

## Numerical Simulation of Double Diffusive Mixed Convection in a Horizontal Annulus with Finned Inner Cylinder

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**Abstract:** The present work relates to a numerical investigation of double diffusive mixed convection around a horizontal annulus with a finned inner cylinder. The solutal and thermal buoyancy forces are sustained by maintaining the inner and outer cylinders at uniform temperatures and concentrations. Buoyancy effects are also considered, with the Boussinesq approximation. The forced convection effect is induced by the outer cylinder rotating with an angular velocity ( $\omega$ ) in an anti-clockwise direction. The studies are made for various combinations of dimensionless numbers; buoyancy ratio number ( $N$ ), Lewis number ( $Le$ ), Richardson number ( $Ri$ ) and Grashof number ( $Gr$ ). The isotherms, isoconcentrations and streamlines as well as both average and local Nusselt and Sherwood numbers were studied. A finite volume scheme is adopted to solve the transport equations for continuity, momentum, energy and mass transfer. The results indicate that the use of fins on the inner cylinder with outer cylinder rotation, significantly improves the heat and mass transfer in the annulus.

**Keywords:** Finned inner cylinder, double diffusive flow, heat and mass transfer, rotating annulus, mixed convection, numerical simulation.

### Nomenclature

$C^*$	concentration	( $\text{kg.m}^{-3}$ )
$C$	dimensionless concentration: $(C^* - C_0^*) / (C_i^* - C_0^*)$	
$D$	mass diffusivity	( $\text{m}^2.\text{s}^{-1}$ )
$e_r, e_\phi$	unit vectors in the radial and angular directions, respectively	
$e$	annulus width: $r_o - r_i$	(m)
$g$	gravitational acceleration	( $\text{m.s}^{-2}$ )
$Gr_s$	solutal Grashof number: $g\beta_s (T_i - T_o) e^3 / \nu^2$	
$Gr_T$	thermal Grashof number: $g\beta_T (T_i - T_o) e^3 / \nu^2$	
$k$	thermal conductivity	( $\text{W.m}^{-1} \text{K}^{-1}$ )
$l_a$	fin length	(m)
$Le$	Lewis number, $Sc/Pr$	
$N$	buoyancy ratio number, $Gr_s/Gr_T$	
$Nu$	local Nusselt number	

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$\overline{Nu}$	average Nusselt number	
$p$ ( $P$ )	dimensional (dimensionless $P=p/\rho_o (\omega r_i)^2$ ) pressure	(Pa)
$Pr$	Prandtl number: $\nu/\alpha$	
$r$ ( $R$ )	dimensional (dimensionless $R=r/e$ ) radius	(m)
$Re$	rotational Reynolds number: $\omega r_i e/\nu$	
$Sc$	Schmidt number: $\nu/D$	
$Sh$	Local Sherwood number	
$\overline{Sh}$	average Sherwood number	
$T$	temperature	(K)
$t$	time	(s)
$\mathbf{v}$ ( $\mathbf{V}$ )	dimensional (dimensionless: $\mathbf{v}/\omega r_i$ ) velocity	(m.s <sup>-1</sup> )
$\mathbf{V}_r, \mathbf{V}_\phi$	dimensionless velocity components in $r$ and $\phi$ directions	
$\omega$	angular velocity (rad/s)	

**Greek symbols**

$\alpha$	thermal diffusivity	(m <sup>2</sup> .s <sup>-1</sup> )
$\beta_T$	solotal expansion coefficient	(m <sup>3</sup> .kg <sup>-1</sup> )
$\beta_T$	thermal expansion coefficient	(K <sup>-1</sup> )
$\mu$	Dynamic viscosity	(N.s.m <sup>-2</sup> )
$\nu$	kinematic viscosity	(m <sup>2</sup> .s <sup>-1</sup> )
$\rho$	density	(kg.m <sup>-3</sup> )
$\theta$	dimensionless temperature: $(T-T_o)/(T_i-T_o)$	
$\tau$	dimensionless time, $\tau=t \omega r_i/e$	
$\phi$	angular coordinate	

**Subscripts**

$i$	inner
$o$	outer
$f$	fluid
$a$	fin

**1 Introduction**

Double-diffusive convection in cylindrical cavities is an important problem in heat and mass transfer; where the fluid flow is induced by the simultaneous presence of two diffusive components temperature and concentration, it is used to simulate a wide range of engineering applications such as technologies involved in chemical deposition processes, in addition, these phenomena appear in many engineering applications. The migration of the species is known to be sensitive to the magnitude of rotational speed, which is a crucial parameter in drying technologies, printing applications and crystal growth. Other technologies including the melting and solidification processes in the rotary kiln are probably candidates for such applications. In addition, convection between two coaxial cylinders in general is a vast area, where many applications have been made on this subject, in 1984 Schiroky et al. [Schiroky and Rosenberger (1984)] made an experimental study on

the natural convection of a gas in a horizontal cylindrical enclosure, for a wide range of Ra number, where they found that only a portion of fluid flows along cold thermode into the lower half of cylinder.

After, studies were made by Lee et al. [Lee, Kang and Son (1999); Lee, Kang and Son (2000)] for the phenomenon study of double-diffusion convection of a stable stratified fluid in a rotating annular space, they showed the effect of rotation on the development and fusion of the multi-layer flow structure and various variables. Then Lykov et al. [Bubnovich and Kolesnikov (1986)] and Kolesnikov et al. [kolesnikov and bubnovich (1988)] made a numerical study of the double-diffusion natural convection in a horizontal annulus and it was solved or they showed the effect of walls on the heat transfer, after Sung et al. [Sung, Cho and Hyun (1993)] made a numerical study of a double-diffusion mixed convection in a rotating annulus in order to see the effect of rotation on the overall structure of the flow, when the buoyancy ratio is moderate the global flux character shows a considerable dependence of the relative strength of the rotation effect, then in 2005 Almiri et al. [Al-Amiri and Khanafer (2006)] have carried out a numerical study of double-diffusion mixed convection in a horizontal annulus where the outer cylinder rotates and which induces forced convection, the rate of heat and mass transfer have been examined for different dimensionless numbers or predictions of the Nusselt and Sherwood number were obtained for the Lewis number ranges and the buoyancy ratio, in the same year the same authors [Al-Amiri and Khanafer (2005)] conducted a numerical study on double-diffusion convection heat transfer in a cylindrical horizontal annulus with a cylinder heated interior with sinusoidal temperature, after study they concluded that the amplitude and frequency of the heated inner cylinder were found to cause a significant increase in the rate of heat transfer, then another numerical study of the dual-diffusion laminar convection was made by Teamah [Teamah (2007)]. In a horizontal cylindrical annulus with a rotating inner cylinder where the inner and outer cylinders are maintained at uniform temperatures and concentrations, the stream function, isothermal lines, iso-concentrations and numbers of Nusselt and Sherwood were studied. Recently, Chen et al. [Chen, Tölke and Krafczyk (2010)] have numerically studied double-diffusion convection in an annulus with opposite temperatures and concentration gradients, in this study the authors have shown interest in Rayleigh numbers greater than  $10^7$  which is rare. Recently, Sorour et al. [Sorour, Teamah, and Maghlany (2013)] numerically studied double-diffusion mixed convection in a rotating horizontal annulus, the latter concerned the selection of better direction of rotation of the inner cylinder to improve the rate of heat and mass transfer.

