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Role of Thermal Radiation Effect on Unsteady Dissipative MHD Mixed Convection of Hybrid Nanofluid over an Inclined Stretching Sheet with Chemical Reaction

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ABSTRACT: Magnetohydrodynamic (MHD) radiative chemically reactive mixed convection flow of a hybrid nanofluid (Al_2O_3-Cu/H_2O) across an inclined, porous, and stretched sheet is examined in this study, along with its unsteady heat and mass transport properties. The hybrid nanofluid's enhanced heat transfer efficiency is a major benefit in high-performance engineering applications. It is composed of two separate nanoparticles suspended in a base fluid and is chosen for its improved thermal properties. Thermal radiation, chemical reactions, a transverse magnetic field, surface stretching with time, injection or suction through the porous medium, and the effect of inclination, which introduces gravity-induced buoyancy forces, are all important physical phenomena that are taken into account in the analysis. A system of nonlinear ordinary differential equations (ODEs) is derived from the governing partial differential equations for mass, momentum, and energy by applying suitable similarity transformations. This simplifies the modeling procedure. The bvp4c solver in MATLAB is then used to numerically solve these equations. Different governing parameters modify temperature, concentration, and velocity profiles in graphs and tables. These factors include radiation intensity, chemical reaction rate, magnetic field strength, unsteadiness, suction/injection velocity, inclination angle, and nanoparticle concentration. A complex relationship between buoyancy and magnetic factors makes hybrid nanofluids better at heat transmission than regular ones. Thermal systems including cooling technologies, thermal coatings, and electronic heat management benefit from these findings.

KEYWORDS: Hybrid nanofluid; viscous dissipation; MHD; thermal radiation; chemical reaction; inclined stretching sheet

1 Introduction

The complex fluid dynamics phenomena of mixed convection arises when forced and natural convection processes combine. In heat exchangers, electronic cooling systems, and building ventilation, buoyancy-driven and externally imposed flow can significantly affect heat transfer rates and temperature distributions [1–4].

In the past few decades, nanofluids have gained attention in scientific and engineering research because of their enhanced thermal characteristics and heat transfer efficiency in energy conversion technologies, cooling units, and heat exchangers. To enhance thermophysical behavior, nanoparticles are dissolved in base



fluids. These nanoparticles can be metals, metal oxides, carbides, or carbon-based substances like nanotubes. Thermal performance has been the subject of much research since Choi and Eastman [5] first presented nanofluids. Nanofluids are intriguing possibilities for improved thermal management, as basic studies, including Choi et al. [6], demonstrated that they can raise thermal conductivity even at low concentrations. Whereas mono nanofluids only contain one nanoparticle, hybrid nanofluids contain two or more. The hybrid technique is increasingly popular because the synergistic effects of particles often lead to superior thermal performance than mono nanofluids. Because of its simplicity and capacity to capture important thermal processes, the single-phase model put forth by Tiwari and Das [7] is widely used to investigate nanofluid flow and heat transfer. Increases in nanofluid thermal conductivity due to nanoparticle volume fraction have been measured by a number of experimental and numerical studies, including Yoo et al. [8] and Kang et al. [9].

It is well-established argument that the thermal conductivity of nanomaterials is higher than that of corresponding ordinary liquids [10]. Hybrid nanofluids are a modified version of conventional or single nanofluids in which there are two or more distinct nanoparticles. Different nanoparticles are dispersed in nanomaterials to produce an ordered arrangement of liquid molecules in order to increase heat conductivity [11]. The investigations [12–14] have shown that binary hybrid nanofluids have better thermophysical characteristics than single nanofluids. Many researchs have explored the volume percentage of nanoparticles [15], particle size [16], base fluids [17], and particle type [18], as well as comparing heat conductivity and heat transfer coefficients of various nanomaterials. Kumar et al. [19] considered an Al_2O_3 nanofluid and evaluated its thermal conductivity. When compared to water, their results demonstrated an improvement in convective heat transfer up to 28%. Alatawi [20] explored nanofluid analysis for an enhancement of heat transfer efficiency in a cavity by employing multi-walled carbon nanotube. Nasrin et al. [21] developed a regression model of nanoliquid for thermal enhancement in the interconnected regime.

Thermal science continues to research nanofluid flow on stretched surfaces to improve heat transfer for modern engineering. The inclined stretching sheet is useful in industrial processes like polymer extrusion, metal forming, and surface coating, as noted by Sakiadis [22], Magyari and Keller [23], and Bachok et al. [24]. When physical events are added, such configurations' fluid dynamics become more complex. Magnetic fields, Joule heating, viscous dissipation, and surface mass transfer (by suction or injection) are important. These mechanisms change the boundary layer's velocity and temperature distributions and determine thermal system efficiency. Thus, nanofluid models must include them to accurately simulate real-world heat transport scenarios and optimise industrial processes (Mahdy [25]). Magnetohydrodynamics (MHD), the study of electrically conducting fluids in magnetic fields, is important in this field. The interaction creates Lorentz forces, which oppose fluid motion and alter flow structure and thermal gradients (Alfvén [26]; Kabeel et al. [27]). Butt et al. [28] conceptualisation of a novel design intelligent computing paradigm based on artificial neural networks by utilizing radial basis function to analyse magnetohydrodynamic Williamson nanofluid 2-dimensional flow along a stretchy sheet under the impact of chemical reaction and thermal radiation in a porous medium. These effects contribute to nanofluid systems' complicated thermal and hydrodynamic behavior and should be studied to develop next-generation thermal technologies.

