

# Towards a Numerical Benchmark for MHD Flows of Upper-Convected Maxwell (UCM) Fluids over a Porous Stretching Sheet

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**Abstract:** The present research gathers an accurate numerical study of the laminar flow induced in an incompressible upper-convected Maxwell (UCM) fluid by a linear stretching of a flat, horizontal and porous sheet in the presence of a transverse magnetic field. The governing partial differential equations are converted into an ordinary differential equation by a similarity transformation. The effects on the velocity field over the sheet of the parameters like elasticity number, suction/blowing velocity, and magnetic parameter are also studied. It has also been attempted to show capabilities and wide-range applications of the 4<sup>th</sup> order Runge-Kutta method in comparison with the homotopy analysis method. Knowing in boundary layer theory that velocity profiles approach the ambient fluid conditions asymptotically, our numerical solutions were carried out under the simultaneous boundary conditions at infinity:  $f' \rightarrow 0$  as  $\eta \rightarrow \infty$ , and  $f'' \rightarrow 0$  as  $\eta \rightarrow \infty$  ( $f$  being the non-dimensional stream function). In this manner, a remarkable accuracy for the missed skin friction coefficient  $f''(0)$  can be achieved.

**Keywords:** MHD flow; UCM fluid; Permeable stretching sheet

## 1 Introduction

Boundary layer behaviour over a moving continuous solid sheet is an important type of flow occurring in several engineering processes. Specifically these include production of plastic sheets, drilling operations, glass-fiber and paper production, cooling of metallic and plastic sheets, liquid films in condensation processes and many others. In general, the involved fluid in these cases is treated as non-Newtonian fluid. Therefore, numerical analysts encounter actually a wide variety of challenges in obtaining suitable algorithms for computing flow and heat transfer of viscoelastic fluids. Very recently, boundary layer flow for non-Newtonian fluids

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with or without heat transfer has been studied by Massoudi (2003), Massoudi, Tran and Wulandana (2009), Cortell (1994), Cortell (2006a), and Bataller (2007a) for a second grade fluid, by Bhatnagar, Gupta and Rajagopal (1995) for an Oldroyd-B fluid, by Abbas, Sajid and Hayat (2006) and Hayat, Abbas and Ali (2008) for an upper-convected Maxwell (UCM) fluid, and also Bataller (2008a) has analyzed flow and heat transfer in the presence of thermal radiation by modelling viscoelastic properties with the help of the FENE-P (finitely extensible nonlinear elastic fluids) constitutive equation.

Flow kinematics can be modified with the help of a sufficiently strong magnetic field applied to an electrically conducting fluid surrounding a stretching sheet, Cortell (2005a), (2006b). Specifically, in flows involving heat transfer, Bird, Curtiss, Armstrong and Hassager (1987), it has been shown that there might be some advantages if the fluid surrounding the sheet can be made viscoelastic by using polymeric additives: Dandapat and Gupta (1989), Cortell (2007a). On the other hand, one can also resort to suction/blowing in order to modify flow kinematics: Cortell (2005b).

In the case of fluids of differential type (see Rajagopal (1995)), the equations of motion are in general one order higher than the Navier-Stokes equations and, in general, need additional boundary conditions to determine the solution completely. These important issues were studied in detail by Rajagopal (1984), Rajagopal and Gupta (1984) and Rajagopal and Kaloni (1989).

Usually, our proposed problems in the present area are solved by using boundary-layer theory along with the concept of similarity solution. The obtained ODE still presents a difficult problem to solve due to the lack of enough physical boundary conditions, as was commented. Nowadays, we currently use for problems of Sakiadis/Crane type (i.e.,  $f' \rightarrow 0$  as  $\eta \rightarrow \infty$ ), the augmented conditions  $f'' \rightarrow 0$  as  $\eta \rightarrow \infty$  ( $f$  being the non-dimensional stream function) for momentum transfer problems, and  $\theta' \rightarrow 0$  as  $\eta \rightarrow \infty$  ( $\theta$  being dimensionless temperature) for heat transfer problems, respectively. See, for example, Cortell (2006c), (2007b), Bataller (2007b). It is worth mentioning here that the aforementioned conditions at infinity have been applied (or obtained) in our studies since early 1990s: Cortell (1994). The problem under investigation (i.e., MHD flows of an UCM fluid) is highly nonlinear and these classes of problems are not easy to examine. Therefore, many applied mathematicians and numerical analysts have also recently paid much attention in developing suitable algorithms for solving these problems. In fact, a systematic analysis of the stagnation-point flow of UCM fluids has been recently carried out by Sadeghy, Hajibeygi and Taghavi (2006). Moreover, suction/blowing effects on MHD flows of an Maxwellian fluid were very recently considered by several researchers (see, for example, Amir, Aliakbar, Farzad and Sadeghy (2009), Hayat, Abbas, and Sajid (2006) who by means of the powerful and contrasted ho-

motopy analysis method (HAM) gave an analytical solution for the problem under investigation. Also, Hayat and his co-workers have considered UCM analyses with an  $x$  dependent free stream velocity far away from the stretching sheet, Hayat, Abbas and Sajid (2009).

