

## An Innovative Approach of Salt Separation Using the Soret Effect

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**Abstract:** In this paper, we discuss a new technique for desalination based on the principles of the Soret effect. In particular, the method stems from the peculiar behavior of the solute in a solution of sodium chloride (i.e. salty water layer) of uniform concentration being less or equal to 5wt% under the effect of an imposed temperature gradient with average temperature equal to 12°C. The Soret coefficient may be positive or negative according to whether the temperature of the solution is above or below 12°C. As two diffusion processes (one parallel to the temperature gradient and the other anti-parallel) can occur at the same time in the solution when heating produces an average temperature of 12°C, this mechanism can be used to induce migration of the solute into the middle part of the salty water-layer forming a bulb of brine. The idea underlying the present theoretical study is to remove the brine that is formed in the center of the solution. A fundamental theory for this new principle of desalination is elaborated (to determine the distribution of the concentration in the medium). Moreover, a possible design of a desalination system based on this phenomenon is proposed accordingly.

**Keywords:** Soret Effect, Desalination, Dispersion, Desalination system.

### Nomenclature

$d$	salty solution thickness
$D_m$	salt diffusivity
$g$	gravitational acceleration
$N$	salt mass fraction
$p$	pressure
$R_a$	critical Rayleigh number
$S_T$	Soret coefficient
$T$	Temperature

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$\Delta T = T_h - T_c$	temperature difference
%Wt	mass per percent weight
cr	critique
i	initial

## Greek Symbols

$\alpha$	thermal diffusivity
$\beta_s$	coefficient of mass expansion
$\beta_T$	coefficient of thermal expansion
$\nu$	kinematic viscosity
$\tau$	relaxation time

## Subscript

$c$	cold
$h$	hot
$max$	maximum
$min$	minimum

## 1 Introduction

The conversion of saline water into fresh potable water and water for industrial purposes has been practiced worldwide over the last forty to fifty years.

Desalination is a process that removes dissolved minerals (including but not limited to salt) from seawater, brackish water, or treated wastewater. A number of technologies have been developed for desalination including reverse osmosis (RO), distillation: Multistage Flash (MSF), Multi Effect Distillation (MED), and electro-dialysis: Reversal (EDR) and vacuum freezing: Vapor Compression (VC).

Oceans make up 97% of the world's supply of water. Desalination using seawater reverse osmosis (SWRO) membrane technology [see Lachish (2007) and Van't Hoff (1887)] has become a viable option for the development of new regional water supplies. However, new ideas of desalination can be elaborated and tested, such as separation of constitution components of a binary solution by using a temperature gradient.

A temperature gradient applied to a liquid mixture not only causes heat flux but also gives rise to a diffusion current of the constituent particles or molecules, which drift along temperature gradients, some condensing in the hotter and some in the colder. The resulting separation of the molecules causes the gradient parallel or anti-parallel with respect to the temperature gradient. Generally, a particle moves from hot to cold, but the reverse is also seen under some conditions. For example: concentration rate or the heating gradient. This phenomenon is widely involved in some sectors, for example the industrial sector [see Furry, Jones and Onsager (1939)].

This microscopic phenomenon, known as Ludwig (1856) effect, Soret (1879) effect, or thermodiffusion, has been studied by DE Groot (1945) [also see Chanu (1994), Luettmer (2005), De Groot and Mazur (1983)], and has been observed in a variety of systems [see Caldwell (1973), Reith and Müller (2000), Hirota and Kimura (1943)].

Because of the combined actions of two diffusing quantities an impressive onset of convection has been observed. Rayleigh-Bénard instability in solution with Soret effect appears as an oscillatory mode. This phenomenon has been extensively studied both experimentally and theoretically [see Platten and Legros (1984), Platten (2006), Saghri, Jiang, Derawi, Stengby and Kawaji (2004)].