Morphology of scattered nanoparticles affects nanofluid thermal performance but is generally overlooked. The effective thermal conductivity and viscous behavior of hybrid nanofluids depend on nanoparticle form and geometry, whether spherical, cylindrical, platelet-like, or irregular. Non-spherical particles can create more efficient thermal routes and change flow resistance, improving heat transfer (Kumar et al. [29], Bakar et al. [30], Lavanya et al. [31]). These shape-dependent properties emphasize the importance of nanoparticle geometry selection for constructing nanofluids for electronic device cooling, thermal energy storage, and high-efficiency heat exchangers. Beyond nanoparticle shape, surface suction at a stretching sheet's edge modulates boundary layer behaviour. Many engineering systems use suction to postpone

boundary layer separation, reduce skin friction, and increase convective heat transfer. Suction reduces momentum and thermal boundary layer thicknesses by pushing fluid toward the surface, typically resulting in steeper thermal gradients and higher Nusselt numbers, which facilitate heat transfer (Elbashbeshy [32], Anjali Devi and Devi [33]). Additional temperature regulation is possible with suction in hybrid nanofluid flow across sloped stretching surfaces. This is useful in advanced applications, including jet impingement cooling, aerodynamic surface control, and adaptive thermal management. Thus, optimizing current heat transfer systems requires assessing suction in such settings. Upreti et al. [34], Mallesh et al. [35], Sen et al. [36], and Shaw et al. [37] show the importance of studying hybrid nanofluid thermal boundary layer phenomena. These studies emphasize the need to study nanoparticle morphology, boundary layer control mechanisms, and external factors to improve thermal efficiency in modern engineering.

Heat transfer increase and heat loss reduction in electrically conducted hybrid nanofluids rely heavily on thermal radiation. Heat radiation impacts mixed convection, making it important in physics and engineering, especially in building, gas turbines, electronics, and aerospace engineering. Investigators initiated research on a variety of hybrid nanofluid flows under different conditions with thermal radiation. Some attempts about studies in this direction can be seen through the attempts [38–42]. The effects of chemical reactions and thermal radiation on hybrid nanofluids have been investigated in a number of research [43–46].

This study analyses unsteady magnetohydrodynamic (MHD) flow of an Al_2O_3 –Cu/water hybrid nanofluid over an inclined, permeable surface. The study focuses on Joule heating, viscous dissipation, surface suction, and nanoparticle shape factor. The single-phase technique described by Tiwari and Das [7] is used to model the hybrid nanofluid's thermal and hydrodynamic dynamics, capturing nanoparticle dispersion's thermal enhancements. To appropriately characterize the hybrid suspension's thermophysical properties, the Brinkman model [47] estimates effective viscosity, and the Hamilton-Crosser correlation [48] accounts for the particle shape's effect on heat conductivity. These models show the hybrid nanofluid's response to flow and thermal conditions more realistically. Similarity transformations convert the partial differential equations regulating mass, motion, and energy conservation into nonlinear ordinary differential equations. Numerically solving these equations under proper boundary conditions yields velocity, temperature, and concentration, skin friction, Nusselt, and Sherwood number profiles. This study carefully evaluates how unsteadiness, magnetic field intensity, suction rate, nanoparticle form, and inclination angle affect system flow and heat transfer. This research has significant implications for designing and optimizing advanced thermal systems, such as next-generation cooling, thermal energy storage, and high-efficiency heat exchangers which are essential in modern energy, electronics, and industry.

2 Mathematical Model

A two-dimensional unsteady MHD boundary layer flow of a hybrid nanofluid along an inclined stretching sheet has been considered. The flow configuration is given in Figs. 1 and 2. The sheet is making an angle γ with the vertical and the flow is caused by the stretching velocity $U_w(x, t) = ax(1 - ct)$ of the sheet. The x -axis is taken along the sheet and the y -axis is normal to it. A uniform external magnetic field of strength B_0^2 is taken along the y -axis. To control the boundary layer separation, it is assumed that the sheet is permeable, so that suction/injection of fluid through the sheet may be possible. The Joule heating effect due to the external magnetic field is also taken into account. It is also assumed that in the flow field, the velocity and temperature difference $(T_w - T_\infty)$ are large enough so that the viscous dissipation effect can be considered in the problem. Since the present study aims to show the flow and heat transfer characteristics of hybrid nanofluid (Al_2O_3 , Cu) nanoparticles and water base fluid) to deal with the nanoparticles Tiwari-Das model has been used.

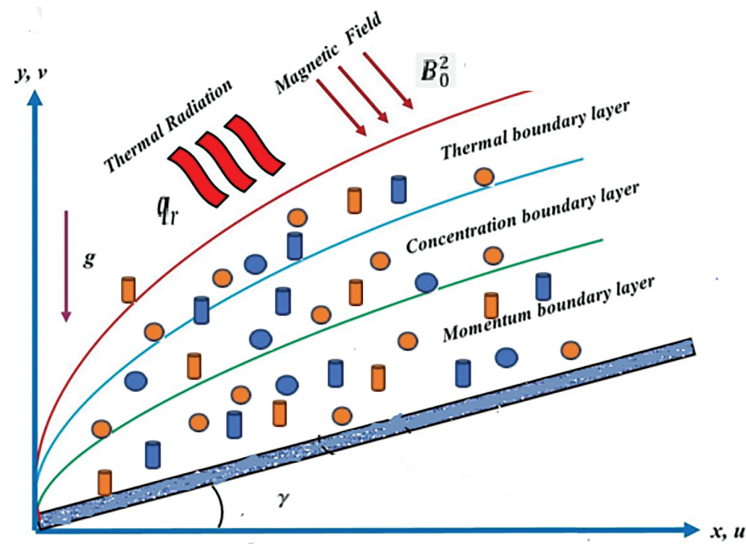


Figure 1: Flow diagram

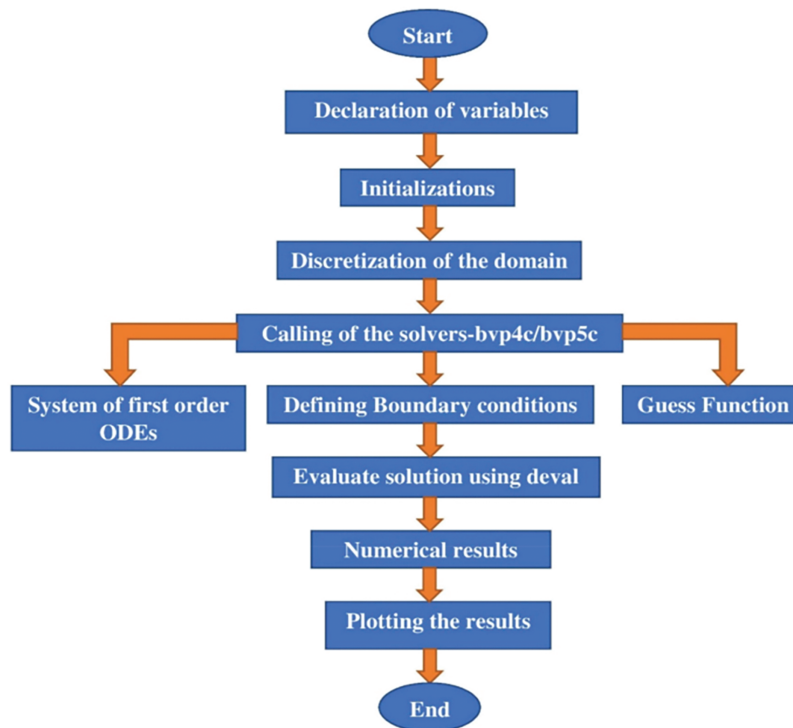


Figure 2: Flow chart bvp4c technique

Based on the assumptions outlined in the previous section, the governing equations of the system namely the continuity, momentum, and energy equations are given as follows (Ref. Mahdy et al. [25]).

