Soret driven thermosolutal convection in an inclined porous layer: search of optimum conditions of separation and validity of the boundary layer theory

A. Rtibi¹, M. Hasnaoui¹ and A. Amahmid¹

Abstract: In this paper we present an analytical and numerical study of Soret convection in an inclined rectangular porous layer saturated with a binary fluid and subject to uniform heat fluxes. In the problem formulation, the Darcy model is considered and the results are presented for wide ranges of $R_T(50 < R_T < 1000)$, $\theta(0^{\circ} \le \theta \le 180^{\circ})$ and $\varphi(-1 \le \varphi \le 1)$ for Le = 10, where R_T , θ , φ , and Le are the thermal Darcy-Rayleigh number, the cavity inclination, the separation parameter, and the Lewis number, respectively. An analytical solution, derived on the basis of the parallel flow approximation, is validated numerically by solving the full governing equations with a finite difference method. It is found that the heat transfer is more sensitive to the variation of the cavity inclination than to the separation parameter while the mass transfer sensitivity is essentially related to positive values of φ in a short range of θ . The thresholds of R_T and φ from which the boundary layer approximations can be considered valid (with a maximum deviation of 5% for the numerical results) are determined; they are found to be dependent on θ . For positive values of the separation parameter, the boundary layer regime is reached at relatively lower values of R_T .

Keywords: Heat transfer, Mass transfer, Natural convection, Inclined porous layer, Soret effect, Boundary layer.

1 Introduction

Combined effects of natural convection and thermodiffusion may be encountered in many engineering applications including underground diffusion of nuclear waste, oil reservoir analysis, petroleum extraction, mineral and material migration and separation of mixtures. This phenomenon occurs when a temperature gradient induces a transfer of solute regardless of whether a solute concentration gradient

¹ Faculty of Sciences Semlalia, Department of Physics LMFE, Laboratory associated to CNRST (URAC 27) BP 2390, Marrakesh - Morocco

already exists or not.

The literature review shows that most of the existing experimental works on the thermodiffusion phenomenon were devoted to the measurement of the Soret coefficient [Platten (2006); Blanco, Polyakov, Bou-Ali, and Wiegand (2008); Vaerenbergh and Legros (1998); Lenglet, Bourdon, Bacri, and Demouchy (2002); Rosanne, Paszkuta, Tevissen, and Adler (2003)]. Based on these experiments, several models were developed to predict the Soret coefficient [Eslamian and Saghir (2009)].

The existing theoretical studies were generally concerned with horizontal or vertical rectangular enclosures. In the former case (horizontal enclosures), several efforts have been devoted to the stability problem. For example, convective instabilities of a fluid mixture in a porous medium heated from below or from above were investigated theoretically by Brand and Steinberg (1983a) using constant temperatures at the boundaries. The convection state was found to be either stationary or oscillatory depending on the sign and the magnitude of the thermodiffusion ratio. The mechanisms of the stationary and oscillatory instability were investigated and the energy balance was used to derive the threshold conditions. The same authors [Brand and Steinberg (1983b)] investigated finite amplitude convection near the thresholds for both stationary and over stable convection. The temporal evolution of the heat and mass transfer rates was predicted.

Karcher and Müller (1994) studied Bénard convection of a binary liquid in a porous medium in the presence of Soret effect. A two-parameter perturbation analysis was used to examine the Soret effect on the stability of the basic state and finite amplitude convection. Their results show that a non linear density temperature relation has a destabilizing effect characterized by the decrease of the critical Rayleigh numbers for the onset of oscillatory and steady-state convection.

Ryskin, Müller, and Pleiner (2003) studied the Soret effect on thermo-convection in a horizontal infinite layer of binary liquid mixtures with weak concentration diffusivity and large separation numbers. By considering the classical Rayleigh Bénard problem, they showed that both linear and nonlinear convective behaviors were significantly altered by the concentration field as compared to single-component systems. An expression for the difference of solute concentration induced by the Soret effect, between the top and the bottom of a vertical porous cavity heated isothermally from the sides was derived analytically by Lorenz and Emery (1959). This problem was reconsidered four decades later by Dutrieux, Chavepeyer, Platten, and Itasse (1999), for the case of a porous medium modelled on the basis of the Brinkman equation. In this way it is possible to take into account the influence of the cavity boundaries 'no-slip conditions' on the separation effects.

In a 3D configuration, Platten, Marcoux, and Mojtabi (2007) examined the linear

stability of a liquid layer heated from below and laterally confined between four vertical rigid and adiabatic boundaries. The Soret effect on the onset of Marangoni convection in a non-reactive binary fluid layer in the presence of through flow was investigated by Saravanan and Sivakumar (2009) for different boundary conditions. Bourich, Hasnaoui, Amahmid, and Mamou (2005) carried out a numerical and analytical investigation of the Soret-driven convection using Brinkman-extended Darcy model for a sparsely packed porous medium. The flows in a shallow enclosure heated from below were studied for the case of fixed heat flux at the boundaries. The critical Rayleigh number was found to be strongly dependent on the separation parameter. Different types of perturbations (monotonic, oscillating and subcritical) were observed at different separation parameter value.