On the same geometry, many numerical applications have been made on heat transfer in a horizontal rotating annulus [Lee (1984); Fusegi, Farouk and Ball (1986); Lee (1992); Yoo (1998); Tzeng (2006); Cheng, Liao and Huang (2008); Hsu (2008)], all their results showed that high rotating Reynolds numbers tend to diffuse thermal convection currents and increasing the absolute value of the number of buoyancy ratio also increases the estimated number of Nusselt number, all of the above studies have shown that the heat transfer between the horizontal rings is limited by the surface of the inner cylinder, hence the use of the fins are sometimes used to increase the heat transfer surface, resulting in increased heat transfer between the cylinders, deep studies were done on natural convection and forced into a ring with fins fixed on the inner cylinder [Sparrow and Preston (1986); Watel, Harmand and Desmet (1998); Rahnama and Farhadi (2004); Fabbri (2005); Chen and Hsu (2007); Haldar,

Kochhar, Manohar et al. (2007); Chen and Hsu (2008); Kiwan and Zeitoun (2008); Huisseune, T'Joen, De Jaeger et al. (2010); Sultan (2014); Arbaban and Salimpour (2014)]. States illustrate that the presence of internal fins themselves contributes very little to the total heat transfer, but their presence considerably modifies the flow profiles and the temperature of the fluid adjacent to the surface of the cylinder, therefore, the transfer of heat. Heat increases in the uncovered area of the cylinder when the buoyancy effects are not negligible.

The main objective of this work is to study in more details numerically the effect of the finned inner cylinder and outer cylinder rotation, on the characteristics of heat and mass transfer of a binary fluid inside the horizontal annulus. As such, the purpose of this article is to examine the effects of relevant dimensionless parameters such as Richardson number (Ri), Grashof number (Gr), Lewis number (Le) and Buoyancy ratio (N). These parameters will be study over a wide range to present the basic flow patterns, isoconcentration and isotherms as well as both local and average Sherwood and Nusselt numbers in cylindrical cavity.

## 2 Problem formulation and mathematical model

The schematic diagram of the horizontal annulus and the relevant boundary conditions are shown in Fig. 1, the inner cylinder of the radius  $r_i$  and the outer cylinder  $r_o$  are kept at constant and uniform temperatures ( $T_i$  and  $T_o$ ) and concentrations ( $C_i^*$  and  $C_o^*$ ), respectively, with  $T_i > T_o$  and  $C_i^* > C_o^*$ . The inner cylinder with fins is fixed while the outer cylinder is driven by a rotational speed ( $\omega$ ). On the other hand the flux in the annular region is supposed to be two-dimensional, stable and laminar. Also, all the physical properties of fluid are considered constant except for the variation of the density in the term of buoyancy, where the approximation of Boussinesq is considered as linearly proportional with temperature and concentration such that:

$$\rho = \rho_0 \left[ 1 - \beta_T (T - T_0) - \beta_S (C^* - C_o^*) \right] \quad (1)$$

where  $\beta_T$  and  $\beta_S$  are the coefficients for thermal and concentration expansions respectively.

The dimensionless form of the governing equations by taking the characteristic length, velocity, pressure and temperature as  $e=(r_o-r_i)$ ,  $(\omega r_i)$ ,  $\rho_o (\omega r_i)^2$  and  $(T_i-T_o)$  are cast in their dimensionless form as:

$$\nabla \cdot (\mathbf{V}) = 0 \quad (2)$$

$$\frac{\partial \mathbf{V}}{\partial \tau} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla(P) + \frac{1}{Re} \Delta \mathbf{V} + \frac{Gr}{Re^2} (\theta + NC) \left[ \cos(\varphi) e_r + \sin(\varphi) e_\varphi \right] \quad (3)$$

$$\frac{\partial \theta}{\partial \tau} + (\mathbf{V} \cdot \nabla) \theta = \frac{1}{Pr Re} \Delta \theta \quad (4)$$

$$\frac{\partial C}{\partial \tau} + (\mathbf{V} \cdot \nabla) C = \frac{1}{Sc Re} \Delta C \quad (5)$$

The dimensionless quantities appearing in above Eqs. (2)-(5) are the Grashof (Gr), Prandtl (Pr), Reynolds (Re), buoyancy ratio (N) and Schmidt numbers (Sc) respectively. Further,

these quantities allow us to define the Richardson number used to distinguish between different types of convection:  $Ri=Gr/Re^2$ . In the above equations,  $\tau$ ,  $P$ ,  $\theta$ ,  $V$ ,  $C$  are respectively the dimensionless time, pressure, temperature, velocity and the concentration. Assuming the non-slip flow, the relevant dimensionless boundary conditions can be written as follows:

$$V_r=0, \quad V_\varphi=0 \quad \theta = 0 \quad C=0 \quad \text{at } \tau = 0 \quad (6)$$

$$V_r=0, \quad V_\varphi=0 \quad \theta = 1 \quad C=1 \quad \text{at } R = R_i \quad (\text{inner cylinder}) \quad (7)$$

$$V_r=0, \quad V_\varphi=1 \quad \theta = 0 \quad C=0 \quad \text{at } R = R_o \quad (\text{outer cylinder}) \quad (8)$$

$$V_r=0, \quad V_\varphi=0 \quad \theta = 1 \quad C=1 \quad \text{at } (R_i \leq R \leq R_i+l_o/e) \quad (\text{fin: tip and lateral surface}) \quad (9)$$

The Nusselt number, which is of a greater interest in engineering applications, is used to evaluate the heat transfer rate at the annulus surfaces. To determine heat transfer characteristics at the enclosure walls, contributions of convection should be taken into account. Thus the local and average Nusselt numbers through the rotating finned cylinder are given by:

The dimensionless heat transfer rate in conduction in the absence of motion of fluid is:

$$Nu_{cond} = \frac{1}{\ln\left(\frac{R_o}{R_i}\right)} \quad (10)$$

The number of local Nusselt is defined as the real heat flux as divided by the  $Nu_{cond}$ :

$$Nu_i(\varphi) = -\left(R \frac{\partial \theta}{\partial R}\right) / Nu_{cond} \quad \text{at } R=R_i \quad (11)$$

$$Nu_o(\varphi) = -\left(R \frac{\partial \theta}{\partial R}\right) / Nu_{cond} \quad \text{at } R=R \quad (12)$$

And the average number of Nu is given by:

$$\overline{Nu}_i = \frac{1}{2\pi} \int_0^{2\pi} Nu_i(\varphi) \partial \varphi \quad (13)$$

$$\overline{Nu}_o = \frac{1}{2\pi} \int_0^{2\pi} Nu_o(\varphi) \partial \varphi \quad (14)$$

Similarly, we can calculate local and average Sherwood numbers as follows:

$$\overline{Sh}_i = \frac{1}{2\pi} \int_0^{2\pi} Sh_i(\varphi) \partial \varphi \quad (15)$$

$$\overline{Sh}_o = \frac{1}{2\pi} \int_0^{2\pi} Sh_o(\varphi) \partial \varphi \quad (16)$$

where

$$Sh_i(\varphi) = \ln \frac{R_i}{R_0} \left( R \frac{\partial C}{\partial R} \right) \quad \text{at } R=R_i \quad (17)$$

$$Sh_0(\varphi) = \ln \frac{R_i}{R_0} \left( R \frac{\partial C}{\partial R} \right) \quad \text{at } R=R_0 \quad (18)$$

The derivative of the local Nusselt number is calculated according to the difference formul by using the temperature obtained on the first annulus of elements around the inner cylinder. An accurate calculus of the integral is obtained by linear interpolation according to the step size at each node of the required elements.