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 u + g \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} (T - T_0) \cos \gamma \quad (2)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_{hnf}}{(\rho C_p)_{hnf}} B_0^2 u^2 - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial q_r}{\partial y} + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

Concentration equation:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_r (C - C_\infty) \quad (4)$$

here Radiation flux q_r is specified by using Roseland approximation as

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}.$$

Now expanding Taylor series about T_∞

$$T^4 \approx 4T_\infty^3 - 3T_\infty^4$$

Boundary Conditions:

$$\begin{aligned} u = U_w(x, t) &= \frac{ax}{1-ct}, v = v_w(t) = \frac{v_0}{\sqrt{1-ct}}, T = T_w(x, t) = T_\infty + \frac{bx}{(1-ct)^2}, \\ C = C_w(x, t) &= \frac{dx}{(1-ct)^2} \quad \text{at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty &\quad \text{as } y \rightarrow \infty \end{aligned} \quad (5)$$

where the symbols have their usual meanings and have been mentioned in the nomenclature. For the computation of the effective value of measures of hybrid nanofluid, a number of models are available. One can apply these models by choosing an appropriate solid volume fraction of nanoparticles in the base fluid and the shape factor of nanoparticles.

Similarity Transformations:

$$\eta = \left(\frac{a}{v_f(1-ct)} \right)^{1/2} y, \quad \psi = \left(\frac{av_f}{(1-ct)} \right)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

where ψ is the stream function such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (6)$$

here, for effective dynamic viscosity and thermal conductivity, Brinkman [47] and Hamilton-Crosser [48] models have been used. Eq. (5) shows the expressions for the effective values of various measures (Devi & Devi [33]). Thermal characteristics of the base fluid and different nano particles are given in Table 1.

$$\begin{aligned}\mu_{hnf} &= \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}, \quad \rho_{hnf} = \rho_f \left[(1-\phi_2) \left\{ (1-\phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f} \right\} \right] + \phi_2 \rho_{s2} \\ (\rho C_p)_{hnf} &= \left[(1-\phi_2) \left\{ (1-\phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} \right\} \right] + \phi_2 (\rho C_p)_{s2} \\ (\rho \beta)_{hnf} &= \left[(1-\phi_2) \left\{ (1-\phi_1) (\rho \beta)_f + \phi_1 (\rho \beta)_{s1} \right\} \right] + \phi_2 (\rho \beta)_{s2} \\ k_{bf} &= k_f \left[\frac{k_{s1} + (s-1)k_f - (s-1)\phi_1(k_f - k_{s1})}{k_{s1} + (s-1)k_f + \phi_1(k_f - k_{s1})} \right], \quad k_{hnf} = k_{bf} \left[\frac{k_{s2} + (s-1)k_{bf} - (s-1)\phi_2(k_{bf} - k_{s2})}{k_{s2} + (s-1)k_{bf} + \phi_2(k_{bf} - k_{s2})} \right], \\ \frac{\sigma_{hnf}}{\sigma_f} &= 1 + \frac{3(s-1)\left(\frac{\sigma_{s1}}{\sigma_f} - 1\right)\phi_1}{\left(\frac{\sigma_{s1}}{\sigma_f} + 2\right) - (s-1)\left(\frac{\sigma_{s1}}{\sigma_f} - 1\right)\phi_1} + \frac{3(s-1)\left(\frac{\sigma_{s2}}{\sigma_f} - 1\right)\phi_2}{\left(\frac{\sigma_{s2}}{\sigma_f} + 2\right) - (s-1)\left(\frac{\sigma_{s2}}{\sigma_f} - 1\right)\phi_2};\end{aligned}\quad (7)$$

Table 1: Thermophysical properties of nanoparticles and base fluid water (H_2O)

Properties	Al_2O_3	Cu	H_2O
ρ (kg/m ³)	3970	8933.0	997.0
C_p (J/kg · K)	765	385.0	4180.0
k (W/m · K)	40.0	400.0	0.6071
σ (S/m)	5.96×10^6	3.69×10^7	5.5×10^{-6}
$\beta \times 10^5$ (K ⁻¹)	0.85	1.67	21.0

The main objective of this study is to find the shear stress and heat transfer at the sheet along with the velocity and temperature profiles. The expressions for shear stress (τ_w), flux (q_w) and non-dimensional form of mass transfer (q_m) at the wall are given by (Sandeep et al. [49])

$$\tau_w = \mu_f \frac{\partial u}{\partial y}_{y=0}, \quad q_w = -k_f \frac{\partial T}{\partial y}_{y=0}, \quad q_m = -\frac{\partial C}{\partial y}_{y=0}. \quad (8)$$

By Using Eq. (6) in Eqs. (1)–(4), the equation of continuity identically satisfied which validates the corrections of the transformation used. Eqs. (2) and (3) take the following form:

$$D_1 f'''' - D_2 \left(A \left(f'(\eta) + \frac{\eta}{2} f''(\eta) \right) + (f'(\eta))^2 - f(\eta) f''(\eta) \right) + D_3 \lambda \cos \gamma \theta(\eta) - D_4 M f'(\eta) = 0 \quad (9)$$

$$\frac{(D_5 + (4/3) Rd)}{Pr} \theta''(\eta) + D_4 M Ec (f'(\eta))^2 + \left(\frac{D_1 D_6}{D_2} \right) Ec f''^2 - D_6 \left(A \frac{\eta}{2} \theta' + 2A\theta + f'\theta - f\theta' \right) = 0 \quad (10)$$

$$\frac{\phi''}{Sc} - (2A + f' + Rc) \phi + \left(\frac{\eta A}{2} - f \right) \phi' = 0 \quad (11)$$