Most of the researches who investigated this phenomenon [see Prigogine, Debrouckere, Amand (1950)] have focused on the determination of the transport coefficient ( $D_m$ ), the thermodiffusion coefficient ( $D_T$ ), and the Soret coefficient ( $S_T$ ). It has been observed that the Soret coefficient [see Caldwell (1976) and (1073), Carlos, Draghi and Avalos (2003) and Würger (2006)] varies its sign.

Some peculiar behavior has been observed due to the change of the Soret sign. For salty water of concentration less than 5wt%, it has been shown that the Soret effect is zero at 12°C; being negative below this temperature and positive above 12°C.

In some sense, the solute in the upper half of the salty layer migrates down, unlike of what happens in the lower half. We find comprehensive reviews on this area in Platten, Chavepeyer and Abdeljabar (1995).

Such arguments may raise many questions on the possibility to use this phenomenon in problems of practical and technological interest, such as desalination.

In this article, we propose a new idea of desalination using the Soret effect. We believe that this technique deserves theoretical attention as well as experimental assessment.

## 2 Formulation of the Problem and Analysis

### 2.1 System considered

We consider a horizontal layer of uniform salinity, which is initially at a temperature of  $\bar{T} = 12^{\circ}C$  and a mass fraction  $N \leq 5wt\%$ . This layer is confined between impervious and conductive two plates which are separated by a distance  $d$  (see Fig.1).

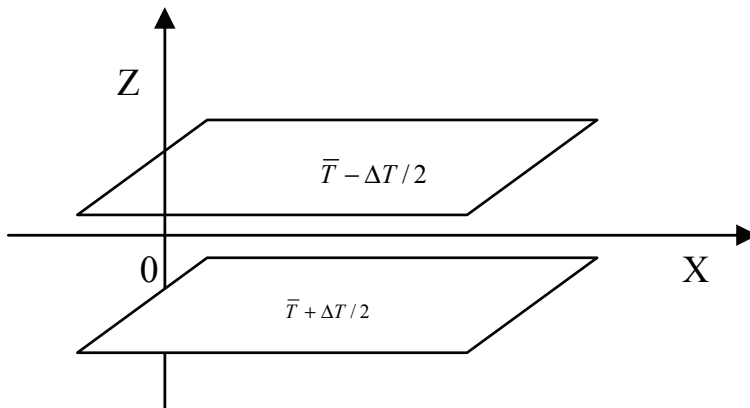


Figure 1: Sketch of salty layer conditions

The lower and upper plates are heated and cooled down at fixed temperatures  $T_h = \bar{T} + \frac{\Delta T}{2}$  and  $T_c = \bar{T} - \frac{\Delta T}{2}$  respectively.

### 2.2 Relaxation times

Herein, the time for which a distribution of salt occurs and remains stationary is called relaxation time. It is defined as the sum of the relaxation time needed for the establishment of the thermal profile and the salt concentration profile. They are deduced from the analytical solution of diffusion equations of salt and heat [see Abdeljabar (2009)].

In such case, the total relaxation time is given by the following equation.

$$\tau \approx \frac{d^2}{\xi \cdot (1 - N_m) \cdot D_m \cdot \Delta T^2} + \frac{d^2}{16\pi^2 \cdot \alpha} \tag{1}$$

where  $\alpha$ : is the coefficient of thermal diffusivity.

### 2.3 Conductive state

In the motionless basic state, a linear temperature distribution is established throughout the salty layer. It is defined by the following equation:

$$T(z) = \varepsilon \Delta T \cdot z^* + \bar{T} \tag{2}$$

where  $\varepsilon = -1$ ,  $z^* = \frac{z}{d}$ , and  $z^*$  varies between its extreme values  $z^* = \pm \frac{1}{2}$ .

We assume that the thermal conductivity is constant (i.e.  $\lambda(c,T) = \text{constant}$ ), thus the temperature profile remains linear across the solution.

Accordingly, the dispersion of the solute by Soret effect leads to a distribution of concentration in the salty layer. When the equilibrium state is reached (for a time  $t = \tau$  that is given in section 2.2), the salt distribution equation is defined by:

$$\frac{\partial N}{\partial z^*} + S_T \cdot Ni(1 - Ni) \frac{\partial T}{\partial z^*} = 0 \tag{3}$$

In such case the Soret coefficient is zero, being negative below and positive above [see Platten, Chavpeyer and Abdeljabar (1995)].