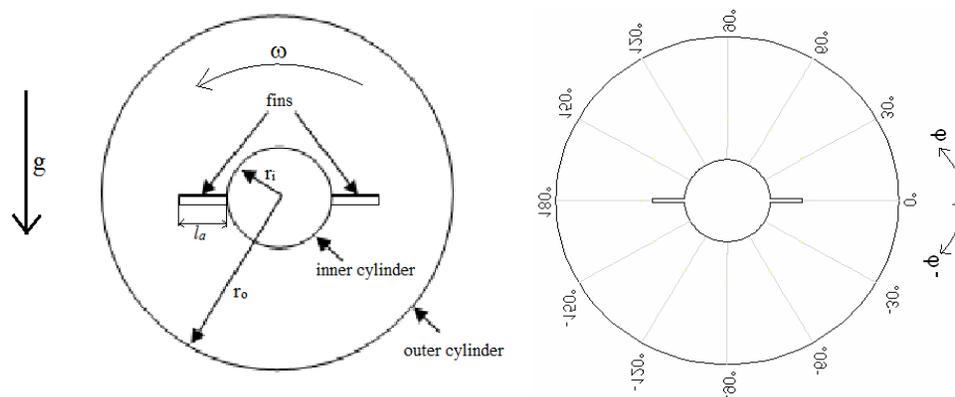


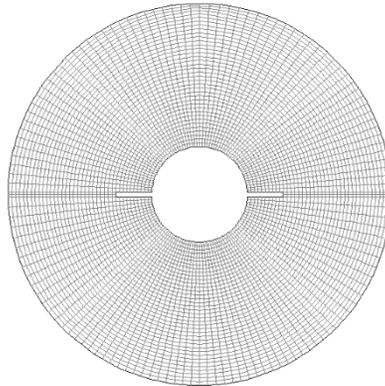
Figure 1: Physical model

### 3 Numerical method and grid independency

The above Eqs. (2)-(5) with corresponding boundary conditions, have been solved numerically by the finite volume method approach developed by Patankar [Patankar (1980)], this method consists in the discretization of the governing equations using central differentiation in space. To allow grid-independent examination, the numerical procedure has been conducted for various grid resolutions ( $40 \times 40$  to  $120 \times 120$ ). Results presented in Tab. 1 in term of the averaged Nusselt number on the inner cylinder show that the values remain almost constant for grids finer than ( $120 \times 120$ ) and heavily depend on the grid size for less fine grids. Consequently, considering both the accuracy and the computational costs, most computations reported in the current work were performed with a multiple grid system of  $80 \times 80$  control volumes. Nevertheless, similar tests were conducted for others Gr and Re and the grid size were adjusted accordingly. To check the convergence of the sequential iterative solution, normalized residual respectively for the mass, momentum and energy equation is calculated, convergence is obtained when the residual becomes smaller than  $10^{-9}$ .

Table 1: Grid dependency

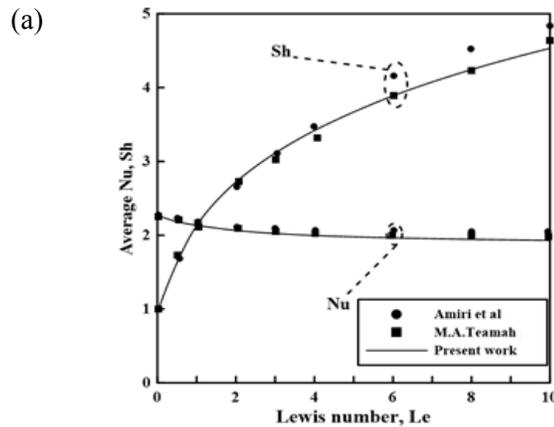
Grid point	40×40	60×60	80×80	100×100	120×120
$\overline{Nu}$	5.36692	5.38447	5.39087	5.39088	5.390902

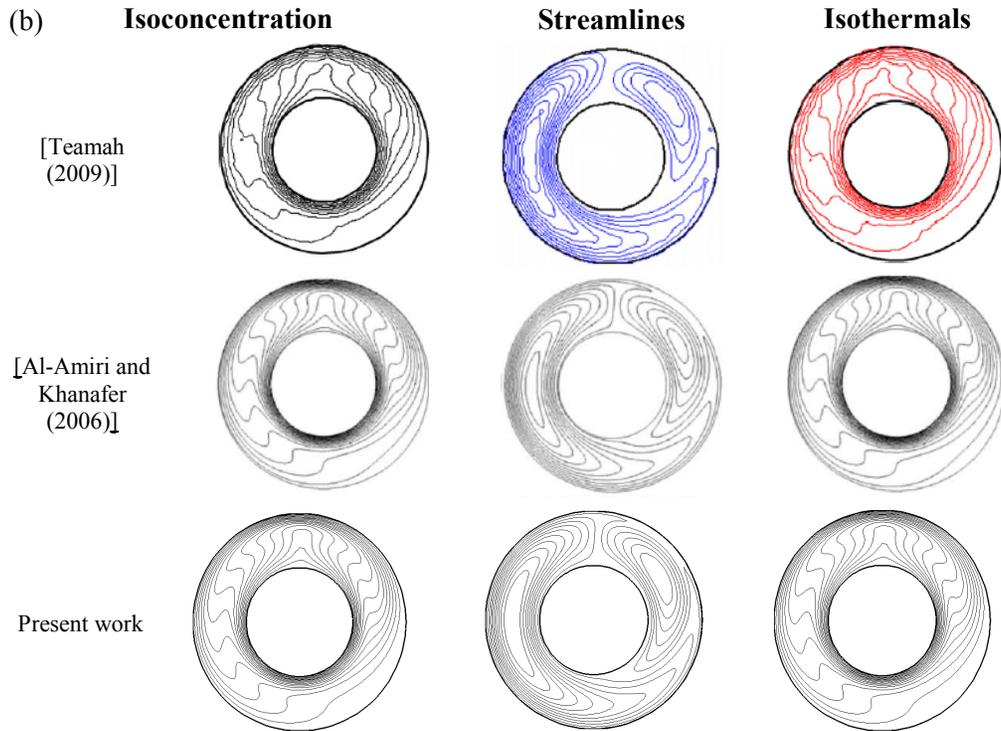


**Figure 2:** Detail of the computational grid within the annular space

**4 Validation**

The subject of this study is a very important area of research, in order to verify the accuracy of the present results, our code was validated with results published in the literature on the double-diffusive mixed convection in rotating annulus. Fig. 3 illustrates a comparison with the results obtained by Al-Amiri et al. [Al-Amiri and Khanafer (2006)] and Teamah [Teamah (2007)], this on the effect of Lewis number on the average Nusselt and Sherwood numbers Fig. 3(a). This figure shows us an excellent agreement for the values of average Nusselt number. On the other hand, the values of average Sherwood number obtained was in good agreement with those found by Teamah [Teamah (2007)], for the high values of Lewis, a maximum difference was less than five percent.



