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Article

New Ranking of Generalized Quadrilateral Shape Fuzzy Number Using Centroid Technique

A. Thiruppathi* and C. K. Kirubhashankar

Department of Mathematics, Sathyabama Institute of Science and Technology, Chennai, 600119, Tamilnadu, India
*Corresponding Author: A. Thiruppathi. Email: thiruppathiap@gmail.com
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Abstract: The output of the fuzzy set is reduced by one for the defuzzification procedure. It is employed to provide a comprehensible outcome from a fuzzy inference process. This page provides further information about the defuzzification approach for quadrilateral fuzzy numbers, which may be used to convert them into discrete values. Defuzzification demonstrates how useful fuzzy ranking systems can be. Our major purpose is to develop a new ranking method for generalized quadrilateral fuzzy numbers. The primary objective of the research is to provide a novel approach to the accurate evaluation of various kinds of fuzzy integers. Fuzzy ranking properties are examined. Using the counterexamples of Lee and Chen demonstrates the fallacy of the ranking technique. So, a new approach has been developed for dealing with fuzzy risk analysis, risk management, industrial engineering and optimization, medicine, and artificial intelligence problems: the generalized quadrilateral form fuzzy number utilizing centroid methodology. As you can see, the aforementioned scenarios are all amenable to the solution provided by the generalized quadrilateral shape fuzzy number utilizing centroid methodology. It's laid out in a straightforward manner that's easy to grasp for everyone. The rating method is explained in detail, along with numerical examples to illustrate it. Last but not least, stability evaluations clarify why the Generalized quadrilateral shape fuzzy number obtained by the centroid methodology outperforms other ranking methods.

Keywords: Fuzzy numbers; quadrilateral fuzzy number; ranking methods; fuzzy risk analysis

1 Introduction

One definition of a fuzzy number (FN) is a real number whose fuzzy set (FS) is both normal and convex. The FN approach is fantastic since it allows for the combination of subjective and objective information. Ranking fuzzy numbers (RFNs) is necessary for many fuzzy application systems, including those dealing with linguistic decision-making, risk management, Industrial engineering, optimization, medicine, artificial intelligence etc. We shall investigate the ordering of quadrilateral fuzzy number generalizations (GQFNs). For a problem with four points and two levels of representational depth, GQFNs can be used. The ranking of FNs is arguably the most essential part, and there are many different ways to do it using techniques from data analysis, optimization, and other disciplines. In the future, it may be used for problems in decision-making, data analysis, optimization, engineering, and technology. Decisions can be aided by RFN measurements (DMP). Since 1976, many systems for categorizing ranking events have been created. Zadeh [1] is credited with coming up with the concept of an FS. An original FRA method for handling risk uncertainty is described by Lee et al. [2]. Fuzzy integers of varying sizes and variances form the basis of the process. Fuzzy numbers are crucial to conveying uncertain values. In this research, we propose many mathematical operations on generalized quadrilateral fuzzy numbers (GQFNs) using the defuzzification technique. Additionally, several features of the defuzzification method are presented, along with relevant numerical examples to back up the features. In the beginning, Jain [3] predicted the positions of FNs for the DMP. Dubois et al. [4]. This article employs a fuzzification approach to allow conventional algebraic operations to be applied to fuzzy numbers. The work of Mizumoto et al. [5,6]. When it comes to the difficulty of making decisions, several approaches have been developed, such as the Pythagorean fuzzy Dombi aggregation operators, the Extended TOPSIS technique, and the Intuitionistic Fuzzy Rough Frank Aggregation Operator-Based EDAS Method.

2 Literature Review

By comparing the rankings obtained by using the FRA problem with the positive, negative, and centroid of the generalized numbers, Ming Chen et al. [7] suggested a hypothesis. Using the counter example of Lee and Chen, Singh [8] advocated a new way of RFN based on the expectation of centroid predictions of varying heights and demonstrated that the sorting method is erroneous. In [9], Pathinathan et al. organized a pentagonal QFN using mathematical computation. Using the mathematical function CEFM, Stephen Dinagar et al. [10] presented a GQF transportation problem. In their essay "defuzzification of QFN and arithmetic operation of CEFM," authors Dinagar et al. examined the characteristics of CEFM. There is a GQF optimum solution to Assignment issues, and it was proposed by Stephen Dinagar et al. [11]. The GQF economic inventory model, which takes into account backorders, was created by Stephen Dinagar et al. [12,13]. Fuzzy Assignment Tricky makes use of a technique proposed by Thiruppathi et al. [14,15] called Ranking of Hexagonal Fuzzy Using Centroids. Using the center of gravity, Barazandeh et al. [16] calculated the score of a generalized fuzzy trapezoidal number (GFTN) with a range of left and right heights. A better system of rankings was created by Jiang et al. [17]. The strengths and disadvantages of current fuzzy ranking systems are investigated, and the elements of FN regions, including the upsides and downsides, and the spreads, are discussed. Hajjari [18] offered a fresh magnitude method for sorting trapezoidal fuzzy numbers (TFNs) using lowest and maximum points and the value of FNs, and he compared and approved the benefits of the supplied strategy comparison. In this study, Rezvani [19] presented a method for using Euclidean distance and central centroids. Results from the proposed method are checked against those of other popular approaches to ensure they are reliable. Yu Vincent et al. [20] proposes a novel method of rating FNs that displays their updated left, right and total integral values. After that, we utilize the median value ranking approach to identify FNs that have area compensation. This article by Ming wang et al. [21] discusses an optional ranking method for FNs based on positive and negative ideal foci, and calls it "area ranking." Predicting the efficacy and modesty of non-preemptive priority fuzzy queues necessitates the use of performance measurements, and Vinnarasi et al. presented the Bell-shaped fuzzy number using the centroid of centroids method [22]. To rate items, only a select few authors have turned to generalized quadrilateral fuzzy numbers. The suggested technique, however, is unique among existing methods and provides a straightforward identification ranking strategy for fuzzy numbers in a quadrilateral form.

The work is coordinated as follows Section 3 provides a fundamental depiction of a GQFN. The method proposed for ordering the integers that can be found in any generalized quadrilateral is discussed in Section 4. (RGQNs). Equivalent Properties in the Classical Period It has been proven that fuzzy arithmetic operations do exist. The sixth section provides an overview of one of the problems with Lee and Chen's RA has been highlighted. In Section 7, we see the implementation of mathematical operations on GQFNs. The Algorithm for Comparing Two Fuzzy Quantities is discussed in Section 8. Existing approaches are compared to the new GQSFN-CT methods in Section 9. This is the tenth section. This section offers guidance on how to use RA to the FRA problem. The final section summarises the entire paper.

