepts [6] in different applications and integrate the concepts of rough and fuzzy sets. Topological structures were applied in rough sets to improve evolutionary-based feature selection technique using the extension of knowledge [7], decision making of COVID-19 [8,9], and enhanced feature selection based on integration containment neighborhoods rough set approximations and binary honey badger optimization [10]. Several articles [11–19] extended the application fields of Pawlak's model. Popa et al. introduced the minimal structure spaces [20], as a generalization of topological spaces to analyze information systems. Various consequences of minimal spaces can be viewed in [21]. EL-Sharkasy [22,23] studied several sorts of sets in minimal structure spaces with some of their characterizations. The notion of the ideal $\mathcal{J}$, which is a nonempty class of sets of Λ fulfills the finite additivity and the hereditary property, is famed in the study of topological problems and their other usages defined by Kuratowski [24]. The advantages of utilizing an ideal in rough set theory are reducing the boundary region and improving the accuracy degree. Accordingly, the study of this theory with ideals is an interesting subject that has delivered the attention of many researchers [25–27]. As an important base for modulation of knowledge extraction and processing, this work will combine the notions of minimal structure approximation spaces and ideals ($\mathcal{J}MSAS$), where these concepts are not only in most fields of mathematics but also in several real-life problems.

The main contributions of this work are constructing ideal minimal structure approximation space $\mathcal{J}MSAS$ from minimal structure approximation space using the notion of ideal and applying them in the decision-making of the classification of amino acids. In addition, some sorts of near open and closed sets via $\mathcal{J}MSAS$ view are proposed and verified.

2 Preliminaries

Herein, some vital concepts and results are introduced, which are helpful in the sequel.

Definition 1 [13] Consider the binary relation η on the nonempty finite set Λ (named a universe set). Thus, the pair (Λ, η) is indicated as a generalized approximation space (briefly, an approximation space).

Definition 2 [13] Suppose that (Λ, η) is an approximation space. Thus, the right neighborhood $N_r(s)$ of a point $s \in \Lambda$ is defined by $N_r(s) = \{y \in \Lambda : s\eta y\}$. Moreover, the class of all right neighborhoods in (Λ, η) is given by $N_r(\Lambda) = \{N_r(s) : s \in \Lambda\}$.

Definition 3 [22,23] For any approximation space (Λ, η), then:

(i) the minimal structure of (Λ, η) is the class $(\Lambda) = \{\varnothing, \Lambda\} \cup N_r(\Lambda)$. Accordingly, the triple $(\Lambda, \eta, MS(\Lambda))$ is titled a minimal structure approximation space (briefly, $MSAS$).

(ii) the elements of $MS(\Lambda)$ are named $MS(\Lambda)$-open sets and their complements are $MS(\Lambda)$-closed sets.

(iii) the class of all $MS(\Lambda)$-closed sets, symbolized by $(MS(\Lambda))^c$, is proposed by:

$(MS(\Lambda))^c = \{B : B^c \in MS(\Lambda)\},$ where B^c represents a complement of B in Λ.

Definition 4 [22,23] Let B be a subset of an MSAS $(\Lambda, \eta, MS(\Lambda))$.Then:

(i) the minimal lower approximation of B is $LMS(B) = \cup\{U \in MS(\Lambda) : U \subseteq B\}$.

(ii) the minimal upper approximation of B is $UMS(B) = \cap\{V \in (MS(\Lambda))^c : B \subseteq V\}$.

(iii) the minimal accuracy of B is $\sigma_r(B) = \dfrac{|LMS(B)|}{|UMS(B)|}$, where $|UMS(B)| \neq 0$.

Proposition 1 [22] If η is a dominance (reflexive and transitive) relation on a universe Λ, therefore *MS(*Λ*)* represents a base for a topology on Λ.

Definition 5 [23] A subset B of an Λ is called:

(i) MS-regular open, if $B = LMS(UMS(B))$ (resp. MS-regular closed if $B = UMS(LMS(B))$).

(ii) MS-semi open, if $B \subseteq UMS(LMS(B))$ (resp. MS-semi closed if $LMS(UMS(B)) \subseteq B$).

(iii) MS-pre open, if $B \subseteq LMS(UMS(B))$ (resp. MS-pre closed if $UMS(LMS(B)) \subseteq B$).

The sets of all MS-regular open, MS-regular closed, MS-semi open, MS-semi closed, MS-pre open and MS-pre closed sets of $(\Lambda, \eta, MS(\Lambda))$ are symbolized as $MS\text{-}RO(\Lambda)$, $MS\text{-}RC(\Lambda)$, $MS\text{-}SO(\Lambda)$, $MS\text{-}SC(\Lambda)$, $MS\text{-}PO(\Lambda)$ and $MS\text{-}PC(\Lambda)$, respectively.

3 Generalized Rough Approximations Relying on Minimal Structure and Ideals

This section aims to introduce the ideal minimal structure approximation space $\mathcal{J}MSAS$ as a generalization of minimal structure approximation space $MSAS$ via ideal. Some characteristics of the proposed method are obtained. Furthermore, the relations between the new approach $\mathcal{J}MSAS$ and the previous ones in [6,22,23] are studied.

Definition 6 Let $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$ be an $\mathcal{J}MSAS$. If $B \subseteq \Lambda$, then:

(i) a minimal lower approximation $LMS^{\mathcal{J}}(B)$ of B with respect to an ideal $\mathcal{J}$ is

$$LMS^{\mathcal{J}}(B) \;=\; \underline{\eta}_r^{\mathcal{J}}(B) \;\cap\; B, \;\text{ where } \underline{\eta}_r^{\mathcal{J}}(B) = \cup\{U \in MS(\Lambda) : U - B \in \mathcal{J}\}.$$

(ii) a minimal upper approximation $UMS^{\mathcal{J}}(B)$ of B with respect to an ideal $\mathcal{J}$ is

$$UMS^{\mathcal{J}}(B) = \;\bar{\eta}_r^{\mathcal{J}}(B) \;\cup\; B, \text{ where } \bar{\eta}_r^{\mathcal{J}}(B) \;=\; \cap\{V \in (MS(\Lambda))^c : B - V \in \mathcal{J}\}.$$

(iii) a minimal accuracy $\sigma_r^{\mathcal{J}}(B)$ of B with respect to an ideal $\mathcal{J}$ is

$$\sigma_r^{\mathcal{J}}(B) = \frac{\left|LMS^{\mathcal{J}}(B)\right|}{\left|UMS^{\mathcal{J}}(B)\right|}, \text{ where } \left|UMS^{\mathcal{J}}(B)\right| \neq 0.$$

Remark 1 *When $\mathcal{J} = \{\varnothing\}$ in Definition 6, the existing approximations are the same as the previous one in Definition 4.*

To study the prime properties of $LMS^{\mathcal{J}}(.)$, and $UMS^{\mathcal{J}}(.)$ approximations, firstly the properties of $\underline{\eta}_r^{\mathcal{J}}(.)$, and $\bar{\eta}_r^{\mathcal{J}}(.)$ will be investigated.