More recently, Lyubimov, Gavrilov, and Lyubimova (2011) studied two-dimensional Soret-driven convection in a porous cavity with perfectly conducting boundaries heated from below. The scenario of the convection onset is discussed in this study. The boundaries of the diffusive state instability to the small-amplitude and finiteamplitude monotonous and oscillatory perturbations have been determined. It has been found that any weak thermodiffusion effect destroys the degeneracy existing in the case of single-component fluid. For small values of the separation parameter, the spatial pattern with even temperature distribution arises in the system. For finite values of the separation parameter the linear stability of the diffusive state was studied numerically by finite-difference method. The instability thresholds were calculated for monotonic and oscillatory perturbations.

Concerning vertical enclosures, Benano-Melly, Caltagirone, Faissat, Montel, and Costeseque (2001) studied numerically and experimentally the problem of thermodiffusion in an initially homogeneous mixture submitted to a horizontal thermal gradient induced by constant but different temperatures imposed on the vertical boundaries. It was found that the theory can represent well the solute behavior only when the solutal buoyancy force is negligible. The observed discrepancy between numerical and experimental results while reproducing thermogravitational experiments by numerical means was attributed by the authors to the dispersion as a possible cause of this difference. Er-Raki, Hasnaoui, Amahmid, and Mamou (2006) studied the Soret effect on the boundary layer flows induced by doublediffusive convection in a vertical porous layer subject to horizontal heat and mass fluxes. The thermo-diffusion effect on the boundary layer thickness was discussed for a wide range of the governing parameters. It was demonstrated analytically that the thickness of the boundary layer could either increase or decrease when the Soret parameter was varied depending on the sign of the buoyancy ratio. Recently, Davarzani and Marcoux (2011) have studied numerically the influence of thermal properties on the separation rate in a model of packed thermogravitational column saturated by a binary mixture. They have reported that the presence of the porous matrix leads to optimal conditions of separation associated to higher Rayleigh numbers than without the porous media. Taking into account heat transfer in the solid matrix leads to separation greater than the maximum values available in the free case.

Most recently, Srinivasan and Saghir (2011) and Parsa and Saghir (2012) investigated the influence exerted on such phenomena by vibrations (g-jitters, see, e.g., also Monti, Savino, and Lappa (2001) and Savino and Lappa (2003)).

The main goal of the present work is to study thermosolutal natural convection in inclined porous layers saturated with a binary mixture and subject to constant fluxes of heat. The combined influence of the separation parameter and cavity inclination is examined and the limits of applicability of the boundary layer approximation are determined. The search of optimum conditions of species separation is also among the objectives of the study.



Figure 1: Schematic diagram of the studied problem.

2 Mathematical formulation

We consider a two dimensional inclined rectangular porous medium saturated with a binary mixture. The inclined cavity, sketched in Fig. 1, has height L' and width H' with long sides subjected to uniform heat fluxes, q', and short sides insulated and impermeable to mass transfer. The mixture saturating the porous medium is assumed to be homogeneous, isotropic and is modeled as a Boussinesq-incompressible fluid. The remaining physical properties are considered constant. The dimension-

less governing equations describing the conservation of momentum, energy and species in the saturated Darcy porous medium and in the presence of Soret effect are written as follows:

$$\nabla^2 \Psi = -R_T \left(\cos \theta \frac{\partial}{\partial x} - \sin \theta \frac{\partial}{\partial y} \right) (T + \varphi S) \tag{1}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T$$
(2)

$$\varepsilon \frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{Le} (\nabla^2 S - \nabla^2 T)$$
(3)

$$u = \frac{\partial \Psi}{\partial y}, \qquad v = -\frac{\partial \Psi}{\partial x} \tag{4}$$

In the above equations, Ψ , *T* and *S* are the dimensionless stream function, temperature and solute concentration, respectively. The boundary conditions associated to the governing equations are:

$$y = \pm 1/2: \quad \Psi = 0, \quad \frac{\partial T}{\partial y} = -1, \quad \frac{\partial S}{\partial y} = -1 \\ x = \pm A_r/2: \quad \Psi = 0, \quad \frac{\partial T}{\partial x} = 0 \quad \frac{\partial S}{\partial x} = 0 \end{cases}$$
(5)

In addition to the inclination θ of the cavity, the problem is governed by four other dimensionless parameters which are the separation parameter, φ , the thermal Darcy-Rayleigh number, R_T , the Lewis number, Le, and the cavity aspect ratio, A_r , defined respectively as:

$$\varphi = -\frac{\beta_S S'_0 (1 - S'_0) D_T}{\beta_T D_{eff}}, \quad R_T = \frac{g \beta_T K \Delta T' H'}{\alpha \upsilon}, \quad Le = \frac{\alpha}{D_{eff}}, \quad A_r = \frac{L'}{H'}$$
(6)

The parameter $D_{eff} = D\varepsilon'$ is the effective mass diffusivity, and D and D_T are respectively the mass diffusivity and the thermodiffusion coefficient.