Transformed boundary conditions are:

$$\begin{aligned} f'(\eta) = 1, f(\eta) = S, \theta(\eta) = 1, \phi(\eta) = 1 \text{ at } \eta = 0; \\ f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \text{ as } \eta = \infty. \end{aligned} \quad (12)$$

Used non dimensional parameters are defined as:

The mixed convection parameter $\lambda = \frac{Gr_x}{Re_x^2} = \frac{gb\beta_f}{a^2}$, The unsteadiness parameter $A = \frac{c}{a}$, The local Grashof number $Gr_x = \frac{g\beta_f(T_w - T_\infty)x^3}{\nu^2}$, The local Reynolds number $Re_x = \frac{U_w(x,t)}{\nu}$, The Magnetic parameter $M = \frac{\sigma_f B_0^2}{(1 - ct)a\rho_f}$, The Eckert number $Ec = \frac{U_w^2}{C_p(T_w - T_\infty)}$, The Schmidt number $Sc = \frac{\nu_f}{D_m}$, The Chemical reaction $Rc = \frac{K_r}{a(1 - ct)}$, The Suction/Injection parameter $S = \frac{v_0}{\sqrt{a\nu_f}}$ where $S < 0$ for injection, $S > 0$ for Suction.

Where,

$$\begin{aligned} D_1 &= \frac{1}{(1 - \phi_1)^{25} (1 - \phi_2)^{25}}, & D_2 &= \left[(1 - \phi_2) \left\{ (1 - \phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f} \right\} \right] + \phi_2 \rho_{s2}, \\ D_3 &= \left[(1 - \phi_2) \left\{ (1 - \phi_1) (\rho\beta)_f + \phi_1 (\rho\beta)_{s1} \right\} \right] + \phi_2 (\rho\beta)_{s2}, & D_4 &= \frac{\sigma_{hnf}}{\sigma_f}, \\ D_5 &= \frac{k_{hnf}}{k_f}, & D_6 &= \left[(1 - \phi_2) \left\{ (1 - \phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} \right\} \right] + \phi_2 (\rho C_p)_{s2}. \end{aligned}$$

The dimensionless form of shear and rate of heat transfer at the surface, known as the skin friction coefficient (C_f), Nusselt number (Nu_x) and Sherwood number Sh_x respectively are given by (Sandeep et al. [49])

$$C_f = Re_x^{-1/2} D_1 f''(0), \quad Nu_x = -Re_x^{1/2} \left(D_5 + \frac{4Rd}{3} \right) \theta'(0) \text{ and } Sh_x = -Re_x^{1/2} \phi'(0).$$

Numerical Procedure:

Due to the nonlinear and coupled nature of Eqs. (9)–(11) along with the associated boundary conditions outlined in Eq. (12), obtaining an analytical solution is highly challenging. To address this complexity, the numerical solver bvp4c, developed by Kierzenka and Shampine [50], is employed. This built-in MATLAB function is specifically designed to handle boundary value problems using a R-K shooting method, which ensures high accuracy and stability in solving stiff systems. Since bvp4c operates on systems of first-order ordinary differential equations, the original higher-order Eqs. (9)–(11) are transformed into an equivalent system of first-order equations using appropriate substitutions, as detailed below:

$f = y_1, \theta = y_4$, and $\phi = y_6$, Then the system of ordinary differential equations becomes

$$y_1' = y_2, \quad y_2' = y_3,$$

$$y_3' = \frac{1}{D_1} \left(D_2 \left(A \left(y_2 + \frac{\eta}{2} y_3 \right) + y_2^2 - y_1 y_3 \right) - D_3 \lambda \cos \gamma y_4 + D_4 M y_2 \right) \quad (13)$$

$$y_4' = y_5$$

$$y_5' = \frac{Pr}{\left(D_5 + \frac{4Rd}{3} \right)} \left(D_6 \left(A \frac{\eta}{2} y_5 + 2A y_4 + y_2 y_4 - y_1 y_5 \right) - D_4 M Ec y_3^2 - \left(\frac{D_1 D_6}{D_2} \right) Ec y_4^2 \right) \quad (14)$$

$$y_6' = y_7$$

$$y_7' = Sc \left((2A + f' + Rc) \phi + \left(\frac{\eta A}{2} - f \right) \phi' \right) \quad (15)$$

The boundary conditions are

$$\begin{aligned} y_1(0) = S, y_2(0) = 1, y_4(0) = 1, y_6(0) = 1; \\ y_2(\infty) = 0, y_4(\infty) = 0, y_6(\infty) = 0. \end{aligned} \quad (16)$$

3 Results and Discussion

There has been a comprehensive investigation of the impact of essential influencing parameters on the distributions of concentration, temperature, and velocity, as well as on the skin friction coefficient, Nusselt, and Sherwood number by considering both spherical and cylindrical nanoparticle shapes. Figs. 3–15 depict the corresponding results, while Table 2 provides a quantitative summary. The intricate relationship between boundary layer behavior alongside external physical factors like surface suction, chemical reaction, magnetic field intensity, inclined angle, mixed convection parameter, thermal radiation, mixed convection parameter, and unsteady parameter is illustrated graphically in the results.