Proposition 2 For an $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$. If $B, \grave{B} \subseteq \Lambda$, then the following conditions are satisfied:

(i) $\underline{\eta}_r^{\mathcal{J}}(B) = \bar{\eta}_r^{\mathcal{J}}(B^c))^c$, and $\bar{\eta}_r^{\mathcal{J}}(B) = (\underline{\eta}_r^{\mathcal{J}}(B^c))^c$.

(ii) $\underline{\eta}_r^{\mathcal{J}}(\Lambda) = \Lambda$, and $\bar{\eta}_r^{\mathcal{J}}(\varnothing) = \varnothing$.

(iii) if $B \subseteq \grave{B}$, then $\underline{\eta}_r^{\mathcal{J}}(B) \subseteq \underline{\eta}_r^{\mathcal{J}}(\grave{B})$, and $\bar{\eta}_r^{\mathcal{J}}(B) \subseteq \bar{\eta}_r^{\mathcal{J}}(\grave{B})$.

(iv) $\underline{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) \subseteq \underline{\eta}_r^{\mathcal{J}}(B) \cap \underline{\eta}_r^{\mathcal{J}}(\grave{B})$, and $\underline{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) \supseteq \underline{\eta}_r^{\mathcal{J}}(B) \cup \underline{\eta}_r^{\mathcal{J}}(\grave{B})$.

(v) $\bar{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) \supseteq \bar{\eta}_r^{\mathcal{J}}(B) \cup \bar{\eta}_r^{\mathcal{J}}(\grave{B})$, and $\bar{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) \subseteq \bar{\eta}_r^{\mathcal{J}}(B) \cap \bar{\eta}_r^{\mathcal{J}}(\grave{B})$.

Proof. The first item will be proved, and the others similarly.

$$(\bar{\eta}_r^{\mathcal{J}}(B^c))^c = (\cap\{V \in (MS(\Lambda))^c : B^c - V \in \mathcal{J}\})^c = \cup\{V^c \in MS(\Lambda) : V^c - B \in \mathcal{J}\} = \underline{\eta}_r^{\mathcal{J}}(B).$$

Example 1 *Let $\Lambda = \{a, b, c, d\}$, and $\eta = \{(a, a), (a, b), (b, b), (b, c), (c, d), (d, c)\}$ be a binary relation on Λ. Then, $N_r(a) = \{a, b\}$, $N_r(b) = \{b, c\}$, $N_r(c) = \{d\}$, and $N_r(d) = \{c\}$.*

Hence, $MS(\Lambda) = \{\varnothing, \Lambda, \{c\}, \{d\}, \{a, b\}, \{b, c\}\}$, and $MS(\Lambda)^c = \{\varnothing, \Lambda, \{c, d\}, \{a, d\}, \{a, b, d\}, \{a, b, c\}\}$.

Remark 2 *Example 1 represents the following:*

(i) the inversion of statement (iii) from Proposition 2 is not true. Suppose $\mathcal{J} = \{\varnothing, \{b\}\}$, and let $B = \{b\}$, $\grave{B} = \{a\}$. Accordingly, $\underline{\eta}_r^{\mathcal{J}}(B) = \varnothing$, $\underline{\eta}_r^{\mathcal{J}}(\grave{B}) = \{a, b\}$, $\bar{\eta}_r^{\mathcal{J}}(B) = \varnothing$, $\bar{\eta}_r^{\mathcal{J}}(\grave{B}) = \{a, b\}$. Hence, $\underline{\eta}_r^{\mathcal{J}}(B) \subseteq \underline{\eta}_r^{\mathcal{J}}(\grave{B})$ and $\bar{\eta}_r^{\mathcal{J}}(B) \subseteq \bar{\eta}_r^{\mathcal{J}}(\grave{B})$ although $B \nsubseteq \grave{B}$.

(ii) the inclusion of the first part of the statement (iv) of Proposition 2 cannot be exchanged by an equality symbol. Suppose $\mathcal{J} = \{\varnothing, \{b\}\}$, *and let* $B = \{a\}$, $\grave{B} = \{c\}$. *Consequently,* $\underline{\eta}_r^{\mathcal{J}}(B) = \{a, b\}$, $\underline{\eta}_r^{\mathcal{J}}(\grave{B}) = \{b, c\}$, *and* $\underline{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) = \varnothing$. *Hence,* $\underline{\eta}_r^{\mathcal{J}}(B) \cap \underline{\eta}_r^{\mathcal{J}}(\grave{B}) \nsubseteq \underline{\eta}_r^{\mathcal{J}}(B \cap \grave{B})$.

(iii) the inclusion of the second part of the statement (iv) of Proposition 2 cannot be exchanged by equality symbol. Suppose $\mathcal{J} = \{\varnothing, \{b\}\}$, *and let* $B = \{a, b, d\}$, $\grave{B} = \{b, c, d\}$. *Thus,* $\bar{\eta}_r^{\mathcal{J}}(B) = \{a, d\}$, $\bar{\eta}_r^{\mathcal{J}}(\grave{B}) = \{c, d\}$, *and* $\bar{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) = \Lambda$. *Hence,* $\bar{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) \nsubseteq \bar{\eta}_r^{\mathcal{J}}(B) \cup \bar{\eta}_r^{\mathcal{J}}(\grave{B})$.

(iv) the inclusion of the first part of the statement (v) of Proposition 2 cannot be exchanged by equality symbol. Consider $\mathcal{J} = \{\varnothing, \{c\}\}$, *and let* $B = \{a\}$, $\grave{B} = \{b\}$. *Therefore,* $\underline{\eta}_r^{\mathcal{J}}(B) = \{c\}$, $\underline{\eta}_r^{\mathcal{J}}(\grave{B}) = \{b, c\}$, *and* $\underline{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) = \{a, b, c\}$. *So,* $\underline{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) \nsubseteq \underline{\eta}_r^{\mathcal{J}}(B) \cup \underline{\eta}_r^{\mathcal{J}}(\grave{B})$.

(v) the inclusion of the second part of the statement (v) of Proposition 2 cannot be exchanged by equality symbol. Consider $\mathcal{J} = \{\varnothing, \{c\}\}$, *and let* $B = \{a, c, d\}$, $\grave{B} = \{b, c, d\}$. *As a result,* $\bar{\eta}_r^{\mathcal{J}}(B) = \{a, d\}$, $\bar{\eta}_r^{\mathcal{J}}(\grave{B}) = \{a, b, d\}$, *and* $\bar{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) = \{d\}$. *Hence,* $\bar{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) \nsupseteq \bar{\eta}_r^{\mathcal{J}}(B) \cap \bar{\eta}_r^{\mathcal{J}}(\grave{B})$.

The succeeding proposition is realized depending on Proposition 1.