In the present research, numerical and stable results are also obtained at large elasticity and/or magnetic number by using a 4<sup>th</sup> order Runge-Kutta (RK4) method along with shooting method and the mechanical characteristics of the flow are analyzed. Also, it will be underlined that the role of the velocity gradient at infinity (i.e.,  $f''(\infty)$ ) is of key relevance for our results from which the entrainment velocity  $f_\infty = f(\infty)$  can also be analyzed for the first time.

RK4 method by applying to an initial value problem (IVP) usually necessitates a choose for the limited integral region (i.e., the  $\eta_\infty$  value) instead of infinity for numerical integration, but our iterative shooting procedure does not need this because it acts only onto  $f''(0)$  (the missed skin friction coefficient) controlling, at the same time, the additional condition at infinity:  $f''(\eta_\infty) \cong 0$ . In this manner, the  $f'(\eta)$  function (velocity profile) tends to zero at infinity ( $\eta = \eta_\infty$ ) in an asymptotical fashion as must be required according boundary layer theory.

For each numerical solution in the iterative process the size of the integration domain is obtained as a natural part of the solution, and there is no necessity to select the extent of the integration domain before calculation. Therefore, our governing momentum transfer equation can be solved (numerically) by marching freely from the origin as it also is common for boundary layers involving interesting problems in the area of fluid dynamics, Bataller (2008a,b,c). This innovative way of integration prevents us from unphysical behaviours of the solution and provides high accuracy of the results.

## 2 Flow analysis

Let us suppose a steady, laminar and two-dimensional flow of an incompressible, electrically conducting and Boussinesq viscoelastic UCM fluid subject to a transverse uniform magnetic field  $B_0$  which is applied in the positive  $y$ -direction past a flat, horizontal and porous sheet coinciding with the plane  $y = 0$ , the flow being confined to  $y > 0$ . The motion of the fluid is generated due to linear stretching of the sheet with the application of two equal and opposite forces, which are applied along the  $x$ -axis so that the wall is stretched keeping the origin fixed. The magnetic Reynolds number is considered to be small and so we have a negligible induced magnetic field. The system of continuity and momentum equations can be written, in the usual notation, as (see Amir, Aliakbar, Farzad and Sadeghy (2009), Hayat,

Abbas and Sajid (2006))

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \beta \left[ u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] - \frac{\sigma_0 B_0^2}{\rho} u, \tag{2}$$

We take  $x$ -axis along the surface, the  $y$ -axis being normal to it and  $u$  and  $v$  are the velocity components in  $x$  and  $y$  directions, respectively,  $\nu$  is the kinematic viscosity,  $\rho$  is the density and  $\beta$  is the relaxation time of the fluid. Further,  $B_0$  is the uniform magnetic field along the  $y$ -axis and  $\sigma_0$  is the electric conductivity. The boundary conditions to the problem are

$$u = cx, \quad v = \pm v_w \quad \text{at } y = 0, \quad c > 0 \tag{3}$$

$$u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \quad \text{as } y \rightarrow \infty. \tag{4}$$

The second condition (4) is the augmented condition since the flow is in an unbounded domain (see Cortell (2006a)).  $v = v_w$  in the second condition (3) is the blowing velocity across the stretching sheet, whereas  $v = -v_w$  is the suction velocity.

Defining new variables

$$u = cx f'(\eta), \quad v = -(c \cdot \nu)^{1/2} f(\eta) \tag{5}$$

where

$$\eta = \left( \frac{c}{\nu} \right)^{1/2} y, \tag{6}$$

and substituting in (2) gives

$$(f')^2 - f f'' + M f' = f''' - K [f'''(f)^2 - 2f f' f''], \tag{7}$$

where  $K = \beta c$  is the elasticity parameter and a prime denotes differentiation with respect to  $\eta$ . Further,  $M = \frac{\sigma_0 B_0^2}{\rho c}$  is the magnetic field parameter. The boundary conditions (3) and (4) becomes

$$f = R, \quad f' = 1 \quad \text{at } \eta = 0, \tag{8}$$

$$f' \rightarrow 0, \quad f'' \rightarrow 0 \quad \text{as } \eta \rightarrow \infty.$$

where  $R = \frac{v_w}{(c\nu)^{1/2}}$  is the suction (>0)/blowing(<0) parameter.

