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Characteristics of a Hydromagnetic Non-Newtonian Squeeze Film Between Wide Parallel Rectangular Plates

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Abstract: The characteristics of a hydromagnetic non-Newtonian squeeze film formed between parallel rectangular plates under the application on an external magnetic field are investigated. A specific hydromagnetic non-Newtonian Reynolds equation is derived via pplication of the hydromagnetic flow theory together with the micro-continuum theory. It is found that the coupled effects of electrically conducting fluids and micropolar fluids result in a higher load capacity and a longer approaching time with respect to the non-conducting Newtonian case. These improved characteristics become more pronounced as the magnetic Hartmann parameter, the coupling parameter, and the fluid-gap parameter are increased.

Keywords: hydromagnetic flows, micropolar fluids, rectangular plates, squeeze film characteristics.

1 Introduction

Study of squeeze film behaviors plays an important role in bio-lubrication and applied sciences Traditional, the analyses of squeeze films focus on the use of a Newtonian viscous lubricant, such as the works by Hays (1963), Wu (1972) and Ramanaiah and Kesavan (1982). Because of the development of modern industries, the use of non-Newtonian complex fluids as lubricants has gained great interests. To describe the flow behavior of these kinds of complex fluids, a micro-continuum theory of micropolar fluids by Erigen (1965, 1966) has been generated. The theory of micropolar fluids takes into account the influences of micro-rotation, the couple stresses, and the body couples of microstructures. Many investigators have applied the micro-continuum theory of micropolar fluids of Erigen (1966) to study various kinds of squeeze film bearings, such as the hemispherical bearings by Sinha

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and Singh (1982), the sphere-plate mechanisms by Al-Fadhalah and Elsharkawy (2008), and the parallel plates by Prakash and Sinha (1976). From their results, the non-Newtonian effects of micropolar fluids provide an increased load capacity, and prolong the elapsed time to prevent the contact of approaching surfaces.

In order to prohibit the viscosity variation of lubricants with temperature, the use of electrically conducting fluids as lubricants has also received a great attention. According to the hydromagnetic flow theory of Cowling (1957), several studies in hydromagnetic squeeze films have been investigated, such as the sphere-plate mechanisms by Chou *et al.* (2003) the circular disks by Zueco and Beg (2010), and the rectangular plates by Lin (2003) and Bujurke and Kudenatti (2007). From their results, the squeeze film characteristics are improved by the use of electrically conducting fluids. Recently, Walicka and Walicki (2004) have applied the non-Newtonian micropolar conducting fluid model to investigate the pressure distribution in a curvilinear hydrostatic bearing.

In the present study, the hydromagnetic non-Newtonian squeeze film characteristics of parallel rectangular plates under the application on an external magnetic field are investigated Applying the hydromagnetic flow theory of Cowling (1957) together with the micro-continuum theory of Erigen (1966), a hydromagnetic non-Newtonian Reynolds equation will be derived. Comparing with the non-conducting Newtonian case, the coupled effects of electrically conducting fluids and micropolar fluids are discussed through the variation of the magnetic Hartmann parameter, the coupling parameter, and the fluid-gap parameter.

2 Analysis

On the basis of the hydromagnetic flow theory of Cowling (1957) together with the micro-continuum theory of Erigen (1966) in the absence of body forces and body couples, the equations governing the flow of an electrically non-Newtonian micropolar conducting fluid with an external magnetic field can be described by:

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + (2\mu + k)\nabla^2 \mathbf{V} + k\nabla \times \mathbf{\Omega} + \mathbf{J} \times \mathbf{B} \tag{2}$$

$$\rho j \frac{D\mathbf{\Omega}}{Dt} = \gamma \nabla(\nabla \cdot \mathbf{\Omega}) - \gamma \nabla \times (\nabla \times \mathbf{\Omega}) + k\nabla \times \mathbf{V} - 2k\mathbf{\Omega} \tag{3}$$

where $\boldsymbol{V}$ and $\mathbf{\Omega}$ is the linear velocity and micro-rotational velocity vectors, p is the pressure, ρ is the density, μ is the dynamic viscosity, j is the micro-inertia constant, γ and k are the spin gradient viscosity and vortex viscosity, $\mathbf{J}$ and $\mathbf{B}$ are the current density and applied magnetic field vectors.