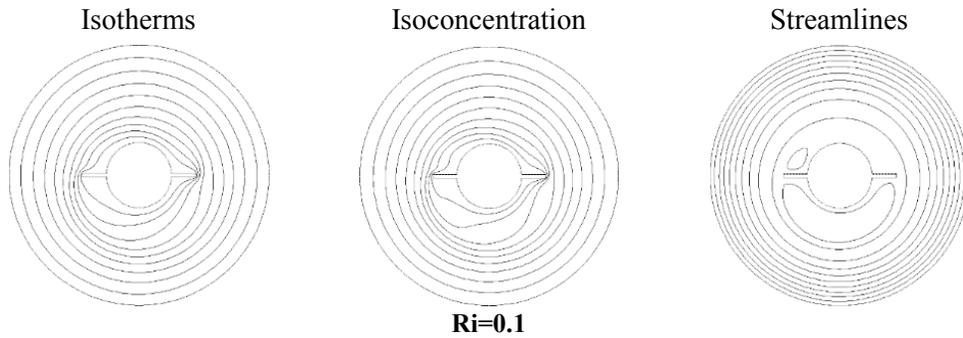


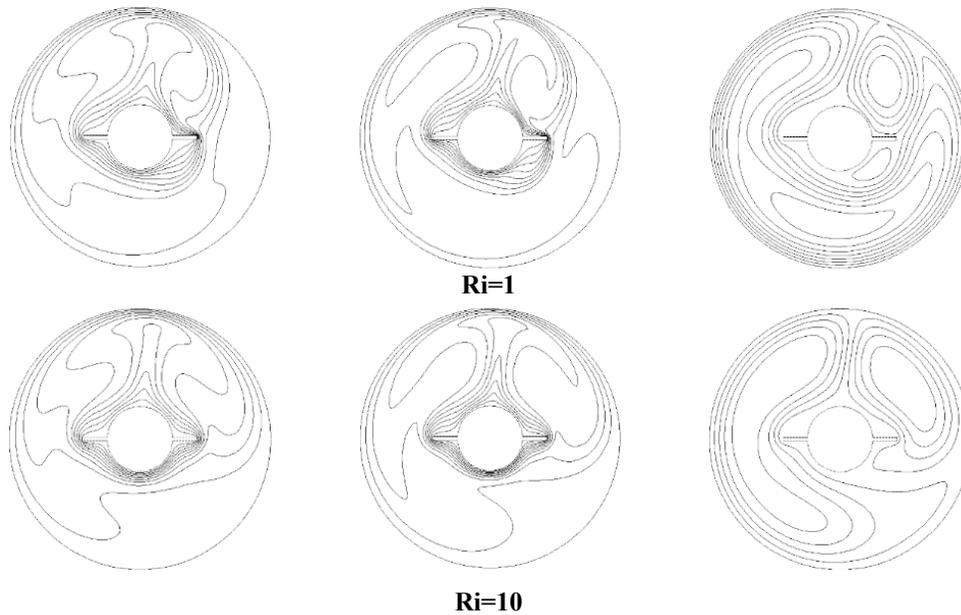
**Figure 3:** Comparison with Amiri et al. and Teamah, Outer cylinder rotates,  $N=1$ ,  $Re=100$  and  $Ra_T=10^4$ , (a) Average Nusselt and Sherwood number, (b) Isoconcentration, streamlines and isothermals, for  $Le=1$

**5 Results and discussion**

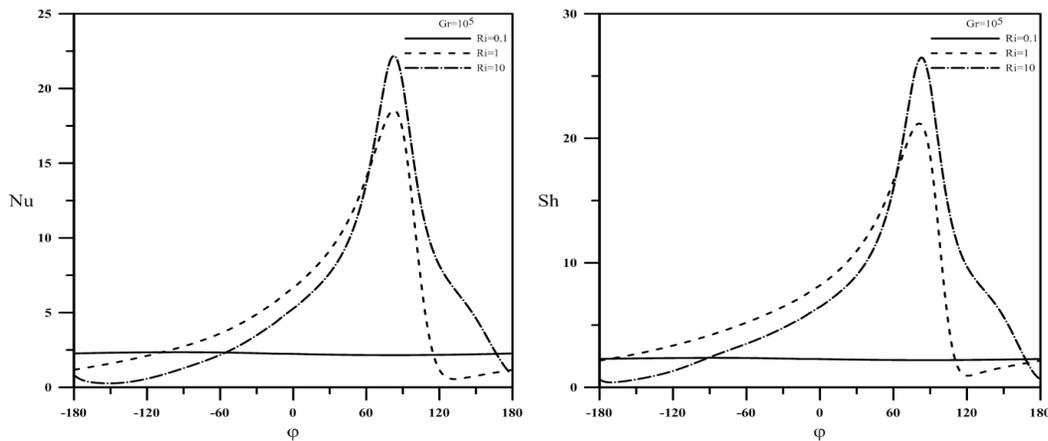
Different cases were made for a good understanding of the Richardson number (0.1 to 10), Lewis numbers (0.01 to 10) and the buoyancy ratio (-20 to 20) effects on the studied phenomenon. The results are presented in terms of the contour lines for streamlines, temperature and concentration likewise, local Nusselt and Sherwood numbers, average Nusselt and Sherwood numbers predictions are also presented.

