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Design Optimization of a Conical Annular Centrifugal Contractor

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Abstract: The present work is concerned with a numerical study of the performance of a conical annular centrifugal contractor through the analysis of the flow properties when the apex angle is changed for different imposed axial flows. The calculations revealed the advantage of using conical annular centrifugal contractors compared to the cylindrical annular centrifuges. The study is conducted by a comparison analysis of the hydrodynamics of fluid flow in both conical and cylindrical contractors where moderate axial flows are imposed. In both systems the outer body is stationary while the inner rotor is maintained at constant speed. The calculations are achieved by a finite differences formulation using the Simplified Marker and Cell algorithm.

Keywords: Annular Contractor, Shear Migration, Taylor Vortices, Conical Contractor, Rotating Systems

1 Introduction

In the chemical and petroleum industries, separation and filtration equipments were used since many decades without major improvement in the fundamental aspects of operational conditions of these separation devices. Separation and filtration is achieved either by hydrocyclone desanders or desilters or using classical cylindrical centrifuges. The need for inline highly efficient separation system for instance requires the development of new filtration and separation equipments able to handle large quantities of produced fluids with high separation efficiency. Recent studies have shown a growing interest in the separation efficiency of the annular centrifugal contractor commonly known as Taylor-Couette flow system (Ohmura *et al.*, 2005; Vedantam and Joshi, 2006).

This type of centrifuges is used as a liquid-liquid extractor, a bioreactor, a filtration apparatus, a chemical reactor and as a heat and mass exchanger (Kataoka and Taki-

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gawa, 1981). This system was found able to both mix and separate species within a liquid (Holeschovsky and Cooney, 1991; Tsao *et al.*, 1994; Ohmura *et al.*, 2005; Henderson *et al.* , 2007). Most of the studies dedicated to this flow system in the past century were focused of the flow regimes and flow instabilities depending on the physical, geometrical and dynamical flow properties. Studies related to specific technological applications of the annular centrifuge are relatively rare compared to fundamental works.

In the annular centrifuge system, an axial flow is imposed to the rotating one operated as an open flow. Several authors have shown that the flow becomes complex when an axial flow and/or a radial heating is imposed to the initial rotating flow (Ali and Weidman, 1990; Chen and Kuo, 1990). When a weak axial flow is imposed on the Taylor-Couette flow, flow instabilities known as Taylor vortices, counter-rotating two by two and filling the whole fluid column, move axially driven by the imposed axial flow. In this axial drift many authors found that convection dominates near the vortex boundaries while diffusion dominates near the vortex centre (Desmet et al., 1996). Recently, Ohmura *et al.* (2005) found that the axial flow results in a by-pass flow around the vortices and enhances the separation of larger particles from smaller ones. The by-pass flow phenomenon was also experimentally observed by Wereley and Lueptow (1999) who confirmed the existence of a strong stream of fluid winding around vortices in the axial flow direction by a PIV technique. This stream does not fill the annular gap and appear alternately displaced toward the inner and outer cylinder. Later on, Hwang and Yang (2004) confirmed numerically these results.

This effect results in keeping the smaller particles in the centre of the vortex while the larger ones move outwards to the vortex boundary. The largest particles are then taken by the axial by-pass flow. This phenomenon was called shear particle migration by Ohmura *et al.* (2005).

From these observations, it seemed that shear particle migration plays an important role in particle segregation especially when the particles have densities closer to the carrier fluid and have different size ranges. Enhancement of the shear particle migration through geometrical optimization would enhance furthermore the separation efficiency and shorten processing time. In this regards, the present work is concerned with the study of the characteristics of another type of centrifuges which is a conical annular contractor. A study of the performance of this conical centrifuge based on its hydrodynamic properties is conducted by numerical calculations of primitive variables.

2 Formulation

The mathematical formulation used in the present work was previously described in details by Noui-Mehidi *et al.* (2002) for the study of the flow between rotating conical cylinders without an axial flow. The present formulation is a hybrid formulation in which the effect of an axial flow is added. In Fig. 1, a representation of the centrifuge configuration is sketched in a cylindrical coordinate system. The coaxial conical cylinders defining the centrifuge walls have inner and outer radii r_i and r_o at the top of the flow system and height L.