We assume that the Soret effect varies linearly with temperature in following form:

$$S_T = \xi(T - 12) \tag{4}$$

Thereby, the salt profile (i.e. at the steady state) is described by:

$$N(z) = N_{\max} - \xi_T \cdot Ni \cdot (1 - Ni) \varepsilon^2 \frac{\Delta T^2}{2} z^{*2} \tag{5}$$

where  $N_{\max}$ : maximum salt concentration, and  $\varepsilon^2 = +1$ .

For a concentration of  $N = 28.5 \cdot 10^{-3}$  (or 0.5 Normal see Caldwell (1974)), the Soret coefficient is given by:

$$S_T = 10^{-5} * [-119 + 11.05 * T - 0.064 * T^2 + 10^{-4} * p * (3.20 - 8.48 * T + 0.0532 * T^2)] \tag{6}$$

The Eq.6 is valid for the following conditions:

$$2^\circ\text{C} \leq T \leq 52^\circ\text{C}$$

$$\text{and } 0 \leq p \leq 70\text{bar}$$

The standard deviation of the deviation from the Eq.6 is  $5.10^{-5} \text{K}^{-1}$  [see Caldwell (1975)].

We assume that the effect of the pressure and the temperature of second power are negligible in Eq.6. Hence, the Soret coefficient can be reduced to the following form:

$$S_T \approx 11.05 * 10^{-5}(T - 10.77) \tag{7}$$

If we equate this equation to  $S_T = \xi(T - \zeta)$ , thereby we have:  
 $\xi = 1.105 * 10^{-4}$  and  $\zeta=10.77$ .

Thus, Eq.7 is approximately equal to Eq.3 that is considered in this paper.

**2.4 Steady state Salt distribution**

The maximum concentration of salt is evidently located in the zone where the Soret effect vanishes. Definitely, it is the zone of the temperature equal to 12°C that is located in the centre of the solution. The concentration at the edges of the salty layer (i.e. for  $z^* = \pm \frac{1}{2}$ ), and after a time  $\tau$ , is:

$$N(z = \pm \frac{d}{2}) = N_{max} - \xi_T.Ni.(1 - Ni)\frac{\Delta T^2}{8} = N_{min} \tag{8}$$

To determine the value of  $N_{max}$  and  $N_{min}$ , we use the fact that the average salt concentration in the salty layer is independent of time that is equal to  $Ni$ . Hence, it is given by the following equation:

$$Ni = \int_{-\frac{1}{2}}^{\frac{1}{2}} (N_{max} - 4(N_{max} - N_{min})z^{*2})dz^*$$

By integration, we have:

$$3Ni. = 2N_{max} + N_{min} \tag{9}$$

Combining Eqs. 8 and 9, we deduce the values of  $N_{max}$  and  $N_{min}$  as follows:

$$N_{max} = Ni + \xi_T.Ni.(1 - Ni)\frac{\Delta T^2}{24} \tag{10}$$

and

$$N_{min} = Ni - \xi_T.Ni.(1 - Ni)\frac{\Delta T^2}{12} \tag{11}$$

For the maximum separation (i.e. after a time  $\tau$  or at steady state), the concentration at the center of the salty layer and the edges of salty layer are, respectively:

$$N(z = 0) = N_{max}, \quad N(z = \pm \frac{1}{2}) = N_{min} \tag{12}$$

The difference of concentration between the extreme values is given by:

$$\Delta N = N_{\max} - N_{\min} = \xi_T \cdot Ni \cdot (1 - Ni) \frac{\Delta T^2}{8} \quad (13)$$

The Eq.13 shows that the dispersion of solute depends on the temperature gradient across the liquid layer. As it is seen in the example of section (2-3), the value of  $\xi$  is small ( $\xi = 1.105 \cdot 10^{-4}$ ), this makes the effect of this phenomenon small. For enhancing its efficiency,  $\Delta T$  must be maximized.