In practical applications (such as in oil reservoir where the Soret effect could be important) the porosity is usually around 0.2. However, the present study concerns the analysis of steady flows which are independent of ε . The parameter φ can assume positive or negative values depending on whether the solutal and thermal buoyancy forces are cooperating ($\varphi > 0$) or opposing ($\varphi < 0$) each other.

3 Numerics

In the present study, numerical simulation has been undertaken essentially with the purpose to validate an analytical solution expressly derived to support the analysis (as illustrated in Sect. 4). The governing equations have been discretized according to the central difference scheme. The iterative procedure has been performed using the alternate direction implicit method (ADI). The stream function field has been determined from Eq. (4) using the point successive over relaxation method (PSOR). The calculation domain was divided into three regions; the grid has been refined in the vicinity of the walls while a coarse uniform grid was employed for the central region of the cavity. In the present study, the grid of 121×101 was found enough to ensure grid-independence of the results. Comparative results concerning the grid's effect and the deviation between analytical and numerical results are presented in Tables 1 and 2 respectively for $(R_T, \varphi, \theta) = (200, 0.5, 45^\circ)$ and $(10^3, -0.5, 45^\circ)$ 105°). In these tables, we can observe that, with the selected grid, there is a satisfactory agreement between analytical and numerical results. Numerous numerical tests have been performed by solving the full governing equations to determine the minimum aspect ratio above which the effect of the short confining sides is negligible. In view of the results obtained (not presented here), it can be stated that the aspect ratio effect becomes negligible from $A_r = 12$ for the worst cases. Hence, the numerical results reported here were obtained with $A_r = 12$.

Table 1:	Grid	effect of	n the	results	for	$R_T =$	200,	Le = 1	10, q	$\rho = 0.5,$	$\boldsymbol{\theta}$	$= 45^{\circ}$	and
$A_r = 12.$													

		Numerical results for various grids and their					
	Analytical	deviations from the analytical results (%)					
		101×61	101×81	121×101	201×101		
Ψ_c	4.3189	4.3298	4.3266	4.3238	4.3227		
		(0.2524%)	(0.1783%)	(0.1134%)	(0.0879%)		
Nu	5.7614	5.7464	5.7459	5.7525	5.7553		
		(0.2603%)	(0.2690%)	(0.1544%)	(0.1058%)		
Sh	10.0279	9.9981	9.9929	10.0198	10.0266		
		(0.2972%)	(0.3490%)	(0.0807%)	(0.0129%)		

	Analytical	Numeric deviatio	Numerical results for various grids and their deviations from the analytical results (%)						
		101×61	101×81	121×101	201 × 101				
Ψ_c	3.104	3.1216	3.1209	3.1085	3.0994				
		(0.5670%)	(0.5444%)	(0.1449%)	(0.1481%)				
Nu	5.2549	5.2495	5.2492	5.2546	5.2573				
		(0.1027%)	(0.1084%)	(0.0057%)	(0.0456%)				
Sh	15.033	15.0431	15.0332	15.1662	15.2538				
		(0.0671%)	(0.0013%)	(0.8860%)	(1.4647%)				

Table 2: Grid effect on the results for $R_T = 10^3$, Le = 10, $\varphi = -0.5$, $\theta = 105^\circ$ and $A_r = 12$.

4 Analytical solution

The analytical solution has been developed by using the parallel flow approximation, valid for slender cavities ($A_r \gg 1$) and allowing the following simplifications:

$$\Psi(x,y) = \Psi(y), T(x,y) = C_T x + \theta_T(y) \text{ and } S(x,y) = C_S x + \theta_S(y)$$
(7)

The parameters C_T and C_S are respectively the unknown constant temperature and concentration gradients in the x direction. They are determined by imposing zero heat and mass fluxes across any transversal section of the cavity. The approximations of Eqs. (7) were combined with the steady state Eqs. (1)-(4) to obtain simplified governing equations for which the analytical solution is obtained as follows:

$$\Psi(y) = -B\Omega\cosh(\Omega y) + G \tag{8}$$

$$u(y) = -B\Omega^2 \sinh(\Omega y) \tag{9}$$

$$T(x,y) = C_T x - C_T B \sinh(\Omega y) + (C_T G - 1)y$$
(10)

$$S(x,y) = C_S x - (C_T + C_S Le)B\sinh(\Omega y) + ((C_T + C_S Le)G - 1)y$$
(11)