Note: Throughout the paper the following abbreviations are used

FS	Fuzzy Sets
FN	Fuzzy Number
RA	Ranking algorithm
RFN	Ranking of Fuzzy Number
QFN	Quadrilateral fuzzy number
GQFNs	Generalized of quadrilateral fuzzy numbers
DMP	Decision-making process
FRA	Fuzzy ranking algorithm
GQSFN-CT	Generalized of quadrilateral shape fuzzy numbers-centroid methods
RGQNs	Ranking generalized quadrilateral numbers
TOPSIS	Technique for order of preference by similarity to ideal solution
EDAS	Evaluation based on distance from average solution
CEFM	Classical equivalent fuzzy mean

3 Preliminaries

3.1 Fuzzy Set

Let $Q(A) = \{(x, \mu_A(X)/x\epsilon Q(A), \mu_A(X)\epsilon(0, 1)\}$ First element x belongs to set Q (A), second element $\mu_A(X)$ belongs to the interval [0, 1], is called membership function. Fig. 1 represent the membership function of Generalized quadrilateral fuzzy numbers.

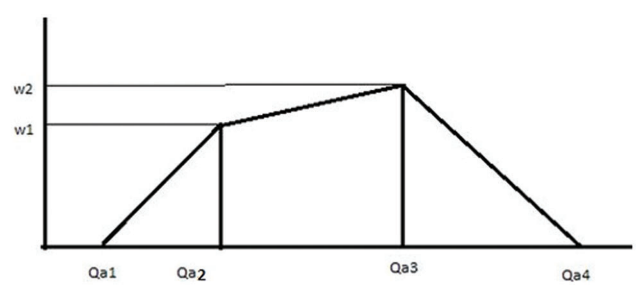


Figure 1: Generalizations of quadrilateral fuzzy number

3.2 Definition: [9–11]

A FN that is GQFNs is defined as $\breve{Q} = [q_{a_1}, q_{a_2}, q_{a_3}, q_{a_4}; \omega_1, \omega_2]$ where $q_{a_1} < q_{a_2} < q_{a_3} < q_{a_4}$ the membership function is defined by

$$\mu_{\breve{Q}}(x) = \begin{cases} \omega_1\left(\frac{x-q_{a_1}}{q_{a_2}-q_{a_1}}\right) & if \quad q_{a_1} \le x \le q_{a_2} \\ \frac{(x-q_{a_2})\omega_2+(q_{a_3}-x)\omega_1}{(q_{a_3}-q_{a_2})} & if \quad q_{a_2} \le x \le q_{a_3} \\ \omega_2\left(\frac{x-q_{a_4}}{q_{a_3}-q_{a_4}}\right) & if \quad q_{a_3} \le x \le q_{a_4} \\ 0 & otherwise \end{cases} \tag{1}$$

When $\omega_1 = \omega_2 = \omega$, the GQFN is turned into the trapezoidal fuzzy number (TFN) and when $q_{a_1} = q_{a_2}$ or $q_{a_3} = q_{a_4}$ the GQFN is formed into the TFN. The trapezoid is the conjecture of the quadrilateral, because the flat line is not parallel in this case, a trapezoidal is inappropriate, as shown by the graphical representation provided by the aforementioned FN. If, however, the two horizontal lines here are brought to equality, we get a trapezoid. These lines may also be referred to as QFNs due to the topographical character of the particular FN in question.

4 Ranking Approach

A technique suggested for ranking generalized fuzzy number (RGFN) of quadrilateral form. $(\omega_1 < \omega_2)$. Three figures were created by vertically dividing the GQFN. Draw the horizontal line parallel to ω_1, now we get three triangles namely triangle ABE, Triangle CDG and triangle EFG. Fig. 2 represent the ranking approach of Generalized quadrilateral fuzzy numbers. Fig. 3 represent the ranking of Generalized quadrilateral fuzzy numbers.

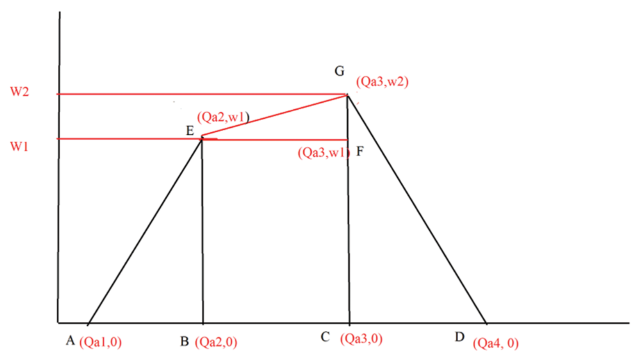


Figure 2: Diagram for Ranking GQFN

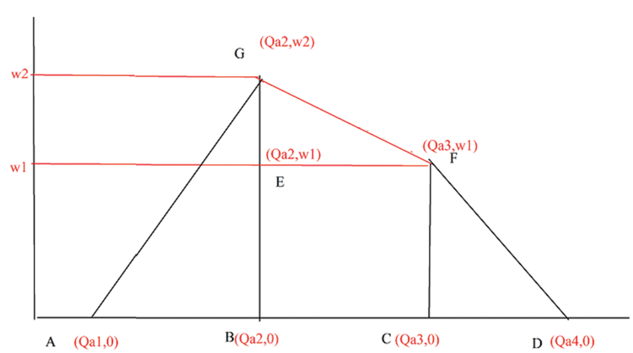


Figure 3: Diagram for Ranking GQFN

$\breve{Q} = [q_{a_1},\ q_{a_2},\ q_{a_3},\ q_{a_4};\ \omega_1,\ \omega_2]$ Here heights $\omega_1 < \omega_2$

The centroid point triangle ABE, Triangle CDG and triangle EFG respectively,

$$Q(\mathbb{G}_1) = \left(\frac{q_{a_1} + 2q_{a_2}}{3}, \frac{\omega_1}{3}\right)$$

$$Q(\mathbb{G}_2) = \left(\frac{2q_{a_3} + q_{a_4}}{3}, \frac{\omega_2}{3}\right)$$

$$Q(\mathbb{G}_3) = \left(\frac{q_{a_2} + 2q_{a_3}}{3}, \ \frac{2\omega_1 + \omega_2}{3}\right)$$

