Fluid Dynamics & Materials Processing

Tech Science Press

Doi:10.32604/fdmp.2025.062000

ARTICLE

Effect of Libration on Fluid Flow and Granular Medium Dynamics in a Rotating Cylindrical Annulus

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Received: 08 December 2024; Accepted: 22 January 2025; Published: 30 May 2025

ABSTRACT: The dynamics of fluid and non-buoyant particles in a librating horizontal annulus is studied experimentally. In the absence of librations, the granular material forms a cylindrical layer near the outer boundary of the annulus and undergoes rigid-body rotation with the fluid and the annulus. It is demonstrated that the librational liquefaction of the granular material results in pattern formation. This self-organization process stems from the excitation of inertial modes induced by the oscillatory motion of liquefied granular material under the influence of the gravitational force. The inertial wave induces vortical fluid flow which entrains particles from rest and forms eroded areas that are equidistant from each other along the axis of rotation. Theoretical analysis and experiments demonstrate that a liquefied layer of granular material oscillates with a radian frequency equal to the angular velocity of the annulus and interacts with the inertial wave it excites. The new phenomenon of libration-induced pattern formation is of practical interest as it can be used to control multiphase flows and mass transfer in rotating containers in a variety of industrial processes.

KEYWORDS: Fluid; granular medium; rotation; librations; inertial waves; pattern formation

1 Introduction

Flows in a rotating cylinder have applications across a range of research areas, including the physics of granular matter, hydrodynamics of suspensions and pure liquid coating flows. A detailed examination of the phenomena associated with rotating cylinders has led to a better understanding related subjects, such as avalanches in granules, segregation in suspensions, and steady flow in pure liquids [1]. The diversity of observed effects in such a simple geometry can be attributed to the multitude of factors that influence phase motion, including rotational speed, surface tension, the density ratio of the media, and others.

One of the most intriguing phenomena is an axial segregation of non-neutrally buoyant particles in diluted suspensions (for example, [2]). This leads to the formation of bands of high particle concentration, which are spaced at regular intervals. The phenomenon occurs when the rotation rate of the cylinder is less than the critical rate at which the centrifugal force becomes dominant, resulting in the formation of a uniform layer of particles adjacent to the cylinder surface. The results of numerous experiments conducted with low-viscosity fluids and negatively buoyant particles indicate that the distance between bands of high particle concentration is dependent on both the radius of the cylinder and the size of the particles. At least two theoretical approaches have been proposed to explain the axial segregation of particles.

According to Lee et al. [3], axial segregation is caused by an attractive interaction between the suspended particles. Specifically, axial perturbations in the concentration field lead to faster settling where particles are closer together, which draws more particles toward the high-concentration regions.

In a second approach, the authors consider the collective action of moving particles on fluid dynamics, rather than the interaction between individual particles [4]. In a rotating cylinder, the combined action of gravitational force, buoyancy force, and viscous force causes particles to move in a circular trajectory with the centre of rotation displaced from the cylinder's axis. The rotation rate of the suspended particle is found to be close to that of the cylinder. The collective effect of the suspended particles can be viewed as a persistently perturbing driving force acting on the fluid. Assuming that both the Rossby number and Ekman number are significantly smaller than one, the action of the driving force results in the excitation of inertial waves in the fluid [5]. The predictions of this theoretical approach are found to be in good agreement with observations under the assumption that inertial waves are excited with a frequency equal to the frequency of gravity-induced motion of the suspended particles [4].

This theoretical approach was applied to containers of varying geometries. For example, Borcia et al. [6] derived an equation to predict the excitation of inertial waves with varying wave numbers in an annulus. Inertial waves occur in a variety of natural systems related to bounded rotating fluids, ranging from the Earth's interior to atmosphere waves. For this purpose, they are thoroughly examined from both theoretical and experimental perspectives (a review can be found in [7]). In particular, researchers investigate the propagation of inertial waves in rotating and librating containers of various geometries filled with isothermal [8–10] or non-isothermal fluids [11,12]. A further strand of research examines the effect of inertial waves on the stability of Couette flow in geometries that may be spherical [13] or cylindrical [14] in nature.