The inner and outer conical cylinders have the same apex angle α resulting in a constant gap width d. The flow analysis of a viscous incompressible fluid of kinematic viscosity ν in the Taylor vortex flow regime is supposed to be axisymmetric according to the previous observations of Wimmer (1995, 2000) when the outer conical cylinder is fixed and the inner one is rotated.

The non-dimensional governing equations for an axisymmetric flow in cylindrical coordinates have been given in different ways by many authors, as reference the formulation given by Hwang and Yang, (2004) written as follows is used:

$$\begin{aligned} &\nabla . U = 0 \\ &\frac{\partial U}{\partial t} + (U.\nabla)\, U = -\frac{1}{\rho}\nabla P + \nu \nabla^2 U \end{aligned} \tag{1}$$

Here primitive variables are to be solved and where U is the fluid velocity, ρ the fluid density, P the pressure field and ν the fluid kinematic viscosity. The velocity field U is decomposed in its components (u, v, w) given in the cylindrical coordinates directions (r, θ, z) respectively. The variables are made dimensionless using the scale d, Ω_i and Ω^{-1} for length, speed and time respectively, where Ω is the angular velocity of the inner conical cylinder. In treating the flow regimes in the case of an axial flow imposed to the rotating annulus, the two non-dimensional numbers defining the flow states are the Taylor number $Ta = \frac{ri\Omega d}{\nu}$ describing the rotation of the flow and the Reynolds number $Re = \frac{Vd}{\nu}$ describing the axial flow intensity, where V is the average axial velocity.

3 Numerical method

The obtained non- dimensional equations are discretized on a rectangular mesh grid by the use of a conformal coordinate transformation which transforms the computational domain to a rectangular domain for any apex angle value between the conical walls. This transformation thus allows the implementation of finite differences in the computational domain. The detailed implementation of the transformation to the governing equations was described in details by Noui-Mehidi *et al.* (2002) for

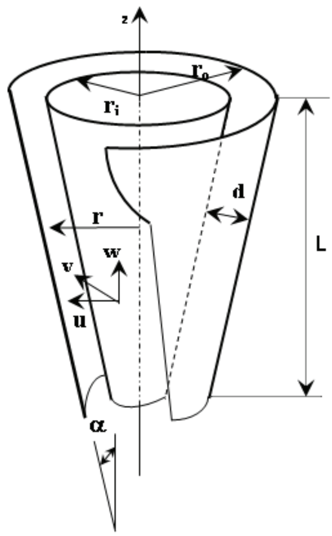

Figure 1: Centrifuge geometry and variable notations. The axial flow is imposed at the lower base of the centrifuge parallel to the conical walls.

the study of flow between rotating conical cylinders. The new transformed coordinates system is obtained by applying η= r-z.tanα and ξ=z. The transformed domain (η, ξ), which is the computational space, has a rectangular form for any value of the conical apex angle α, which only appears in the governing equations. This analytic transformation is unique and mathematically conformal as reported in the previous work (Noui-Mehidi *et al.*, 2002). Applying the transformation to the governing equations explicitly exhibit the value of the apex angle after implementing the derivatives obtained from the coordinate transformation. In the resolved transformed equations, the apex angle α permits to investigate different cases including the case of circular cylinders, since α=0 corresponds to the case of cylindrical parallel walls.

No-slip boundary conditions are applied at the walls while a constant flow flux is imposed at both inlet and outlet of the system. The integration method together with the boundary and initial conditions implementing a finite-difference formulation was previously described by (Ohmura *et al.*, 1994). The numerical domain is discretized using staggered mesh grid, in which the different unknown variables are not located at the same grid points. The Pressure is defined at the centers of the numerical cells. The implementation of a staggered grid permits to avoid the appearance of oscillatory solutions, particularly for the pressure P (Fletcher, 1970).