Proposition 3 Let η, and $MS(\Lambda)$ be a dominance relation, and a minimal structure on Λ, respectively. If $B, \grave{B} \subseteq \Lambda$, then for any ideal $\mathcal{J}$ the following results hold: $\underline{\eta}_r^{\mathcal{J}}(B \cap \grave{B}) = \underline{\eta}_r^{\mathcal{J}}(B) \cap \underline{\eta}_r^{\mathcal{J}}(\grave{B})$, and $\bar{\eta}_r^{\mathcal{J}}(B \cup \grave{B}) = \bar{\eta}_r^{\mathcal{J}}(B) \cup \bar{\eta}_r^{\mathcal{J}}(\grave{B})$.

The next remark is devoted to clarifying the differences between the existing approximations and the preceding one of Propositions 2 & 3 in [22].

Remark 3 *Let* B, $\grave{B}$ *be subsets of an* $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$. *Then Example 1 with* $\mathcal{J} = \{\varnothing, \{c\}\}$ *confirms the following:*

(i) $\underline{\eta}_r^{\mathcal{J}}(\varnothing) \neq \varnothing$. *As* $\underline{\eta}_r^{\mathcal{J}}(\varnothing) = \{c\}$.

(ii) $\bar{\eta}_r^{\mathcal{J}}(\Lambda) \neq \Lambda$. *As* $\bar{\eta}_r^{\mathcal{J}}(\Lambda) = \{a, b, d\}$.

(iii) $\underline{\eta}_r^{\mathcal{J}}(B) \nsubseteq B$. *As* $B = \{a\}$, $\underline{\eta}_r^{\mathcal{J}}(B) = \{c\}$.

(iv) $B \nsubseteq \bar{\eta}_r^{\mathcal{J}}(B)$. *As* $B = \{b, c, d\}$, $\bar{\eta}_r^{\mathcal{J}}(B) = \{a, b, d\}$.

Proposition 4 Let $\mathcal{J}, \mathcal{J}^{\bullet}$ be two ideals on *an* $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda))$, and $B \subseteq \Lambda$. Then, the next results hold:

(i) if $\mathcal{J} \subseteq \mathcal{J}^{\bullet}$, then $\underline{\eta}_r^{\mathcal{J}}(B) \subseteq \underline{\eta}_r^{\mathcal{J}^{\bullet}}(B)$.

(ii) if $\mathcal{J} \subseteq \mathcal{J}^{\bullet}$, then $\bar{\eta}_r^{\mathcal{J}^{\bullet}}(B) \subseteq \bar{\eta}_r^{\mathcal{J}}(B)$.

(iii) $\bar{\eta}_r^{(\mathcal{J} \cup \mathcal{J}^{\bullet})}(B) = \bar{\eta}_r^{\mathcal{J}}(B) \cup \bar{\eta}_r^{\mathcal{J}^{\bullet}}(B)$ and $\bar{\eta}_r^{(\mathcal{J} \cap \mathcal{J}^{\bullet})}(B) = \bar{\eta}_r^{\mathcal{J}}(B) \cap \bar{\eta}_r^{\mathcal{J}^{\bullet}}(B)$.

(iv) $\underline{\eta}_r^{\mathcal{J}}(B) \cup \underline{\eta}_r^{\mathcal{J}^{\bullet}}(B) = \underline{\eta}_r^{(\mathcal{J} \cup \mathcal{J}^{\bullet})}(B)$ and $\underline{\eta}_r^{(\mathcal{J} \cap \mathcal{J}^{\bullet})}(B) = \underline{\eta}_r^{\mathcal{J}}(B) \cap \underline{\eta}_r^{\mathcal{J}^{\bullet}}(B)$.

(v) $B \in \mathcal{J}$ iff $\bar{\eta}_r^{\mathcal{J}}(B) = \varnothing$.

(vi) $B^c \in \mathcal{J}$ iff $\underline{\eta}_r^{\mathcal{J}}(B) = \Lambda$.

Proof. The first item will be proved, and the others similarly.

Let $w \in \underline{\eta}_r^{\mathcal{J}}(B)$, and then there exists $U \in MS(\Lambda)$ such that $w \in U$ and $U - B \in \mathcal{J}$. Since $\mathcal{J} \subseteq \mathcal{J}^{\bullet}$, then $w \in \underline{\eta}_r^{\mathcal{J}^{\bullet}}(B)$ and so $\underline{\eta}_r^{\mathcal{J}}(B) \subseteq \underline{\eta}_r^{\mathcal{J}^{\bullet}}(B)$.

Example 2 *Let* η *be a binary relation on* $\Lambda = \{a, b, c\}$ *such that* $N_r(a) = \{a\}$, $N_r(b) = \Lambda$, *and* $N_r(c) = \{a, c\}$. *Then* $MS(\Lambda) = \{\varnothing, \Lambda, \{a\}, \{a, c\}\}$, *and* $MS(\Lambda)^c = \{\varnothing, \Lambda, \{b\}, \{b, c\}\}$.

Remark 4 *Suppose* $\mathcal{J} = \{\varnothing, \{a\}\}$, *and* $\mathcal{J}^{\bullet} = \{\varnothing, \{b\}\}$ *in Example 2. Then,*

(i) *the inversion of statement (i) from Proposition 4 is false. Suppose* $B = \{b\}$, *then* $\underline{\eta}_r^{\mathcal{J}^\bullet}(B) = \varnothing$, $\underline{\eta}_r^{\mathcal{J}}(B) = \{a\}$. *Hence,* $\underline{\eta}_r^{\mathcal{J}^\bullet}(B) \subseteq \underline{\eta}_r^{\mathcal{J}}(B)$, *while* $\mathcal{J}^\bullet \nsubseteq \mathcal{J}$.

(ii) *the inversion of statement* (ii) *from Proposition 4 is not true. Suppose* $B = \{a, b\}$, *then* $\overline{\eta}_r^{\mathcal{J}^\bullet}(B) = \Lambda$, $\overline{\eta}_r^{\mathcal{J}}(B) = \{b\}$. *Hence,* $\overline{\eta}_r^{\mathcal{J}}(B) \subseteq \overline{\eta}_r^{\mathcal{J}^\bullet}(B)$, *while* $\mathcal{J}^\bullet \nsubseteq \mathcal{J}$.

Consequently, for any relation, the new kind of rough approximations $\mathcal{J}$*MSAS is more accurate than the preceding ones* [6,22,23].

Remark 5 Table 1 *compare between lower approximations, upper approximations, and the accuracy and* Table 2 *compare between boundary, internal edge, and external edge defined in Definitions 4 and 6 which are given by the relation* η *of Example 1 and ideal* $\mathcal{J} = \{\varnothing, \{c\}\}$.

Table 1: Comparison between lower approximations, upper approximations, and the accuracy defined in Definitions 4 and 6 which are given by the relation η of Example 1, and an ideal $\mathcal{J} = \{\varnothing, \{c\}\}$.