For this study, the squeeze film geometry between wide parallel rectangular plates with an incompressible non-Newtonian micropolar conducting fluid is shown in Figure 1, where the upper plate is approaching the lower fixed plate with a squeezing velocity: $-d(2h)/dt$. An external magnetic field B_0 is applied along the $y-$direction. Assume that the thin-film lubrication theory is applicable. Then the governing equations can be written as following:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$0=-\frac{\partial p}{\partial x}+\frac{1}{2}(2\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial \Omega}{\partial y}-\sigma B_0^2 u \tag{5}$$

$$\gamma\frac{\partial^2 \Omega}{\partial y^2}-k\frac{\partial u}{\partial y}-2k\Omega=0 \tag{6}$$

where Ω is the micro-rotational velocity and σ represents the electrical conductivity of the fluid.

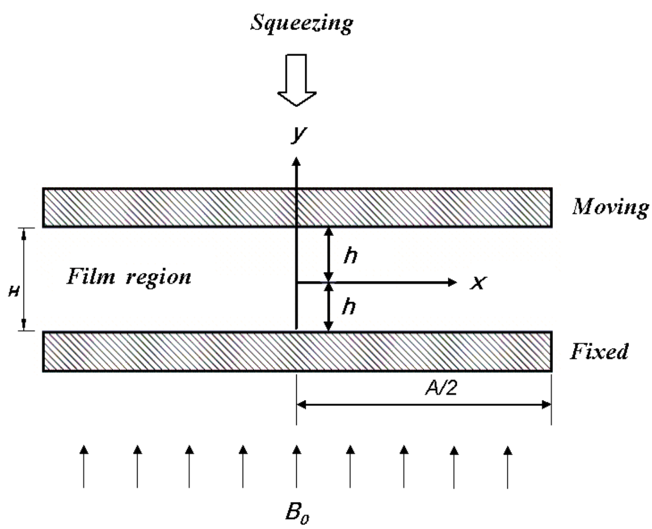

Figure 1: Squeeze film configuration for micropolar conducting fluid-lubricated rectangular plates.

The corresponding boundary conditions for the velocities at the plate surfaces are:

$$u=0, \quad \Omega=0 \quad \text{at} \quad y=-h \tag{7}$$

$$u=0, \quad \Omega=0 \quad \text{at} \quad y=+h \tag{8}$$

$$v=0 \quad \text{at} \quad y=-h \tag{9}$$

$$v=d(2h)/dt \quad \text{at} \quad y=+h \tag{10}$$

Eliminating Ω from equations (5) and (6) one can obtain the differential equation governing the velocity component u

$$\frac{\partial^4 u}{\partial y^4}-a\frac{\partial^2 u}{\partial y^2}+bu=f(x) \tag{11}$$

where

$$a=\frac{2(2\mu k+\gamma\sigma B_0^2)}{\gamma(2\mu+k)}, \quad b=\frac{4k\sigma B_0^2}{\gamma(2\mu+k)}, \quad f(x)=-\frac{4k}{\gamma(2\mu+k)}\frac{\partial p}{\partial x} \tag{12}$$

Integrating equation (11) yields

$$u=c_1\cosh(r_1y)+c_2\sinh(r_1y)+c_3\cosh(r_2y)+c_4\sinh(r_2y)+\frac{f}{b} \tag{13}$$

where c_1, c_2, c_3 and c_4 are integration constants to be determined. Using the expression of u and integrating equation (5) lead to

$$\begin{aligned}\Omega =&c_1F_1[\sinh(r_1y)+\sinh(r_1h)]+c_2F_1[\cosh(r_1y)-\cosh(r_1h)]\\&+c_3F_2[\sinh(r_2y)+\sinh(r_2h)]+c_4F_2[\cosh(r_2y)-\cosh(r_2h)]\end{aligned} \tag{14}$$

