

Effects of Non-Newtonian Ferrofluids on the Performance Characteristics of Long Journal Bearings

J.R. Lin¹, P.J. Li² and T.C. Hung³

Abstract: On the basis of the Shliomis ferrofluid model (1972) together with the micro-continuum theory of Stokes (1966), the influences of non-Newtonian ferrofluids on the steady-state performance of long journal bearings have been investigated in the present paper. Analytical solutions for bearing performances are obtained from the non-Newtonian ferrofluid Reynolds-type equation. Comparing with the Newtonian non-ferrofluid case, the effects of non-Newtonian ferrofluids with applied magnetic fields provide an increase in the zero pressure-gradient angle and the load capacity, and a decrease in the friction parameter, especially for a larger non-Newtonian couple stress parameter and magnetic Langevin's parameter. For the long journal bearing lubricated with non-Newtonian ferrofluids and operating at larger couple stress parameter and magnetic Langevin's parameter (for example, $N=0.3$, $\xi=50$), slight variation is observed for the friction parameter, but higher bearing loads are obtained with increasing values of the volume concentration parameter.

Keywords: long journal bearings, couple stress fluids, ferrofluids, magnetic fields.

1 Introduction

Ferrofluids are made from a stable colloidal suspension of tiny magnetic particles in a liquid such as water or oil. Since NASA was looking for different methods for controlling liquids in space and discovered ferrofluids in the 1960's, there has been a tendency for increasing the application of ferrofluids. It is found that as the magnetic fields are applied to the ferrofluids, each ferromagnetic particle experiences a

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force that changes the angular momentum and influences the rotation of the spacecraft. Goldowsky (1980) proposed many new methods of ferrofluid applications for sealing, filtering and lubricating in thermal converter. The new and successful applications of ferrofluid lubrication solve the difficult problems in operating life and high reliability for thermal converter. In medicine, ferrofluids can also be applied for cancer detection by using contrast agents for magnetic resonance imaging. Several studies have been reported in the fields of ferrofluid lubrication. For instance, Urreta *et al.* (2010) has shown the experiment results of hydrodynamic bearing lubricated with the magnetic fluids in comparison with the theoretical results. It is found that the theoretical results agree well with the experimental data which achieve good performance as using active magnetorheological fluids in the bearing. Based upon the ferrofluid model of Neuringer-Rosensweig (1964), Osman *et al.* (2001) found that journal bearing characteristics are significantly improved when the magnetic influences are comparable with the hydrodynamic effects. In order to compare with the ferrofluid model of Neuringer-Rosensweig (1964), Jenkins (1972) and Shliomis (1972), the squeeze film behaviors in ferrofluid lubricated long journal bearings in the presence of uniform and non-uniform magnetic fields were investigated by Shah and Bhat (2004). According to their results, the consideration of rotational viscosity of ferromagnetic particles in Shliomis model (1972) could affect the squeeze film bearing characteristics. In slider bearings, Shukla and Kumar (1987) derived the lubrication equation for ferrofluid lubrication using Shliomis model (1972). It is found that increasing the magnetic fields and Brownian relaxation time can increase the load capacity and reduce the friction parameter. In addition, the effects of ferrofluid lubrication on the bearing performances are apparent as compared to the conventional lubrication. On the other hand, several works have investigated the effect of non-Newtonian couple stresses by applying the Stokes micro-continuum theory (1966) on the performance of the journal bearings, such as the works in short bearings by Lin (1997 a) and Naduvinamani *et al.* (2001); long bearings by Lin (1997 b) and Naduvinamani *et al.* (2005); and finite bearings by Lin (1997c), Elsharkawy and Guedouar (2001) and Elsharkawy (2005) According to their results, the non-Newtonian lubricated bearing characteristics are better than those of the Newtonian lubrication. Recently, based upon the ferrofluid model of Neuringer-Rosensweig (1964) and the Stokes micro-continuum theory (1966), the coupled effects of non-Newtonian couple stresses and ferrofluids on the journal bearing performances were investigated by Nada and Osman (2007). It is concluded that the performances of journal bearings lubricated by non-Newtonian ferrofluids can be improved. However, the effects of rotational viscosity of ferromagnetic particles are not considered. Since the inclusion of rotational viscosity of ferromagnetic particles using Shliomis model (1972) may influence the squeeze film bearing performances, an investigation on the short journal bearing

is carried out by Lin *et al.* (2013). According to their results, the consideration of non-Newtonian couple stresses provides an improvement in performance characteristics of ferrofluid journal bearings. However, we have no idea that how the non-Newtonian ferrofluids affect the lubrication performances of journal bearings when the bearing length tends to be long. Therefore, a further study is motivated in the long journal bearings