Subsequently, connect the points $Q(\mathbb{G}_1),\ Q(\mathbb{G}_2)$ and $Q(\mathbb{G}_3)$. They come together to build a triangle. The centre of the triangle (CoT) formed by the vertices $Q(\mathbb{G}_1),\ Q(\mathbb{G}_2)$ and $Q(\mathbb{G}_3)$ is now

$$Q(\mathbb{G}) = \left(\frac{q_{a_1} + 3q_{a_2} + 4q_{a_3} + q_{a_4}}{9}, \frac{3\omega_1 + 2\omega_2}{9}\right) \tag{2}$$

The GQFN ranking function $\breve{Q} = [q_{a_1},\ q_{a_2},\ q_{a_3},\ q_{a_4};\ \omega_1,\ \omega_2]$ translates the set of all FNs to a set of real numbers. It is well-defined as follows

$$R(Q) = \frac{q_{a_1} + 3q_{a_2} + 4q_{a_3} + q_{a_4}}{9} \times \frac{3\omega_1 + 2\omega_2}{9}$$

GQFN $Q(R)$ assigns real numbers for each FN in $Q(R)$ given in the following new ranking GQSFN-CT method,

$$\breve{Q}_1 = [q_{a_1},\ q_{a_2},\ q_{a_3},\ q_{a_4};\ \omega_1,\ \omega_2] \in Q(R),\quad \omega_1 < \omega_2$$

$$R(Q) = \frac{q_{a_1} + 3q_{a_2} + 4q_{a_3} + q_{a_4}}{9} \times \frac{3\omega_1 + 2\omega_2}{9} \tag{3}$$

Ranking approach method

A technique suggested for RGFN of quadrilateral form. $\omega_1 > \omega_2$

Three fig were created by vertically dividing the GQFN. Draw the horizontal line parallel to ω_1, now we get three triangles namely triangle ABE, Triangle CDG and triangle EFG.

$\breve{Q} = [q_{a_1},\ q_{a_2},\ q_{a_3},\ q_{a_4};\ \omega_1\ \ \omega_2]$ Here heights $\omega_1 > \omega_2$

The centroid point triangle ABE, Triangle CDG and triangle EFG respectively

$$Q(\mathbb{G}_1) = \left(\frac{q_{a_1} + 2q_{a_2}}{3}, \frac{\omega_1}{3}\right)$$

$$Q(\mathbb{G}_2) = \left(\frac{2q_{a_3} + q_{a_4}}{3}, \frac{\omega_2}{3}\right)$$

$$Q(\mathbb{G}_3) = \left(\frac{2q_{a_2} + q_{a_3}}{3}, \frac{2\omega_1 + \omega_2}{3}\right)$$

Subsequently, connect the points $Q(\mathbb{G}_1)$, $\mathrm{Q}(\mathbb{G}_2)$ and $Q(\mathbb{G}_3)$. They come together to create a triangle. The CoT formed by the vertices $Q(\mathbb{G}_1)$, $\mathrm{Q}(\mathbb{G}_2)$ and $Q(\mathbb{G}_3)$ is now

$$Q(\mathbb{G}) = \left(\frac{\mathfrak{q}_{a_1} + 4\mathfrak{q}_{a_2} + 3\mathfrak{q}_{a_3} + \mathfrak{q}_{a_4}}{9}, \frac{3\omega_1 + 2\omega_2}{9}\right) \tag{4}$$

The GQFN ranking function $\breve{\mathcal{Q}} = [\mathfrak{q}_{a_1}, \mathfrak{q}_{a_2}, \mathfrak{q}_{a_3}, \mathfrak{q}_{a_4}; \omega_1, \omega_2]$. translates the set of all FNs to a set of real numbers. It is defined as follows

$$R(Q) = \frac{\mathfrak{q}_{a_1} + 4\mathfrak{q}_{a_2} + 3\mathfrak{q}_{a_3} + \mathfrak{q}_{a_4}}{9} \times \frac{3\omega_1 + 2\omega_2}{9}$$

GQFN $Q(R)$ assigns real numbers for each FN in $Q(R)$ given in the following new ranking GQSFN-CT method,

$$\breve{\mathcal{Q}}_1 = [\mathfrak{q}_{a_1}, \mathfrak{q}_{a_2}, \mathfrak{q}_{a_3}, \mathfrak{q}_{a_4}; \omega_1, \omega_2] \in Q(R) \quad \omega_1 > \omega_2$$

$$R(Q) = \frac{\mathfrak{q}_{a_1} + 4\mathfrak{q}_{a_2} + 3\mathfrak{q}_{a_3} + \mathfrak{q}_{a_4}}{9} \times \frac{3\omega_1 + 2\omega_2}{9} \tag{5}$$

5 Properties of Classical Equivalent Fuzzy Arithmetic Operation; [10–12]

Property: 5.1

Let $\breve{\mathcal{Q}}_1$ and $\breve{\mathcal{Q}}_2$ be two GQFN and $R(\breve{\mathcal{Q}}_1)$ and $R(\breve{\mathcal{Q}}_2)$ be the respective fuzzy ranking values (FRVs). Using Classical equivalent Fuzzy mean arithmetic operation. FRV of sum of GQFNS $\breve{\mathcal{Q}}_1$, $\breve{\mathcal{Q}}_2$ is the RS-FN of $\breve{\mathcal{Q}}_1$, $\breve{\mathcal{Q}}_2$.

i.e., $R(\breve{\mathcal{Q}}_1 + \breve{\mathcal{Q}}_2) = R(\breve{\mathcal{Q}}_1) + R(\breve{\mathcal{Q}}_2)$ (6)

Property: 5.2

Let $\breve{\mathcal{Q}}_1$, $\breve{\mathcal{Q}}_2$ and $\breve{\mathcal{Q}}_3$ be three GQFN and $R(\breve{\mathcal{Q}}_1)$, $R(\breve{\mathcal{Q}}_2)$ and $R(\breve{\mathcal{Q}}_3)$ be the respective FRVs. Using Classical equivalent Fuzzy mean arithmetic operation. FRV of sum of GQFNS $\breve{\mathcal{Q}}_1$, $\breve{\mathcal{Q}}_2$, $\breve{\mathcal{Q}}_3$ is the RS-FN of $\breve{\mathcal{Q}}_1$, $\breve{\mathcal{Q}}_2$, $\breve{\mathcal{Q}}_3$.

i.e. $R(\breve{\mathcal{Q}}_1 + \breve{\mathcal{Q}}_2 + \breve{\mathcal{Q}}_3) = R(\breve{\mathcal{Q}}_1) + R(\breve{\mathcal{Q}}_2) + R(\breve{\mathcal{Q}}_3)$ (7)