This solution is valid for Ω real; with $\Omega B = G/\cosh\left(\frac{\Omega}{2}\right)$, $\Omega^2 = R_T \sin(\theta)[C_T + \varphi(C_T + C_S Le)]$ and $G\Omega^2 = R_T[(C_T + \varphi C_S)\cos(\theta) + (1 + \varphi)\sin(\theta)]$. According to Eq. (9), it appears that the velocity could not vanish for lyl varying in the range]0, 1/2[. This means that only mono-cellular flow is possible. The analytical expressions of the parameters C_T and C_S are obtained as follows:

$$C_{T} = \alpha_{1}B^{2}C_{T} + B(1 - GC_{T})\alpha_{2}$$

$$C_{S} = C_{T} + Le[\alpha_{1}B^{2}(LeC_{S} + C_{T}) + B\alpha_{2}(1 - G(C_{T} + LeC_{S}))]$$
(12)

where $\alpha_1 = \frac{\Omega}{2} [\sinh(\Omega) - \Omega]$ and $\alpha_2 = \Omega \cosh(\frac{\Omega}{2}) - 2 \sinh(\frac{\Omega}{2})$. Therefore, Eqs. (10)-(11) are used to obtain the expressions of *Nu* and *Sh*, characterizing respectively heat and solute transfer rates across the layer:

$$Nu = \frac{1}{T(x, -1/2) - T(x, 1/2)} = \frac{1}{1 - BC_T \alpha_2}$$

$$Sh = \frac{1}{S(x, -1/2) - S(x, 1/2)} = \frac{1}{[1 - B(C_T + C_S Le)\alpha]}$$
(13)

The system of Eqs. (12) was solved by using the Newton-Raphson iterative procedure. In this way, the temperature and concentration gradients C_T and C_S can be obtained for any combination of R_T , θ , *Le* and φ . Then, the velocity, temperature and concentration profiles are calculated using Eqs. (9)-(11) while Nusselt and Sherwood numbers were deduced from Eq. (13).

It is to note that, in the case of imaginary Ω , the resulting solution can be deduced by substituting $\Omega = i\omega$ in the above equations where $\omega = (|R_T \sin(\theta)(C_T + (C_T + C_S Le))|)^{1/2}$ is real and *i* is the imaginary number $(i^2 = -1)$. Using $\sinh i\omega = i\sin\omega$ and $\cosh i\omega = \cos\omega$, the solution for imaginary Ω is similar to that given by Eqs. (8)-(11) in the case of real Ω . In other words, the solution for imaginary Ω is obtained by merely replacing the hyperbolic functions by circular ones.

4.1 Solution for the particular case $\Omega = 0$ and inclined cavity

The parameter Ω becomes zero for a given inclination θ of the cavity, other than those corresponding to its horizontal positions ($\theta = 0^{\circ}$ and 180°), for $C_T = -\varphi(C_T + C_S Le)$. This particular situation, leads to the following solution:

$$\Psi(y) = -\frac{E}{2}(y^2 - \frac{1}{4}) \tag{14}$$

$$u(y) = -Ey \tag{15}$$

$$\theta_T(y) = -\frac{C_T E}{6} y^3 + (C_T \frac{E}{8} - 1)y$$
(16)

$$\theta_S(y) = (C_T + LeC_S)\frac{E}{6}y^3 + ((C_T + LeC_S) - 1)y$$
(17)

where

$$E = R_T \lfloor \cos(\theta) (C_T + \varphi C_S) + (1 + \varphi) \sin(\theta) \rfloor$$
(18)

In the above equations, the values of C_T and C_S can be obtained by substituting Eqs. (14)-(18) into their integral forms (not given here), which yields

$$C_T = \frac{10E}{120 + E^2} \text{ and } C_S = C_T \frac{120 - LeE^2}{120 + Le^2 E^2} + \frac{10LeE}{120 + Le^2 E^2}$$
 (19)

Substituting Eqs. (19) into Eqs. (13), the analytical expressions of Nu and Sh for this particular case are obtained as

$$\left.\begin{array}{l}
Nu = 6\frac{E^2 + 120}{E^2 + 720} \\
Sh = \frac{12}{(C_T + LeC_S)E - 12}
\end{array}\right\}$$
(20)

The same expression was obtained in the past by Mamou, Vasseur, Bilgen, and Gobin (1995) for Nu in the case of pure double-diffusive problem but the difference observed in the expressions of *Sh* is attributed to the Soret effect which was not considered in the ref. by Mamou, Vasseur, Bilgen, and Gobin (1995).

The final expression of *E* can be obtained by using relations (19) and the condition $C_T = -\varphi(C_T + C_S Le) \text{ which leads to } \frac{10E}{120+E^2} \left(\frac{120-LeE^2}{120+Le^2E^2} + \frac{1+\varphi}{\varphi Le}\right) + \frac{10LeE}{120+Le^2E^2} = 0.$ This equation for *E* can also be obtained under another form by using Eq. (18) to obtain $E = R_T \left[(1+\varphi)\sin\theta + \frac{10E}{120+E^2} \left(1 - \frac{1+\varphi}{Le}\right)\cos\theta \right].$