***Effect of Richardson number***





**Figure 4:** Isotherms, isoconcentration and streamlines for  $Le=1$ ,  $N=1$ ,  $Gr=10^5$  and different  $Ri$

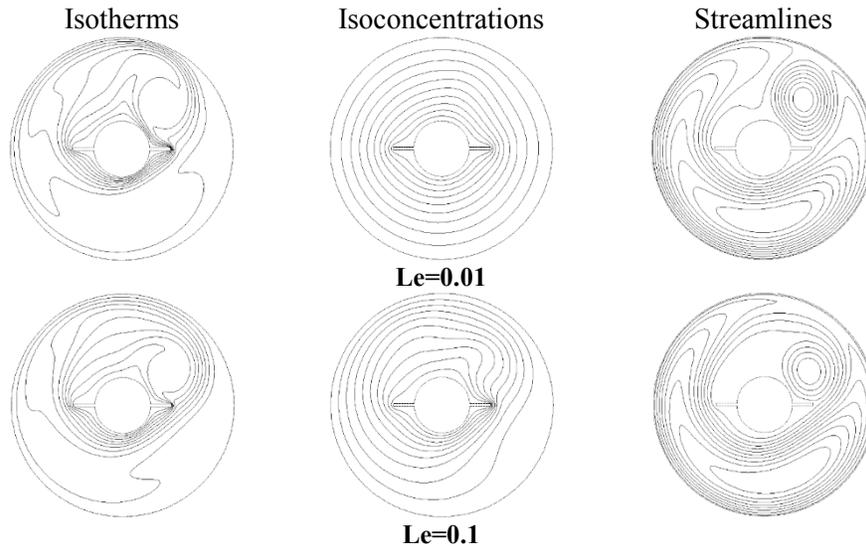


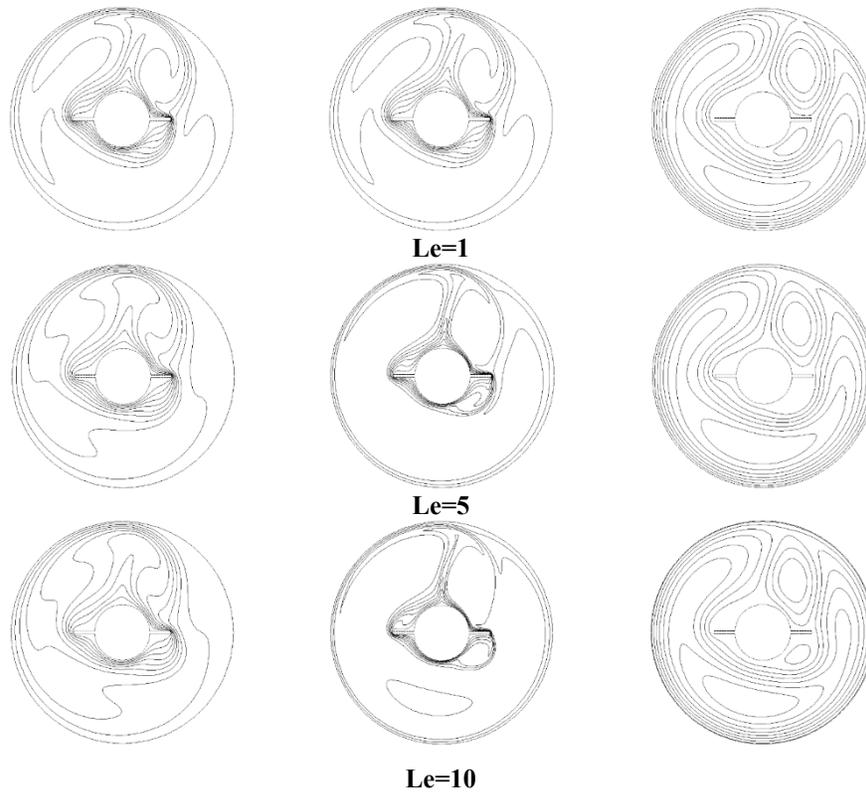
**Figure 5:** Effect of Richardson number on local Nusselt and Sherwood numbers for  $Le=1$ ,  $N=1$  and  $Gr=10^5$

For a good understanding of the effect of Richardson number on isotherms and streamlines, Fig. 4 shows the different cases for  $Ri$  ranging from 0.1 to 10 and  $Le=1$ ,  $N=1$  and  $Gr=10^5$ . For a high value of Richardson number ( $Ri=10$ ), the flow is supposed to be induced by buoyancy force sustained by the temperature gradient. The formation of one pair of cells in streamlines, one covers  $3/4$  of the annular space and the other one take this portion which remains, at the top of the right fin, so therefore the two cells do not have the same shape, since in the left portion, the forced flow caused by the rotation of outer cylinder

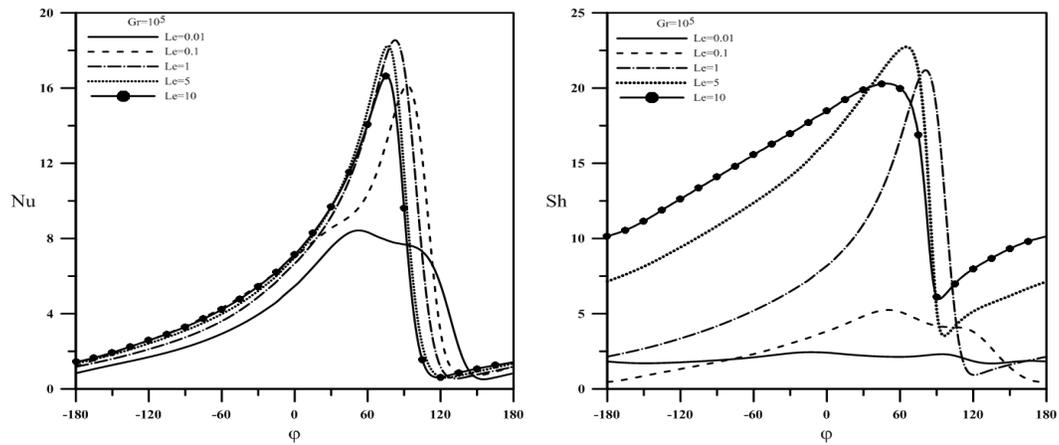
helps the natural flow, all this means that fins and rotation of the outer cylinder allow a more large path of the flow, knowing that the two vortices circulate in the opposite direction of each other. When  $Ri$  decrease, the effect of forced flow is deep, in the upper right prtion the vortex of this region is tightened and becomes stronger and bigger, in addition the lines of separation encompass the small vortex. Moreover  $Ri$  decreases, the viscous force leads the two cells in the direction of rotation of the cylinder, at  $Ri=0.1$  the vortices disappear and the forced flow dominates the flow and the diagram of the flow stratifies in annulus and formation of two cells near the fins, this distribution is semilary to that of Couette flow, on the other hand for high value of  $Ri$  isotherms are represented by a thermal plume which appear on the inner cylinder and which are closed under the fins. In order to know the importance of natural convection compared to the forced convection effect induced by the rotation of the outer cylinder, the Richardson number gives us a good idea about that. The Lewis and buoyancy ratio numbers are kept to unity and  $Gr=10^5$ , this will highlight implications of  $Ri$  alone as shown in Fig. 5, knowing that Richardson number is varied from 0.1 to 10. For  $Ri=0.1$ , the flow configurations are characterized by Concentric lines, except in the region near the fins, the regime is convective, indicating that the forced convection is the dominant mode, the local Sherwood or Nusselt numbers are represented by the horizontal line, so they are constant around the outer cylinder. As the  $Ri$  increases, the natural flow becomes deep and the local  $Nu$  or  $Sh$  increases. The local numbers of  $Sh$  or  $Nu$  reach maximum values for  $\Phi=85$  (position of the thermal plume).