$B \subseteq \Lambda$	A method of Definition 4			Current method of Definition 6		
	$LMS(B)$	$UMS(B)$	$\sigma_r(B)$	$LMS^{\mathcal{J}}(B)$	$UMS^{\mathcal{J}}(B)$	$\sigma_r^{\mathcal{J}}(B)$
$\{a\}$	$\varnothing$	$\{a\}$	0	$\varnothing$	$\{a\}$	0
$\{b\}$	$\varnothing$	$\{a, b\}$	0	$\{b\}$	$\{a, b\}$	0.5
$\{c\}$	$\{c\}$	$\{c\}$	1	$\{c\}$	$\{c\}$	1
$\{d\}$	$\{d\}$	$\{d\}$	1	$\{d\}$	$\{d\}$	1
$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	1	$\{a, b\}$	$\{a, b\}$	1
$\{a, c\}$	$\{c\}$	$\{a, b, c\}$	0.33	$\{c\}$	$\{a, c\}$	0.5
$\{a, d\}$	$\{d\}$	$\{a, d\}$	0.5	$\{d\}$	$\{a, d\}$	0.5
$\{b, c\}$	$\{b, c\}$	$\{a, b, c\}$	0.67	$\{b, c\}$	$\{a, b, c\}$	0.67
$\{b, d\}$	$\{d\}$	$\{a, b, d\}$	0.33	$\{b, d\}$	$\{a, b, d\}$	0.67
$\{c, d\}$	$\{c, d\}$	$\{c, d\}$	1	$\{c, d\}$	$\{c, d\}$	1
$\{a, b, c\}$	$\{a, b, c\}$	$\{a, b, c\}$	1	$\{a, b, c\}$	$\{a, b, c\}$	1
$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	1	$\{a, b, d\}$	$\{a, b, d\}$	1
$\{a, c, d\}$	$\{c, d\}$	Λ	0.5	$\{c, d\}$	$\{a, c, d\}$	0.67
$\{b, c, d\}$	$\{b, c, d\}$	Λ	0.75	$\{b, c, d\}$	Λ	0.75
Λ	Λ	Λ	1	Λ	Λ	1

Table 2: Comparison between the boundary, internal edge, and external edge defined in Definitions 4 and 6 which are given by the binary relation η of Example 1, and an ideal $\mathcal{J} = \{\varnothing, \{c\}\}$.

$B \subseteq \Lambda$	A method of Definition 4			Current method of Definition 6		
	MS-$b_r(B)$	LMS-edg(B)	UMS-edg(B)	MS-$b_r^{\mathcal{J}}(B)$	$LMS^{\mathcal{J}}$-edg(B)	$UMS^{\mathcal{J}}$-edg(B)
{a}	{a}	{a}	$\varnothing$	{a}	{a}	$\varnothing$
{b}	{a, b}	{b}	{a}	{a}	$\varnothing$	{a}
{c}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
{d}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

(Continued)

Table 2 (continued)

$B \subseteq \Lambda$	A method of Definition 4			Current method of Definition 6		
	$MS\text{-}b_r(B)$	$LMS\text{-edg}(B)$	$UMS\text{-edg}(B)$	$MS\text{-}b_r^{\mathcal{J}}(B)$	$LMS^{\mathcal{J}}\text{-edg}(B)$	$UMS^{\mathcal{J}}\text{-edg}(B)$
{a, b}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
{a, c}	{a, b}	{a}	{b}	{a}	{a}	$\varnothing$
{a, d}	{a}	{a}	$\varnothing$	{a}	{a}	$\varnothing$
{b, c}	{a}	$\varnothing$	{a}	{a}	$\varnothing$	{a}
{b, d}	{a, b}	{b}	{a}	{a}	$\varnothing$	{a}
{c, d}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
{a, b, c}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
{a, b, d}	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$
{a, c, d}	{a, b}	{a}	{b}	{a}	{a}	$\varnothing$
{b, c, d}	{a}	$\varnothing$	{a}	{a}	$\varnothing$	{a}
Λ	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$

The next result exhibits the connections among the lower, and upper approximations and the degree of accuracy that were offered in both Definitions 4, and 6.

Theorem 1 Let $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$ be an $\mathcal{J}MSAS$. If $B \subseteq \Lambda$. Then, the next properties are held:

(i) $LMS(B) \subseteq LMS^{\mathcal{J}}(B)$.

(ii) $UMS^{\mathcal{J}}(B) \subseteq UMS(B)$.

(iii) $\sigma_r(B) < \sigma_r^{\mathcal{J}}(B)$.

Proof. The first item will be proved, and the others similarly.

Let $w \in LMS(B)$, and then there exists $U \in MS(\Lambda)$ such that $w \in U$ and $U \subseteq B$. Hence, $U - B = \varnothing \in \mathcal{J}$. i.e., $w \in \underline{\eta}_r^{\mathcal{J}}(B)$. So, $LMS(B) \subseteq \underline{\eta}_r^{\mathcal{J}}(B)$. Since $LMS(B) \subseteq B$, therefore $LMS(B) \subseteq LMS^{\mathcal{J}}(B)$.

Remark 6 *It must be perceived that the suggested approximations* $LMS^{\mathcal{J}}(.)$, *and* $UMS^{\mathcal{J}}(.)$ *in Definition 6 have equal characteristics of* $\underline{\eta}_r^{\mathcal{J}}(.)$ *and* $\bar{\eta}_r^{\mathcal{J}}(.)$ *identified in Propositions 1 and 3. Moreover, it fulfills the next properties:*

(i) $LMS^{\mathcal{J}}(\varnothing) = \varnothing$ *and* $UMS^{\mathcal{J}}(\Lambda) = \Lambda$.

(ii) $LMS^{\mathcal{J}}(B) \subseteq B \subseteq UMS^{\mathcal{J}}(B)$.

Definition 7 For an $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$. If $B \subseteq \Lambda$, then

(i) a minimal positive of B with respect to an ideal $\mathcal{J}$ is $MS\text{-}Po_r^{\mathcal{J}}(B) = LMS^{\mathcal{J}}(B)$,

(ii) a minimal exterior (negative) of B with respect to an ideal $\mathcal{J}$ is $MS\text{-}Ex_r^{\mathcal{J}}(B) = \Lambda - UMS^{\mathcal{J}}(B)$,

(iii) a minimal boundary of B with respect to ideal $\mathcal{J}$ is $MS\text{-}b_r^{\mathcal{J}}(B) = UMS^{\mathcal{J}}(B) - LMS^{\mathcal{J}}(B)$,

(iv) a minimal internal edge of B with respect to an ideal $\mathcal{J}$ is $LMS^{\mathcal{J}}\text{-edg}(B) = B - LMS^{\mathcal{J}}(B)$, and

(v) a minimal external edge of B with respect to an ideal $\mathcal{J}$ is $UMS^{\mathcal{J}}\text{-edg}(B) = UMS^{\mathcal{J}}(B) - B$.