4.2 Solution for the particular case $\Omega = 0$ and horizontal cavity

The function Ω becomes also zero when the cavity is horizontal. This position corresponds either to $\theta = 0^{\circ}$ (cavity heated from below) or $\theta = 180^{\circ}$ (cavity heated from the top). For such a situation, $\frac{d^2\Psi}{dy^2} = -E$ with $E = \pm R_T(C_T + \varphi C_S)$; (-) being for $\theta = 0^{\circ}$ and (+) for $\theta = 180^{\circ}$. Same expressions (as those reported above with $\Omega = 0$ and $\theta \neq 0^{\circ}$ and 180°) are obtained for Ψ , θ_T , θ_S , C_T , C_S , u, Nu and Sh but the expression of E is now different and obtained as

$$E = \pm R_T \left[\frac{10E}{120 + E^2} \left(1 + \varphi \frac{120 - LeE^2}{120 + Le^2 E^2} \right) + \varphi \frac{10LeE}{120 + Le^2 E^2} \right]$$
(21)

4.3 Boundary layer regime

The boundary layer regime is expected to appear for large values of Ω (high values of R_T). Contour lines of stream function, isotherms, iso-cocentrations and density presented in Fig. 2 together with velocity, temperature and concentration profiles presented in Figs. 3-4 show clearly that the velocity exhibits a classical boundary layer behavior with zero gradient far from the active walls. However, it can be noticed that the boundary layer profile of concentration and temperature is characterized by a linear variation and thereby a constant gradient outside the boundary layer. To quantify the importance of the latter gradient we illustrate in Fig. 5 the profile of $\frac{\partial S}{\partial x}$ and $\frac{\partial T}{\partial x}$ (i.e. transverse gradient of concentration and temperature) for $R_T = 1000$, Le = 10, $\varphi = 1$ and $\theta = 45^\circ, 75^\circ, 90^\circ, 105^\circ$. This figure shows that the profiles of $\frac{\partial S}{\partial x}$ and $\frac{\partial T}{\partial x}$ exhibit clearly a behavior of boundary layer type. Furthermore, the nearly constant gradients of *S* and *T* observed outside the boundary layer



Figure 2: From left to right, contour lines of stream function (a), isotherms (b), iso-concentrations (c) and density (d) for $R_T = 1000$, Le = 10, $\theta = 45^\circ$ and $\varphi = 1$.



Figure 3: u-velocity (a), temperature (b) and concentration (c) profiles along y at mid-height of the cavity for $R_T = 200$, Le = 10 and $\varphi = 0.5$ and different values of cavity inclination.



Figure 4: u-velocity (a), temperature (b) and concentration (c) profiles along *y* at mid-height of the cavity for $R_T = 200$, Le = 10 and $\theta = 45^\circ$ and different values of φ .



Figure 5: Temperature and concentration gradients at mid-height of the cavity for $R_T = 1000$, Le = 10, $\varphi = 1$ and $\theta = 45^\circ$, 75° , 90° and 105°

are small compared with those in the vicinity of the active walls. This is the reason for which the boundary layer approximations developed below give satisfactory results (see the next sections). In the case of very large values of Ω , the parallel flow solutions can be simplified to obtain:

$$\Psi(y) = G(1 - \exp\Omega(|y| - 1/2))$$
(22)

$$u(y) = -\frac{|y|}{y}G\Omega\exp\Omega(|y| - 1/2)$$
(23)

$$T(x,y) = C_T x + (C_T G - 1)y + \frac{|y|}{y} \frac{C_T G}{\Omega} \exp \Omega(|y| - 1/2)$$
(24)

$$S(x,y) = C_S x + ((C_T + LeC_S)G - 1)y + \frac{|y|}{y} \frac{(C_T + LeC_S)G}{\Omega} e^{\Omega(|y| - \frac{1}{2})}$$
(25)

$$Nu = \frac{1 + \alpha_4 G^2}{1 + \alpha_5 G^2 / \Omega} \tag{26}$$

$$Sh = \frac{(1 + \alpha_4 G^2)(1 + \alpha_4 G^2 L e^2)}{1 + \frac{1}{\Omega}\alpha_5 G^2 L e^2 + \alpha_4 G^2 + \alpha_4 \alpha_5 G^4 L e^2 / \Omega - (1 + L e)\alpha_3^2 G^2}$$
(27)

Where

$$C_T = \frac{\alpha_3 G}{1 + \alpha_4 G^2} \text{ and } C_S = \frac{(1 + Le)\alpha_3 G}{1 + (1 + Le^2)\alpha_4 G^2 + \alpha_4^2 G^4 Le^2}$$
(28)

with $\alpha_3 = 1 - 2/\Omega$, $\alpha_4 = 1 - 3/\Omega$ and $\alpha_5 = 1 - 4/\Omega$.

5 Results and discussion

In this section, the attention is mainly focused on the effects of the inclination of the cavity, the separation parameter and the Rayleigh number on Ψ_c , Nu and Sh. The study is conducted for Le = 10 and $A_r = 12$. The examination of the validity of the boundary layer approximations counts among the objectives of the present study (see sub-sections 5.2 and 5.3).

5.1 Effect of the inclination angle

In the present study the tilt angle was varied within the range $[0^{\circ}, 180^{\circ}]$. Figs. 6(a)-(c) illustrate the variations of the flow intensity, Ψ_c (in the centre of the cavity), and heat and mass transfer, characterized by Nu and Sh, respectively, versus the inclination angle, θ , for various values of the separation parameter φ and $R_T = 200$. Different trends are observed in the evolutions of Ψ_c , Nu and Sh when the inclination



Figure 6: Effect of the inclination θ on $\Psi_c(a)$, Nu(b), Sh(c) and $\Delta C(d)$ for Le = 10 and $R_T = 200$.