Proof:

$$\breve{\mathcal{Q}}_1 = [\mathfrak{q}_{a_1}, \mathfrak{q}_{a_2}, \mathfrak{q}_{a_3}, \mathfrak{q}_{a_4}; \ell_1, \ell_2]$$

$$\breve{\mathcal{Q}}_2 = [\mathfrak{q}_{b_1}, \mathfrak{q}_{b_2}, \mathfrak{q}_{b_3}, \mathfrak{q}_{b_4}; m_1, m_2]$$

$$\breve{\mathcal{Q}}_3 = [\mathfrak{q}_{c_1}, \mathfrak{q}_{c_2}, \mathfrak{q}_{c_3}, \mathfrak{q}_{c_4}; n_1, n_2]$$

$$\omega_l = 3\ell_1 + 2\ell_2$$

$$\omega_m = 3m_1 + 2m_2$$

$$\omega_n = 3n_1 + 2n_2$$

$$\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2+\breve{\mathcal{Q}}_3=\begin{bmatrix}\frac{3(\mathfrak{q}_{a_1}\omega_l+\mathfrak{q}_{b_1}\omega_m+\mathfrak{q}_{c_1}\omega_n)}{\omega_l+\omega_m+\omega_n},\frac{3(\mathfrak{q}_{a_2}\omega_l+\mathfrak{q}_{b_2}\omega_m+\mathfrak{q}_{c_2}\omega_n)}{\omega_l+\omega_m+\omega_n}\frac{3(\mathfrak{q}_{a_3}\omega_l+\mathfrak{q}_{b_3}\omega_m+\mathfrak{q}_{c_3}\omega_n)}{\omega_l+\omega_m+\omega_n},\\ \frac{3(\mathfrak{q}_{a_4}\omega_l+\mathfrak{q}_{b_4}\omega_m+\mathfrak{q}_{c_4}\omega_n)}{\omega_l+\omega_m+\omega_n};\ \frac{(\ell_1+m_1+n_1)}{3},\frac{(\ell_2+m_2+n_2)}{3}\end{bmatrix}$$

$$R(\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2+\breve{\mathcal{Q}}_3)=\frac{\frac{3(\mathfrak{q}_{a_1}\omega_l+\mathfrak{q}_{b_1}\omega_m+\mathfrak{q}_{c_1}\omega_n)}{\omega_l+\omega_m+\omega_n}+\frac{3\times3(\mathfrak{q}_{a_2}\omega_l+\mathfrak{q}_{b_2}\omega_m+\mathfrak{q}_{c_2}\omega_n)}{\omega_l+\omega_m+\omega_n}}{9}$$

$$+\frac{\frac{4\times3(\mathfrak{q}_{a_3}\omega_l+\mathfrak{q}_{b_3}\omega_m+\mathfrak{q}_{c_3}\omega_n)}{\omega_l+\omega_m+\omega_n}+\frac{3(\mathfrak{q}_{a_4}\omega_l+\mathfrak{q}_{b_4}\omega_m+\mathfrak{q}_{c_4}\omega_n)}{\omega_l+\omega_m+\omega_n}}{9}$$

$$\times\left(\frac{\frac{3(\ell_1+m_1+n_1)}{3}+\frac{2(\ell_2+m_2+n_2)}{3}}{9}\right)$$

$$=\frac{\frac{3\mathfrak{q}_{a_1}\omega_l+3\mathfrak{q}_{b_1}\omega_m+3\mathfrak{q}_{c_1}\omega_n}{\omega_l+\omega_m+\omega_n}+\frac{9\mathfrak{q}_{a_2}\omega_l+9\mathfrak{q}_{b_2}\omega_m+9\mathfrak{q}_{c_2}\omega_n}{\omega_l+\omega_m+\omega_n}}{9}$$

$$+\frac{\frac{12\mathfrak{q}_{a_3}\omega_l+12\mathfrak{q}_{b_3}\omega_m+12\mathfrak{q}_{c_3}\omega_n}{\omega_l+\omega_m+\omega_n}+\frac{3\mathfrak{q}_{a_4}\omega_l+3\mathfrak{q}_{b_4}\omega_m+3\mathfrak{q}_{c_4}\omega_n}{\omega_l+\omega_m+\omega_n}}{9}$$

$$\times\left(\frac{\frac{(3\ell_1+3m_1+3n_1)}{3}+\frac{(2\ell_2+2m_2+2n_2)}{3}}{9}\right)$$

$$=\frac{3\ \ (\mathfrak{q}_{a_1}\omega_l+3\mathfrak{q}_{a_2}\omega_l+4\mathfrak{q}_{a_3}\omega_l+3\mathfrak{q}_{a_4}\omega_l)+(\mathfrak{q}_{b_1}\omega_m+3\mathfrak{q}_{b_2}\omega_m+4\mathfrak{q}_{b_3}\omega_m+\mathfrak{q}_{b_4}\omega_m)}{9(\omega_l+\omega_m+\omega_n)}$$

$$+\frac{((\mathfrak{q}_{c_1}\omega_n+3\mathfrak{q}_{c_2}\omega_n+4\mathfrak{q}_{c_3}\omega_n+3\mathfrak{q}_{c_4}\omega_n))}{9(\omega_l+\omega_m+\omega_n)}\times\left(\frac{\frac{(3\ell_1+2\ell_2)+(3m_1+2m_2)+(3n_1+2n_2)}{3}}{9}\right)$$

$$=\left[\left(\frac{\frac{3(\mathfrak{q}_{a_1}+3\mathfrak{q}_{a_2}+4\mathfrak{q}_{a_3}+3\mathfrak{q}_{a_4})\omega_l+(\mathfrak{q}_{b_1}+3\mathfrak{q}_{b_2}+4\mathfrak{q}_{b_3}+\mathfrak{q}_{b_4})\omega_m+(\mathfrak{q}_{c_1}+3\mathfrak{q}_{c_2}+4\mathfrak{q}_{c_3}+\mathfrak{q}_{c_4})\omega_n}{\omega_l+\omega_m+\omega_n}}{9}\right)\times\left(\frac{\frac{\omega_+\omega_m+\omega_n}{3}}{9}\right)\right]$$

$$=\left[\left(\frac{(\mathfrak{q}_{a_1}+3\mathfrak{q}_{a_2}+4\mathfrak{q}_{a_3}+3\mathfrak{q}_{a_4})\omega_1+(\mathfrak{q}_{b_1}+3\mathfrak{q}_{b_2}+4\mathfrak{q}_{b_3}+\mathfrak{q}_{b_4})\omega_m+(\mathfrak{q}_{c_1}+3\mathfrak{q}_{c_2}+4\mathfrak{q}_{c_3}+\mathfrak{q}_{c_4})\omega_n}{9}\right)\times\left(\frac{1}{9}\right)\right]$$