Remark 7 According to Example 1 with $\mathcal{J} = \{\emptyset, \{c\}\}$, Table 2 illustrates the next relationships for $B \subseteq \Lambda$:

(i) $MS\text{-}b_r^{\mathcal{J}}(B) \subseteq MS\text{-}b_r(B)$,

(ii) $LMS^{\mathcal{J}}\text{-edg}(B) \subseteq LMS\text{-edg}(B)$, and

(iii) $UMS^{\mathcal{J}}\text{-edg}(B) \subseteq UMS\text{-edg}(B)$.

Definition 8 For an $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$, a subset B of Λ is categorized as:

(i) MS-definable (exact) with respect to an ideal $\mathcal{J}$, if $MS\text{-}b_r^{\mathcal{J}}(B) = \emptyset$. Otherwise, a subset B is called MS-undefinable (rough) with respect to the $\mathcal{J}$.

(ii) MS-internally definable with respect to an ideal $\mathcal{J}$, if $LMS^{\mathcal{J}}(B) = B$.

(iii) MS-externally definable with respect to an ideal $\mathcal{J}$, if $UMS^{\mathcal{J}}(B) = B$.

Remark 8 *According to* Table 1 *and Example 1 with* $\mathcal{J} = \{\emptyset, \{c\}\}$, *it is noticed that:*

(i) *the sets* $\{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}$, *and* Λ *are MS-definable with respect to an ideal* $\mathcal{J}$.

(ii) the sets $\{b\}, \{b, c\}, \{b, d\}$, *and* $\{b, c, d\}$ *are MS-internally definable with respect to an ideal* $\mathcal{J}$.

(iii) the sets $\{a\}, \{a, c\}, \{a, d\}$, *and* $\{a, c, d\}$ *are MS-externally definable with respect to an ideal* $\mathcal{J}$.

Corollary 1 *For a* $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$. *Then,*

(i) every MS-definable is MS-definable with respect to an ideal $\mathcal{J}$.

(ii) every MS-undefinable with respect to an ideal $\mathcal{J}$ is MS-undefinable.

4 Some Near Open Sets Based on MSAS and Ideals

This section aims to introduce and investigate some sorts of near open (resp. closed) sets via the viewpoint of an $\mathcal{J}MSAS$.

Definition 9 A subset B of an $\mathcal{J}MSAS$ $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$ is called:

(i) $MS^{\mathcal{J}}$-regular open if $B = LMS^{\mathcal{J}}\left(UMS^{\mathcal{J}}(B)\right)$

(ii) $MS^{\mathcal{J}}$-semi open if $B \subseteq UMS^{\mathcal{J}}\left(LMS^{\mathcal{J}}(B)\right)$.

(iii) $MS^{\mathcal{J}}$-pre open if $B \subseteq LMS^{\mathcal{J}}\left(UMS^{\mathcal{J}}(B)\right)$.

Remark 9

(i) *The complement of an* $MS^{\mathcal{J}}$*-regular open (resp.* $MS^{\mathcal{J}}$*-semi open, and* $MS^{\mathcal{J}}$*-pre open) set is known* $MS^{\mathcal{J}}$*-regular closed (resp.* $MS^{\mathcal{J}}$*-semi closed, and* $MS^{\mathcal{J}}$*-pre closed) set.*

(ii) *The set of all* $MS^{\mathcal{J}}$*-regular open (resp.* $MS^{\mathcal{J}}$*-regular closed*, $MS^{\mathcal{J}}$*-semi open*, $MS^{\mathcal{J}}$*-semi closed*, $MS^{\mathcal{J}}$*-pre open and* $MS^{\mathcal{J}}$*-pre closed) sets of* $(\Lambda, \eta, MS(\Lambda))$ *is denoted by* $MS^{\mathcal{J}}\text{-}RO(\Lambda)$ *(resp.* $MS^{\mathcal{J}}\text{-}RC(\Lambda)$, $MS^{\mathcal{J}}\text{-}SO(\Lambda)$, $MS^{\mathcal{J}}\text{-}SC(\Lambda)$, $MS^{\mathcal{J}}\text{-}PO(\Lambda)$ *and* $MS^{\mathcal{J}}\text{-}PC(\Lambda)$*).*

Remark 10 *In a topological space, the class of semi (resp. pre-) open sets is contained in the class of ideal semi (resp. pre-) open sets. While this fact is discussed for minimal structures, surprisingly it is incorrect i.e.,* $MS\text{-}SO(\Lambda)$ *and* $MS^{\mathcal{J}}\text{-}SO(\Lambda)$ *(resp.* $MS\text{-}PO(\Lambda)$ *and* $MS^{\mathcal{J}}\text{-}PO(\Lambda)$*) are incomparable, as shown in Examples 3, and 4.*

Example 3 **(***Continued from Example 2) Consider* $\mathcal{J} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ *is an ideal on* $(\Lambda, \eta, MS(\Lambda))$, *that will result in.*

(i) $MS\text{-}RO(\Lambda) = \{\emptyset, \Lambda\}$.

(ii) $MS\text{-}SO(\Lambda) = \{\emptyset, \{a\}, \{a, c\}, \{a, b\}, \Lambda\}$.

(iii) $MS\text{-}PO(\Lambda) = \{\emptyset, \{a\}, \{a,c\}, \{a,b\}, \Lambda\}$.

(iv) $MS^{\mathcal{J}}\text{-}RO(\Lambda) = \{\emptyset, \{a\}, \{b,c\}, \Lambda\}$.

(v) $MS^{\mathcal{J}}\text{-}SO(\Lambda) = \{\emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}, \Lambda\}$.

(vi) $MS^{\mathcal{J}}\text{-}PO(\Lambda) = \{\emptyset, \{a\}, \{c\}, \{a,c\}, \{b,c\}, \Lambda\}$.

Example 4 *Consider* $\Lambda = \{a,b,c,d\}$, η *is a binary relation on* Λ, *and* $N_r(a) = \{a\}$, $N_r(b) = \{a,b\}$, $N_r(c) = \{c\}$, *and* $N_r(d) = \{b,c,d\}$. *Accordingly*, $MS(\Lambda) = \{\emptyset, \Lambda, \{a\}, \{c\}, \{b,c,d\}, \{a,b\}\}$ *and*

$MS(\Lambda)^c = \{\emptyset, \Lambda, \{a\}, \{c,d\}, \{a,b,d\}, \{b,c,d\}\}$. *If* $\mathcal{J} = \{\emptyset, \{a\}, \{d\}, \{a,d\}\}$, *and* $\mathrm{L} = \{\emptyset, \{\mathrm{b}\}, \{\mathrm{d}\}, \{\mathrm{b},\mathrm{d}\}\}$ *be ideals on* $(\Lambda, \eta, MS(\Lambda))$, *then*

(i) $MS\text{-}RO(\Lambda) = \{\emptyset, \Lambda, \{a\}, \{c\}, \{b,c,d\}, \{a,b\}\}$.