Based upon the ferrofluid model of Shliomis (1972) and the micro-continuum theory of Stokes [9], the objective of this paper is to investigate the effects of non-Newtonian couple stress ferrofluids on the steady-state performance of long journal bearings. Analytical expressions for the zero pressure-gradient angle, load capacity and friction parameter are derived. Comparing with the case of Newtonian non-ferrofluids, the influences of non-Newtonian ferrofluids on the bearing characteristics are presented through the variation of the couple stress parameter, the volume concentration parameter and the magnetic Langevin's parameter.

2 Analysis

According to the ferrofluid model of Shliomis (1972) and the micro-continuum theory (1966), the field equations of an incompressible isothermal non-Newtonian couple stress ferrofluid neglecting the body couples and the second derivative of the internal angular momentum but retaining the magnetic body force can be written as:

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \mu \nabla^2 \vec{V} + \mu_M (\vec{M} \cdot \nabla) \vec{H} + \frac{1}{2\tau_s} \nabla \times (\vec{S} - I\vec{\Omega}) - \eta \nabla^4 \vec{V} \tag{2}$$

where

$$\vec{S} = I\vec{\Omega} + \mu_M \tau_s (\vec{M} \times \vec{H}) \tag{3}$$

$$\vec{M} = M_0 \frac{\vec{H}}{H_0} + \frac{\tau_B}{I} (\vec{S} \times \vec{M}) \tag{4}$$

In these equations, \vec{V} is the velocity vector, t is the time, ρ is the fluid density, p is the pressure, μ is the viscosity of the suspension, μ_M is the permeability of free space, η is a material constant responsible for the couple stress fluids property, τ_s is the magnetic moment relaxation time, τ_B is the Brownian relaxation time, I is the sum of moments of inertia of the particles per unit volume, $\vec{\Omega} = (1/2)\nabla \times \vec{V}$, H_0 is the magnitude of the applied magnetic field \vec{H} , and M_0 is the equilibrium magnetization of the magnetization vector \vec{M} .

Figure 1 shows the physical configuration of a long journal bearing lubricated with an incompressible isothermal non-Newtonian couple stress ferrofluid in the presence of a uniform applied magnetic field $\vec{H} = (0, H_0, 0)$. The journal is rotating with a uniform tangential velocity U within a rigid bearing housing. The film thickness is $h = C + e \cos \theta$, where C is the radial clearance denoting the difference between the bearing radius and the journal radius, e is the eccentricity. Assume that (i) the length of the bearing L is long as compared to the radius of journal R , (ii) the film is thin, (iii) there is no variation of pressure across the fluids film, and (iv) the inertia forces are negligible. Then equations (1) and (2) reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

$$\frac{\partial p}{\partial x} = \mu(1 + \tau) \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \quad (6)$$

$$\frac{\partial p}{\partial y} = 0 \quad (7)$$

where τ is the rotational viscosity parameter,

$$\tau = \frac{3}{2} \phi \frac{\xi - \tanh \xi}{\xi + \tanh \xi} \quad (8)$$

The detailed derivation of equation (6) is similarly derived as in the work of Lin *et al.* (2013). And ϕ is the volume concentration of ferrofluid particles, ξ is the Langevin's parameter measuring the strength of applied magnetic fields as Shliomis (1972).

$$\xi = \frac{\mu_M m H_0}{k_B T} \quad (9)$$

where m is the magnetic moment of a particle, k_B is the Boltzmann constant, and T is the temperature. In addition, the viscosity of the suspension can be approximated by the Einstein formula according to Shliomis (1972)

$$\mu = \mu_0 (1 + 2.5\phi) \quad (10)$$

where μ_0 is the viscosity of the main liquid. The boundary conditions for the velocity components of the lubricant at the bearing surfaces and at the journal surfaces are