$$=\frac{(\mathfrak{q}_{a_1}+3\mathfrak{q}_{a_2}+4\mathfrak{q}_{a_3}+3\mathfrak{q}_{a_4})}{9}\times\frac{\omega_1}{9}+\frac{(\mathfrak{q}_{b_1}+3\mathfrak{q}_{b_2}+4\mathfrak{q}_{b_3}+\mathfrak{q}_{b_4})}{9}\times\frac{\omega_m}{9}+\frac{(\mathfrak{q}_{c_1}+3\mathfrak{q}_{c_2}+4\mathfrak{q}_{c_3}+\mathfrak{q}_{c_4})}{9}\times\frac{\omega_n}{9}$$

$$= \frac{(\mathfrak{q}_{a_1} + 3\mathfrak{q}_{a_2} + 4\mathfrak{q}_{a_3} + 3\mathfrak{q}_{a_4})}{9} \times \frac{3\ell_1 + 2\ell_2}{9} + \frac{(\mathfrak{q}_{b_1} + 3\mathfrak{q}_{b_2} + 4\mathfrak{q}_{b_3} + \mathfrak{q}_{b_4})}{9} \times \frac{3m_1 + 2m_2}{9}$$

$$+ \frac{(\mathfrak{q}_{c_1} + 3\mathfrak{q}_{c_2} + 4\mathfrak{q}_{c_3} + \mathfrak{q}_{c_4})}{9} \times \frac{3n_1 + 2n_2}{9}$$

$$= R(\breve{\mathcal{Q}}_1) + R(\breve{\mathcal{Q}}_2) + R(\breve{\mathcal{Q}}_3)$$

Property: 5.3

Let $\breve{\mathcal{Q}}_1,\ \breve{\mathcal{Q}}_2,\ \breve{\mathcal{Q}}_3 \ldots\ \breve{\mathcal{Q}}_n$ be three GQFN and $R(\breve{\mathcal{Q}}_1),\ R(\breve{\mathcal{Q}}_2)$ …, $R(\breve{\mathcal{Q}}_n)$ be the respective FRVs. Using Classical equivalent Fuzzy mean arithmetic operation. FRV of sum of GQFNS $\breve{\mathcal{Q}}_1,\ \breve{\mathcal{Q}}_2,\ \breve{\mathcal{Q}}_3 \ldots\ \breve{\mathcal{Q}}_n$ is the ranking summation of the fuzzy data (RS-FD) of $\breve{\mathcal{Q}}_1,\ \breve{\mathcal{Q}}_2,\ \breve{\mathcal{Q}}_3 \ldots\ \breve{\mathcal{Q}}_n$.

i.e., $$R(\breve{\mathcal{Q}}_1 + \breve{\mathcal{Q}}_2 + \breve{\mathcal{Q}}_3 + \ldots, +\breve{\mathcal{Q}}_n) = R(\breve{\mathcal{Q}}_1) + R(\breve{\mathcal{Q}}_2) + R(\breve{\mathcal{Q}}_3) + \ldots, +R(\breve{\mathcal{Q}}_n) \tag{8}$$

Similar to the above proof

6 A Shortcoming of Lee and Chen's Ra

Wang and Keere proposed the following acceptable conditions for ranking function validation.

$\breve{\mathcal{Q}}_1 = [\mathfrak{q}_{a_1},\ \mathfrak{q}_{a_2},\ \mathfrak{q}_{a_3},\ \mathfrak{q}_{a_4};\ \ell_1,\ \ell_2]$ and $\breve{\mathcal{Q}}_2 = [\mathfrak{q}_{b_1},\ \mathfrak{q}_{b_2},\ \mathfrak{q}_{b_3},\ \mathfrak{q}_{b_4};\ m_1,\ m_2]$ are two normal FS then $Q_1 > Q_2\ \Rightarrow\ (Q_1 \oplus Q_3) > (Q_1 \oplus Q_3)$

$$Q_1 < Q_2\ \Rightarrow\ (Q_1 \oplus Q_3) < (Q_1 \oplus Q_3)$$

$$Q_1 : Q_2\ \Rightarrow (\ Q_1 \oplus Q_3):\ (Q_1 \oplus Q_3) \tag{9}$$

where $\breve{\mathcal{Q}}_3 = [\mathfrak{q}_{c_1},\ \mathfrak{q}_{c_2},\ \mathfrak{q}_{c_3},\ \mathfrak{q}_{c_4};\ n_1,\ n_2]$ is normal FS. Where

$$(n_1,\ n_2) \leq (\min(\ell_1,\ m_1)\ \ \min(\ell_2,\ m_2)$$

Example 1. [12] Let $\breve{\mathcal{Q}}_1 = (1, 2, 3, 4; 0.6, 0.4)$, $\breve{\mathcal{Q}}_2 = (0, 3, 4, 5; 0.4, 0.2)$ and, $\breve{\mathcal{Q}}_3 = (1, 3, 4, 5; 0.4, 0.2)$ GQFNS, Then then according to Lee and Chen's RA $\breve{\mathcal{Q}}_1 < \breve{\mathcal{Q}}_2 \not\Rightarrow \breve{\mathcal{Q}}_1 \oplus \breve{\mathcal{Q}}_3 > \breve{\mathcal{Q}}_2 \oplus \breve{\mathcal{Q}}_3$

Example 2. [12] Let $\breve{\mathcal{Q}}_1 = (2, 5, 6, 7; 0.6, 0.4)$, $\breve{\mathcal{Q}}_2 = (3, 4, 5, 6; 0.8, 0.6)$ and, $\breve{\mathcal{Q}}_3 = (4, 3, 7, 8; 0.6, 0.4)$ GQFNS, Then then according to Lee and Chen's RA $\breve{\mathcal{Q}}_1 > \breve{\mathcal{Q}}_2 \not\Rightarrow \breve{\mathcal{Q}}_1 \oplus \breve{\mathcal{Q}}_3 < \breve{\mathcal{Q}}_2 \oplus \breve{\mathcal{Q}}_3$

Our new RA has been implemented. Mathematical operations on GQFNS have been proposed as follows [9–11].