For the sake of brevity we concentrate on latest results of Section 2 of Hayat and Sajid (2007) regarding Eqs. (7), (8), which govern the problem under consideration. For a more detailed discussion of the model we refer to Sadeghy, Hajibeygi and Taghavi (2006); Atalik (2008).

Quantities of relevant physical interest are:

1. The entrainment velocity of the fluid  $f_\infty$  defined here as  $f_\infty = f(\eta_\infty)$  with  $f'(\eta_\infty) \approx 10^{-4}$ . The corresponding  $\eta_\infty$  values are also given here in tabular form. Realize that from the second Eq.(5) we obtain  $v_\infty = -(c\nu)^{1/2}f_\infty$  and these quantities are related with the amount of fluid dragged by the sheet.
2. The thickness of the boundary layer  $\delta$  defined as the value of the  $y$  coordinate for which  $f'(\eta_\delta) = \frac{f'(0)}{100}$  holds. The corresponding  $\eta_\delta$  values are also given here in tabular form.

### 2.1 Numerical method and results

The best approximate for solving Eq. (7) that can be used is RK4 method. It is often used to solve differential equations systems. To this end, Eq. (7) can easily be written as the equivalent first-order system

$$w'_1 = w_2, \quad w'_2 = w_3$$

$$w'_3 = \frac{1}{1 - Kw_1^2} [w_2^2 - w_1w_3 - 2Kw_1w_2w_3 + Mw_2] \quad (9)$$

where the prime indicates differentiation with respect to  $\eta$ , and  $w_1 = f$ . In accordance with boundary conditions (8) we obtain

$$w_1(0) = R; \quad w_2(0) = 1, \quad (10)$$

$$w_2(\infty) = 0; \quad w_3(\infty) = 0. \quad (11)$$

Using numerical methods of integration and disregarding temporarily the boundary conditions (11), a family of solutions of {(9)-(10)} can be obtained for arbitrarily chosen values of  $w_3(0) = f''(0)$ . Tentatively we assume that a special value of  $|w_3(0)|$  yields a solution for which  $f'(\eta)$ ,  $|f''(0)|$  simultaneously vanishes at a certain  $\eta = \eta_\infty$  (see boundary conditions (11)) and satisfies the additional condition

$$w_2(\eta_\infty) = 0; \quad |w_3(\eta_\infty)| = 0, \quad (12)$$

where the solution reach its asymptotic state.

We guess  $w_3(0)$  and integrate Eq. (9) and boundary conditions (10) as an initial value problem by employing a RK4 algorithm for high-order initial value problems (Cortell (1993)) with the additional conditions (12). The iterative procedure is stopped to give the velocity and velocity-gradient distributions when the boundary conditions (12) at infinity are simultaneously reached. Those boundary conditions at infinity correct unphysical behaviours of the solution and seem to play an important role in this class of boundary problems. Equivalent step sizes  $\Delta\eta$  of 0.01 and 0.001 are used throughout the paper. It is worth mentioning that, for each numerical solution, the  $\eta_\infty$  value, namely, the integration domain, depends on the non-dimensional parameters which govern the momentum boundary layer problem, and it is obtained (no fixed before calculation as usually is made) as a natural part of the numerical solution. In other words, our iterative procedure acts only on the missed skin friction coefficient  $f''(0)$ . For a selected set of parameters entering the problem, Table 1 gathers some numerical results for the governing Eq. (7) along with boundary conditions given by Eq. (8).

Table 1: Some numerical results for the momentum transfer solution at  $K = 2$ ,  $M = 0.5$  and  $R = 0.3$  with  $\Delta\eta = 0.01$ .

$\eta$	$f$	$f'$	$-f''$
0.0	0.3	1.0	2.9498859
0.02	0.3194196	0.9424382	2.8067710
0.04	0.3377164	0.8877077	2.6668640
0.06	0.3549464	0.8357387	2.5306960
0.08	0.3711639	0.7864522	2.3986820
0.10	0.3864217	0.7397617	2.2711340
0.15	0.4206982	0.6337825	1.9730970
0.20	0.4500355	0.5419378	1.7058460
0.30	0.4964895	0.3945467	1.2612740
0.50	0.5546882	0.2069984	0.6718130
1.00	0.6057701	0.0405178	0.1324193
1.41	0.6149218	0.0106139	0.0346507
1.42	0.6150262	0.0102730	0.0335354
1.43	0.6151273	0.0099431	0.0324559
3.03	0.6181933	0.0001006	0.0001421
3.04	0.6181943	0.0000980	0.0001364
3.45	0.6182285	0.0000761	0.0000031

From this Table it is clear that at  $K = 2$ ,  $M = 0.5$  and  $R = 0.3$  (suction), we get  $\eta_\delta =$

1.42,  $\eta_\infty = 3.03$  and  $f_\infty = 0.6181933$ .