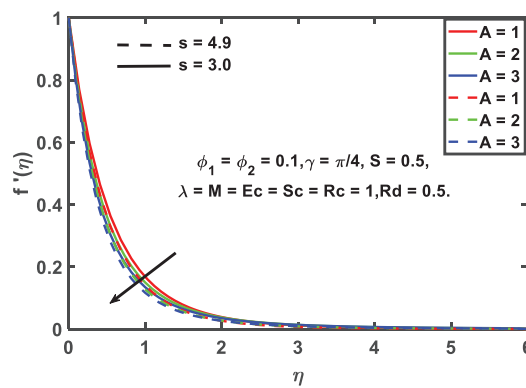


Figure 3: Velocity profile for unsteady parameter A

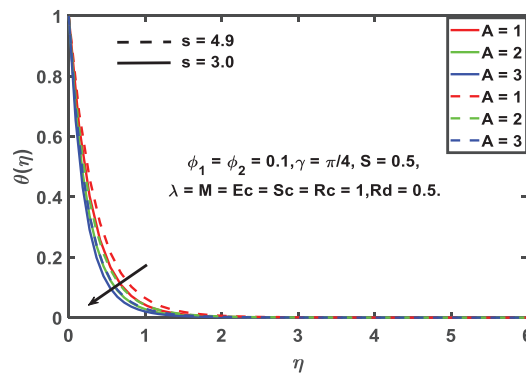


Figure 4: Temperature profile for unsteady parameter A

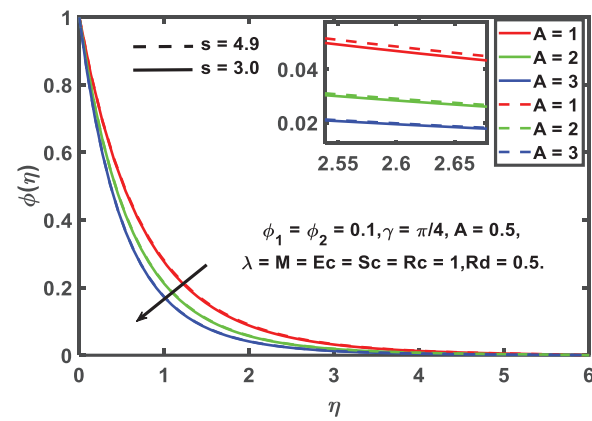


Figure 5: Concentration profile for unsteady parameter A

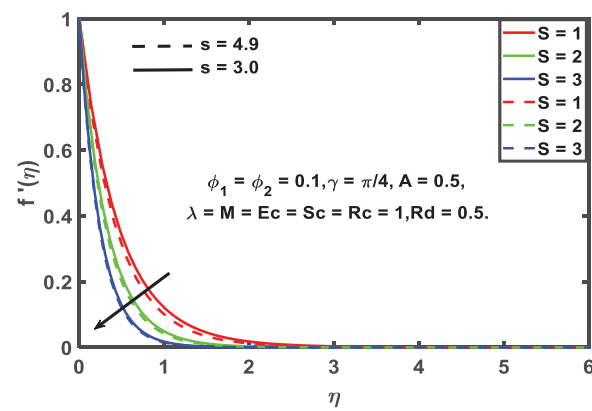


Figure 6: Velocity profile for Suction S

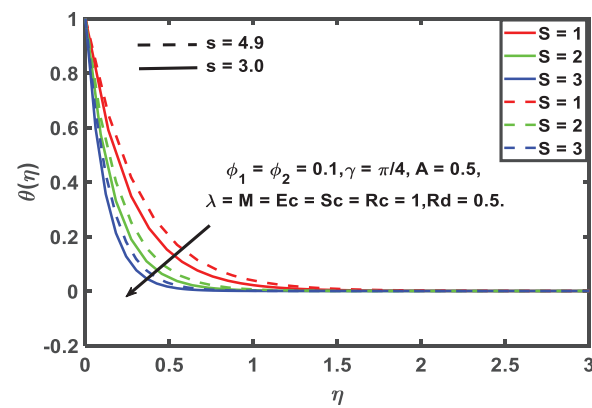


Figure 7: Temperature profile for Suction S

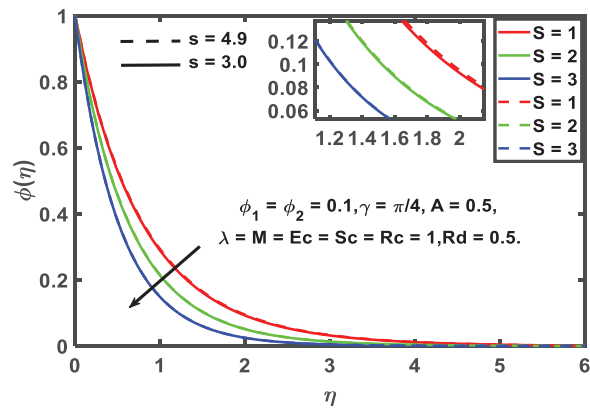


Figure 8: Concentration profile for Suction S

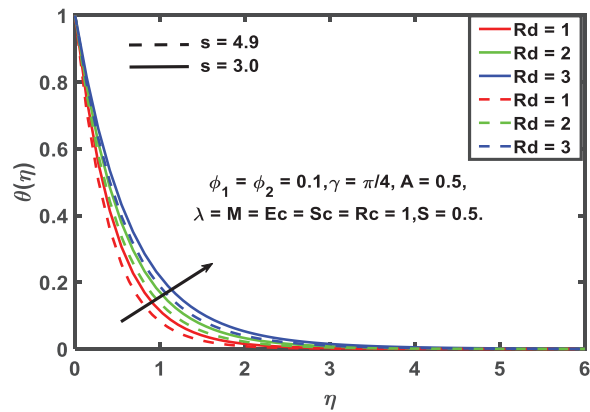


Figure 9: Temperature profile for Thermal radiation Rd

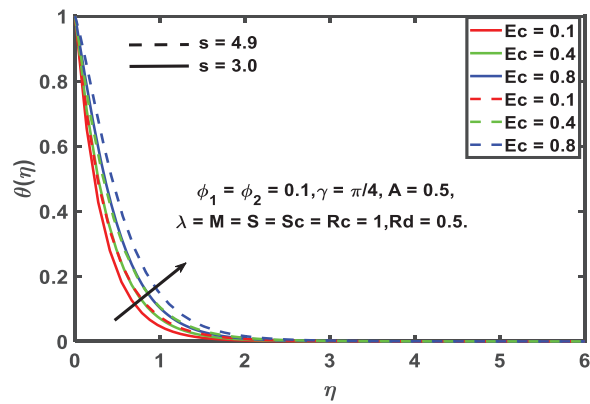


Figure 10: Temperature profile for Eckert number Ec

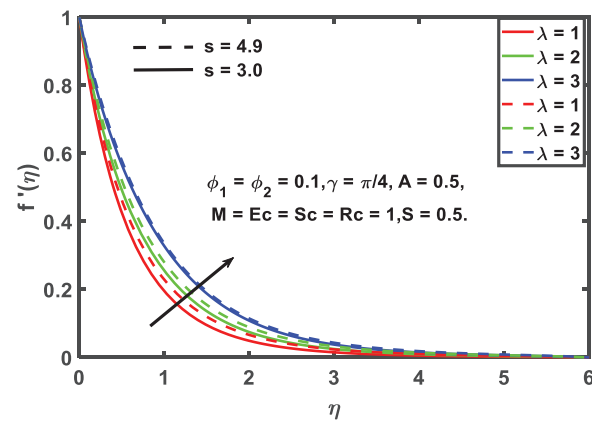


Figure 11: Velocity profile for Mixed convection parameter λ

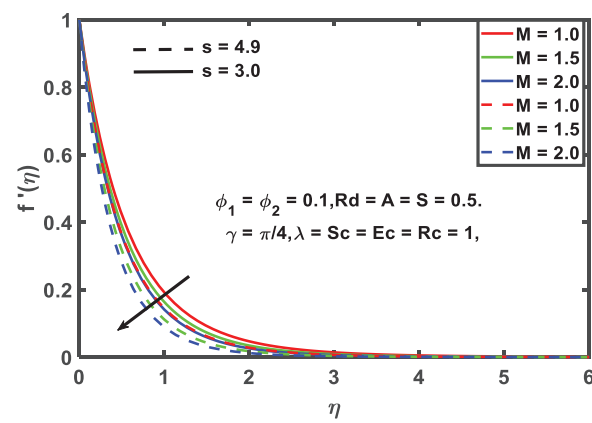


Figure 12: Velocity profile for Magnetic parameter M

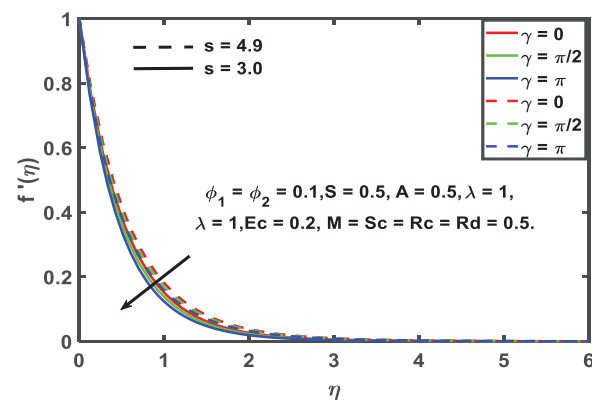


Figure 13: Velocity profile for aligned angle γ

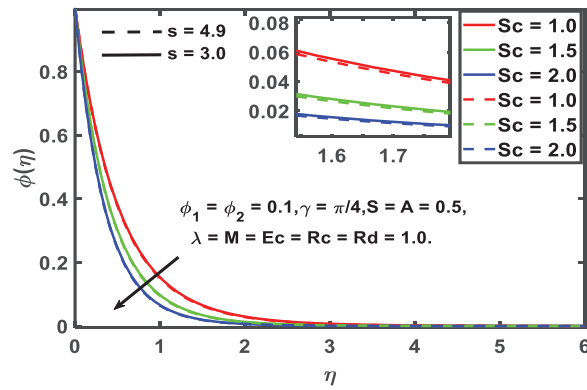


Figure 14: Concentration profile for Schmidt number Sc

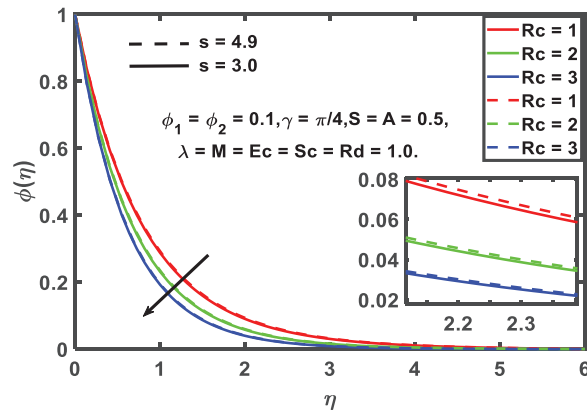


Figure 15: Concentration profile for Chemical Reaction Rc

Table 2: Skin friction coefficient C_f , Nusselt number Nu_x and Sherwood number Sh_x impact on Al_2O_3-Cu/H_2O hybrid nanofluid

		Spherical nanoparticles ($s = 3$)			Cylindrical nanoparticles ($s = 4.9$)		
		$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
<i>M</i>	0.10	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.11	-2.135986	6.299894	0.320894	-2.228934	10.042973	0.310306
	0.12	-2.141999	6.337043	0.321043	-2.234740	10.107983	0.313013