(ii) $MS\text{-}SO(\Lambda) = \{\emptyset, \Lambda, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$.

(iii) $MS\text{-}PO(\Lambda) = \{\emptyset, \Lambda, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$.

(iv) $MS^{\mathcal{J}}\text{-}RO(\Lambda) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c,d\}, \Lambda\}$.

(v) $MS^{\mathcal{J}}\text{-}SO(\Lambda) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\},$

$\{a,c,d\}, \{b,c,d\}, \Lambda\ \}$.

(vi) $\mathrm{MS}^{\mathrm{L}}\text{-}PO(\Lambda) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \Lambda\}$.

(vii) $\mathrm{MS}^{\mathrm{L}}\text{-}RO(\Lambda) = \{\emptyset, \{a\}, \{a,b\}, \{c,d\}, \{b,c,d\}, \Lambda\}$.

($viii$) $\mathrm{MS}^{\mathrm{L}}\text{-}SO(\Lambda) = \{\emptyset, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \Lambda\}$.

(ix) $MS^{L}\text{-}PO(\Lambda) = \{\emptyset, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{c,d\}.\{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \Lambda\}$.

According to Definition 9, the proof of the proposition 5 is obvious.

Proposition 5 Let $(\Lambda, \eta, MS(\Lambda), \mathcal{J})$ be an $\mathcal{J}MSAS$, then $MS^{\mathcal{J}}\text{-}RO(\Lambda) \subseteq MS^{\mathcal{J}}\text{-}SO(\Lambda) \cap MS^{\mathcal{J}}\text{-}PO(\Lambda)$.

The next example shows that the converse of Proposition 5 is incorrect.

Example 5 *In Example 4*, $\{b,c\}$ *is an* $MS^{\mathcal{J}}$*-pre open set and it is not* $MS^{\mathcal{J}}$*-regular open. In addition*, $\{b,d\}$ *is* $MS^{\mathcal{J}}$*-semi open and it is not* $MS^{\mathcal{J}}$*-regular open.*

Remark 11 *In view of Example 4, the following results are noticed:*

(i) *the union of* $MS^{\mathcal{J}}$*-regular open sets is not* $MS^{\mathcal{J}}$*-regular open. Consider* $\{b\}, \{c\} \in MS^{\mathcal{J}}\text{-}RO(\Lambda)$. *Clearly*, $\{b,c\} \notin MS^{\mathcal{J}}\text{-}RO(\Lambda)$.

(ii) *the intersection of* $MS^{\mathcal{J}}$*-regular open sets is not* $MS^{\mathcal{J}}$*-regular open. Consider* $\{a,b\}, \{b,c,d\} \in MS^{\mathcal{J}}\text{-}RO(\Lambda)$. *Clearly*, $\{b\} \notin MS^{\mathcal{J}}\text{-}RO(\Lambda)$.

(iii) *the intersection of* $MS^{\mathcal{J}}$*-semi open sets is not* $MS^{\mathcal{J}}$*-semi open. Consider* $\{a,b,d\}, \{a,c,d\} \in MS^{\mathcal{J}}\text{-}SO(\Lambda)$. *Clearly*, $\{a,d\} \notin MS^{\mathcal{J}}\text{-}SO(\Lambda)$.

(iv) *the intersection of* $MS^{\mathcal{J}}$*-pre open sets is not* $MS^{\mathcal{J}}$*-pre open. Consider* $\{a,b\}, \{b,c\} \in MS^{\mathcal{J}}\text{-}PO(\Lambda)$. *Clearly*, $\{b\} \notin MS^{\mathcal{J}}\text{-}PO(\Lambda)$.

(v) *the intersection of* $MS^{\mathcal{J}}$*-regular open set and* $MS^{\mathcal{J}}$*-pre open set is not* $MS^{\mathcal{J}}$*-pre open set. Consider* $\{a,b\} \in MS^{\mathcal{J}}\text{-}RO(\Lambda)$, *and* $\{b,c,d\} \in MS^{\mathcal{J}}\text{-}PO(\Lambda)$. *Clearly*, $\{b\} \notin MS^{\mathcal{J}}\text{-}PO(\Lambda)$.

(vi) *the intersection of* $MS^{\mathcal{J}}$*-regular open set and* $MS^{\mathcal{J}}$*-semi open set is not* $MS^{\mathcal{J}}$*-semi open set. Consider* $\{a,b\} \in MS^{\mathcal{J}}\text{-}RO(\Lambda)$, *and* $\{b,c,d\} \in MS^{\mathcal{J}}\text{-}SO(\Lambda)$. *Clearly*, $\{b\} \notin MS^{\mathcal{J}}\text{-}SO(\Lambda)$.

5 Biochemical Applications

An example in the area of chemistry is provided by utilizing the actual approximation in Definition 6 to clarify the notions practically.

Example 6 [26] *Let* $\Lambda = \{m_1, m_2, m_3, m_4, m_5\}$ *be five amino acids (AAs). The (AAs) is qualified by the attributes* A_1, A_2, A_3, A_4, A_5 *such that* A_1 *refers to PIE,* A_2 *refers to SAC (surface area),* A_3 *refers to MR (molecular refractivity),* A_4 *refers to LAM (the side chain polarity), and* A_5 *refers to Vol (molecular volume).* Table 3 *shows all quantitative attributes of AAs.*

Table 3: Quantitative attributes of AAs

	A_1	A_2	A_3	A_4	A_5
m_1	0.23	254.2	2.126	−0.02	82.2
m_2	−0.48	303.6	2.994	−1.24	112.3
m_3	−0.61	287.9	2.994	−1.08	103.7
m_4	0.45	282.9	2.933	−0.11	99.1
m_5	−0.11	335.0	3.458	−0.19	127.5

Presently, it shall be investigated five relations on Λ determined by $\eta_l = \{(m_i, m_j) \in \Lambda \times \Lambda : m_i(A_l) - m_j(A_l) < \frac{\delta_l}{2}, i, j, l = 1, 2, 3, 4, 5\}$, where δ_l symbolizes the standard deviation of the quantitative attributes $A_l, l = 1, 2, 3, 4, 5$. Thus, the right neighborhoods for all members of Λ according to the relations $\eta_l, l = 1, 2, 3, 4, 5$ are tabulated in Table 4.