### ***Effect of Lewis number***





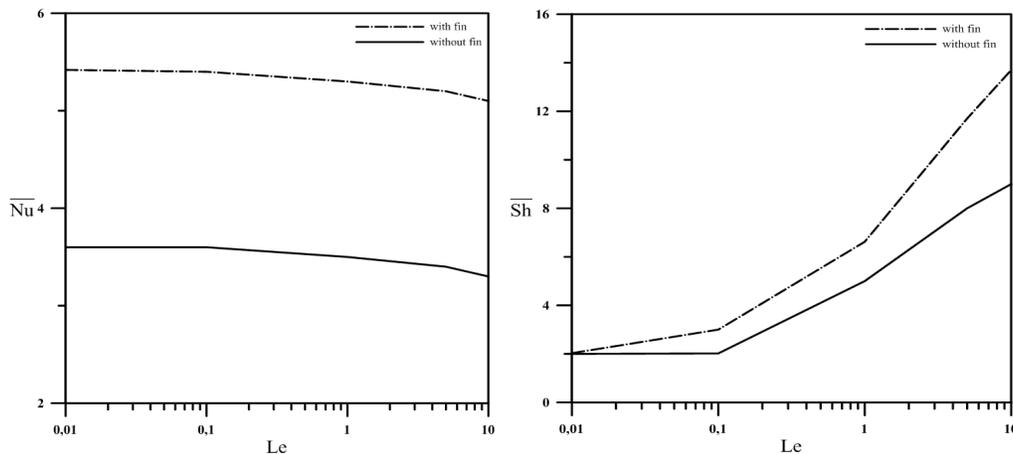
**Figure 6:** Isotherms, isoconcentration and streamlines for  $N=1$ ,  $Gr=10^5$ ,  $Ri=1$  and different  $Le$



**Figure 7:** Effect of Lewis number on local Nusselt and Sherwood numbers for  $Gr=10^5$ ,  $Ri=1$  and  $N=1$

The Lewis number gives us a measure of thermal diffusivity of a fluid at its mass diffusivity, Fig. 6 shows the dependence of heat and mass transfer on Lewis number. At  $Le=0.01$  (low

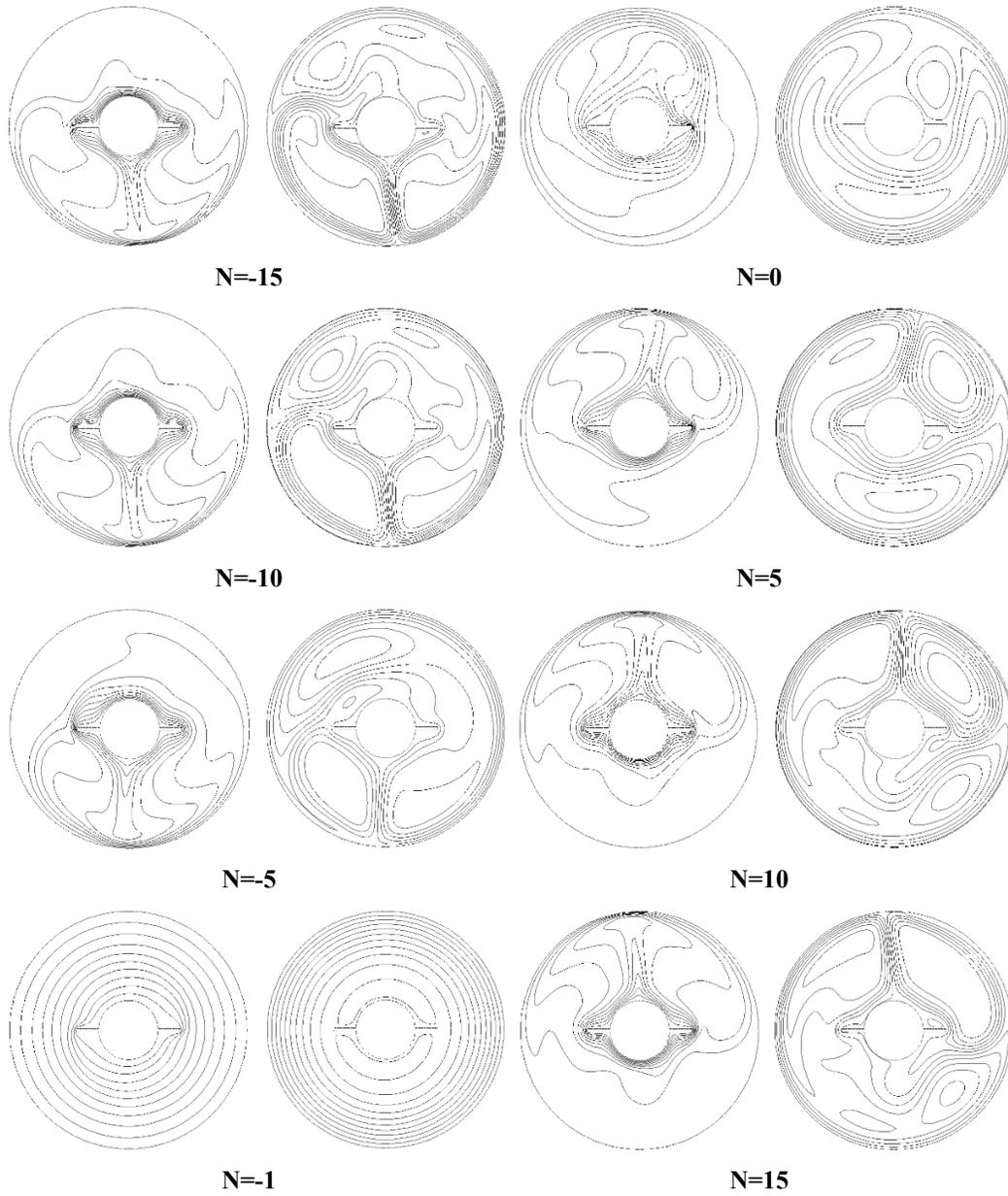
number of Lewis) the mass diffusion rate is stratified in the radial direction, at  $Le=0.1$  a slight increase in mass transfer is noticed and just after at  $Le=1$ , the isoconcentrations and isotherms have the same contour profiles since they have the same diffusion characteristics. For Lewis numbers greater than 1 a concentration plume emerging above the inner cylinder which indicates an increase in mass transfer and thinner solutal boundary layers are clustered under the fins and inner cylinder, this indicates a considerable increase mass transfer rate, this is a big advantage in the drying process, more than Lewis increases the root of the plume concentration becomes very thin, here the dominant regime is mass transfer and Lewis effect is insignificant on isotherms. The effect of Lewis number on local Nusselt and Sherwood numbers over the outer cylinder is illustrated in Fig. 7. For Richardson numbers and buoyancy ratio numbers are maintained to unity and  $Gr=10^5$ . For a low Lewis number  $Le=0.01$  the local Sherwood number is almost constant unlike the local Nusselt, as the  $Le$  increased, local Sherwood and Nusselt numbers increase. Local  $Nu$  reaches a maximum value for  $Le=1$ , after which it decreases slightly knowing that the maximum values are reached at the location where the thermal plume is located. The effect of Lewis number on the average Sherwood and Nusselt numbers is presented in Fig. 8, in order to encompass all the remarks made on Lewis number effect and at the same time, a comparison between the configuration with and without fin. All this to clearly see the improvement brought by the use of fins. Indeed, these considerations essentially allow us to focus on the enhancement provided by the use of fins.