$$u = 0, \quad v = 0 \quad \text{at} \quad y = 0 \quad (11)$$

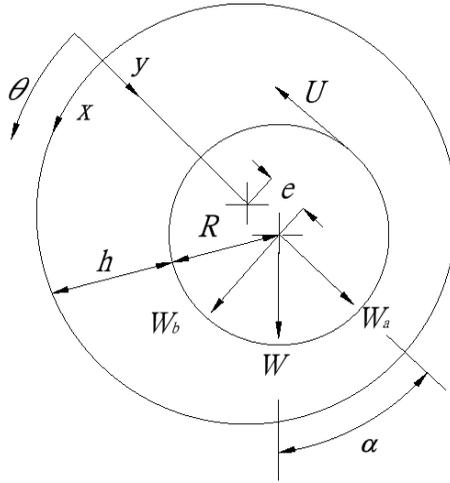


Figure 1: Physical configuration of a long journal bearing lubricated with a non-Newtonian ferrofluid

$$\frac{\partial^2 u}{\partial y^2} \Big|_{y=0} = 0 \tag{12}$$

$$u = U, \quad v = 0 \quad \text{at} \quad y = h \tag{13}$$

$$\frac{\partial^2 u}{\partial y^2} \Big|_{y=h} = 0 \tag{14}$$

Integrating equation (6) with respect to y by applying the related boundary conditions, the velocity component u can be derived.

$$u = \frac{y}{h}U + \frac{1}{2\mu_0(1+\tau)(1+2.5\varphi)} \frac{\partial p}{\partial x} \cdot \left\{ y^2 - hy + \frac{2l_c^2}{(1+\tau)(1+2.5\varphi)} \left[1 - \frac{\cosh\left(\frac{(2y-h)\sqrt{(1+\tau)(1+2.5\varphi)}}{2l_c}\right)}{\cosh\left(\frac{h\sqrt{(1+\tau)(1+2.5\varphi)}}{2l_c}\right)} \right] \right\} \tag{15}$$

where $l_c = \sqrt{\eta/\mu_0}$. By replacing the expression of u and integrating the continuity equation (5) with respect to y with the related boundary conditions, one can obtain the non-Newtonian ferrofluid Reynolds-type equation for the long journal bearing.

$$\frac{\partial}{\partial x} \left[f(h, l_c, \varphi, \tau) \frac{\partial p}{\partial x} \right] = 6\mu_0(1+\tau)(1+2.5\varphi)U \frac{\partial h}{\partial x} \tag{16}$$

where $f = f(h, l_c, \varphi, \tau)$ is given by

$$f = h^3 - \frac{12l_c^2}{(1 + \tau)(1 + 2.5\varphi)}h + \frac{24l_c^3}{(1 + \tau)^{3/2}(1 + 2.5\varphi)^{3/2}} \tanh\left(\frac{\sqrt{(1 + \tau)(1 + 2.5\varphi)}}{2l_c}h\right) \tag{17}$$

Expressing in a non-dimensional form leads to

$$\frac{\partial}{\partial \theta} \left[F(h^*, N, \varphi, \xi) \frac{\partial p^*}{\partial \theta} \right] = 6(1 + 2.5\varphi)(1 + \tau) \frac{\partial h^*}{\partial \theta} \tag{18}$$

where $\tau = \tau(\varphi, \xi)$ is defined in equation (8) and $F = F(h^*, N, \varphi, \xi)$ is given by

$$F = h^{*3} - \frac{12N^2}{(1 + \tau)(1 + 2.5\varphi)}h^* + \frac{24N^3}{(1 + \tau)^{3/2}(1 + 2.5\varphi)^{3/2}} \tanh\left(\frac{\sqrt{(1 + \tau)(1 + 2.5\varphi)}}{2N}h^*\right) \tag{19}$$

In addition, the non-dimensional variables and parameters are

$$p^* = \frac{pC^2}{\mu_0UR}, \quad \theta = \frac{x}{R}, \quad h^* = \frac{h}{C} = 1 + \varepsilon \cos \theta, \quad \varepsilon = \frac{e}{C}, \quad N = \frac{l_c}{C} \tag{20}$$

where N denotes the non-Newtonian couple stress parameter.