Table 4: Right neighborhoods of five relations

	$m_l\eta_1$	$m_l\eta_2$	$m_l\eta_3$	$m_l\eta_4$	$m_l\eta_15$
m_1	$\{m_1, m_4\}$	Λ	Λ	$\{m_1, m_4, m_5\}$	Λ
m_2	Λ	$\{m_2, m_5\}$	$\{m_2, m_3, m_4, m_5\}$	Λ	$\{m_2, m_5\}$
m_3	Λ	$\{m_2, m_3, m_4, m_5\}$	$\{m_2, m_3, m_4, m_5\}$	Λ	$\{m_2, m_3, m_4, m_5\}$
m_4	$\{m_4\}$	$\{m_2, m_3, m_4, m_5\}$	$\{m_2, m_3, m_4, m_5\}$	$\{m_1, m_4, m_5\}$	$\{m_2, m_3, m_4, m_5\}$
m_5	$\{m_1, m_4, m_5\}$	$\{m_5\}$	$\{m_5\}$	$\{m_1, m_4, m_5\}$	$\{m_5\}$

The intersection of all right neighborhoods of all elements $l = 1, 2, 3, 4, 5$ is computed as follows: $m_l\eta = \cap_{l=1}^{5} m_l\eta_1 = \{m_1, m_4\}$, $m_2\eta = \cap_{l=1}^{5} m_2\eta_1 = \{m_2, m_5\}$, $m_3\eta = \cap_{l=1}^{5} m_3\eta_1 = \{m_2, m_3, m_4, m_5\}$, $m_4\eta = \cap_{l=1}^{5} m_4\eta_1 = \{m_4\}$, and $m_5\eta = \cap_{l=1}^{5} m_5\eta_1 = \{m_5\}$. Then, it will be obtained $MS(\Lambda) = \{\varnothing, \Lambda, \{m_4\}, \{m_5\}, \{m_1, m_4\}, \{m_2, m_5\}, \{m_2, m_3, m_4, m_5\}\}$.

$(MS(\Lambda))^c = \{\varnothing, \Lambda, \{m_1\}, \{m_1, m_3, m_4\}, \{m_2, m_3, m_5\}, \{m_1, m_2, m_3, m_4\}, \{m_1, m_2, m_3, m_5\}\}$.

$MS\text{-}RO(\Lambda) = \{\varnothing, \Lambda, \{m_1, m_4\}, \{m_2, m_5\}\}$.

$MS\text{-}PO(\Lambda) = \{\varnothing, \Lambda, \{m_4\}, \{m_5\}, \{m_1, m_4\}, \{m_4, m_5\}, \{m_2, m_5\}, \{m_1, m_4, m_5\}, \{m_2, m_4, m_5\}, \{m_3, m_4, m_5\}, \{m_1, m_2, m_4, m_5\}, \{m_1, m_3, m_4, m_5\}, \{m_2, m_3, m_4, m_5\}\}$.

$MS\text{-}SO(\Lambda) = \{\varnothing, \Lambda, \{m_4\}, \{m_5\}, \{m_1, m_4\}, \{m_4, m_5\}, \{m_2, m_5\}, \{m_3, m_4\}, \{m_3, m_5\}, \{m_1, m_3, m_4\}, \{m_1, m_4, m_5\}, \{m_2, m_3, m_5\}, \{m_3, m_4, m_5\}, \{m_2, m_4, m_5\}, \{m_1, m_2, m_4, m_5\}, \{m_1, m_3, m_4, m_5\}, \{m_2, m_3, m_4, m_5\}\}$.

If $\mathcal{J} = \{\emptyset, \{m_5\}\}$ is an ideal on Λ, then the minimal accuracy measure $\sigma_r^{\mathcal{J}}(B)$ of B is calculated (see Table 5). Also, it will be shown

$MS^{\mathcal{J}}$-$RO(\Lambda) = \{\emptyset, \Lambda, \{m_2\}, \{m_5\}, \{m_1, m_4\}, \{m_2, m_5\}, \{m_1, m_4, m_5\}, \{m_1, m_2, m_3, m_4\}\}$.

$MS^{\mathcal{J}}$-$PO(\Lambda) = \{\emptyset, \Lambda, \{m_2\}, \{m_4\}, \{m_5\}, \{m_1, m_4\}, \{m_2, m_4\}, \{m_2, m_5\}, \{m_4, m_5\}, \{m_1, m_2, m_4\}, \{m_1, m_4, m_5\}, \{m_2, m_3, m_4\}, \{m_2, m_4, m_5\}, \{m_1, m_2, m_4, m_5\}, \{m_1, m_2, m_3, m_4\}, \{m_2, m_3, m_4, m_5\}\}$.

$MS^{\mathcal{J}}$-$SO(\Lambda) = \{\emptyset, \Lambda, \{m_2\}, \{m_4\}, \{m_5\}, \{m_1, m_4\}, \{m_2, m_3\}, \{m_2, m_4\}, \{m_2, m_5\}, \{m_4, m_5\}, \{m_3, m_4\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}, \{m_1, m_4, m_5\}, \{m_2, m_3, m_4\}, \{m_2, m_3, m_5\}, \{m_2, m_4, m_5\}, \{m_3, m_4, m_5\}, \{m_1, m_2, m_4, m_5\}, \{m_1, m_2, m_3, m_4\}, \{m_1, m_3, m_4, m_5\}, \{m_2, m_3, m_4, m_5\}\}$.

The ideal minimal accuracy measure $\sigma_r^{\mathcal{J}}(.)$ that is calculated by using the current approximation in Definition 6 increased more than the minimal accuracy measure $\sigma_r(.)$ due to Definition 4, for any subset of Λ as tabulated in Table 5.

Table 5: Comparison between $\sigma_r^{\mathcal{J}}()$ in Definition 6 and $\sigma_r()$ due to Definition 4, for any subset of Λ, $\mathcal{J} = \{\emptyset, \{m_5\}\}$.

$B \subseteq \Lambda$	$\sigma_r()$	$\sigma_r^{\mathcal{J}}()$	$B \subseteq \Lambda$	$\sigma_r()$	$\sigma_r^{\mathcal{J}}()$
$\{m_1\}$	0	0	$\{m_1, m_2, m_3\}$	0	0.33
$\{m_2\}$	0	0.5	$\{m_1, m_2, m_4\}$	0.5	0.75
$\{m_3\}$	0	0	$\{m_1, m_2, m_5\}$	0.5	0.5
$\{m_4\}$	0.33	0.33	$\{m_1, m_3, m_4\}$	0.67	0.67
$\{m_5\}$	0.33	1	$\{m_1, m_3, m_5\}$	0.25	0.33
$\{m_1, m_2\}$	0	0.33	$\{m_1, m_4, m_5\}$	0.6	0.75
$\{m_1, m_3\}$	0	0	$\{m_2, m_3, m_4\}$	0.25	0.75
$\{m_1, m_4\}$	0.67	0.67	$\{m_2, m_3, m_5\}$	0.67	0.67
$\{m_1, m_5\}$	0.25	0.5	$\{m_2, m_4, m_5\}$	0.6	0.6
$\{m_2, m_3\}$	0	0	$\{m_3, m_4, m_5\}$	0.4	0.5
$\{m_2, m_4\}$	0.25	0.5	$\{m_1, m_2, m_3, m_4\}$	0.5	1
$\{m_2, m_5\}$	0.67	0.67	$\{m_1, m_2, m_3, m_5\}$	0.5	0.5
$\{m_3, m_4\}$	0.33	0.33	$\{m_1, m_2, m_4, m_5\}$	0.8	0.8
$\{m_3, m_5\}$	0.33	0.5	$\{m_1, m_3, m_4, m_5\}$	0	0.75
$\{m_4, m_5\}$	0.4	0.5	$\{m_2, m_3, m_4, m_5\}$	0.8	0.8

6 An Algorithm and Framework

This section provides an algorithm and a framework for decision-making problems. The suggested algorithm is checked with fictitious data and compared to existing methods. This technique represents a simple tool that can be used in MATLAB.