The numerical calculations are achieved using the concept of the Simplified Marker and Cell (SMAC) method developed by Amsden and Harlow (1970) and which is a type of operator-splitting method that separates the solutions of velocity and pressure fields with an iterative procedure. A first-order time integration scheme is applied with a central finite differences method at the second order for spatial discretization. This choice does not affect the precision of the calculations since relatively small Reynolds numbers are only investigated the final flow state being still laminar. For the no time derivative terms mainly the pressure, the use of the Poisson equation for calculating the potential function as an implicit method combined with the staggered mesh grid permits to avoid any numerical instability and make the SMAC formulation a semi-implicit method. The choice of the optimal over-relaxation parameter in the SOR method has been done through numerical experimentation for optimal convergence conditions. It has been chosen in the range $1.5 \leq \omega \leq 1.8$. The convergence criteria ε imposed for the time integration has been fixed to $\varepsilon \leq 10^{-7}$ and the final stable solution is obtained when the condition $|V^{n+1} - V^{n}| \leq 10^{-12}$ is satisfied for the velocity field. As vortices eventually occupy the entire annulus both axially and radially in the present problem, fixed grid spacing was used for good resolution everywhere in the fluid field. The numerical simulations for the studied flow configuration were in good accordance with the experimental results (Noui-Mehidi *et al.*, 2002). In the present study, Taylor number Ta was fixed to 128 and Reynolds number Re to 1.28 and 2.56 corresponding to a ratio of Re/Ta of Vx=0.01 and Vx=0.02. In order to keep our investigation in the Taylor vortex regime analysis and according to the study of Hoffman and Busse (1999) we have chosen to work with relatively small values of the apex angle between 0 and 8^o corresponding geometrically to conical cylinders for the case of a wide gap with r_i / r_o =0.83. The aspect ratio L/d was kept constant for all calculations and fixed to 10.

4 Numerical results and flow analysis

Usually the analysis of flow within annular centrifugal contractors is conducted by first validating the results obtained by the numerical scheme with those known in the case of a pure rotational fluid column without an axial flow. There are many references in the literature to conduct the validation. The flow field in the conical annular contractor without an axial flow was previously studied in details by Noui-Mehidi *et al.* (2002). A brief summary is given in the following, describing the main features of the flow structures observed between two conical cylinders without an axial flow. In the previous work by Noui-Mehidi *et al.* (2002) the results obtained in the case of α=0 using the present numerical scheme have shown a good agreement with the results presented in the literature (Ohmura *et al.*, 1994 for example). On the other hand, numerical calculations presented by Noui-Mehidi et al. (2002) have shown that the number of Taylor vortices decreases when α increases with no-axial flow.

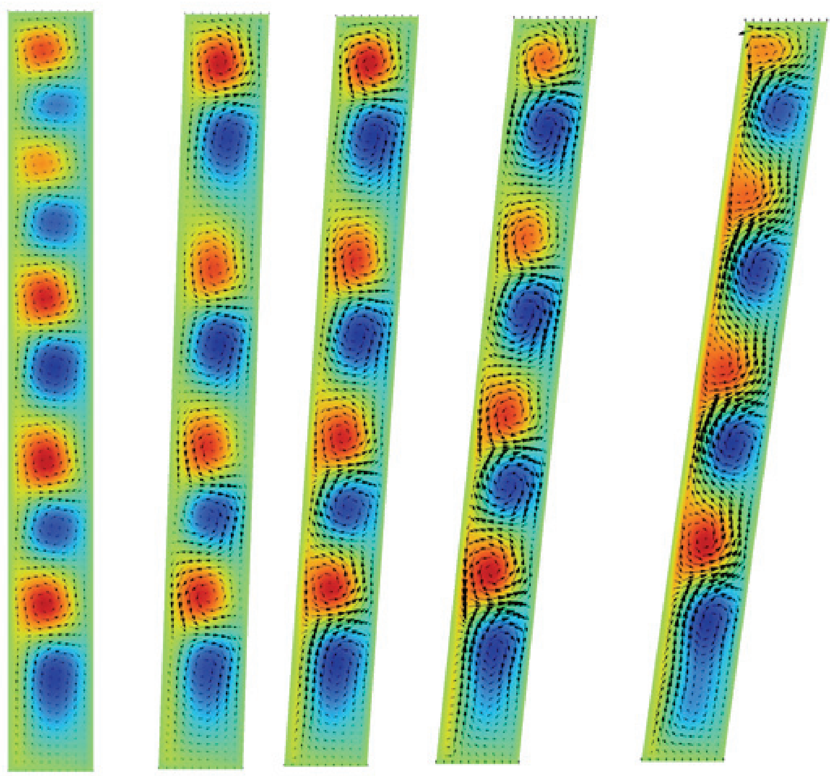

Figure 2: Coloured streamlines and velocity vector representation of the flow structures for different apex angles (Ta=128, Re=1.28)