The objective of this paper is to examine the dynamics of fluid and negatively buoyant particles in a non-uniformly rotating (librating) annulus. We consider a rapidly rotating annulus in which particles of granular material are pressed against the outer wall of the annulus by centrifugal force of inertia. A significant difference between the present study and prior studies is a large amount of granular material which forms a layer of several particle diameters in thickness near the rotating wall.

Librations have an advantage over uniform rotation. The management of both the amplitude and frequency of oscillations allows for the control of both the frequency and amplitude of particle oscillations. This enables the excitation of inertial waves with various frequencies and wavenumbers while the gravity-induced motion of particles in a uniformly rotating container results in the excitation of inertial waves with a single frequency that is equal to the rotation rate.

2 Experimental Technique

The experiments are carried out with a liquid and a granular medium in a rotating cylindrical annulus (Fig. 1). Two cavities are used in the experiments. Their dimensions are given in Table 1. Each cavity is a horizontal cylindrical annulus, with a metal cylinder forming the inner boundary and a transparent Plexiglas tube forming the outer boundary.

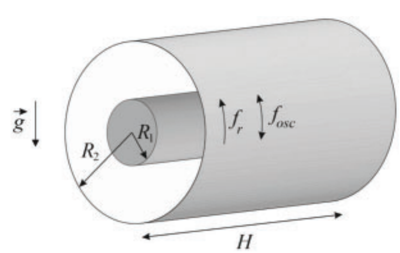


Figure 1: Scheme of the annulus

Table 1: Dimensions of annuli

Cavity	R_1, cm	R_2, cm	H, cm	h, cm	R_s, cm
1	1.1	3.7	18.2	0.2	3.5
2	1.0	4.0	31.8	0.04	3.96

The experiments are conducted at a room temperature of 20°C, with temperature fluctuations of ±2°C. A typical experiment is conducted over a period of one to two hours, during which temperature fluctuations are negligible. The working fluid is water with a density of ρ_l = 1.0 g/cm^3 and a viscosity of ν = 0.01 cm/s^2.

The granular material is composed of glass spheres with a diameter of d = 0.020 ± 0.002 cm and a density of ρ_s = 2.5 g/cm^3. The volume of the granular material V_s is selected to be considerably smaller than the volume of the annulus V_a. In the experiments conducted with cavities #1 and #2, the relative volume $q = V_s/V_a$ is equal to 10% and 2%, respectively. Additionally, the amount of granular material is described by the thickness h of the cylindrical layer of particles, which is formed under the force of centrifugal inertia in the vicinity of the Plexiglas wall (Table 1).

The cavity is driven by a stepper motor. The rotation rate of the cavity varies with time according to the law $f = f_r\left[1 + \varepsilon \sin\left(2\pi f_{osc} t\right)\right]$. Here f_r is the mean cavity rotation rate, ε is the amplitude of the rotation rate modulation, f_{osc} is the oscillation frequency, and t is time. The motor is managed by a ZETLAB digital signal generator which enables the independent regulation of f_r, f_{osc}, and ε with high precision (the relative error does not exceed 0.1%). The rotation rate f_r ranges from 0 to 7 rps, the oscillation frequency f_{osc} varies from 1 to 2 Hz, and the amplitude ε varies from 0 to 0.45.

The granular material is observed through the cylindrical wall of the cavity. For this purpose, a digital camera Canon 600D with a lens EF 50 mm f/1.8 STM is positioned against the rotating cavity. An Light Emitting Diode (LED) is used to illuminate the layer. It is mounted opposite the photo camera, with the rotating cavity between them. The exposure time is selected to be no longer than 1/2000 s in order to achieve optimal image sharpness.

The protocol for the experiments is as follows. The cylindrical annulus is slowly accelerated from rest to seven revolutions per second. At this rotation rate, the granular material forms a layer of uniform thickness near the cylindrical surface. The two-phase system undergoes a rigid-body rotation: the rotation rate of the fluid and the granular material is equal to the rotation rate of the cavity. In the absence of rotation rate modulation, this state is maintained indefinitely. Once an axisymmetric interface is established between the granular layer and the fluid, the selected rotation rate f_r is set and azimuthal oscillations with a frequency f_{osc} are initiated. The modulation amplitude ε gradually increases in steps of 0.01–0.05. Each step takes 10–15 min, during which time photos and videos are taken.