Require: Initiate an information table generated from the given data such that the first column contains a set of objects Λ, and the set of attributes as a first row.

Output: An accurate decision for exact and rough sets.

Step 1: Input a finite set of data as a universal set Λ, and A_l a set of attributes from the information table.

Step 2: Define the binary relations $\eta_l = \{(m_i, m_j) \in \Lambda \times \Lambda: m_i(A_l) - m_j(A_l) < \frac{\delta_l}{2}, i, j, l = 1, 2, 3, 4, 5\}$.

Step 3: Compute all right neighborhoods of all elements by $m_i\eta = \cap_{l=1}^{5} mi\eta_1$, for each $i, l = 1, 2, 3, 4, 5$

Step 4: Construct the class of minimal structure by **Step 3**.

Step 5: Using the ideal $\mathcal{J}$ (which is given by an expert), compute $LMS^{\mathcal{J}}(B)$ and $UMS^{\mathcal{J}}(B)$ of $B \subseteq \Lambda$ with respect to ideal $\mathcal{J}$ as follows: $LMS^{\mathcal{J}}(B) = \underline{\eta}_r^{\mathcal{J}}(B) \cap B$, where $\underline{\eta}_r^{\mathcal{J}}(B) = \cup\{U \in MS(\Lambda): U - B \in \mathcal{J}\}$, and $UMS^{\mathcal{J}}(B) = \overline{\eta}_r^{\mathcal{J}}(B) \cup B$, where $\overline{\eta}_r^{\mathcal{J}}(B) = \cap\{V \in (MS(\Lambda))^c : B - V \in \mathcal{J}\}$.

Step 6: Using the ideal $\mathcal{J}$ (which is given by an expert), compute the minimal accuracy of the approximations in Step 5 of all subsets in Λ by $\sigma_r^{\mathcal{J}}(B) = \frac{|LMS^{\mathcal{J}}(B)|}{|UMS^{\mathcal{J}}(B)|}$, where $|UMS^{\mathcal{J}}(B)| \neq 0$.

Step 7: If $\sigma_r^{\mathcal{J}}(B) = 1$, then B is an exact set. Else, B is a rough set.

The following figure (Fig. 1) illustrates a simple flowchart for calculating the degree of accuracy induced from the above algorithm.

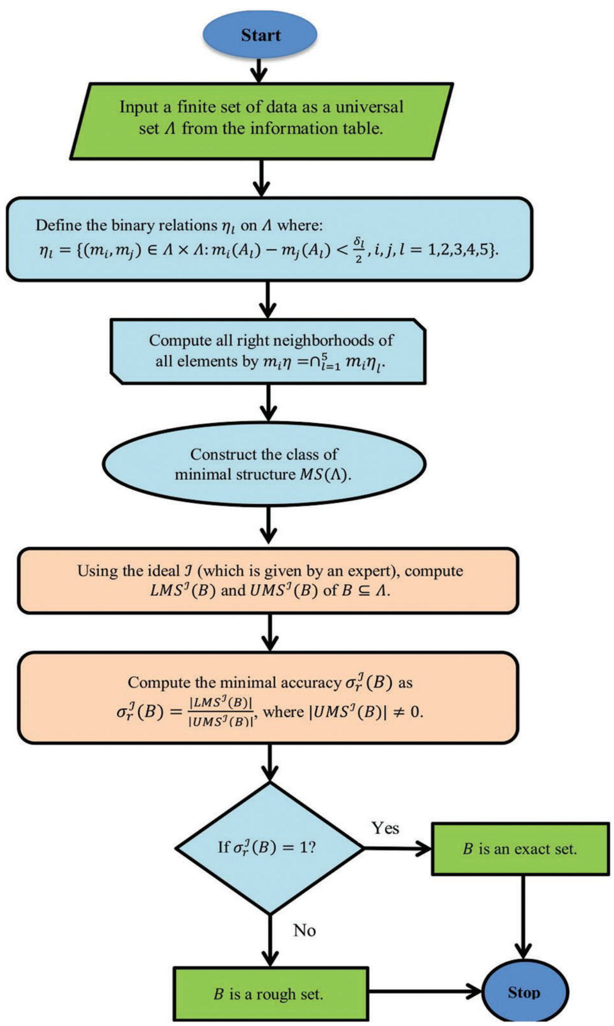

Figure 1: A flowchart for decision making using an $\mathcal{J}MSAS$

7 Conclusions and Discussions

The novel rough approximation space $\mathcal{J}MSAS$ generated by the minimal structure and ideal concepts were proposed, and their principal characteristics were verified. The best approximations and degrees of accuracy have been achieved. A novel approach has been compared with the other approaches in the references [6,22,23] via counterexamples that has been examined as indicated in Theorem 1 and Table 1. From Remark 3, some principal properties of rough sets concerning $\underline{\eta}_r^{\mathcal{J}}(\ )$, and $\overline{\eta}_r^{\mathcal{J}}(\ )$ were deduced. By increasing the number of elements of ideals, the lower approximation would increase and the upper approximation would decrease and hence the measure of accuracy becomes more accurate as given in Proposition 4. In addition, the minimal internal edge, minimal external edge, and the degree of accuracy were described by using minimal boundary. Several sorts of near open and near closed sets by the $\mathcal{J}MSAS$ view were studied.

One of the challenges in daily problems, as in the medical diagnosis, is making an accurate decision. Therefore, the applied example in biochemistry offers a clear vision that the expansion using the ideal gives better results. Thus, by the $\mathcal{J}MSAS$ different sorts of mathematical tools, which may help experts in studying amino acids, were suggested. A simple approach is used in MATLAB, and the proposed method in an algorithm form was demonstrated. In reality, that approach may be useful in solving some future real-life problems.

In the forthcoming, the $\mathcal{J}MSAS$ approach will be extended to a variety of other concepts, such as fuzzy sets and soft rough sets. Also, it is planned to benefit from the $\mathcal{J}MSAS$ approach to apply them to the problems in [4,8,14,15] to improve their accuracy values.

Acknowledgement: We appreciate the reviewers for their invaluable time in reviewing our paper and providing thoughtful and valuable comments. It was their insightful suggestions that led to sensible improvements in the current version.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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