3 Performance and Characteristics

The pressure boundary conditions for the lubricant in the film region are the well-known Reynolds boundary conditions as Hamrock (1994).

$$p^* = 0 \quad \text{at} \quad \theta = 0 \tag{21}$$

$$p^* = 0 \quad \text{at} \quad \theta = \theta_c \tag{22}$$

$$\frac{\partial p^*}{\partial \theta} = 0 \quad \text{at} \quad \theta = \theta_c \tag{23}$$

where θ_c is the zero pressure-gradient angle for the lubricant film. Integrating the non-dimensional modified Reynolds equation once and applying the above boundary conditions, the zero pressure-gradient angle can be found from the following relationship.

$$\cos \theta_c = \frac{\int_{\theta=0}^{\theta=\theta_c} \frac{\cos \theta}{F(h^*, N, \varphi, \xi)} d\theta}{\int_{\theta=0}^{\theta=\theta_c} \frac{1}{F(h^*, N, \varphi, \xi)} d\theta} \tag{24}$$

Integrating the non-dimensional modified Reynolds equation twice and applying the above boundary conditions, one can obtain the film pressure.

$$p^* = 6\varepsilon(1 + 2.5\varphi)(1 + \tau) \int_{\theta=0}^{\theta=\theta_c} \frac{\cos \theta - \cos \theta_c}{F(h^*, N, \varphi, \xi)} d\theta \quad (25)$$

Integrating the film pressure over the journal surface, one can obtain the load components along and perpendicular to the line of the journal center.

$$W_a = W \cos \alpha = -LR \int_{\theta=0}^{\theta=\theta_c} p \cos \theta d\theta \quad (26)$$

$$W_b = W \sin \alpha = LR \int_{\theta=0}^{\theta=\theta_c} p \sin \theta d\theta \quad (27)$$

Using a non-dimensional form,

$$W^* = \frac{WC^2}{\mu_0 UR^2 L} \quad (28)$$

gives the non-dimensional load components.

$$W_a^* = W^* \cos \alpha = -6\varepsilon(1 + 2.5\varphi)(1 + \tau) \int_{\theta=0}^{\theta=\theta_c} \frac{\cos \theta - \cos \theta_c}{F(h^*, N, \varphi, \xi)} \sin \theta d\theta \quad (29)$$

$$W_b^* = W^* \sin \alpha = -6\varepsilon(1 + 2.5\varphi)(1 + \tau) \int_{\theta=0}^{\theta=\theta_c} \frac{\cos \theta - \cos \theta_c}{F(h^*, N, \varphi, \xi)} \cos \theta d\theta \quad (30)$$

As a result, the non-dimensional load capacity can be obtained.

$$W^* = \sqrt{W_a^{*2} + W_b^{*2}} \quad (31)$$

The friction force acting on the journal surface is

$$F_h = LR \int_{\theta=0}^{\theta=2\pi} \tau_h d\theta \quad (32)$$

where the shear stress is

$$\tau_h = \left[\mu \frac{\partial u}{\partial y} - \eta \frac{\partial^3 u}{\partial y^3} \right]_{y=h} \quad (33)$$

Using a non-dimensional form results in the friction parameter f_p

$$f_p = \frac{F_h R}{W C} = \frac{F_h^*}{W^*} \quad (34)$$

where the non-dimensional friction force is given by

$$F_h^* = \int_0^{2\pi} \left\{ \frac{1+2.5\varphi}{h^*} + \frac{1}{(1+\tau)} \cdot \left[\frac{h^*}{2} + \frac{N\tau}{\sqrt{(1+\tau)(1+2.5\varphi)}} \cdot \tanh\left(\frac{h^* \sqrt{(1+\tau)(1+2.5\varphi)}}{2N}\right) \right] \frac{\partial p^*}{\partial \theta} \right\} d\theta \quad (35)$$

Equations (24), (25), (31) and (34) can be calculated by the numerical method of Gaussian quadrature.

4 Results and Discussion

Based upon the above analysis, the steady-state performance of long journal bearings lubricated with a non-Newtonian couple stress ferrofluid under the application of a magnetic field are dominated by three parameters: the non-Newtonian couple stress parameter N , the volume concentration φ and the magnetic Langevin's parameter ξ .