Figure 7: u-velocity along y at mid-height of the cavity for Le = 10 and $R_T = 200$.



Figure 8: Iso-concentrations for $\theta = 45^{\circ}$, Le = 10, $R_T = 200$ and (a): $\varphi = 1$, (b): $\varphi = 0.5$ and (c): $\varphi = -0.1$

 θ is varied. In the case of Ψ_c , Fig. 6a shows that the evolution is characterized by a continuous decrease when θ is increased from 0° (horizontal cavity heated from below) to 180° (horizontal cavity heated from above). All the trends observed are not affected by the Soret parameter but, depending on the inclination of the cavity, the impact of the latter could be important on the flow intensity. Globally, the inclination angle is seen to have an important quantitative effect on the flow intensity and heat and mass transfer characteristics. In a previous study, the case of $\theta = 0^\circ$ was considered by Bourich, Hasnaoui, Amahmid, and Mamou (2005). The authors have demonstrated the existence of three regions with specific behaviors and the critical Rayleigh numbers corresponding to the onset of subcritical and supercritical flows were determined explicitly versus the governing parameters. The other limit value of $\theta(\theta = 180^\circ)$ corresponds to a cavity heated from above. This case



Figure 9: Iso-concentrations for Le = 10, $R_T = 200$ and (a): $(\varphi, \theta) = (1, 156^\circ)$, (b): $(\varphi, \theta) = (0.5, 163^\circ)$ and (c): $(\varphi = -0.1, \theta = 177^\circ)$.

corresponds to a stable situation only in the absence of Soret effect or in the case of positive separation parameter. For this situation the convective flow is cancelled $(\Psi_c = 0)$ and heat and mass transfers are those of the diffusive regime. However, even for $\theta = 180^\circ$, we can observe a small circulation ($\Psi_c \neq 0$) in the case of negative φ . By increasing θ from 0° to 180° , Nu (Fig. 6b) and Sh (Fig. 6c) behave differently with this increase. In fact, Nu increases with θ up to a maximum near $\theta = 40^\circ$ and then decreases towards the conductive regime with further increase in θ . It is observed that positive values of the separation parameter are favourable to heat transfer within the system for moderate inclinations of the cavity. These observations, compared to the variations of Ψ_c , indicate that the heat transfer is rather controlled by the intensity of the local flows but not by their global intensities. In fact, we can observe complex and different behaviors when we compare the velocity profiles in Fig. 7 obtained for $\theta = 45^{\circ}$ and 135° . For $\theta = 45^{\circ}$, the intensity of the local flow is seen to decrease by decreasing φ in the vicinity of the thermally active wall. The change in the tendency is observed for $y \approx 0.09$ where the local effect of the flow intensity becomes probably negligible on heat transfer. This behavior is a possible explanation of the decrease of Nu observed for moderate inclinations of the cavity (for $\theta < 90^{\circ}$) when φ is varied. On the other hand, we can observe in the same figure that the decrease of u engendered by the decrease of φ in the case of $\theta = 135^{\circ}$ is restricted to the immediate vicinity of the wall while the change in the trend is observed earlier. Probably this earlier change in the trend overcomes the local effects of the flow in the control of heat transfer in the immediate vicinity of the boundary and could be behind the increase of the Nusselt number when the separation parameter φ is decreased in the case of $\theta = 135^{\circ}$ (for $\theta > 90^{\circ}$ in general). In the case of Sh, it is seen from Fig. 6c that the inclination effect remains limited for $\varphi \leq 0$ while for positive values of this parameter this effect becomes important in some range of θ with maxima (very sensitive to the increase of φ) observed around $\theta = 135^{\circ}$. Fig. 6d illustrates the species separation evolution, ΔC , with the inclination angle of the cavity for $R_T = 200$, Le = 10 and $\varphi = 1$, $\varphi = 0.5$ and $\varphi = -0.1$. The difference ΔC is defined as the difference of solute concentration between the two lateral walls of the cavity normalized with the aspect ratio. It can be seen that separation goes through a maximum value when the inclination angle is increased from 0° to 180°. Furthermore, the location of the maximum is a function of φ . For all the values of φ considered in this study, the maximum of separation is located beyond 150° and it is seen to change slightly with φ . but sharper picks are obtained at smaller φ . Consequently, noticeable values of separation are obtained in a small range of θ in the case of $\varphi = -0.1$. On the basis of the results presented in Fig. 6d, the observations concerning the species separation are verified for $R_T = 200$ and Le = 10 by illustrating numerically a case where the separation of species is absent (case illustrated in Fig. 8 for $\theta = 45^{\circ}$ and various φ) and another case where the separation of species is maximum (case illustrated in Fig. 9 for three combinations of $\theta - \varphi$ leading to ΔC_{max}). In fact, in Fig. 8d, both analytical and numerical results show that the separation of species is absent for $\theta = 45^{\circ}$ whatever is the value of the parameter φ . This observation is in agreement with the iso-concentrations of Fig. 9 that show an almost uniform concentration distribution within the cavity for the three values of φ . On the other hand, the combinations $\theta - \varphi$ selected in Fig. 9 are those leading to maximum ΔC_{max} in Fig. 6d. Each of the three combinations leads to maximum of species separation for a specific θ in the range 150°-180°. More precisely, for $\varphi = 1/0.5/-0.1$, the corresponding inclination leading to ΔC_{max} is $\theta = 156^{\circ}/163^{\circ}/177^{\circ}.$