Using the expression of u and integrating equation (6) give

$$\begin{aligned}\Omega =&c_1\frac{r_1}{2k}\sinh(r_1y)\left[\gamma F_1r_1-k\right]+c_2\frac{r_1}{2k}\cosh(r_1y)\left[\gamma F_1r_1-k\right]\\&+c_3\frac{r_2}{2k}\sinh(r_2y)\left[\gamma F_2r_2-k\right]+c_4\frac{r_2}{2k}\cosh(r_2y)\left[\gamma F_2r_2-k\right]\end{aligned} \tag{15}$$

where

$$r_1=\sqrt{\frac{a+\sqrt{a^2-4b}}{2}}, \quad r_2=\sqrt{\frac{a-\sqrt{a^2-4b}}{2}} \tag{16}$$

$$F_1=\frac{2\sigma B_0^2-(2\mu+k)r_1^2}{2kr_1}, \quad F_2=\frac{2\sigma B_0^2-(2\mu+k)r_2^2}{2kr_2} \tag{17}$$

Applying the four boundary conditions (7) and (8) to equations (13), (14) and (15), the four integration constants can be determined. After arranging the equation, the expression of u can derived.

$$u = -\frac{\partial p}{\partial x}\frac{F_2\sinh(r_2h)\,[\cosh(r_1h)-\cosh(r_1y)]-F_1\sinh(r_1h)\,[\cosh(r_2h)-\cosh(r_2y)]}{\sigma B_0^2[F_2\sinh(r_2h)\cosh(r_1h)-F_1\sinh(r_1h)\cosh(r_2h)]} \tag{18}$$

Now integrating the continuity equation (4) across the film thickness and applying the corresponding boundary conditions, one can derive a hydromagnetic non-Newtonian Reynolds equation for the parallel squeeze film plates.

$$\frac{r_2F_2\sinh(r_2h)[r_1h\cosh(r_1h)-\sinh(r_1h)]-r_1F_1\sinh(r_1h)[r_2h\cosh(r_2h)-\sinh(r_2h)]}{r_1r_2\sigma B_0^2[F_2\sinh(r_2h)\cosh(r_1h)-F_1\sinh(r_1h)\cosh(r_2h)]}\frac{d^2p}{dx^2} = \frac{dh}{dt} \tag{19}$$

Introduce the non-dimensional variables and parameters.

$$x^* = \frac{x}{A}, \quad H^* = \frac{H}{H_0} = \frac{2h}{H_0}, \quad p^* = \frac{pH_0^3}{\mu A^2(-dH/dt)}, \quad \Phi(H^*,N,L,M) = \frac{\Phi_U}{\Phi_D} \tag{20}$$

$$\Phi_U = 24\Big\{r_2{}^*F_2{}^*\sinh(0.5r_2{}^*H^*)\,[0.5r_1{}^*H^*\cosh(0.5r_1{}^*H^*)-\sinh(0.5r_1{}^*H^*)] - r_1{}^*F_1{}^*\sinh(0.5r_1{}^*H^*)\,[0.5r_2{}^*H^*\cosh(0.5r_2{}^*H^*)-\sinh(0.5r_2{}^*H^*)]\Big\} \tag{21}$$

$$\frac{\Phi_D}{M^2r_1^*r_2^*} = F_2^*\sinh(0.5r_2^*H^*)\cosh(0.5r_1H^*) - F_1^*\sinh(0.5r_1^*H^*)\cosh(0.5r_2^*H^*) \tag{22}$$

$$F_1^* = F_1H_0 = \frac{M^2(1-N^2)-r_1^{*2}}{2N^2r_1^*}, \quad F_2^* = F_2H_0 = \frac{M^2(1-N^2)-r_2^{*2}}{2N^2r_2^*} \tag{23}$$

$$r_1^* = r_1H_0 = \sqrt{\frac{a^*+\sqrt{a^{*2}-4b^*}}{2}}, \quad r_2^* = r_2H_0 = \sqrt{\frac{a^*-\sqrt{a^{*2}-4b^*}}{2}} \tag{24}$$

$$a^* = aH_0^2 = \frac{N^2+M^2(1-N^2)L^2}{L^2}, \quad b^* = bH_0^4 = \frac{N^2M^2}{L^2} \tag{25}$$

$$N = \left(\frac{k}{2\mu + k}\right)^{1/2}, \quad L = \frac{(\gamma/(4\mu))^{1/2}}{H_0}, \quad M = B_0 H_0 \left(\frac{\sigma}{\mu}\right)^{1/2} \tag{26}$$

where H_0 is the initial total film thicknesses, A denotes the length of the plates, N represents the non-Newtonian coupling parameter, L is the fluid-gap interacting parameter and M is the magnetic Hartmann parameter. Then the hydromagnetic non-Newtonian Reynolds equation can be expressed in a non-dimensional form.