3 Experimental Results

When the cylindrical annulus is rotated at a high rate, the fluid and the granular material undergo a rigid-body rotation, with a uniform distribution of granules near the outer boundary of the annulus (Fig. 2a). When modulation is applied, the initially axisymmetric interface becomes unstable. It is demonstrated that as the amplitude of azimuthal oscillations increases, the banding pattern appears at the surface of the granular layer (Fig. 2b). The alternating light and dark stripes, with a width of a few millimeters, show changes in the thickness of the granular layer. A further increase in the oscillation amplitude results in the appearance of wavy patterns elongated along the rotation axis superimposed on a background of banding patterns (Fig. 2c).

These patterns are regular in the azimuthal direction but not parallel. This may indicate azimuthal time-averaged flows with inhomogeneous velocity along the rotation axis.

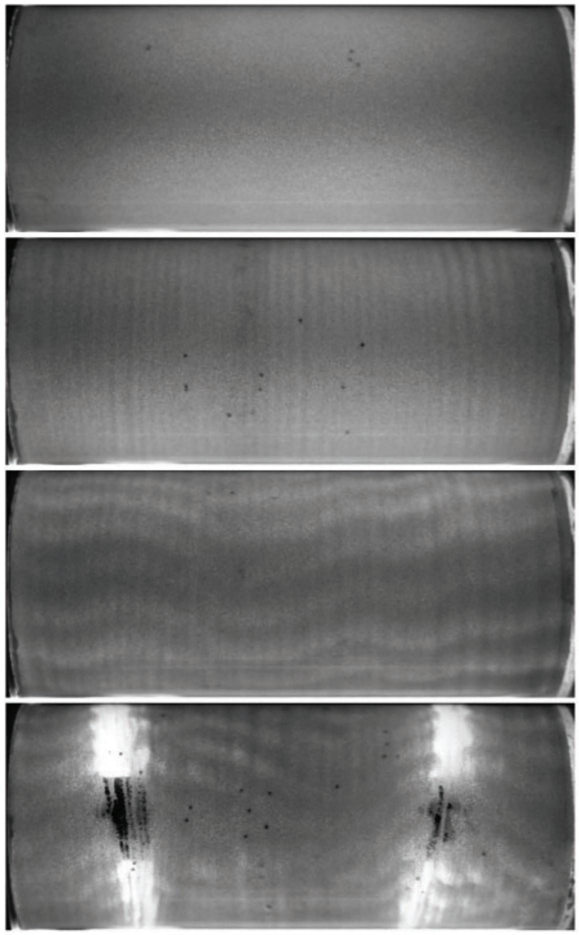
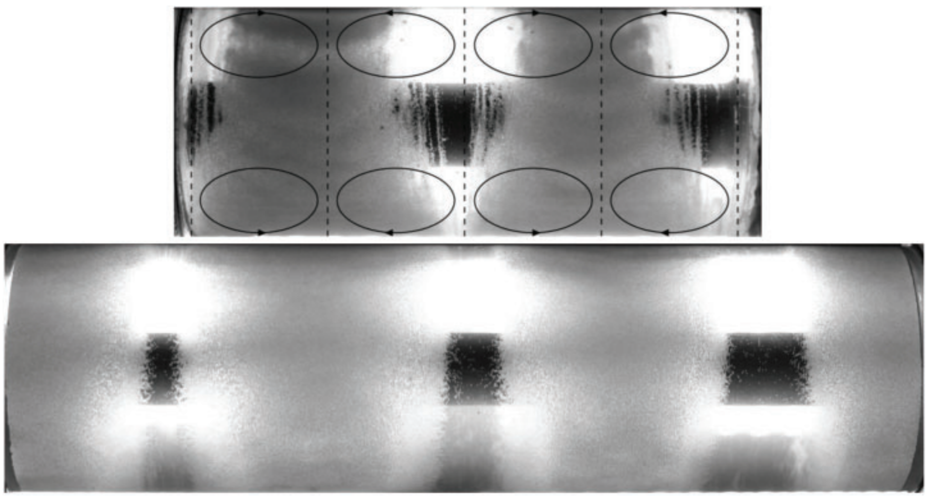