7 Numerical Examples

Example-1

Based on Classical equivalent Fuzzy mean mathematical operations on GQFNS.

$$\breve{\mathcal{Q}}_1 = [3,\ 4,\ 5,\ 7;\ 0.2,\ 0.4] \Rightarrow R(\breve{\mathcal{Q}}_1) = 0.73$$

$$\breve{\mathcal{Q}}_2 = [7,\ 8,\ 10,\ 11;\ 0.3,\ 0.6] \Rightarrow R(\breve{\mathcal{Q}}_2) = 2.13$$

Addition

$$\begin{aligned}&\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2=[10.80,\ 12.80,\ 16,\ 18.80;\ .025,\ 0.50]\\&R(\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2)=2.86\\&R(\breve{\mathcal{Q}}_1)+R(\breve{\mathcal{Q}}_1)=0.73+2.13=2.86\\&\therefore R(\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2)=R(\breve{\mathcal{Q}}_1)+R(\breve{\mathcal{Q}}_1)\end{aligned}\tag{10}$$

Example-2

Based on Classical equivalent Fuzzy mean mathematical operations on GQFNS.

$$\breve{\mathcal{Q}}_1=[8,\ 11,\ 13,\ 15\ ;\ 0.2,\ 0.4]\Rightarrow R(\breve{\mathcal{Q}}_1)=1.87$$

$$\breve{\mathcal{Q}}_2=[6,\ 8,\ 10,\ 12\ ;\ 0.3,\ 0.6]\Rightarrow R(\breve{\mathcal{Q}}_2)=2.13$$

Addition

$$\begin{aligned}&\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2=[13.60,\ 18.40,\ 22.40,\ 26.40.025,\ 0.50]\\&R(\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2)=3.99\\&R(\breve{\mathcal{Q}}_1)+R(\breve{\mathcal{Q}}_1)=1.87+2.13=3.99\\&\therefore R(\breve{\mathcal{Q}}_1+\breve{\mathcal{Q}}_2)=R(\breve{\mathcal{Q}}_1)+R(\breve{\mathcal{Q}}_1)\end{aligned}\tag{11}$$

Example-3

Based on Classical equivalent Fuzzy mean mathematical operations on GQFNS.

$$\breve{\mathcal{Q}}_1=[3,\ 4,\ 5,\ 7;\ 0.2,\ 0.4]\Rightarrow R(\breve{\mathcal{Q}}_1)=0.73$$

$$\breve{\mathcal{Q}}_2=[7,\ 8,\ 10,\ 11;\ 0.3,\ 0.6]\Rightarrow R(\breve{\mathcal{Q}}_2)=2.13$$

Subtraction

$$\begin{aligned}&\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2=[-6.00,\ -6.40,\ \ -8.00,\ \ -7.60;\ .025,\ 0.50]\\&R(\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2)=-1.40\\&R(\breve{\mathcal{Q}}_1)-R(\breve{\mathcal{Q}}_1)=0.73-2.13=-1.40\\&\therefore R(\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2)=R(\breve{\mathcal{Q}}_1)-R(\breve{\mathcal{Q}}_1)\end{aligned}\tag{12}$$

Example-4

Using arithmetic operation on GQFNS based on CEFM

$$\breve{\mathcal{Q}}_1=[8,\ 11,\ 13,\ 15\ ;\ 0.2,\ 0.4]\Rightarrow R(\breve{\mathcal{Q}}_1)=1.87$$

$$\breve{\mathcal{Q}}_2=[6,\ 8,\ 10,\ 12\ ;\ 0.3,\ 0.6]\Rightarrow R(\breve{\mathcal{Q}}_2)=2.13$$

Subtraction

$$\begin{aligned}&\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2=[-0.80,\ -0.80,\ -1.60,\ -2.40;\ \ 025,\ 0.50]\\&R(\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2)=-0.26\\&R(\breve{\mathcal{Q}}_1)-R(\breve{\mathcal{Q}}_1)=1.87-2.13=-0.26\\&\therefore R(\breve{\mathcal{Q}}_1-\breve{\mathcal{Q}}_2)=R(\breve{\mathcal{Q}}_1)-R(\breve{\mathcal{Q}}_1)\end{aligned}\tag{13}$$

Example-5

Based on Classical equivalent Fuzzy mean mathematical operations on GQFNS.

$$\breve{\mathcal{Q}}_1 = [8,\ 11,\ 13,\ 15\ ;\ 0.2,\ 0.4] \Rightarrow R(\breve{\mathcal{Q}}_1) = 1.87$$

$$\breve{\mathcal{Q}}_2 = [6,\ 8,\ 10,\ 12\ ;\ 0.3,\ 0.6] \Rightarrow R(\breve{\mathcal{Q}}_2) = 2.13$$

Multiplication if $R(\breve{\mathcal{Q}}_1) > 0$

$$\begin{aligned}
&\breve{\mathcal{Q}}_1 \times \breve{\mathcal{Q}}_2 = [13.61,\ \ 18.71,\ \ 22.11,\ 25.51;\ \ 025,\ 0.50] \\
&R(\breve{\mathcal{Q}}_1 \times \breve{\mathcal{Q}}_2) = 3.97 \\
&R(\breve{\mathcal{Q}}_1) \times R(\breve{\mathcal{Q}}_1) = 1.87 \times 2.13 = 3.97 \\
&\therefore R(\breve{\mathcal{Q}}_1 \times \breve{\mathcal{Q}}_2) = R(\breve{\mathcal{Q}}_1) \times R(\breve{\mathcal{Q}}_1)
\end{aligned} \tag{14}$$

Example-6

Based on Classical equivalent Fuzzy mean mathematical operations on GQFNS.

$$\breve{\mathcal{Q}}_1 = [8,\ 11,\ 13,\ 15;\ 0.2,\ 0.4] \Rightarrow R(\breve{\mathcal{Q}}_1) = 1.87$$

$$\breve{\mathcal{Q}}_2 = [6,\ 8,\ 10,\ 12;\ 0.3,\ 0.6] \Rightarrow R(\breve{\mathcal{Q}}_2) = 2.13$$

Multiplication if $R(\breve{\mathcal{Q}}_1) > 0$

$$\begin{aligned}
&\frac{\breve{\mathcal{Q}}_1}{\breve{\mathcal{Q}}_2} = [3.010,\ 4.139,\ 4.892,\ 5.645;\ 025,\ 0.50] \\
&R\left(\frac{\breve{\mathcal{Q}}_1}{\breve{\mathcal{Q}}_2}\right) = .88 \\
&\frac{R(\breve{\mathcal{Q}}_1)}{R(\breve{\mathcal{Q}}_1)} = 1.87 \times 2.13 = 3.97 \\
&\therefore R\left(\frac{\breve{\mathcal{Q}}_1}{\breve{\mathcal{Q}}_2}\right) = \frac{R(\breve{\mathcal{Q}}_1)}{R(\breve{\mathcal{Q}}_1)}
\end{aligned} \tag{15}$$