For a flow system with a value of α=8^o, the vortices located at the bottom of the system are larger than those above as was observed experimentally. The numerical simulations performed for values of α lower than 8^o have confirmed this tendency for all investigated values of a conical gap, i.e. $\alpha \neq 0$. The large vortices observed at the bottom of the fluid column in the case of α=8^o for example rotate in the same direction as the basic three-dimensional meridional flow i.e. upwards with the rotating conical cylinder and downwards with the fixed conical cylinder. These

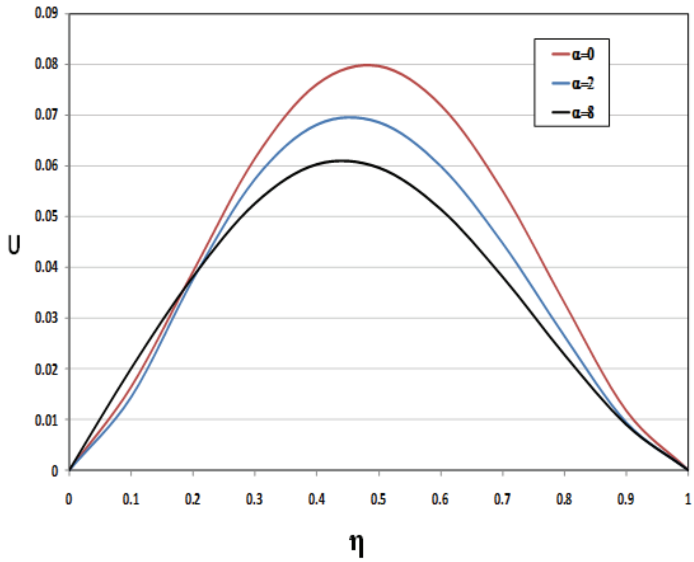

Figure 3: Radial velocity distribution at the last outflow boundary before the system exit compared for different apex angle, the value of α is in degrees (Ta=128, Re=1.28).

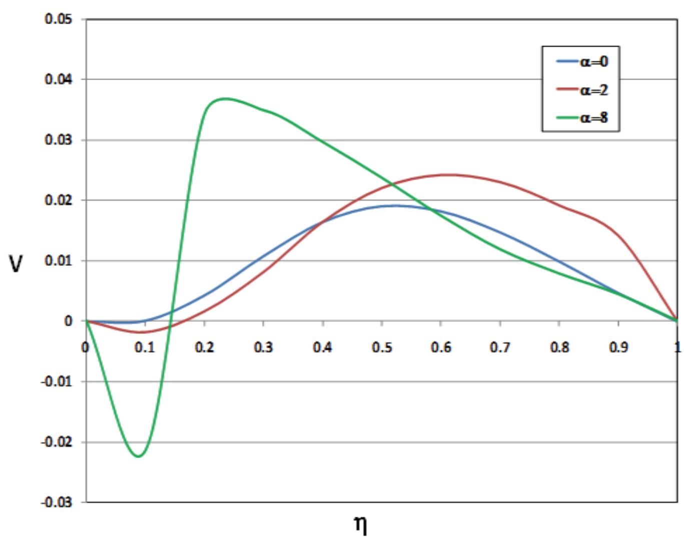

Figure 4: Axial velocity distribution at the last outflow boundary before the system exit compared for different apex angle, the value of α is in degrees (Ta=128, Re=1.28)

large vortices have also a strong vorticity compared to the adjacent counter-rotating smaller ones. The variation in Taylor vortices size in the conical system configuration has an important consequence on shear mechanism around the vortices. The vortices which have the same rotational sense as the basic meridional flow are the largest vortices as previously stated and contribute to the enhancement of the shear mechanism between the vortices as the apex angle α increases.