Figure 2: Photos of granular medium and liquid obtained at rotation rate f_r = 5 rps and oscillation frequency f_{osc} = 1 Hz in the cavity #1 (a–d): ε = 0, 0.17, 0.24, and 0.32. The black horizontal band in photo (d) is the inner boundary of the cylindrical annulus which is formed by a black-painted metal tube

As the amplitude of oscillation is further increased, the formation of azimuthal bands with a zero concentration of granular material is observed (Fig. 2d). These areas are hereafter referred to as eroded areas. Two eroded areas are located symmetrically relative to the centre of the rotating annulus. The distance between these areas is approximately twice the distance between the eroded areas and the end walls of the cavity. So, the spatial period (axial distance between neighboring eroded areas) is half the length of the annulus. Fig. 2d indicates that the eroded areas are about 1–2 cm wide and their edges are sloped in relation to the direction of the oscillatory motion of the fluid. The origin of the unique geometry of the eroded areas is yet to be determined.

Thus, in the experiments with a fixed rotation rate and oscillation frequency, and an increasing oscillation amplitude, three distinct patterns are observed: band patterns, azimuthally periodic distribution of granular material, and eroded areas. The present paper is devoted to an examination of eroded areas. Two other phenomena of scientific interest will be discussed in detail in subsequent papers.

Pattern formation in the layer of granular material in response to an increasing modulation amplitude has been observed at varying rotation rates and oscillation frequencies in cavities of different lengths. The development of patterns follows a process that is analogous to the aforementioned scenario. As illustrated in Figs. 2d and 3a, the location of eroded areas can vary in different experiments. In some instances, these areas are located in the middle of the cavity, while in others they are located near the end walls. It is noteworthy that in both instances, the length of the pattern formation remains unaltered, with the pattern extending to a length equal to half that of cavity #1. The different arrangement of the eroded areas (Figs. 2d and 3a) in cavity #1 clearly indicates that both the eroded areas and the granular material bands can be located in pressure antinodes. Thus, the wavelength of the patterns is twice the distance between adjacent pressure antinodes. Fig. 3b shows that there are three eroded areas in the cavity #2 instead of two as in the cavity #1. It is noteworthy that the ratio of the radii of the cylindrical boundaries in both cavities is approximately equal, and that the relative length of cavity #2 is approximately one and a half times the length of cavity #1. This demonstrates that the pattern formation observed in the two cavities is of a similar nature.



(a)

(b)

Figure 3: Photos of granular medium and liquid obtained (a) in the short cavity #1 (f_r = 4 rps, f_{osc} = 1 Hz, ε = 0.36) and (b) in the long cavity #2 (f_r = 4 rps, f_{osc} = 1.82 Hz, ε = 0.45). The dashed lines indicate the location of pressure antinodes. The vertical pairs of ellipses illustrate the cross-section of the vortices (arrows show the direction of steady rotation)

4 Discussion

Here we consider the action of gravity on a two-phase system in a rapidly rotating annulus. The centrifugal force of inertia induces the distribution of the granular material in the form of a thin axisymmetric layer near the outer cylindrical wall of the cavity. The gravitational force rotates in the reference frame of the cavity with a velocity equal to the rotation rate, acting as a periodic external force. When the cavity rotates rapidly, the gravitational force is negligible in comparison to the centrifugal force of inertia. Consequently, the granular material is stationary in the rotating frame of reference in the absence of azimuthal oscillations. Once a critical level of azimuthal oscillations is reached, the granular medium undergoes liquefaction resulting in the formation of a quasi-liquid layer of high density and viscosity. The fluid and the liquefied granular material respond to the alternating action of gravitational force and tangential force caused by librations. It is known that oscillations with a frequency equal to the rotation rate or libration frequency can give rise to inertial waves in a rotating fluid. Let us now consider these waves.

Currently, research interest in inertial waves and inertial oscillations is largely driven by the need to understand the dynamics of liquid cores in rotating planets [15]. Experimental studies [16,17] of various wave modes in a rotating cylinder under vibrations confirmed the theoretical predictions concerning the dynamics of inertial modes in the limit of small Ekman and Rossby numbers.