Next in the Section (2-7), we discuss the settlement for which we can increase the  $\Delta T$ .

Now substituting  $N_{\max}$  by its value in Eq.4, it yields to the equation describing the distribution of concentration:

$$N(z) = Ni \left[ 1 + \xi \cdot (1 - Ni) \frac{\Delta T^2}{8} \left( \frac{1}{3} - 4z^*{}^2 \right) \right] \quad (14)$$

According to Eq.14, it is obvious that the salt distribution depends on the value of initial concentration and the temperature gradient.

This study leads to determine the salt distribution profile throughout the solution, which is crucial for practical processing.

## 2.5 Typical separation scenarios

In such arrangement, the same phenomenon appears in each half thickness of the salty layer but it happens in the opposite direction. In the half thickness of salty layer for which  $T > 12^\circ\text{C}$ , the solute migrates upward, while in the other half, the solute migrates downward. However, at the central zone of the salty layer where  $T = 12^\circ\text{C}$ , we have no Soret effect. This phenomenon carries on until a steady state is reached. It is reached at the relaxation time  $\tau$  (see Section 2.2) for which the distribution of salt remains stationary.

In Fig.2, the curve of the distribution of salt and the brine at steady state are sketched by a dashed curve and by an elliptical pattern, respectively.

This distribution of salt can be affected by the onsets of convection. Thereby, for the feasibility of the method of desalination, the convection must be avoided from the medium.

In the next part, we analyze the ability to avoid convection in the medium for preserving the distribution of salt in the medium.

## 2.6 Control parameters

It is worthwhile to preserve the structure of the salt distribution in order to extract the liquid of high concentration. Such condition is achieved if the convection is

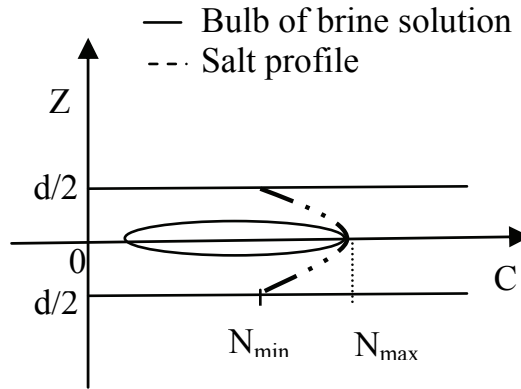


Figure 2: Profile of salt concentration due to Soret effect

avoided. The best way to do is to control the temperature gradient (i.e.  $\Delta T$ ) that must be less than the critical temperature gradient  $\Delta T_{cr}$  ( $\Delta T < \Delta T_{cr}$ ) if the heating is supplied from below. Obviously, the critical gradient is calculated from the value of the critical thermal Rayleigh number:

$$\Delta T_{cr} = \frac{Ra_{cr} \cdot \nu \cdot \alpha}{\beta_T \cdot g \cdot d^3},$$

where  $Ra_{cr}$  is the critical Rayleigh number,  $\nu$  is the cinematic viscosity,  $\alpha$  is the thermal diffusivity,  $\beta_T$  is the thermal expansion coefficient,  $g$  is the gravity acceleration and  $d$  is the thickness of the salty solution.

In such case the critical Rayleigh number can be obtained by resolving the classical governing equations using linear theory. It is described in earlier work of Platten (2006) and it is as follows.

$$\lambda \cdot Ra_{cr}^2 + \xi \cdot Ra_{cr} + \gamma \cdot R_s^2 \cdot S_T^2 + \delta = 0 \quad (15)$$

where  $Ra_{cr} = \frac{\beta_T \cdot g \cdot \Delta T_{cr} \cdot d^3}{\nu \cdot \alpha}$  is the thermal Rayleigh number,  $R_s = \frac{\beta_s \cdot g \cdot N_1^0 \cdot d^3}{\nu \cdot \alpha}$  is looks like salt Rayleigh number,  $S_T = \frac{D_T}{D_m} = \Psi(T)$  is the Soret effect, and  $\lambda = f(\kappa, Pr)$ ,  $\xi = f(\kappa, Pr)$ ,  $\gamma = f(\kappa, Pr)$ ,  $\delta = f(\kappa, Pr)$  are functions of Prandtl number ( $Pr$ ) and wave number ( $\kappa$ ) [see DE Groot (1945)]. When we resolve Eq.15, we obtain the critical thermal Rayleigh number.