<i>Rd</i>	0.1	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.3	-2.129021	6.380663	0.310158	-2.222311	9.886388	0.307615
	0.5	-2.128139	6.503447	0.310191	-2.221546	9.825083	0.307650
<i>Ec</i>	0.1	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.2	-2.128685	5.522703	0.310167	-2.221769	8.625449	0.307629
	0.3	-2.127405	4.784117	0.310209	-2.220418	7.275339	0.307677
<i>S</i>	0.10	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.11	-2.140785	6.317973	0.310371	-2.233918	10.057069	0.307842

(Continued)

Table 2 (continued)

		Spherical nanoparticles ($s = 3$)			Cylindrical nanoparticles ($s = 4.9$)		
		$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
Sc	0.12	-2.151655	6.373780	0.310618	-2.244765	10.137095	0.308104
	0.10	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.11	-2.129967	6.262533	0.323532	-2.223122	9.977537	0.320775
	0.12	-2.129967	6.262533	0.336785	-2.223122	9.977537	0.333823
Rc	0.10	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581
	0.11	-2.129967	6.262533	0.311730	-2.223122	9.977537	0.309194
	0.12	-2.129967	6.262533	0.313328	-2.223122	9.977537	0.310800
λ	0.2	-2.114240	6.268602	0.310333	-2.205465	9.990163	0.307860
	0.3	-2.098543	6.274629	0.310539	-2.187855	10.002673	0.308137
	0.4	-2.082877	6.280614	0.310744	-2.170291	10.015070	0.308412
A	0.01	-2.075563	5.916233	0.284930	-2.170546	9.368325	0.282212
	0.02	-2.081629	5.955643	0.287807	-2.176410	9.437910	0.285109
	0.03	-2.087688	5.994812	0.290665	-2.182270	9.507004	0.287986
γ	0	-2.123449	6.265052	0.310212	-2.215802	9.982781	0.307697
	$\pi/6$	-2.126430	6.263900	0.310172	-2.219149	9.980385	0.307644
	$\pi/4$	-2.129967	6.262533	0.310126	-2.223122	9.977537	0.307581