$$\Phi(H^*, N, L, M) \cdot \frac{d^2 p^*}{dx^{*2}} = -12 \tag{27}$$

The corresponding boundary conditions for the film pressure are: $p^* = 0$ at $x^* = \pm 1/2$ and $dp^*/dx^* = 0$ at $x^* = 0$. Solving the Reynolds equation, one can obtain the film pressure.

$$p^* = \frac{6}{\Phi(H^*, N, L, M)} \cdot \left(\frac{1}{4} - x^{*2}\right) \tag{28}$$

Integrating the film pressure, the load capacity per unit width can be evaluated.

$$\frac{W}{B} = \int_{x=-A/2}^{A/2} p dx \tag{29}$$

where B denotes the width of the parallel rectangular plates. After performing the integration, the non-dimensional load capacity can be obtained.

$$W^* = \frac{W H_0^3}{\mu_0 A^3 B(-dH/dt)} = \frac{1}{\Phi(H^*, N, L, M)} \tag{30}$$

Now introduce the non-dimensional approaching time into the above equation.

$$t^* = \frac{W H_0^2}{\mu_0 A^3 B} \cdot t \tag{31}$$

The differential equation governing the film thickness varying the time can be derived.

$$\frac{dH^*}{dt^*} = -\,\Phi(H^*, N, L, M) \tag{32}$$

The initial condition for the film thickness at initial time is: $H^* = 1$at $t^* = 0$. Integrating the differential equation, one can obtain the height-time relationship.

$$t^* = \int_{H^*}^{H^*=1} \frac{dH^*}{\Phi(H^*, N, L, M)} \tag{33}$$

The non-dimensional approaching time can be numerically calculated by applying the Gaussian quadrature method [Spiegel and Liu (1999)].

3 Results and Discussion

From on the above analysis, the coupled effects of electrically conducting fluids and micropolar fluids on the squeeze film characteristics are dominated by three parameters defined in equation (26): the non-Newtonian coupling parameter N, the fluid-gap interacting parameter L and the magnetic Hartmann parameter M.

For $N = 0$ or $L = 0$, $M = 0$: the Newtonian, non-conducting case. The limiting case of the non-dimensional function Φ reduces to the results of Hamrock (1994).

$$\lim_{N=0(orL=0),M=0} \Phi = H^{*3} \tag{34}$$

For $N \neq 0$ and $L \neq 0$, $M = 0$: the non-Newtonian, non-conducting case. The limiting case of reduces to the results of Prakash and Sinha (1976).

$$\lim_{N\neq 0,L\neq 0,M=0} \Phi = H^{*3} + 12L^2H^* - 6NLH^{*2}\coth(NH^*/2L) \tag{35}$$

where the fluid-gap interacting parameter L has been defined as the present analysis.

For $N = 0$ or $L = 0$, $M \neq 0$: the Newtonian, electrically conducting case. The limiting case of Φ reduces to the derivation of Lin (2003).

$$\lim_{N=0(orL=0),M\neq 0} \Phi = \frac{12MH^* - 24\tanh(0.5MH^*)}{M^3} \tag{36}$$

For this study, the squeeze film performances are illustrated for $N \neq 0$, $L \neq 0$ and $M \neq 0$. Figure 2 presents the film pressure p^* versus the non-dimensional coordinate x^* for different values of N, L and M under H^*=0.4. As compared to the Newtonian non-conducting case (N=0 or L=0,M=0), the non-Newtonian effects of micropolar fluids (N=0.4, L=0.4, M=0) result in a higher squeeze film pressure. When the electrically conducting non-Newtonian fluids (N=0.4, L=0.4, M=5; N=0.8, L=0.8, M=5; N=0.8, L=0.8, M=7.5) are applied, larger increments of the film pressure are predicted.