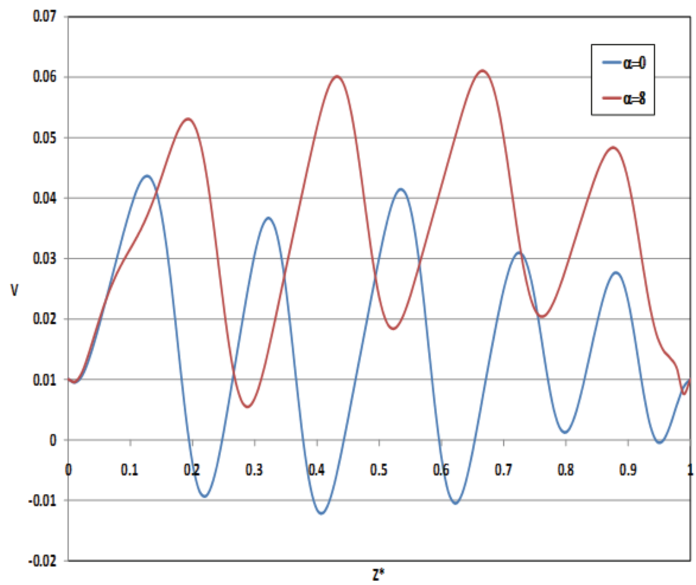

Figure 5: Axial velocity distribution along the middle of the annular gap compared for α=0 and α=8. (Ta=128, Re=1.28)

When an axial flow is imposed, the disposition of Taylor vortices in the axial direction is clearly altered. In Fig. 2 the imposed axial flow enters the flow system at the lower end and is directed upwards. For α=0 corresponding to a cylindrical annulus, at the entrance i.e. at the lower end of the system, the first vortex resulting from both the rotation of the inner cylinder (left wall) and the inlet flow has a size larger than the other vortices above. On the other hand, the by-pass flow winding around the vortices as observed by previous authors (Wereley and Lueptow, 1999; Ohmura *et al.*, 2005) is strongly established even at a value of the axial Reynolds number of 1.38. It can be also noticed that the centre of the vortices are alternately near the inner rotating wall for the vortices rotating counter-clockwise and near the outer fixed wall for the vortices rotating clockwise. This phenomenon was also observed by Hwang and Yang (2004). The total number of vortices for α=0 is 10. When the

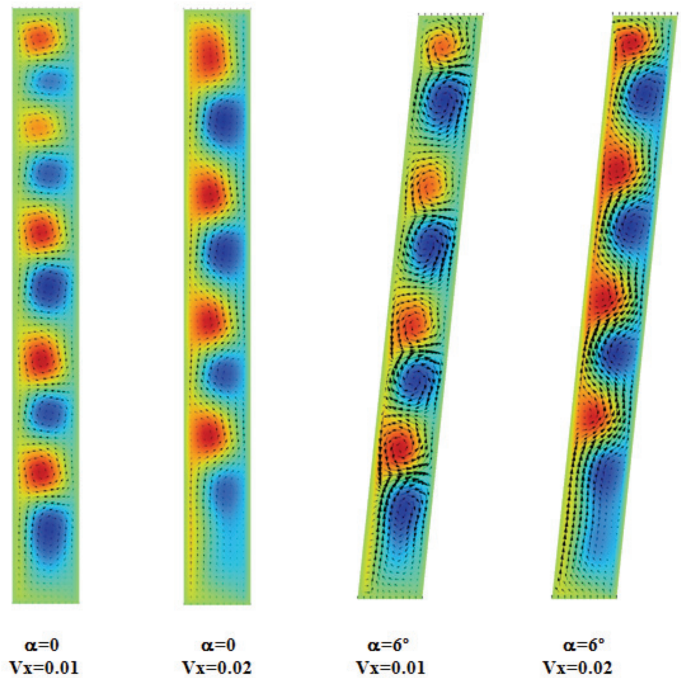

Figure 6: Coloured streamlines and velocity vector representation of the flow structures for different apex angles and different axial flows (Ta=128, Re=1.28 and Re=2.56)

annulus walls are slightly inclined to form a conical annulus as seen in Fig. 2 from the value of $\alpha=2^o$, the number of vortices decreases to 8 for the same radius ratio at the top of system and same Taylor and Reynolds numbers as for $\alpha=0$.