Now we turn our attention to the problem of determining the natural frequencies of inertial oscillations of a fluid (inertial modes) in a rotating horizontal annulus of given aspect ratios $R \equiv R_2/R_1$ and $L = H/R_1$. Borcia et al. [6] determined the eigenspectrum and eigenmodes in a rotating annulus in the limit of low-viscosity fluids (small Ekman numbers) and the approximation of small-amplitude oscillations (small Rossby numbers). Borcia et al. [6] obtained a solution for the pressure in the presence of an inertial wave with a dimensionless frequency of wave oscillation $\omega \equiv f_w/f_r$ in a rotating annulus. If we denote the pressure perturbation in a fluid as $P = F(r)\, e^{ik\theta} \cos n\pi z$ (k and n are the azimuthal and axial wavenumbers, respectively), then the function $F(r)$ is defined by the equation

$$r^2 \frac{\partial^2 F}{\partial r^2} + r \frac{\partial F}{\partial r} + \left[\left(4/\omega^2 - 1\right) n^2 \pi^2 r^2 - k^2\right] F = 0. \tag{1}$$

Here, the unit of distance measurement is the cavity length H. If the radial coordinate is expressed in the form $x = \sigma r$ ($\sigma = n\pi\sqrt{(4/\omega^2) - 1}$), then Eq. (1) is reduced to the Bessel equation. The general solution of Bessel equation of order k is given by

$$F(x) = A\left(J_k(x) + B Y_k(x)\right), \tag{2}$$

where $J_k(x)$ and $Y_k(x)$ are the Bessel functions of the first and second kind. The solution must satisfy the following boundary conditions

$$x\, \partial F/\partial x + k\sqrt{(\sigma/\pi n)^2 + 1}\, F = 0 \tag{3}$$

at $x_1 = \sigma r_1$ and $x_2 = \sigma r_2$. Note that r_1 and r_2 are dimensionless radii of the inner and outer boundaries of the fluid layer, with the unit being the cavity length H. These boundary conditions allow us to calculate the parameter B and mth eigenvalue of a mode with azimuthal wavenumber k and axial wavenumber n denoted by ω_{mnk}. Here, ω_{mnk} is the ratio of the frequency of the wave oscillations to the cavity rotation rate.

In the case under consideration, the wave is excited by the force of gravity, with a velocity equal to the rotation rate of the cavity. A wave with the frequency $|\omega_{mnk}| = 1$ and azimuthal wavenumber $k = 1$ should be expected to fulfill the resonance condition. The mode with the longest wavelength in the radial direction corresponds to $m = 1$. This is determined by two factors: first, m is equal to the number of pressure nodes (velocity antinodes) in the radial direction; second, the radial velocity component is equal to zero at the cylindrical walls of the cavity. It can therefore be anticipated that the mode of inertial oscillations excited by gravitational force will take the form $|\omega_{1n1}| = 1$. Therefore, the determination of the conditions required for the resonant excitation of inertial modes within a known length cavity can be defined as the determination of the relative radius of the outer boundary of the layer, R, which satisfies the condition $\omega_{1n1} = 1$. It can be deduced from the solution of Eq. (3) and boundary conditions that for the relative length $H/R_1 = 16.5$ (cavity #1) $R_{theory} = 3.10$ at $n = 4$ and $R_{theory} = 2.71$ at $n = 5$ fulfill the resonance conditions.

Given that $R_1 = 1.10$ cm and $R_2 = 3.70$ cm for cavity #1, the ratio of radii $R = 3.36$. This value is greater than the theoretical value $R_{theory} = 3.10$ that would be expected under conditions of resonant wave excitation with a wavenumber of $n = 4$.

If we take into account the thickness $h = 0.2$ cm of the granular medium near the cylindrical wall and assume that the outer boundary of the annulus is transferred to the surface of the granular medium, then the resulting dimensions are $R_S = R_2 - h = 3.5$ cm and $R_s/R_1 = 3.18$. This value is approximately equal to R_{theory} at $n = 4$. Figs. 2d and 3a demonstrate that the spatial period of the eroded areas is consistent with the calculated value of the length of the standing inertial wave which incorporates two inertial vortices oscillating in antiphase. Here, two waves (two pairs of vortices) arranged along the length of the annulus correspond to the value $n = 4$.