Figure 3 shows the load capacity W^* versus the film height H^* for different values of N, L and M. The load capacity is observed to increase with decreasing values of the film height. As compared to the Newtonian, non-conducting case (N=0 or L=0, M=0), the influences of non-Newtonian micropolar fluids (N=0.4, L=0.4, M=0) yield an increase in the load capacity. By the application of a non-Newtonian micropolar conducting fluid (N=0.4, L=0.4, M=5; N=0.8, L=0.8, M=5), further increments of the load capacity are obtained. Furthermore, increasing the value of the magnetic Hartmann parameter (N=0.8, L=0.8, M=7.5) increases the effects of non-Newtonian micropolar conducting fluids on the squeeze film load capacity.

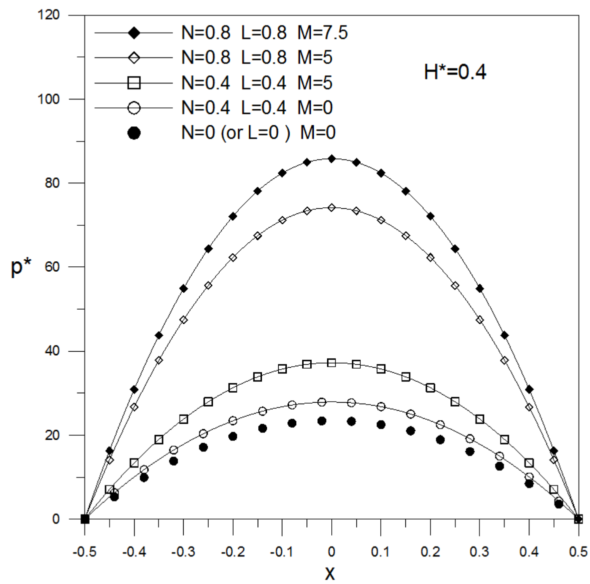

Figure 2: The film pressure p^* versus the coordinate x^* for different N, L and M.

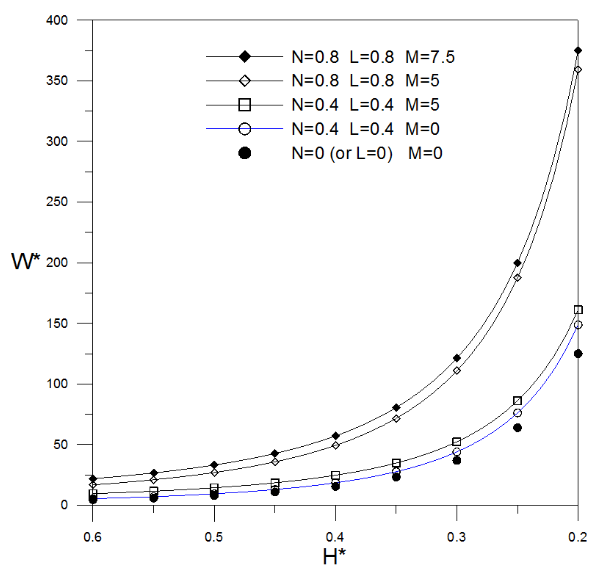

Figure 3: The load capacity W^* versus the film height H^* for different N, L and M.

Figure 4 presents the film height H^* versus the approaching time t^* for different values of N, L and M. Comparing with the Newtonian, non-conducting case (N=0 or L=0, M=0), the effects of micropolar fluids (N=0.4, L=0.4, M=0) increase the approaching time. When the micropolar conducting fluids (N=0.4, L=0.4, M=5; N=0.8, L=0.8, M=5; N=0.8, L=0.8, M=7.5) are applied, further increases of the approaching time are provided. These phenomena can be realized that since the micropolar conducting fluids result in a higher film pressure and then a higher load capacity, a higher film height would be expected for the same time to be taken as compared to the Newtonian, non-conducting case. A longer approaching time is therefore predicted for the squeeze film plates lubricated with a non-Newtonian micropolar conducting fluid.