In such case the impact of the Soret effect is limited, due to the restriction under which the temperature gradient is considered. However such restriction decreases the efficacy of the dispersion of the solute.



In the next, we study how to enhance the rate of separation due to the Soret effect, which it can be achieved if no condition is imposed on the temperature gradient ( $\Delta T$ ).

### **2.7 Adverse heating**

To prevent the onset of convection, an adverse heating can be performed. The profile of temperature is similar to one indicated by Eq.1 case where  $\varepsilon=+1$ . In such case the heating and cooling is strictly the inverse of those cited in Section (2-1). In such arrangement the onset of convection is totally avoided. Therefore the temperature gradient for heating can be greater than those restricted by the onset of convection. The high temperature gradient is valuable for increasing the separation of salt under Soret effect. However an increasing of temperature difference can induce the onset of an oscillatory mode due to the buoyant convection [see Gupta (1999)]. In turn the efficiency of the method can be limited. Therefore, more experimental studies are needed to quantify the behavior of this phenomenon.

## **3 Description of Desalination Process**

The process of desalination, which we are dealing with, is a new form of separating fresh water from salty water by using so-called Soret effect. The dispersion of the components of the solution under heating allows us to separate mechanically the more salty and the less salty.

The solution of the salty water at 12°C is pumped in the rectangular cell (see Fig.4). Then, a temperature gradient is applied on the layer. The gradient can be induced by thermoelectric devices for example (see Fig.3). The top and the bottom faces of the experimental cell (see Fig.3) are heated and cooled down; however, the lateral walls are insulated. The average temperature of the gradient is maintained at 12°C. Besides, we considered the case of temperature gradient for which convection can't appear. These conditions induce the separation of solute by Soret effect in opposite direction. Beyond the zone of temperature of 12°C the solute migrates to this zone, and accumulates in the center forming a bulb of brine water. When we extract, from the layer of salty water, the bulb of brine water and repeat the process on the remaining water, this can yield potable water. This process is what we called "desalination by Soret effect".

The advantages of such a desalination system is the low energy consumption. Moreover, the desalination process can be supplied with the electrical energy produced by solar energy plants, and this in the aim to reduce the energy charge as much as all devices can work with low voltage energy.

The heat and cold supplied at the boundaries of the solution layers can be produced