It should be noted that the nodes of the axial velocity and the antinodes of the radial and azimuthal velocities, as well as the pressure antinodes, are located on the end walls of the cavity. Thus, the fluid flow within an inertial wave has the form of annular vortices located between pressure antinodes. The rotation of neighboring vortices occurs in opposite directions, with the velocity of rotation varying in accordance with a sinusoidal law along the azimuthal coordinate. The rotation of neighboring vortices is opposite, with the velocity of rotation varying in accordance with a sinusoidal law along the azimuth.

The analysis of the results indicates that the inertial wave (mode) may be responsible for the development of a series of eroded areas at the surface of the granular medium. The formation of each eroded area is attributed to the interaction of two neighboring vortices, which results in a wavelength twice the distance between the pressure antinodes. Although neighboring vortices rotate in a coordinated manner, the direction of their rotation can vary and is determined by random factors. This clearly demonstrates that the location of eroded areas in different experiments may differ while the essential feature remains unchanged: They are located in the pressure antinodes. What is the driving force for the excitation of the inertial wave and the axial segregation of granular material?

At a rotation rate of $f_r = 5$ rps, the granular medium and the fluid undergo a rigid-body rotation resulting in the formation of a cylindrical interface between them. This is due to the fact that the effect of gravity is relatively insignificant in comparison to the centrifugal force, that is to say, $\Gamma \equiv \frac{g}{(2\pi f_r)^2 R_2} \approx 0.3$. The addition of azimuthal oscillations results in the water maintaining a rigid-body rotation with a rotation rate f_r while the granular medium librates together with the cavity. This leads to the conclusion that the fluid undergoes azimuthal oscillations at a frequency of f_{osc} and an amplitude of ε in relation to the granular medium. The action of azimuthal oscillations results in the liquefaction of the granular medium when the critical value of the Shields number, defined as the ratio between the viscous shear stress at the interface between the granular medium and the fluid and the apparent weight of a grain, is reached.

Let us check that the band formation occurs only after the granules become mobile. We can calculate the critical value of the Shields number τ_c using the dependence of the Shields number on the Reynolds number $\mathrm{Re} = \frac{u_{osc} d}{\nu}$ [18]. The amplitude of the velocity of the oscillatory motion of the fluid can be calculated by the formula $u_{osc} = 2\pi f_{osc} \varphi_0 R_s$, where φ_0 is the angular amplitude of the fluid oscillations which can be calculated from $\varphi_0 = \varepsilon f_r / f_{osc}$. Then the Reynolds number takes the form

$$\mathrm{Re} = \frac{2\pi f_r \varepsilon R_s d}{\nu}. \tag{4}$$

In the discussed experiment, band patterns are clearly visible at $f_r = 5$ rps, $f_{osc} = 1$ Hz, and $\varepsilon = 0.17$ (Fig. 2b). Then, Eq. (4) gives $\mathrm{Re} \approx 38$. It follows from [18] that $\tau_c = 0.04$ at $\mathrm{Re} \approx 38$.

Now we can determine the experimental value of the Shields number. We will use the data obtained from the study of the pattern formation at the interface between fluid and the granular medium in a short cylinder under librations [19]:

$$\tau = \frac{\nu\varepsilon}{\left(\frac{\rho_s}{\rho_l} - 1\right) 2\pi f_{osc} d\delta}. \qquad (5)$$

Here, $\delta = \sqrt{\nu/\pi f_{osc}}$ is the thickness of the Stokes boundary layer near the interface between the granular medium and the fluid. The Eq. (5) is obtained in the approximation that the particle size is smaller than the thickness of the viscous boundary layer which matches the conditions of the experiment: $\delta = 0.056$ cm while $d = 0.02$ cm. Substituting values into Eq. (5) leads to $\tau = 0.16$. Thus, the formation of bands is confirmed to occur only after the particles at the surface of the granular medium become mobile ($\tau > \tau_c$). Upon reaching the critical value τ_c, the particles at the interface become mobile for a short interval of time, which may be less than the necessary interval to move the particles. This means that for patterns to form, there needs to be a long time (lots of oscillation cycles) during which particles move to a distance enough for regular patterns to form. The observations indicate that, for the same rotation rate f_r and oscillation frequency f_{osc}, but slightly different modulation amplitudes ε in the supercritical region, the time required for the pattern formation is less at higher modulation amplitudes. In the experiment under discussion, the formation of regular band patterns occurs over a period of several minutes. The liquefied granular material oscillates azimuthally relative to the cavity under the action of gravitational force. The oscillations of the liquefied material occur at a frequency equal to the cavity rotation rate and can induce an inertial wave that propagates in the direction opposite to the cavity rotation with azimuthal velocity $\omega = 1$. In the laboratory frame of reference, the wave remains stationary, and thus the wave-induced fluid motion is stationary at any given point.