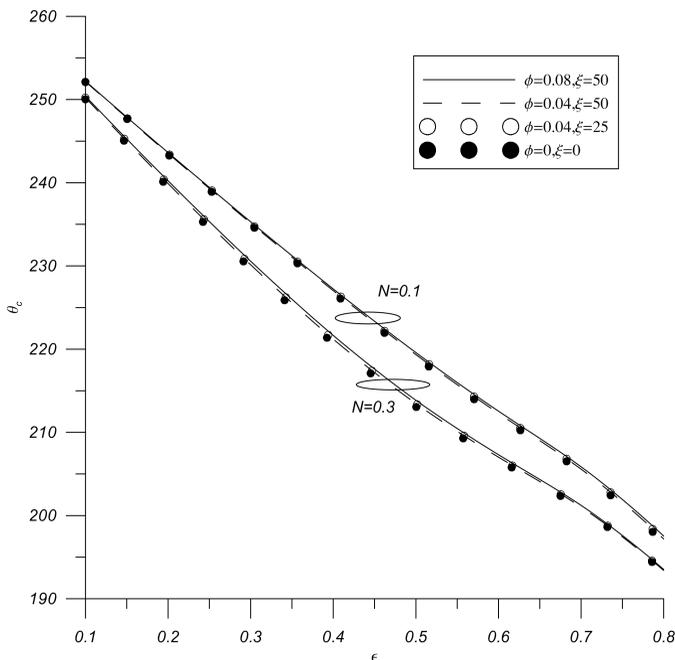


Figure 2: Variation of the zero pressure-gradient angle θ_c with the eccentricity ratio ϵ for different φ and ξ under $N=0.1$ and $N=0.3$.

Figure 2 shows the variation of the zero pressure-gradient angle θ_c with the eccentricity ratio ϵ for different φ and ξ under $N=0.1$ and $N=0.3$. The value of the zero

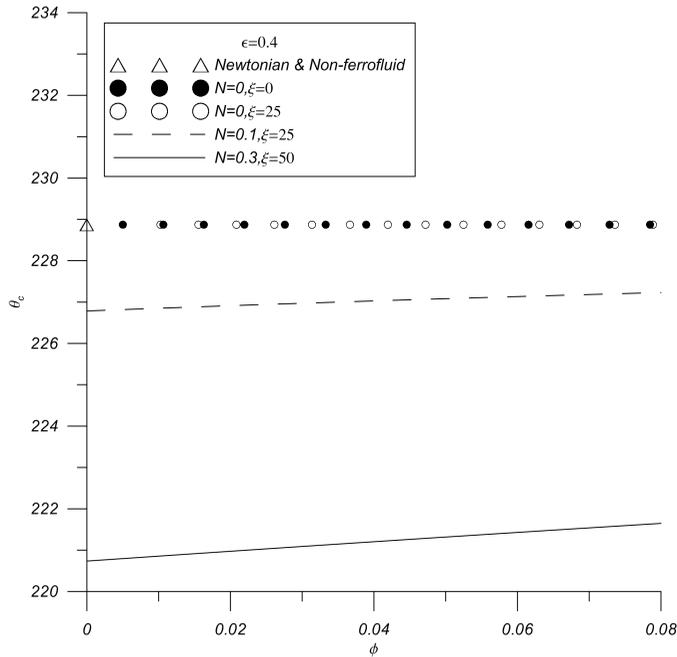


Figure 3: Variation of the zero pressure-gradient angle θ_c with the volume concentration ϕ for different N and ξ under $\epsilon=0.4$

pressure-gradient angle is observed to decrease with increasing values of the eccentricity ratio. Comparing with the non-ferrofluid case ($\phi=0$, $\xi=0$) under $N=0.1$, the ferrofluid lubricated bearing under the application of magnetic fields ($\phi=0.04$, $\xi=25$) results in an increase in the zero pressure-gradient angle. Increasing values of the olume concentration and the magnetic Langevin's parameter ($\phi=0.04$, $\xi=50$; $\phi=0.08$, $\xi=50$) yields a larger increment of the zero pressure-gradient angle. It is also seen that larger reduction for the zero pressure-gradient angle are obtained for a larger non-Newtonian couple stress parameter provided ($N=0.3$). Figure 3 displays the variation of the zero pressure-gradient angle θ_c with the volume concentration ϕ for different N and ξ under $\epsilon=0.4$. Comparing with the Newtonian and non-ferrofluid case, the Newtonian ferrofluids without magnetic fields ($N=0$, $\xi=0$) and the Newtonian ferrofluids with magnetic fields ($N=0$, $\xi=25$) give negligible influences on the zero pressure-gradient angle. However, the effects of non-Newtonian ferrofluids with magnetic fields ($N=0.1$, $\xi=25$; $N=0.3$, $\xi=50$) are observed to increase the zero pressure-gradient angle with increasing values of the volume concentration of ferrofluids.