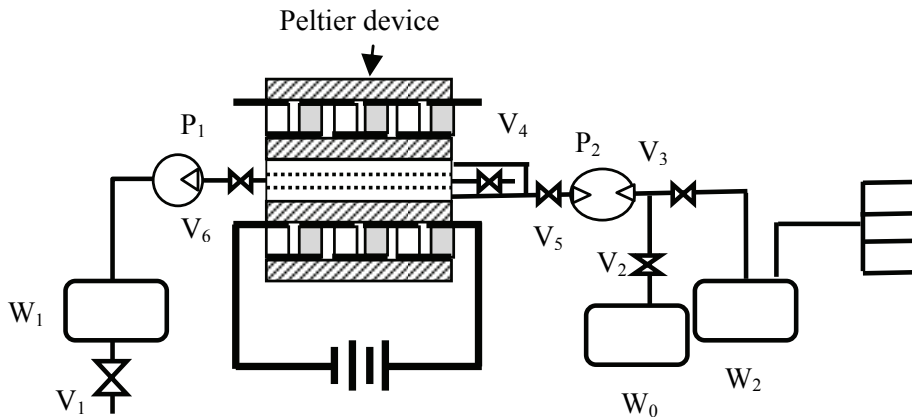


Figure 3: Design of Unit of Desalination System

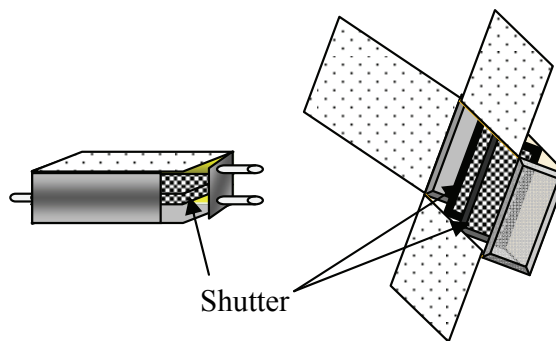


Figure 4: Cut-away of the Cell

by using thermoelectric system (i.e. Peltier device). Such device needs a small current that can be supplied from solar energy by using a Solar Panel. In spite of the low COP of the device that is of order of 0.1, it can produce enough of cold and heat that can maintain an average temperature of  $12^{\circ}\text{C}$  throughout the solution.

The desalination process is intended so to perform a series of separation stages. Each solution obtained should be processed again a certain number of times. Each stage accomplishes the same function; what we call here a unit of desalination.

In Fig.3, we give a sketch of the unit of desalination system. The main component in the desalination unit is the cell (see Fig.4) into which the separation is performed. The particular dimension of the cell is its thickness. It depends on the nature of the

temperature gradient: parallel or anti-parallel to the gravity. In the case where convection can occur, the thickness should satisfy the condition of non-onset of convection ( $Ra < Ra_{cr}$ ). Thanks to the Eq. (15), we can determine the thickness of the cell when we fix the gradient or conversely. Otherwise no condition is compulsory, but it is convenient that the thickness must be selected for the effectiveness of the Soret effect.

The arrangement of the unit of the desalination is as follows. A small pump ( $P_1$ ) is connected to the central zone of the cell which serves to discharge the more salty water from the cells and delivers it to what we call a brine collector tank ( $W_1$ ). The pump ( $P_2$ ) is connected to the cell, which pumps the less salty solution from the cell into the tank ( $W_2$ ), besides it fills the cell up of the brut salty solution carried from the tank ( $W_0$ ). The valve ( $V_1$ ) serves to drain the more salty water, however, the valves ( $V_2$  and  $V_3$ ) serve to control the liquid flow from the cell to the tank ( $W_2$ ) and from the tank ( $W_0$ ) to the cell, respectively. The valve  $V_4$  is used for evacuating air from the cell and the valves ( $V_5$  and  $V_6$ ) are used to isolate or link the cell with the others component of the system. Two shutters are installed symmetrically inside the cell (see Figs.3 and 4).

The cell is divided in three zones: central zone and two peripheral zones. The shutters can isolate the central zone from the two peripheral zones, when the salt is accumulated in the middle. The shutters have been inserted to perform the best solution as a trap of brine accumulated by Soret effect. Notice that all electrical energy needed for the components of the desalination plant (pump, electro-van. . .) can be provided by electrical energy produced by solar energy.

Having set such plant, the distillation process is normally performed according the following steps: The salty water is pumped into the cell, until it will be filled, and then it is preserved of all external connections (i.e. the valve  $V_5$  and  $V_6$  are closed). Subsequently, the thermal gradient is applied on the salty layer. A distribution of salt appears in the medium and evolves until it reaches a steady state. Ultimately, the shutters are closed and the brine can be drawn out from the central zone of the cell. However, the water in the peripheral zones is collected and recycled for a second distillation process. This operation is repeated in the plant, till the quality of the water will be what we call potable.

#### **4 Conclusion**

In this paper, we have studied the possibility of desalination by a process based on the Soret effect. A theoretical analysis of the desalination method has been presented. It has been shown that the desalination by using Soret effect is technically feasible; therefore, a possible design for a plant has been presented accordingly.

The energy-consumption cost of such a system can be reduced by using the solar energy. The main disadvantage of this method may be the relaxation time that is relatively long.

Next step will consist of experimental validation of the proposed method/system.

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