But in counter-part the vortices size are much larger at the top of system where their vorticity is stronger due to the larger centrifugal force resulting from larger radii at the top of the system. This tendency is maintained for values of α up to 6 degrees, where still 8 vortices are observed. But as seen, there is a shift of each vortex centre as well as an increase in the local vorticity resulting from higher centrifugal forces as α increases. It can be clearly noticed that the by-pass flow gains strength as well at the top of the system. These properties can be further analyzed through the radial distribution of both the radial and the axial velocity components shown in Figs. 3 and 4 at the outflow boundary between the top most two vortices. As shown in Fig. 3 the radial velocity intensity decreases as α increases, which means that the outflow boundary jet-like flow shifts towards higher streamline inclinations towards

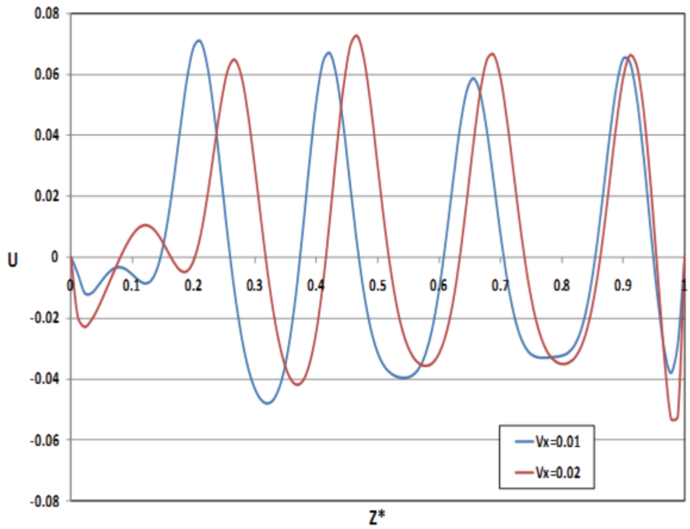

Figure 7: Radial velocity distribution along the middle of the annular gap compared Vx=0.01 and Vx=0.02 (α= 6^o, Ta=128)

the axial direction. In Fig. 4 the axial velocity distribution when α increases exhibits a complicated flow resulting from the imposed axial motion and the increase in the centrifugal force. For α=8^o for instance the by-pass flow intensity can be clearly seen in both the streamline contours and in Fig. 4 where the by-pass flow is the strongest near the rotating wall. On the other hand there is a decrease of the number of vortices for α=8^o to 6, which affects the axial distribution of the axial velocity as shown in Fig. 5. These flow observations reveal a clear advantage in choosing a conical annular contractor rather than a pure cylindrical one since both Taylor vortices vorticity and the by-pass flow are enhanced.

The effect of increasing the axial flow average velocity was also investigated. As illustration, Fig. 6 presents the contour plots of streamlines and velocity vectors for both α=0 and α=6^o for Re/Ta ratios of Vx=0.01 and Vx=0.02. It is first observed that increasing the axial flow intensity decreased the number of Taylor vortices as shown for α=0. A stronger by-pass flow is clearly seen for Vx=0.02. On the other hand surprisingly the number of vortices did not change for α= 6^o but a shrinkage of the vortices can be observed while the by-pass flow is also stronger between the vortices. The effect of the entrance can be clearly seen on the bottom first vortex in all cases. Fig. 7 shows that the axial distribution of the radial velocity component in the case of α= 6^o had only a shift in the axial direction without a major change in its radial amplitude, which means that the vortices conserved their intensity in the presence of a stronger by-pass flow as the axial flow was increased.

5 Conclusions

The flow analysis of a conical annular contractor through numerical simulations has revealed advantages compared to a classical cylindrical annular contractor. It was shown that increasing the apex angle value of the coaxial conical walls resulted in enhancing both centrifugal forces and the by-pass flow around the vortices.

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