We will now proceed to discuss the formation of eroded areas. They could appear if inertial toroidal vortices perform a steady rotation against the background of inertial oscillations. Steady flow is analogous to a Taylor-Couette vortical flow between independently rotating cylinders. What is the nature of the vortical flow?

One potential explanation for this phenomenon is that the granular medium is liquefied under the action of libration and gravity. Then, the degree of liquefaction, and thus the physical properties of the liquefied material (density, viscosity, and granular layer thickness), oscillate with a frequency equal to the rotation rate. This frequency coincides with the frequency $\omega_{1n1} = 1$ of fluid oscillations caused by the inertial mode. This should result in the classical effect of vibrational thermal convection [20], namely the generation of the steady flow and the redistribution of the disperse phase under the action of an averaged mass force acting in the bulk of a density-inhomogeneous fluid. Another possible explanation is the generation of steady streaming in an oscillating viscous boundary layer. A similar steady streaming is observed when axisymmetric inertial modes with wavenumber $k = 0$ are excited in a librating cylinder [16]. The pattern of the steady streaming flow near the cylindrical wall is consistent with the phenomenon observed in the present study. The wavelength of the steady streaming is twice the distance between the pressure antinodes in the standing inertial wave. Thus, the $n = 4$ mode corresponds to two wavelengths of the steady flow (Fig. 3a). It can be reasonably assumed that the experiments indicate the occurrence of both the granular material liquefaction and the excitation of the steady streaming. The effects of granular medium properties and libration parameters on the steady streaming pattern are of considerable interest and are the focus of ongoing research.

The solution of Eq. (1) for cavity #2 indicates that the mode with wavenumbers $n = 6$ and $k = 1$ is excited at a frequency $\omega_{161} = 1.1$. This frequency is approximately equal to the frequency of oscillations resulting from the action of gravitational force. This finding confirms that the origin of the inertial wave excitation and the origin of the steady streaming in both cavities are identical.

The effect of librations on the excitation of inertial modes remains unclear. Does the effect of librations solely result in the liquefaction of the granular material?

As mentioned above, librations can excite axisymmetric inertial modes with azimuthal wavenumber $k = 0$ in a rotating cylinder [16]. The solution of Eq. (1) indicates that there are no inertial modes with $k = 0$ within the investigated region of low libration frequencies (Fig. 4). Here, the solid line shows the results of calculating the axial wavenumber n at $k = 0$ and $m = 1$ using Eqs. (1)–(3) for cavity #1. The symbols illustrate data obtained in the experiments presented in Fig. 2d (circle) and Fig. 3a (square). This finding confirms that the phenomenon of eroded areas is caused by the interaction between liquefied granular material and the inertial wave with azimuthal wavenumber $k = 1$, which is excited by gravitational force. In this context, it is relevant to cite the work [4], which describes the excitation of the mode with azimuthal wavenumber $k = 1$ in a rotating horizontal cylinder filled with fluid and non-neutrally buoyant particles. The particles do not undergo rotation together with the cavity but rather descend from the rising cylindrical wall, thereby introducing perturbations with $k = 1$ and $\omega = 1$. While there are notable differences in the cavity geometry and formulation between the two problems, there are also common features. In our experiments, a liquefied granular material undergoes tidal oscillations under the effect of gravitational force and excites inertial waves with an azimuthal wavenumber $k = 1$ and frequency $\omega = 1$. Also, suspended particle segregation into well-defined periodic axial bands located in interleaving nodal planes of the standing inertial wave in a rotating cylinder is studied in [21]. It is revealed that the formation of bands is determined by the axial pressure gradient associated with an inertial mode excitation in the bounded fluid. This effect is analogous to the series of eroded areas observed at the nodes of axial velocity of the inertial wave detected in the present study.