Figure 10: Effect of the Rayleigh number R_T on $\Psi_c(a)$, Nu(b) and Sh(c) for Le = 104, $\theta = 45^\circ$ and different values of φ .

5.2 Effect of Rayleigh number

In the boundary layer regime, the analytical solutions of Ψ_c , Nu and Sh are concerned by the boundary layer simplifications. To examine the conditions of validity of the boundary layer simplifications established above, we illustrate the evolutions of Ψ_c , Nu and Sh versus the Rayleigh number, within the range of [50, 1000], in Figs. 10 to 12 for Le = 104, different values of φ and $\theta = 45^{\circ}, 75^{\circ}$ and 105° , respectively. The analytical results based on the parallel flow approximation (solid lines) are seen to be in excellent agreement with those obtained numerically (full dots) by solving the full governing equations. In the same figures, the approximate boundary layer results are presented by dashed lines. In all these figures, we can see that, in general, negative values of φ are the least favorable to the development of the boundary layer. The importance of the qualitative and quantitative disagreements observed can be enhanced or attenuated, depending on the angle of inclination θ and the Rayleigh number R_T . More precisely, for $\theta = 45^{\circ}$ and $\varphi = -0.5$, the simplified equations of the boundary layer lead to qualitative and quantitative agreements with the numerical solution in the case of Ψ_c from the



Figure 11: Effect of the Rayleigh number R_T on $\Psi_c(a)$, Nu(b) and Sh(c) for Le = 10, $\theta = 75^{\circ}$ and different values of φ .

threshold $R_{TC} = 550$ (Fig. 10a). By increasing the inclination θ to 75° and maintaining $\varphi = -0.5$ (Fig. 11a), the difference characterizing the deviation of 5% between the boundary layer results and those obtained numerically is observed from $R_{TC} = 260$. Beyond the vertical position, case illustrated in Fig. 12a with $\theta = 105^{\circ}$, the concavity of the Ψ_c curves undergoes changes but quantitatively speaking, for $\varphi = -0.5$, the boundary layer approximation becomes valid even at R_T as low as 250. In the case of Nu, the thresholds of R_T marking the validity of the boundary layer approximations for $\varphi = -0.5$ decrease by increasing θ ; they are 860, 300 et 190 respectively for $\theta = 45^{\circ}, 75^{\circ}$ and 105° . Finally, for this negative value of φ , the Sh boundary layer agree within 5% with the parallel flow solution and the numerical results from $R_T = 994.75, 411.34, 323.23$, respectively for $\theta = 45^\circ, 75^\circ$ and 105°. In general, the combination $(\theta, \phi) = (45^\circ, -0.5)$ remains the worst for the development of the boundary layer for Ψ_c , Nu and also Sh. For both Ψ_c and Nu, the boundary layer approximation reproduces the numerical results from thresholds of R_T lower in fact than 50 for aiding buoyancy forces ($\varphi > 0$) regardless of θ . In the case of Sh, these thresholds of R_T are higher. More precisely for $\varphi = 0.5/(1)$, these thresholds values of R_T are 138.11/(93.60), 79.95/(49.46), and 66.10/(35.06)



Figure 12: Effect of the Rayleigh number R_T on $\Psi_c(a)$, Nu(b) and Sh(c) for Le = 10 and $\theta = 105^\circ$ and different values of φ .

respectively for $\theta = 45^{\circ}, 75^{\circ}$ and 105° . Finally, it is to note in all these figures that, for a given R_T , Ψ_c increases by decreasing φ while a reverse trend is observed in the case of Nu (for $\theta = 45^{\circ}$ and 75°) and Sh (for the three inclinations considered). This behavior can be explained by the fact that Nu is rather controlled by the intensity of the local flows as explained in the previous sub-paragraph.