Figure 4 shows the variation of the load capacity W^* with the eccentricity ratio ϵ

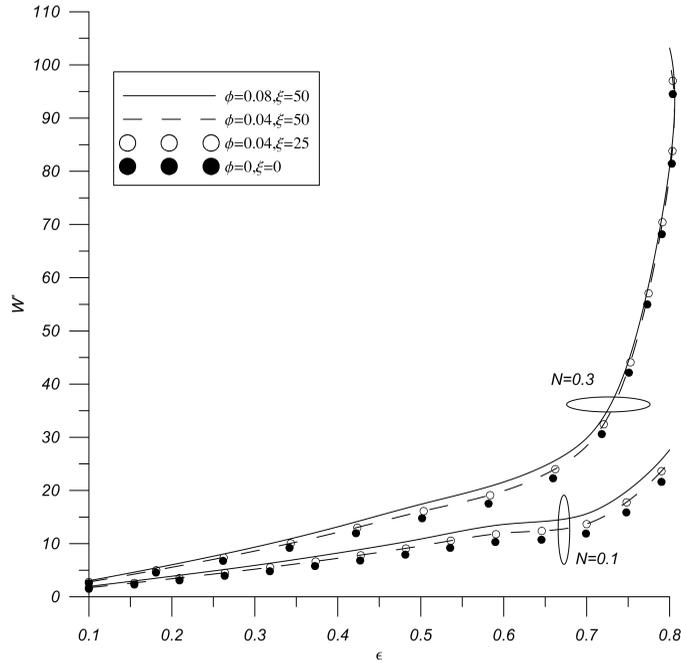


Figure 4: Variation of the load capacity W^* with the eccentricity ratio ε for different φ and ξ under $N=0.1$ and $N=0.3$

for different φ and ξ under $N=0.1$ and $N=0.3$. The load capacity is observed to increase with increasing values of the eccentricity ratio. Comparing with the non-ferrofluid case ($\varphi=0$, $\xi=0$) under $N=0.1$, higher values of the load are predicted for the ferrofluid lubricated bearing in the presence of magnetic fields ($\varphi=0.04$, $\xi=25$; $\varphi=0.04$, $\xi=50$; $\varphi=0.08$, $\xi=50$). Increasing values of the non-Newtonian couple stress parameter ($N=0.3$), further higher load capacities are provided for the non-Newtonian ferrofluid lubricated journal bearing. Figure 5 presents the variation of the load capacity W^* with the volume concentration φ for different N and ξ under $\varepsilon=0.4$. Comparing with the Newtonian, non-ferrofluid case, the values of the load capacity of the Newtonian ferrofluid lubricated bearing without magnetic fields ($N=0$, $\xi=0$) and with magnetic fields ($N=0$, $\xi=25$) are found to increase with the volume concentration of ferrofluids. Increasing the non-Newtonian couple stress parameter and the magnetic Langevin's parameter ($N=0.1$, $\xi=25$; $N=0.3$, $\xi=50$) provide further higher values of the bearing load.

Figure 6 shows the variation of the friction parameter f_p with the eccentricity ratio ε for different φ and ξ under $N=0.1$ and $N=0.3$. It is observed that the friction param-

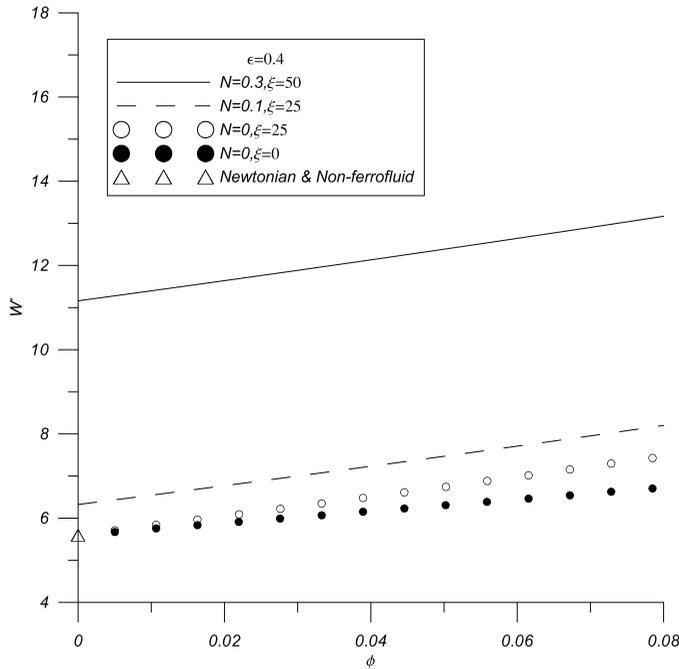


Figure 5: Variation of the load capacity W^* with the volume concentration ϕ for different N and ξ under $\epsilon=0.4$