According illustrated by Figs. 3–5, the velocity, temperature, and concentration properties of the hybrid nanofluid are affected by the unsteadiness parameter, which takes into account both spherical and cylindrical nanoparticle shapes. Fluid flow, temperature, and concentration all go down as the unsteadiness parameter values go up in the figures, suggesting that thermal diffusion is suppressed by transient effects. Pulsed heating in manufacturing and dynamic cooling cycles in electronics are two examples of unstable industrial processes where this is of paramount importance. Using two form variables that correspond to spherical and cylindrical nanoparticles, the impact of nanoparticle shape is additionally examined. Because of their larger surface area and the drag forces acting upon them, cylindrical nanoparticles have lower velocity profiles, whereas spherical nanoparticles display a more consistent distribution of temperatures. These results are directly applicable to the development of hybrid nanofluids for bio-thermal applications, microelectronic thermal management, and enhanced cooling fluids for the aerospace and automotive industries, where the shape of nanoparticles is an important factor in thermal performance.

Figs. 6–8 show a clear trend: for both spherical and cylindrical nanoparticle forms, a rise in the suction parameter significantly lowers the velocity, temperature, and concentration profiles. The removal of fluid from the boundary layer causes the layer to become thinner and the velocity of nanofluids close to the surface to decrease, leading to this behavior. Due to reduced mass and thermal energy close to the wall, convective transport is inhibited and thermal gradients are reduced as suction becomes stronger. The total temperature distribution is reduced due to improved suction, which restricts thermal retention near the surface, because the wall temperature is higher than the ambient fluid temperature. In thermal management applications, where manipulating the boundary layer is critical, this outcome is practically significant. To reduce drag and control surface heat flux, suction is commonly used in thermal insulation systems and aerodynamic surface

treatments. The results of this study lend credence to the idea that surface suction could be a useful tool for engineering systems to manage hydrodynamic and thermal boundary layers, and they provide evidence in favor of such implementations.

This is demonstrated in Fig. 9, which shows that an increase in the thermal radiation parameter (R_d) results in an increase in the distribution of fluid temperature. This effect is to be anticipated due to the fact that thermal radiation functions as an additional source of heat, hence boosting the total mechanism of heat transmission. The thermal boundary layer is the zone that is characterized by the presence of temperature gradients between the heated surface and the fluid that comprises the surrounding environment. An rise in R_d results in a greater amount of radiative energy penetrating deeper into the fluid, which in turn causes the boundary layer to expand. Because of this, the temperature of the diffusion increases, and the heated zone that is contained within the convective region of the convective flow expands.

Fig. 10 examines the influence of the Eckert number, which measures the speed of viscous dissipation, or the transformation of kinetic energy into thermal energy resulting from internal friction within the fluid, on temperature profiles. With an increase in the Eckert number, temperature profiles demonstrate upward trends. This improvement results from increased internal heat generation, leading to a thicker thermal boundary layer and higher fluid temperatures. The implications of these findings are relevant to engineering systems that exhibit high shear rates or rapid fluid motion, including high-speed journal bearings, microchannel cooling arrays, and viscous-dominated microfluidic devices. Dissipative heating is a critical factor in these contexts, affecting both thermal performance and the mechanical stability of the flow regime. However, this temperature increases results in a decrease in the Nusselt number, signifying a reduction in the efficiency of heat transfer from the surface to the fluid.

Fig. 11 demonstrates the beneficial effect of the mixed convection parameter on the velocity field, highlighting the contribution of buoyancy forces to the flow enhancement. With an increase in this parameter, the influence of the thermal gradient becomes more significant, facilitating fluid acceleration. Mixed convection mechanisms are critical in solar collectors, electronic device cooling, and HVAC systems, where accurate control of natural and forced convection is necessary.

Fig. 12 illustrates the impact of the magnetic field parameter on the velocity characteristics of hybrid nanofluid, considering various nanoparticle shape factors, demonstrating the typical behavior of electrically conducting fluids in magnetic fields. An improvement in the magnetic field number is associated with a reduction in the significant velocity segment u . This is because u , the important velocity component, decreases as the magnetic field number increases. As expected initially, a gradual decrease in velocity can be observed as a result of raising the magnetic field number M . This result follows from the standard interpretation, which states that the capacitive field immediately preceding the electrically manipulated liquid's ascent to the opposite force category appears to supply the Lorentz strength. The observation of this phenomenon is what prompted this change. After being hit by the aforementioned strengths, the indicators of the liquid streamlet eventually reached a point where they stalled in the thrust boundary stratum thicknesses. This happened as the fluid stream's velocity decreased. This behavior is particularly significant in magnetohydrodynamic (MHD) applications, where magnetic fields regulate flow and temperature in electrically conductive fluids. Examples include thermal management in nuclear reactors, magnetic guidance in biomedical fluid systems, and temperature control in metallurgical casting or smelting operations. Analyzing the thermal response to different magnetic field intensities is essential for enhancing performance and maintaining stability in these systems.

For both spherical and cylindrical nanoparticle geometries, Fig. 13 displays the way the aligned angle (γ) affects the velocity field. The distinctive drop in velocity away from the surface, observed in boundary layer flows, is indicated by the fact that the fluid velocity decreases with increasing aligned angle in both

instances. Cylindrical nanoparticles with greater inclination angles ($\gamma = \pi$) improve fluid mobility close to the wall by increasing velocity magnitudes.

Fig. 14 provides a description of the behavior of the Schmidt number (Sc) on the concentration contours. There is a variable known as the Schmidt number Sc that is utilized for the purpose of describing the ratio of mass diffusivity to speed diffusivity. It provides a specification of the equivalent relevance of speed and mass transmission by utilizing dispersion in the hydrodynamic boundary layer. This is accomplished by the utilization of water. The mass dispersion of the fluid will decrease in a manner that is proportional to the decrease in the Schmidt numeral. Fig. 15 presents an investigation of the impact that a chemical reaction has on concentration silhouettes. This analysis is illustrated in the figure. The scope of this inquiry includes the discussion of a chemical response that is destructive ($Kr > 0$). With an increase in the number of chemical processes, the diffusions of concentration become more constrained. The chemistry is accompanied by a physical manifestation that is distinguished by a significant number of disturbances with the intention of causing damage. The concentration distributions of fluid flow are reduced as a result of increased molecular mobility, which leads to an increase in conveyance spectacles. This is the consequence of the increased molecular mobility.