8 Algorithm for Comparing Two Fuzzy Quantities $\breve{\mathcal{Q}}_1$ and $\breve{\mathcal{Q}}_2$

Step 1: when heights $w_1 < w_2$. Use formulas (2) to calculate the $Q(\mathbb{G}) = (\mathrm{x},\ \mathrm{y})$

Step 2: when heights $w_1 > w_2$ Use formulas (4) to calculate the $Q(\mathbb{G}) = (\mathrm{x},\ \mathrm{y})$

Step 3: when heights $w_1 < w_2$. Use step (1) and formulas (3) to calculate the $R(\breve{\mathcal{Q}}_1)$ and $R(\breve{\mathcal{Q}}_2)$

Step 4: when heights $w_1 > w_2$ Use step (2) and formulas (5) to calculate the $R(\breve{\mathcal{Q}}_1)$ and $R(\breve{\mathcal{Q}}_2)$

Compare $R(\breve{\mathcal{Q}}_1)$and R($\breve{\mathcal{Q}}_2$)

If, $(\breve{\mathcal{Q}}_1) < R(\breve{\mathcal{Q}}_2)$, then $\breve{\mathcal{Q}}_1 < \breve{\mathcal{Q}}_2$

If, $R(\breve{\mathcal{Q}}_1) >\ R(\breve{\mathcal{Q}}_2)$, then $\breve{\mathcal{Q}}_1 > \breve{\mathcal{Q}}_2$

If, $R(\breve{\mathcal{Q}}_1) \cong R(\breve{\mathcal{Q}}_2)$, then $\breve{\mathcal{Q}}_1 \cong \breve{\mathcal{Q}}_2$

Table 1 specifies, the comparison between the proposed methods and existing benchmark methods tested by various researchers in different times.

Table 1: The comparison between existing and new GQSFN-CT methods

Sets	Lee and chen	Pushpin-der singh	Y Baraz-andeh	D Steph endinagar	New GQSFN-CT methods
$\breve{Q}_1 = [3,\ 4,\ 5,\ 7;\ 0.2,\ 0.4]$ $\breve{Q}_1 = [7,\ 8,\ 10,\ 11;\ 0.3,\ 0.6]$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$
$\breve{Q}_1 = [8,\ 11,\ 13,\ 15;\ 0.2,\ 0.4]$ $\breve{Q}_2 = [6,\ 8,\ 10,\ 12;\ 0.3,\ 0.6]$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$
$\breve{Q}_1 = [1,\ 2,\ 3,\ 4;\ 0.6,\ 0.4]$ $\breve{Q}_2 = [0,\ 3,\ 4,\ 5;\ 0.4,\ 0.2]$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$
$\breve{Q}_1 = [2,\ 5,\ 6,\ 7;\ 0.6,\ 0.4]$ $\breve{Q}_2 = [3,\ 4,\ 5,\ 6;\ 0.8,\ 0.6]$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$
$\breve{Q}_1 = [1,\ 3,\ 4,\ 5;\ 0.4,\ 0.2]$ $\breve{Q}_2 = [4,\ 3,\ 7,\ 8;\ 0.6,\ 0.4]$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$	$R(\breve{Q}_1) > R(\breve{Q}_2)$
$\breve{Q}_1 = [0.1,\ 0.2,\ 0.4,\ 0.5;\ 0.8,\ 1]$ $\breve{Q}_2 = [0.1,\ 0.2,\ 0.4,\ 0.5;\ 1,\ 0.8]$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) \cong R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$
$\breve{Q}_1 = [4,\ 8,\ 9,\ 11;\ 0.6,\ 0.4]$ $\breve{Q}_2 = [5,\ 7,\ 12,\ 16;\ 0.7,\ 0.8]$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$	$R(\breve{Q}_1) < R(\breve{Q}_2)$

9 An Application of RFN

In this section, we will appertain the suggested ranking technique to FRA. Let's assume that there are three manufactories $C_1,\ C_2$ and C_3 to produce the components $A_1,\ A_2$ and A_3 respectively. Each module A_i is formed of three sub-modules $A_{i1},\ A_{i2}$ and A_{i3}, where $1 \le i \le 3$. Given Table 2 shows the extent of loss W_{ik} and the chances of failure R_{ik} of the sub-module A_{ik} made by manufactory C_i, where $1 \le i \le 3$ and $1 \le k \le 3$. In the accompanying, the proposed New GQSFN-CT FRA manner to manage with the FRA problem is shown as follows:

Table 2: Extent of loss and chances of failure

Manufactory	Sub-module	Extent of loss $\breve{W}_{ik}$	Chances of failure R_{ik}
$\breve{c}_1$	$\breve{A}_{11}$	$\breve{W}_{11} = (0.04,\ 0.1,\ 0.18,\ 0.23;\ 0.8,\ 0.9)$	$R_{11} = (0.11,\ 0.22,\ 0.36,\ 0.42;\ 0.9,\ 0.9)$
	$\breve{A}_{12}$	$\breve{W}_{12} = (0.58,\ 0.63,\ 0.80,\ 0.86,;\ 0.65;\ 0.7)$	$R_{12} = (0.32,\ 0.41,\ 0.58,\ 0.65;\ 0.9,\ 0.7)$
	$\breve{A}_{13}$	$\breve{W}_{13} = (0.0,\ 0.\ 0,\ 0.0,\ 0.0,\ ;\ 0.5,\ 0.6)$	$R_{13} = (0.58,\ 0.63,\ 0.80,\ 0.86;\ 0.8,\ 0.9)$
$\breve{c}_2$	$\breve{A}_{21}$	$\breve{W}_{21} = (0.04,\ 0.1,\ 0.18,\ 0.23;\ 0.8,\ 0.7)$	$R_{21} = (0.93,\ 0.98,\ 1.0,\ 1.0;\ 0.85,\ 0.8)$
	$\breve{A}_{22}$	$\breve{W}_{22} = (0.58,\ 0.63,\ 0.80,\ 0.86,;\ 1.0, 0.5)$	$R_{22} = (0.58,\ 0.63,\ 0.80,\ 0.86;\ 0.9,\ 0.9)$
	$\breve{A}_{23}$	$\breve{W}_{23} = (0.0,\ 0.\ 0,\ 0.2,\ 0.07,;\ 0.4,\ 0.8)$	$R_{23} = (0.32,\ 0.41,\ 0.58,\ 0.65;\ 0.7,\ 0.9)$
$\breve{c}_3$	$\breve{A}_{31}$	$\breve{W}_{31} = (0.04,\ 0.1,\ 0.18,\ 0.23;\ 1.0,\ 1.0)$	$R_{31} = (0.17,\ 0.22,\ 0.36,\ 0.42;\ 0.95,\ 0.95)$
	$\breve{A}_{32}$	$\breve{W}_{32} = (0.58,\ 0.63,\ 0.80,\ 0.86,;\ 0.8,\ 0.8)$	$R_{32} = (0.72,\ 0.78,\ 0.92,\ 0.97;\ 0.5,\ 0.6)$
	$\breve{A}_{33}$	$\breve{W}_{33} = (0.0,\ 0.\ 0,\ 0.07,\ 0.2,;\ 0.9,\ 0.8)$	$R_{33} = (0.58,\ 0.63\ ,\ 0.80,\ 0.86;\ 1.0,\ 1.0)$