eter decreases with increasing values of the eccentricity ratio. For fixed value of the couple stress parameter $N=0.1$, the effects of ferrofluid lubricants in the presence of the magnetic fields ($\phi=0.04, \xi=25$; $\phi=0.04, \xi=50$; $\phi=0.08, \xi=50$) are found to decrease the friction parameter in comparison with the case with non-ferrofluid lubricant ($\phi=0, \xi=0$). Increasing values of the couple stress parameter up to $N=0.3$, a reversed trend is obtained. However, the differences of the friction parameter are very small for the ferrofluid lubricated bearing and the non-ferrofluid lubricated bearing operating at the same eccentricity ratio. Figure 7 presents the variation of the friction parameter f_p with the volume concentration ϕ for different N and ξ under $\epsilon=0.4$. Comparing with the Newtonian, non-ferrofluid case, the Newtonian ferrofluid lubricants without magnetic fields ($N=0, \xi=0$) give very slight influences on the friction parameter. In the presence of an external magnetic field ($N=0, \xi=25$), the friction parameter decreases with the volume concentration of ferrofluids. When the effects of couple stresses are included ($N=0.1, \xi=25$; $N=0.3, \xi=50$), further decrements of the friction parameter are provided for the non-Newtonian ferrofluid lubricated bearings.

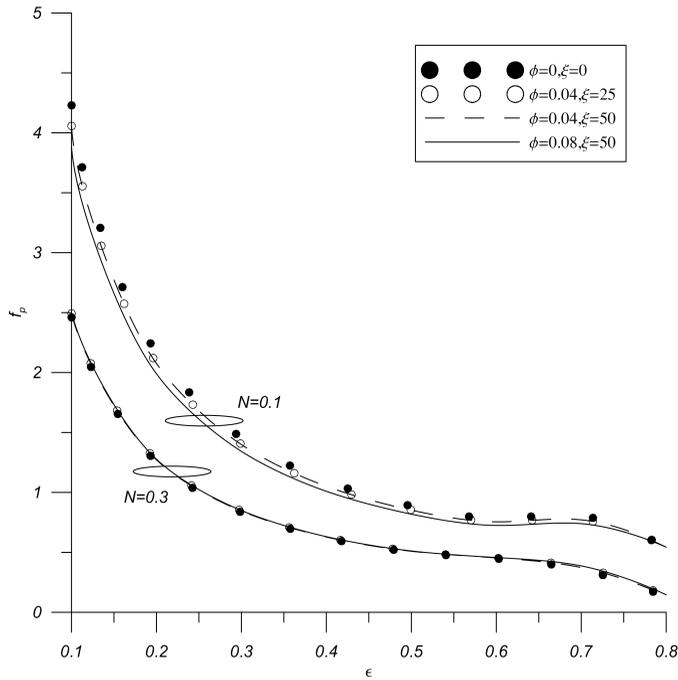


Figure 6: Variation of the friction parameter f_p with the eccentricity ratio ϵ for different φ and ξ under $N=0.1$ and $N=0.3$

Table 1: Numerical values of F_h^* , W^* and f_p under different cases at $\epsilon=0.4$.

| $\epsilon = 0.4$ | | φ | F_h^* | W^* | f_p |
|------------------|---------------------------|-----------|---------|---------|--------|
| Case1 | $N = 0, \quad \xi = 0$ | 0.02 | 7.2705 | 5.8847 | 1.2355 |
| | | 0.06 | 7.9629 | 6.4451 | 1.2355 |
| Case2 | $N = 0, \quad \xi = 25$ | 0.02 | 7.2705 | 6.0476 | 1.2022 |
| | | 0.06 | 7.9629 | 6.9806 | 1.1407 |
| Case3 | $N = 0.1, \quad \xi = 25$ | 0.02 | 7.2705 | 6.7704 | 1.0739 |
| | | 0.06 | 7.9629 | 7.7092 | 1.0329 |
| Case4 | $N = 0.3, \quad \xi = 50$ | 0.02 | 7.2705 | 11.6396 | 0.6246 |
| | | 0.06 | 7.9629 | 12.6438 | 0.6298 |

It is noted that the friction parameter is counted by dividing the friction force by the load capacity, $f_p = F_h^*/W^*$. Table 1 shows the values of F_h^* , W^* and f_p under different cases at $\epsilon=0.4$. It is observed that the values of the friction force under the four Cases are $F_h^* = 7.2705$ and $F_h^* = 7.9629$ for $\varphi=0.02$ and $\varphi=0.06$, respectively.