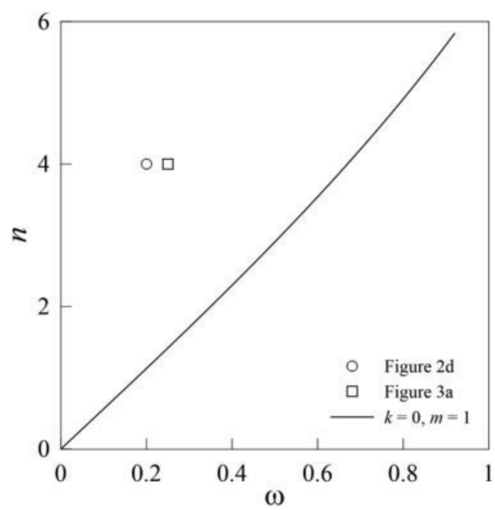


Figure 4: Dependence of axial wavenumber on dimensionless frequency. The results of the calculations (solid line) and experiments (symbols) correspond to cavity #1

5 Conclusion

The dynamics of fluid and non-buoyant particles in a librating horizontal annulus is studied experimentally. It is found that at high rotation rates when the layer of granular material is axisymmetric, modulation of the rotation rate leads to the formation of eroded areas. Theoretical analysis indicates that the pattern formation occurs as a result of the excitation of inertial waves. The excitation of inertial waves is made possible by the liquefaction of the granular material. An inertial standing wave is generated by oscillatory motion of a liquefied granular material which is induced by the rotation of the gravitational force relative to the cavity frame of reference. In the rotating frame of reference, the inertial wave rotates with the angular velocity of the annulus rotation in a direction opposite to that of the annulus rotation. Thus, the wave is stationary in the laboratory frame of reference. It is demonstrated that the distance between the eroded areas is governed by the axial wavenumber of the inertial mode for a given size of an annulus. The experimental results are in accordance with the findings of the theoretical analysis performed under the approximation of low fluid viscosity and rapid rotation (in the limit of small values of Ekman and Rossby numbers).

Acknowledgement: The authors are grateful for the technical assistance of Alexander Selyanin.

Funding Statement: This research is funded by the Ministry of Education of the Russian Federation within the framework of a state assignment, number 1023032300071-6-2.3.1.

Author Contributions: Study conception and design: Victor Kozlov; Data collection: Denis Polezhaev and Alexey Vjatkin; Analysis and interpretation of results: Victor Kozlov and Denis Polezhaev; draft manuscript preparation: Victor Kozlov and Denis Polezhaev. All authors reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: The data that support the findings of this study are available on request from the corresponding author.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

Nomenclature

d	Particle diameter, cm
f	Annulus rotation rate, rps
f_r	Mean rotation rate of the annulus, rps
f_{osc}	Oscillation frequency, Hz
f_w	Frequency of wave oscillations, Hz
h	Granular layer thickness, cm
H	Annulus length, cm
k	Azimuthal wavenumber
L	Relative length of the annulus (ratio of H and R_1)
m	Radial wavenumber
n	Axial wavenumber
R_1	Annulus inner radius, cm
R_2	Annulus outer radius, cm
R_s	Radius of the interface between the granular bed and the fluid, cm
R	Ratio of R_1 and R_2
Re	Reynolds number
q	Relative volume of granular material
u_{osc}	Velocity amplitude of the fluid oscillatory motion, cm/s
V_a	Annulus volume
V_s	Granular material volume
Γ	Ratio of gravitational acceleration and centrifugal acceleration
ε	Libration amplitude
δ	Thickness of the Stokes boundary layer, cm
ν	Fluid kinematic viscosity, cm^2/s
ρ_l	Fluid density, g/cm^3
ρ_s	Particle density, g/cm^3

τ	Shields number
τ_c	Critical value of the Shields number
φ_0	Angular amplitude of the fluid oscillations, rad
ω	Dimensionless frequency

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