5.3 Effect of Separation parameter

Within the range of [-1,1], the effect of the separation parameter on the validity of the boundary layer theory is illustrated in Fig. 13a-c where the respective variations of Ψ_c , Nu and Sh are presented versus φ for $R_T = 200$, Le = 10 and various inclinations ($\theta = 45^\circ, 75^\circ, 90^\circ, 105^\circ$ and 135°). In general, it can be seen that the inclination $\theta = 45^\circ$ is the least favorable for the validity of the boundary layer regime despite the fact that it leads to better fluid circulation and higher Nusselt number but it is the most favourable for the validity of these approximations in the case of Sh despite the fact that it leads to lower Sherwood numbers. In fact, for a given value of φ , it is seen in Fig. 13a-c that both Ψ_c and Nu decrease by increasing the inclination of the cavity from $\theta = 45^\circ$ while Sh increases by increasing it from



Figure 13: Effect of the separation parameter φ on $\Psi_c(a)$, Nu(b) and Sh(c) for Le = 10 and $R_T = 200$.

its lowest value. In addition, by increasing the separation parameter from negative value, it appears that the validity of the boundary layer theory is practically restricted to positive values of φ in the case of $\theta = 45^{\circ}$ while for the remaining inclinations the boundary layer theory is applicable for thresholds of φ starting from negative values of this parameter. More precisely, by increasing φ in its range for the inclination $\theta = 45^{\circ}$, the relative difference between the analytical and numerical results on one hand and those obtained by the boundary layer theory on the other hand becomes less than 5% only from the thresholds $\varphi \approx 0$ and -0.04 for Ψ_c and Nu, respectively. The thresholds of φ for $\Psi_c/(Nu)$ corresponding to the remaining inclinations are -0.35/(-0.41), -0.42/(-0.47), -0.43/ (-0.48) and -0.2/(-0.32) respectively for $\theta = 75^{\circ}$, 90°, 105° and 135°. Hence, the threshold of the dynamic boundary layer regime is seen to decrease by increasing the inclination angle of the cavity in the range $45^{\circ} \le \theta \le 105^{\circ}$ though the global circulation has an inverse tendency, i.e. Ψ_c decreases by increasing θ . This tendency is however inverted when θ passes from 105° to 135° as the threshold undergoes an increase. Such a behavior seems somewhat surprising, but in reality, heat and mass transfer are rather controlled by the local variations than by the global intensity of the fluid as it can be seen by plotting u(y) at mid-height of the cavity in Fig. 14. In fact, we can see



Figure 14: u-velocity along y at mid-height of the cavity for Le = 10 and $R_T = 200$.

in this figure that the velocity increases by decreasing θ in the vicinity of the long boundaries and the tendency in the evolution of *Nu* is rather corroborated by this behavior. It is to note that, in the case of Sh (Fig. 13c), a different deduction can be made as to the validity of the approximations that led to boundary layer approximations. In fact, these approximations are justified from $\varphi = 0.27, -0.22, -0.28$, and -0.11 respectively for $\theta = 45^{\circ}, 75^{\circ}, 105^{\circ}$ and 135° .

6 Conclusion

Thermodiffusion and natural convection (and related heat and mass transfer proceses) in a two dimensional inclined rectangular porous layer impermeable to mass transfer with long sides submitted to uniform fluxes of heat have been studied analytically and numerically. The results obtained show that the heat transfer is essentially governed by the local flow intensity. In addition, it has been observed that the Nusselt number is more sensitive to the variations of the cavity inclination than to the separation parameter. It goes through a maximum value when the separation parameter is increased in the range [-1, 1]. The location of the maximum depends on the inclination angle. The presence of thermodiffusion in the medium may be favourable or unfavourable to heat transfer depending on the inclination angle and the separation parameter value. Separation of species reaches its maximum for a specific value of the inclination angle (which depends on φ) in the range 150°-180°. Thresholds in terms of $R_T/(\varphi)$ from which the boundary layer theory results coincide with those obtained numerically and analytically are strongly dependent on θ and φ/R_T .

Finally, it is also worth mentioning that the Nusselt number goes through a maximum value depending on φ for critical inclinations around 40° (more precisely between 40° and 45° depending on φ) when θ is increased from 0° to 180°.

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Appendix A: Nomenclature

g	gravitational acceleration.
D	mass diffusivity (m^2/s)
Le	Lewis number, α/D
Nu	Nusselt number
R_T	Thermal Darcy-Rayleigh number
S	dimensionless solute concentration, $(S^{'} - S_{0}^{'})/\Delta S^{'}$
S_0	reference solute concentration (at $x = y = 0$)
$\Delta S'$	characteristic solute concentration, $-D_T S_0'(1-S_0')\Delta T'/D_{eff}$
Sh	Sherwood number
t	dimensionless time, $t' \alpha / \sigma H'^2$
Т	dimensionless temperature, $(T^{'} - T^{'}_{0})/\Delta T^{'}$
T_0	reference temperature (at $x = y = 0$)
$\Delta T^{'}$	characteristic temperature, $H'q'/\lambda$
Greeks	
α	thermal diffusivity of the porous medium (m^2/s)

- Ψ dimensionless stream function, Ψ'/α
- β_s solutal expansion coefficient (m^3/kg)

β_T	thermal	expansion	coefficient	(K^{-1})	.)

- ε' porosity of the porous medium
- v Kinematic viscosity of the fluid (m^2/s)
- ρ density of the fluid mixture (kg/m^3)
- θ cavity inclination (°)
- λ thermal conductivity (W/m^2K)
- φ separation parameter; $\varphi = \beta_s \Delta S' / (\beta_T \Delta T')$