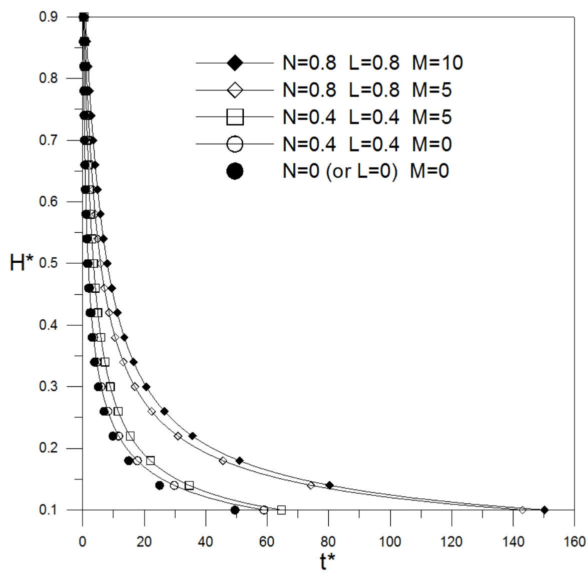

Figure 4: The film height H^* versus the approaching time t^* for different N, L and M.

Figure 5 displays the effect of the coupling parameter N on the load capacity W^* under H^*=0.4 and L=0.8. The variation of coupling parameter provides an increase in the load capacity. Comparing with the non-conducting case ($M = 0$), the use of electrically conducting fluids with external magnetic fields (M=2.5; 5; 7.5; 10) results in further higher values of the load.

Figure 6 shows the effect of the fluid-gap interacting parameter L on the load capacity W^* under H^*=0.4 and N=0.8. It also shows that the load capacity increases with increasing values of the fluid-gap interacting parameter. In addition, the elec-

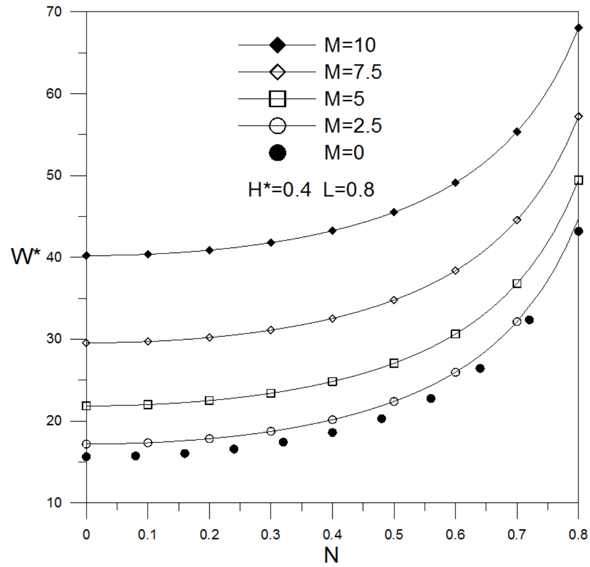

Figure 5: Effect of the coupling parameter N on the load capacity W^* for different M.

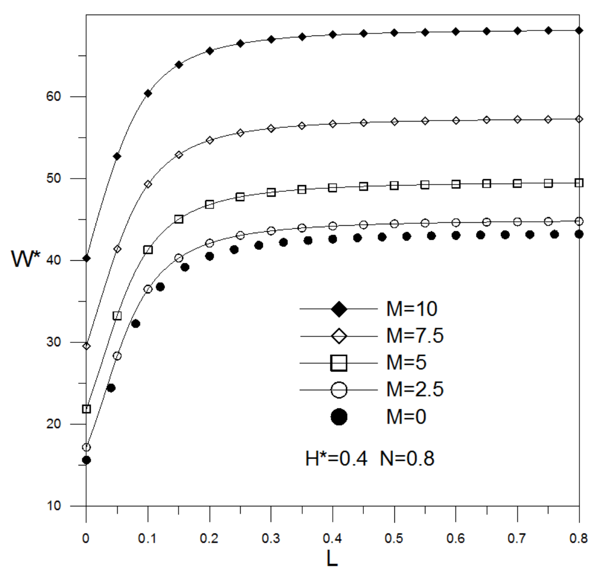

Figure 6: Effect of the fluid-gap interacting parameter L on the load capacity W^* for different M.

trically conducting films (M=2.5; 5; 7.5; 10) signify an increase in the load capacity as compared to the non-conducting case ($M = 0$).