In the final stage, we concentrate our efforts on solving that case using our new ranking GQSFN-CT methods. Now, the probability of failure of each Sub-module $\mathrm{A_i}$ created by the Manufactory $\mathrm{C_i}$ is assumed to be equal to $\tilde{\mathrm{Q}}_i$ for $\mathrm{i} = 1,\ 2,\ 3$.

$\breve{\mathcal{Q}}_1 = (0.1765,\ 0.2860,\ 0.7244,\ 1.0574;\ 0.5;\ 0.6)$
$\breve{\mathcal{Q}}_2 = (0.3221,\ 0.4949,\ 1.1392,\ 1.6373;\ 0.4;\ 0.5)$
$\breve{\mathcal{Q}}_3 = (0.3290,\ 0.4890,\ 1.1737,\ 1.7787;\ 0.5;\ 0.6)$

Using Y. Barazandeh and B. Ghazanfari

Rank $(\breve{\mathcal{Q}}_1) = 0.2167$, Rank $(\breve{\mathcal{Q}}_2) = 0.3269$ and Rank $(\breve{\mathcal{Q}}_3) = 0.3640$.

Using Shyi-Ming Chen,

Rank $(\breve{\mathcal{Q}}_1) = 0.4875$, Rank $(\breve{\mathcal{Q}}_2) = 0.8748$ and Rank $(\breve{\mathcal{Q}}_1) = 0.9248$

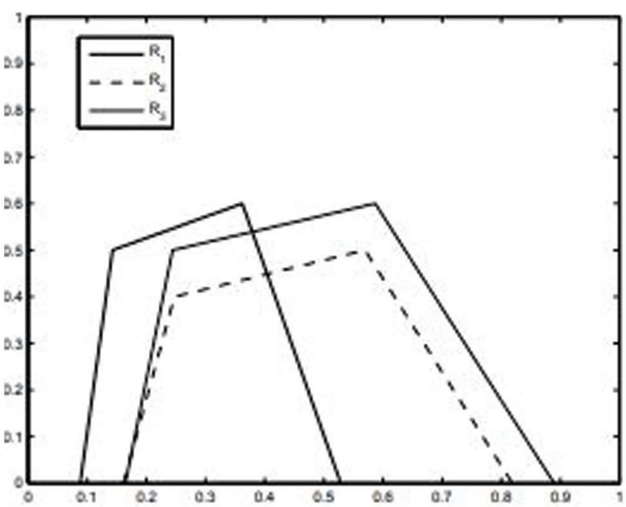
D. Stephen Dinagar

Rank $(\breve{\mathcal{Q}}_1) = 0.30859$, Rank $(\breve{\mathcal{Q}}_2) = 0.40426$ and Rank $(\breve{\mathcal{Q}}_1) = 0.51843$

Our new methods

Rank $(\breve{\mathcal{Q}}_1) = 0.16632$, Rank $(\breve{\mathcal{Q}}_2) = 0.21731$ and Rank $(\breve{\mathcal{Q}}_1) = 0.27565$

Therefore $\breve{\mathcal{Q}}_1 \prec \breve{\mathcal{Q}}_2 \prec \breve{\mathcal{Q}}_3$ this is the order in which factories' risks are ranked. $C_1,\ C_2$ and C_3 is $C_1 < C_2 < C_3$ that is the Sub-module A_3 Generated by manufactory C_3 has a highest Chances of failure then $C_2,\ C_1$ respectively. For the FN $\breve{\mathcal{Q}}_1,\ \breve{\mathcal{Q}}_2,\ \breve{\mathcal{Q}}_3$ Y. Barazandeh and B. Ghazanfari techniques, Shyi-Ming Chen techniques, D. Stephen Dinagar techniques and the proposed techniques get the identical rating order with coincides. However, the Coefficient of variation in our novel GQSFN-CT scheme has the lowest coefficient variation as compared with present techniques.



Set of standard fuzzy numbers R_1, R_2, R_3

10 Stability Analyses

From the above Table 3 shows the effective ranking technique using our new ranking methods compared to others previous existing methods this method reduce the failure time more with other methods.

Table 3: Stability analyses

Sl.no	Y. Barazandeh and B. Ghazanfari method	Shyi-Ming Chen method	D. Stephen Dinagar method	Our new method
Rank ($\breve{Q}_1$)	0.2167	0.4875	0.30859	0.16632
Rank ($\breve{Q}_2$)	0.3269	0.8748	0.40426	0.21731
Rank ($\breve{Q}_3$)	0.3640	0.9248	0.51843	0.27565

11 Conclusion

We've provided a simple technique for ranking generalized quadrilateral shape fuzzy numbers (QFN) by directly transforming them into a suggested crisp quantity. Finally, we compared the performance of our technique to that of other methods designed to achieve similar ends and offered numerical examples to back up our claims. Our innovative GQSFN-CT technique has the lowest coefficient of variance compared to other approaches. Our primary fields of focus were industrial engineering, optimization, medicine, and AI, all of which made extensive use of positive fuzzy numbers. The outcomes of the recommended procedure are verified by comparing them to those of other prevalent approaches. The GQSFN-CT methods we propose here have the potential to forecast correct numbers and might prove valuable to researchers in the future across a wide range of disciplines. This strategy makes use of a modality index that evaluates how important the central value is in comparison to the two extremes and the optimistic decision maker. In addition to producing excellent outcomes for issues with clear parameters, this technique also gives a superior answer for problem ranking. We hope that our GQSFN-CT approach will provide a more realistic way to evaluate GQSFN-CT and its potential future uses in the near future. This technique may also be applied to fuzzy numbers in the spherical shape, such as sine trigonometric spherical form fuzzy numbers.

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