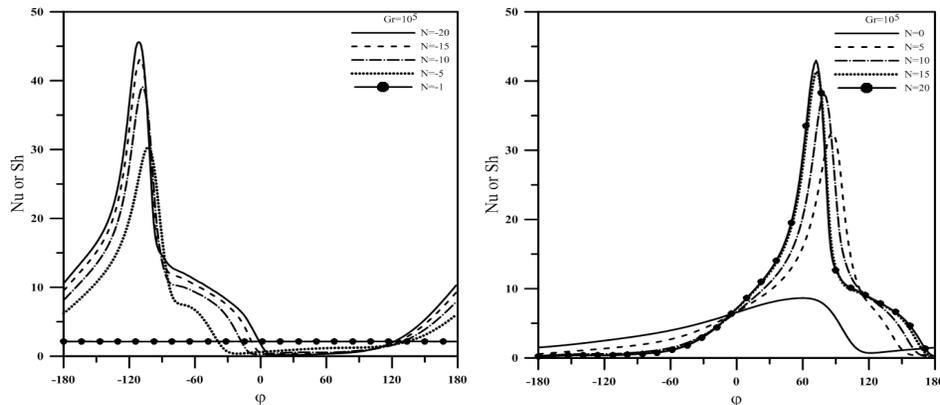
**Figure 8:** Effect of Lewis number on the average Sherwood and Nusselt numbers for  $Ri=1$ ,  $N=1$  and  $Gr=10^5$

According to Fig. 8 the Lewis number doesn't contribute to the improvement of heat transfer, moreover for the large Lewis number, the average  $Nu$  decreases slightly unlike the other case, when Lewis number is increased, the mass transfer regime is dominant. About the use of the fins on the heated inner cylinder, heat and mass transfer are significantly improve.

***Effect of buoyancy ratio number***



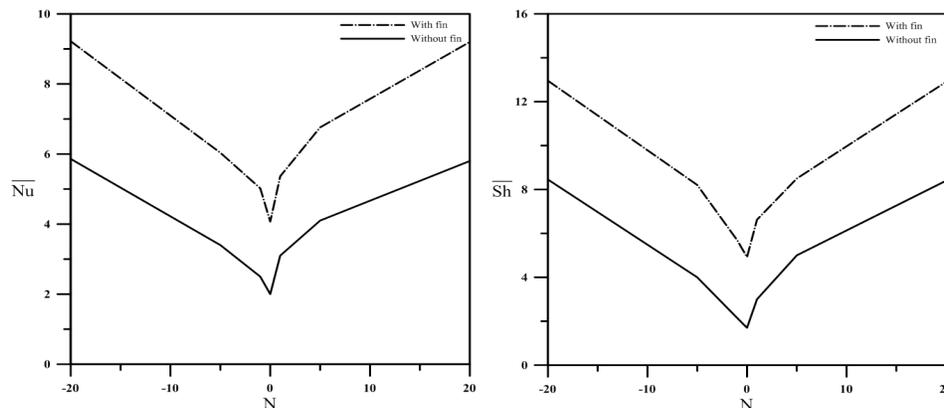
**Figure 9:** Effect of buoyancy ratio on streamlines and isotherms for  $Le=1$ ,  $Ri=1$



**Figure 10:** Effect of buoyancy ratio on local Nusselt and Sherwood numbers for  $Gr=10^5$ ,  $Ri=1$  and  $le=1$

The effect of buoyancy ratio number is shown in Fig. 9, which is defined as the ratio of mass buoyancy to thermal buoyancy forces, in order to elude the effect of positive and negative  $N$  on isotherms and streamlines for  $N$  values (-15 to 15). Negative  $N$  values inform us that the volumetric expansion coefficient with mass fraction takes a negative value, for the prescribed temperature range. At  $N=-15$  the dominant regime is mass transfer, two cells that forms, a small bottom left of the fin and a large one that is encompassed by the separating streamline, which is also remarkable is that the cells of right are trained up on the left. Similarly, for the isotherms, the thermal plume moved towards the lower part for negative values of  $N$ . We then notice for the negative values of  $N$ , basic flow patterns and isotherms undergo an inversion with respect to the median line of positive value of  $N$ . In addition the resistance to flow is reduced more with the increase of  $N$  negative when the cells in the right region continue to grow in size. At  $N=-1$  heat and mass diffusion are opposing each other, all eddies dissociate and become concentric circles and dissipation of thermal plume, in this case the forced convection has overwhelmed the diffusion in the enclosure, which has led to the stratification of isotherms. At  $N=0$  there is reappearance of the two eddies, so we are in the problem of pure mixed thermal convection since  $\beta_s=0$ . After the increase of the size of cells in left portion, which is due to the fact that the forced flow caused by the rotation of the outer cylinder helps the flow induced by the buoyancy force in the upper part on the annulus, which is also remarkable, is that in the contours of isotherms of  $N$  positive the higher diffusion rates have moved down. Whenever  $N$  is increased, the cells on right portion become larger, this is attributed to the reduction in the magnitude of the viscous drag in the left region and all of this means that we get higher flow activities and diffusion rates. The dependence of the local Sherwood and Nusselt numbers on the outer cylinder on the buoyancy ratio number is presented in Fig. 10, for negative and positive values of  $N$  which varies between -20 to 20 by keeping  $Le$  and  $Ri$  at unity. The numbers of local  $Sh$  and  $Nu$  are constant, which means that the conduction mode is predominant for the heat and mass transfer, the local values increase when the absolute value of buoyancy ratio is increased and the mixed convection is pronounced, since negative and positive values of  $N$  have an inverted shape along the horizontal center line of annulus. Furthermore, the absolute value of  $N$  increases. Natural convection is

dominated and the offset of the minimum and maximum values is decreased. The effect of Sherwood and Nusselt numbers is presented in Fig. 11, same as previously with the effect of Lewis number, here again a comparison was made between a case with and without fins. According to these figures, buoyancy ratio number contributes to the improvement of heat and mass transfer. For high absolute values of  $N$ , the  $Sh$  and  $Nu$  average numbers increase. At  $N=0$ , absence of the effect of buoyancy ratio number, which means that we are in pure mixed thermal convection problem where  $Sh$  and  $Nu$  average numbers take a minimal value. As for the comparison between the two cases (with and without fin), it is clearly seen that the use of fins improves in an unspeakable manner the heat and mass transfer.



**Figure 11:** Effect of buoyancy ratio number on the average Sherwood and Nusselt numbers for  $Ri=1$ ,  $Le=1$  and  $Gr=10^5$

## 6 Conclusion

Numerical investigation of heat and mass transfer in annulus from a finned cylinder has been carried out. The numerical results obtained led to the following conclusions:

- As the Richardson number increases, the heat and mass transfer is increased.
- By increasing the Lewis number, the mass transfer is increased but without any influence on heat transfer, that is very sought in the drying processes.
- Increasing the absolute value of the buoyancy ratio number, heat and mass transfer are increased.
- To show the interest of providing inner cylinder with fins, a comparison has been made on the case without fins, according to this comparison, the two fins enhance greatly the heat and mass transfer.