Figure 7 represents the effect of the magnetic Hartmann parameter M on the load capacity W^* for different N and L under H^*=0.4. As compared to the Newtonian (N=0 or L=0) non-conducting case (M=0) case, the load capacity increases with increasing values of the magnetic Hartmann parameter for the plates lubricated with Newtonian (N=0 or L=0) conducting fluids (M >0). As compared to the Newtonian (N=0 or L=0) case, the non-Newtonian effects of micrpolar fluids (N=0.4, L=0.4; N=0.4, L=0.8; N=0.6, L=0.8; N=0.7, L=0.8; N=0.8, L=0.8) provide further increments of the load, especially for a larger magnetic Hartmann parameter. On the whole, the combined effects of non-Newtonian micropolar fluids and electrically conducting fluids provide a higher load capacity and a longer approaching time for the parallel rectangular plates as compared to the Newtonian non-conducting case. Specific numerical results of hydromagnetic non-Newtonian squeeze film characteristics for different values of N, L and M under H^*=0.5 are also included in Table 1 for engineering references.

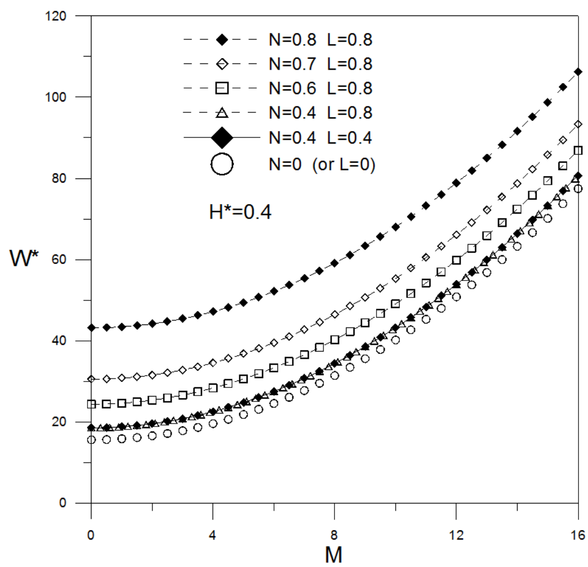

Figure 7: Effect of the Hartmann parameter M on the load capacity W^* for different N and L.

Table 1: Hydromagnetic non-Newtonian load capacity and approaching time for different N, L and M under the film height H^*=0.5.

Load capacity W^*					
	$M=0$	$M=2.5$	$M=5$	$M=7.5$	$M=10$
Newtonian	8.000	9.248	12.965	19.087	27.532
$N=0.4, L=0.4$	9.516	10.765	14.487	20.627	29.111
$N=0.4, L=0.8$	9.522	10.770	14.493	20.632	29.116
$N=0.8, L=0.8$	22.060	23.310	27.052	33.256	41.883
Approaching time t^*					
Newtonian	1.500	1.932	3.209	5.296	8.158
$N=0.4, L=0.4$	1.783	2.215	3.496	5.592	8.471
$N=0.4, L=0.8$	1.785	2.217	3.497	5.594	8.473
$N=0.8, L=0.8$	4.111	4.545	5.838	7.973	10.924

4 Conclusions

The characteristics of a hydromagnetic non-Newtonian squeeze film related to parallel rectangular plates have been investigated by applying the hydromagnetic flow theory together with the micro-continuum theory. Specific Reynolds equations have been obtained for sucha purpose. It is found that the hydromagnetic non-Newtonian effects leads to an increase in the load capacity and in the approaching time as compared to the non-conducting Newtonian case. The improved squeeze film characteristics are more pronounced when the plates work with larger values of the magnetic Hartmann parameter, the coupling parameter, and the fluid-gap parameter. Some numerical results for the load capacity and the approaching time considering the hydromagnetic non-Newtonian effects have been expressly provided in a Table for engineering references.

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