The numerical results for skin friction, Sherwood number, and Nusselt number are shown in Table 2 for spherical and cylindrical nanoparticles of hybrid nanofluids. Increasing the parameters Magnetic field parameter, diminishes the drag coefficient. Conversely, enhancing the parameter missed convective parameter raises the drag coefficient. Higher values of viscous dissipation result in reduced the heat transfer rate (Nusselt number), while larger values of Rd , S , λ , A and lead to an increase in rate of heat transfer. The mass rate of transfer is enhanced by the parameters Sc , and Rc (Fig. 16).

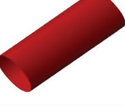
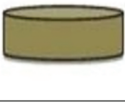
Particle Form	Shapes	Shape Factor	Particle Form	Shapes	Shape Factor
Sphere		3	Tetrahedron		4.06
Bricks		3.7	Hexahedron		3.72
Cylinder		4.9	Octahedron		1.18
Platelet		5.7	Blade		8.9

Figure 16: The appearance of nanoparticles and their shape factor Ali Akbar et al. [51]

Validation of Procedure: The correctness of bvp4c code for the present problem is verified by comparing the values of $-f''(0)$ and $\theta'(0)$ from existing results of Mahdy et al. [25] in Table 3. It is represented in below and there is a good agreement between the results.

Table 3: Comparison of numerical values of $-f''(0)$, and $-\theta'(0)$ for various values of ϕ_2 when $S = M = Ec = break\phi_1 = 0$, $A = 0.5$, $\lambda = 0.5$, $Pr = 6.5$

ϕ_2	s	Present Study		Mahdy et al. [25]	
		$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.00	3.0	1.07443	3.783073	1.07432	3.78318
0.05	3.0	1.20983	3.470990	1.20983	3.47099
0.10	3.0	1.29466	3.197600	1.29466	3.19760

Closing remarks:

An in-depth analysis of the coupled effects of chemical, thermal radiation, viscous dissipation, Joule heating, surface suction, and nanoparticle geometry is described in this study, which examines the unsteady magnetohydrodynamic (MHD) flow of a water-based hybrid nanofluid containing Al_2O_3 and Cu nanoparticles across an inclined permeable stretching sheet. To evaluate the effects of spherical and cylindrical nanoparticle shapes on concentration, temperature, and flow characteristics, the hybrid nanofluid was modeled using a one-phase technique. The impact of important physical parameters on the velocity field, temperature distribution, Sherwood number, skin friction coefficient, and Nusselt number was investigated in a thorough parametric study. These parameters include chemical, thermal, radiation, magnetic field intensity, suction rate, Eckert number, and particle morphology. In order to compare the hybrid nanofluid's thermal efficiency, it was also tested under the same flow circumstances as pure water, Al_2O_3 -water, and Cu -water nanofluids. This is a synopsis of the study's main points:

- The increase in the magnetic parameter and Eckert number leads to higher fluid temperatures, attributed to enhanced Joule heating and viscous dissipation, which both contribute to increased internal energy generation and thermal diffusion.
- An increase in suction strength and magnetic field intensity results in a significant decrease in fluid velocity and wall shear stress, due to enhanced resistance forces within the boundary layer that obstruct near-wall fluid motion.
- The temperature profile is elevated by an increase in the parameters Ec and Rd , whereas it diminishes with a decrease in the parameters A and S .
- Cylindrical nanoparticles exhibit superior thermal conduction compared to spherical nanoparticles, leading to enhanced temperature gradients and more effective heat transport within the fluid domain.
- Among all tested nanofluid configurations, the Al_2O_3 - Cu /water hybrid nanofluid with cylindrical nanoparticles demonstrates the lowest skin friction and Nusselt number, signifying increased flow resistance and diminished convective heat transfer under identical operating conditions.

Future work: Future goals include incorporating non-Newtonian fluid models and complex nanoparticle interactions, exploring time-dependent magnetic and non-linear radiative effects, and extending the model to three-dimensional geometries with experimental validation for applications in industrial solar energy and thermal management systems.

Applications: This research enhances the understanding of hybrid nanofluid dynamics in magnetohydrodynamic flow environments and offers insights for optimizing thermal management systems. The findings have significant implications for the design of advanced cooling technologies, such as high-efficiency heat exchangers, intelligent energy storage systems, and thermal management devices in electronics, aerospace, and process industries.

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Availability of Data and Materials: Requests for data can be made at any time.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

Nomenclature

f	Base fluid
∞	Free Stream Conditions
w	Conditions at the wall
M	Magnetic parameter
Re_x	Reynold approximation
Ec	Eckert Number
ϕ_1	First kind Nanoparticle volume fraction (Al_2O_3)
ϕ_2	Second kind Nanoparticle volume fraction (Cu)
S	Suction/Injection
Rc	Chemical reaction
ν	Kinematic viscosity (m^2/s)
T_∞	Free stream heat
$\psi(x, y)$	Stream function (kg/ms)
C_p	Specific heat
B_0^2	Magnetic field
Sc	Schmidt number
Rd	Thermal radiation
s	Nanoparticle shape factor
v_w	Suction/injection velocity
a, b, c	Constants
σ	Electrical Conductivity
η	Similarity Variable
x, y	Cartesian coordinates
A	Unsteady parameter
Gr_x	Local Grashof number
Pr	Prandtl Number
k	Thermal conductivity
$(\rho C_p)_f Sc$	Heat capacitance of the base fluid (J/K)
u, v	Velocity components along x and y axes resp. (m/s)
U_w	Velocity of stretching the sheet
U_∞	Ambient constant of velocity
γ	Angle of inclination from x -axis
Rc	Chemical reaction parameter
T	Temperature (K)

μ	Dynamic viscosity (Pa s)
ρ	Density (kg/m ³)
τ_w	Shear stress (Pa)
λ	Mixed convection parameter
g	Gravitational acceleration
β_t	Thermal expansion coefficient
q_w	Heat flux
D_m	Mass diffusivity

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