## References

**Arbaban, M.; Salimpour, M. R. (2014):** Enhancement of laminar natural convective heat transfer in concentric annuli with radial fins using nanofluids. *Heat Mass Transfer*, vol. 51, no. 3, pp. 353-362.

**Al-Amiri, A. M.; Khanafer, K. M.** (2005): Unsteady numerical simulation of double diffusive convection heat transfer in a pulsating horizontal heating annulus. *Heat Mass Transfer*, vol. 42, no. 11, pp. 1007-1015.

**Al-Amiri, A. M.; Khanafer, K. M.** (2006): Numerical simulation of double-diffusive mixed convection within a rotating horizontal annulus. *International Journal of Thermal Sciences*, vol. 45, pp. 567-578.

**Bubnovich, V. I.; Kolesnikov, P. M.** (1986): Conjugate transient heat transfer in laminar natural convection in a horizontal cylindrical annulus. *Inzhenerno-Fizicheskii Zhurnal*, Vol. 51, no. 4, pp. 576-583

**Chen, S.; Tölke, J.; Krafczyk, M.** (2010): Numerical investigation of double-diffusive (natural) convection in vertical annuluses with opposing temperature and concentration gradients. *International Journal of Heat and Fluid Flow*, vol. 31, pp. 217-226.

**Cheng, Y. J.; Liao, Y. H.; Huang, C. K.** (2008): Heat transfer on a radially rotating heated cylinder. *International Communications in Heat and Mass Transfer*, vol. 35, pp. 1355-1359.

**Chen, H. T.; Hsu, W. L.** (2007): Estimation of heat transfer coefficient on the fin of annular-finned tube heat exchangers in natural convection for various fin spacings. *International Journal of Heat and Mass Transfer*, vol. 50, pp. 1750-1761.

**Chen, H. T.; Hsu, W. L.** (2008): Estimation of heat-transfer characteristics on a vertical annular circular fin of finned-tube heat exchangers in forced convection. *International Journal of Heat and Mass Transfer*, vol. 51, pp. 1920-1932.

**Fabbri, G.** (2005): Optimum cross-section design of internally finned tubes cooled by a viscous fluid. *Control Engineering Practice*, vol. 13, pp. 929-938.

**Fusegi, T.; Farouk, B.; Ball, K. S.** (1986): Mixed-convection flows within a horizontal concentric annulus with a heated rotating inner cylinder. *Numerical Heat Transfer*, vol. 6, pp. 591-604.

**Hsu, P. T.** (2008): The inverse estimation of the thermal behavior and the viscosity of fluid between two horizontal concentric cylinders with rotating inner cylinder. *Applied Thermal Engineering*, vol. 28, pp. 380-387.

**Haldar, S. C.; Kochhar, G. S.; Manohar, K.; Sahoob, R. K.** (2007): Numerical study of laminar free convection about a horizontal cylinder with longitudinal fins of finite thickness. *International Journal of Thermal Sciences*, vol. 46, pp. 692-698.

**Huisseune, H.; T'Joens, C.; De Jaeger, P.; Willockx, A.; De Paepe, M.** (2010): Study of junction flows in louvered fin round tube heat exchangers using the dye injection technique. *Experimental Thermal and Fluid Science*, vol. 34, pp. 1253-1264.

**Kiwan, S.; Zeitoun, O.** (2008): Natural convection in a horizontal cylindrical annulus using porous fins. *International Journal of Numerical Methods for Heat & Fluid Flow*, vol. 18, pp. 618-634.

**Kolesnikov, P. M.; Bubnovich, V. I.** (1988): Non-stationary conjugate free-convective heat transfer in horizontal cylindrical coaxial channels. *International Journal of Heat and Mass Transfer*, vol. 31, no. 6, pp. 1149-1156.

**Lee, J.; Kang, S. H.; Son, Y. S.** (1999): Experimental study of double-diffusive convection in a rotating annulus with lateral heating. *International Journal of Heat and Mass Transfer*, vol. 42, pp. 821-832.

**Lee, J.; Kang, S. H.; Son, Y. S.** (2000): Numerical study of multilayered flow regime in double-diffusive convection in a rotating annulus with lateral heating. *Numerical Heat Transfer*, vol. 38, pp. 467-489.

**Lee, T. S.** (1984): Numerical experiments with laminar fluid convection between concentric and eccentric heated rotating cylinder. *Numerical Heat Transfer*, vol. 6, pp. 77-87.

**Lee, T. S.** (1992): Numerical computation of fluid convection with air enclosed between the annuli of eccentric heated horizontal rotating cylinders. *Computers Fluids*, vol. 21, pp. 355-368.

**Patankar, S. V.** (1980): *Numerical Heat Transfer and Fluid Flow*. McGraw-Hill, New York.

**Rahnama, M.; Farhadi, M.** (2004): Effect of radial fins on two-dimensional turbulent natural convection in a horizontal annulus. *International Journal of Thermal Sciences*, vol. 43, pp. 255-264.

**Sparrow, E. M.; Preston, C. S.** (1986): Heat transfer from rotating annular fins, Transfert thermique par des ailettes angulaires tournantes. *International Journal of Heat and Mass Transfer*, vol. 29, pp. 831-839.

**Sultan, K. F.** (2014): Numerical appraisal of heat transfer and flow in concentric annuli permeable and impermeable fins by using nanofluids. *International Journal of Engineering Sciences & Research Technology*, vol. 3, pp. 2277-9655.

**Sourour, M.; Teamah, M.; El Maghlany, W.** (2013): Numerical simulation of double diffusive laminar mixed convection in a horizontal rotating annulus. *CFD Letters*, vol. 6, no. 1.

**Sung, H. J.; Cho, W. K.; Hyun, J. M.** (1993): Double-diffusive convection in a rotating annulus with horizontal temperature and vertical solutal gradients. *International Journal of Heat and Mass Transfer*, vol. 36, pp. 3773-3782.

**Schiroky, G. H.; Rosenberger, F.** (1984): Free convection of gases in a horizontal cylinder with differentially heated end walls. *International Journal of Heat Mass Transfer*, vol. 27, no. 4, pp. 587-598.

**Teamah, M. A.** (2007): Numerical simulation of double diffusive laminar mixed convection in a horizontal annulus with hot, solutal and rotating inner cylinder. *International Journal of Thermal Sciences*, vol. 46, pp. 637-648.

**Tzeng, S. C.** (2006): Heat transfer in a small gap between co-axial rotating cylinders. *International Communications in Heat and Mass Transfer*, vol. 33, pp. 737-743.

**Watel, B.; Harmand, S.; Desmet, B.** (1998): Etude des échanges convectifs sur un arbre aileté tournant, soumis à un courant d'air parallèle aux ailettes. *International Journal of Heat and Mass Transfer*, vol. 41, pp. 3741-3757.

**Yoo, J. S.** (1998): Mixed convection of air between two horizontal concentric cylinders with a cooler rotating outer cylinder. *International Journal of Heat and Mass Transfer*, vol. 41, pp. 293-302.