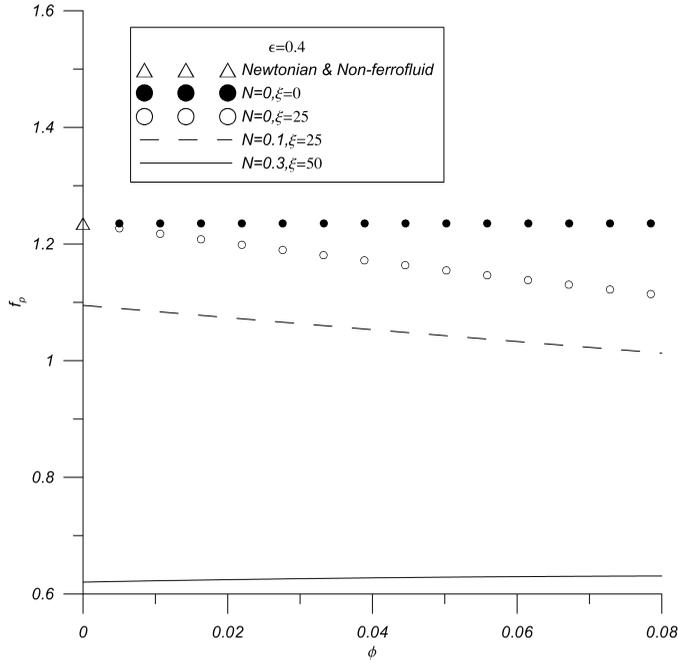


Figure 7: Variation of the friction parameter f_p with the volume concentration ϕ for different N and ξ under $\epsilon=0.4$

However, the values of the load capacity are:

$$(W^*)_{Case4} > (W^*)_{Case3} > (W^*)_{Case2} > (W^*)_{Case1}$$

From the definition of the friction parameter, it is counted by dividing the friction force by the load capacity, $f_p = F_h^*/W^*$. Therefore, the effects of non-Newtonian ferrofluids in the presence of the magnetic fields (Case 4 and Case 3) result in a smaller friction parameter as compared to the cases of Newtonian ferrofluids with magnetic fields (Case 2) and without magnetic fields (Case 1).

Some values used for non-Newtonian ferrofluids could be found in many contributions such as Shliomis (1972), Shah and Bhat (2004) and Naduvinamani *et al.* (2012). To justify the parameters used, an example is illustrated as the following.

$$C = 0.1mm;$$

$$T = 25^\circ C; \quad k_B = 1.38 \times 10^{-23} kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}; \quad \mu_M = 4\pi \times 10^{-7} kg \cdot m \cdot s^{-2} \cdot A^{-2};$$

$$H_0 = 0 \quad A \cdot m^{-1}, \quad 4.1 \times 10^5 A \cdot m^{-1} \quad 8.2 \times 10^5 A \cdot m^{-1};$$

$$m = 2 \times 10^{-19} \text{A} \cdot \text{m}^2; \quad \mu_0 = 0.002 \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1};$$

$$\eta = \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \quad 2 \times 10^{-13} \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \quad 18 \times 10^{-13} \text{kg} \cdot \text{m} \cdot \text{s}^{-1}$$

From the definition of parameters, one can find:

$$\xi \cong 0, \quad 25, \quad 50; \quad N = 0, \quad 0.1, \quad 0.3$$

For $\varepsilon = 0.4$ and $\varphi = 0.02$ and 0.06 , the bearing performance can be obtained as displayed in Table 1.

5 Conclusions

The steady-state performance of long journal bearings lubricated with non-Newtonian ferrofluids has been investigated by applying the ferrofluid model of Shliomis (1972) incorporating the micro-continuum theory of Stokes (1966). Analytical solutions for bearing performances are obtained from the non-Newtonian ferrofluid Reynolds-type equation. Comparing with the Newtonian non-ferrofluid case, the effects of non-Newtonian ferrofluids under external magnetic fields provide increased values of the zero pressure-gradient angle and the load capacity, and result in decreased values of the friction parameter especially for a larger non-Newtonian couple stress parameter and magnetic Langevin's parameter. For the long journal bearing lubricated with non-Newtonian ferrofluids and operating at larger values of the couple stress parameter and magnetic Langevin's parameter (for example, $N=0.3$, $\xi=50$), increasing the volume concentration parameter gives slight change in the friction parameter, but results in the further increments in the bearing load.

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