Table 2: The effect of the elasticity number  $K$  on flow characteristics for the case of suction ( $R = 0.3$  and  $M = 0.5$ ).

$K$	$-f''(0)$		$\eta_\delta$		$\eta_\infty$	$f_\infty$
	Present	Amir et al, 2009	Present	Amir et al, 2009		
0	1.3838958	1.2784	3.32	3.574	6.65	1.0225250
1	2.0863156	1.9959	2.05	2.262	4.16	0.7547567
2	2.9498859	2.8712	1.42	1.509	3.03	0.6181933
3	4.0212689	3.9613	1.04	1.050	2.19	0.5338660
4	5.3857126	5.3171	0.78	0.826	1.59	0.4757220
5	7.1874124	7.1254	0.59	0.614	1.21	0.4327303
6	9.6842483	9.6214	0.445	0.460	0.895	0.3993294
7	13.3850337	13.338	0.325	0.332	0.660	0.3724656
8	19.4525116	19.402	0.227	0.231	0.468	0.3502666
9	31.2523558	31.212	0.144	0.145	0.292	0.3315256
10	64.2650950	64.224	0.070	0.071	0.140	0.3154414

Table 3: The effect of the elasticity number  $K$  on flow characteristics for the case of blowing ( $R = -0.3$  and  $M = 0.5$ ). Parenthesis indicates results given in Amir, Aliakbar, Farzad and Sadeghy (2009).

$K$	$-f''(0)$	$\eta_\delta$	$\eta_\infty$	$f_\infty$	
0	1.0838959	(0.978163)	4.23	8.88	0.6226065
1	1.0535625	(0.962215)	3.66	7.17	0.5318333
2	1.0110784	(0.930555)	3.27	6.28	0.4710189
3	0.9584569	(0.885648)	3.00	5.94	0.4270273
4	0.8968999	(0.830316)	2.78	5.39	0.3932772
5	0.8270765	(0.76491)	2.61	5.01	0.3664440
6	0.7492927	(0.690684)	2.47	4.69	0.3444547
7	0.6635790	(0.609347)	2.36	4.39	0.3260133
8	0.5697243	(0.519199)	2.24	4.25	0.3103083
9	0.4672438	(0.420646)	2.15	4.08	0.2966856
10	0.3551857	(0.311931)	2.07	3.90	0.2847280

In Tables 2-3 we show the comparisons between our numerical solutions and the HAM solutions reported by Amir, Aliakbar, Farzad and Sadeghy (2009), and we can see that the general trends earlier reported are maintained.

Although the solution for the velocity gradient at the wall  $f''(0)$  has already been obtained by several investigators (see Hayat, Abbas, Sajid (2006), Amir, Aliakbar, Farzad and Sadeghy (2009)), the corresponding results for the heat transfer characteristics of engineering interest have been reported very scarcely: Aliakbar, Alizadeh-Pahlavan, and Sadeghy (2009). It is found that our approach provides numerical results for  $f_\infty$ , too. It is further clear from Tables 2-3 that blowing broadens the values of  $\eta_\infty$  and an opposite trend can be seen for the case of suction. Moreover, for fixed  $K$ , the effect of increasing blowing is to diminish the entrainment velocity  $f_\infty$ . The latter augments for stronger suction. The effect of increasing values of the elasticity number  $K$  is to decrease the magnitude of  $f_\infty$  largely in the boundary-layer, and this is true for both suction and injection situations.

In order to more fully characterize the behaviour of the numerical solutions with respect to the involving parameters, that is,  $K$  (elastic parameter),  $M$  (magnetic field parameter) and  $R$  (suction/blowing parameter) which govern this highly non-linear problem, representative dimensionless velocity and velocity gradient profiles at selected values of the elastic parameter  $K$  are shown in Fig.1. This Figure shows that, for  $M = 0.5$  and  $R = 0.3$  (suction), the effects of the fluid's elasticity are to decrease the dimensionless velocity  $f'(\eta)$ . In other words, the momentum boundary-layer thickness becomes thinner as the elastic parameter  $K$  increases.

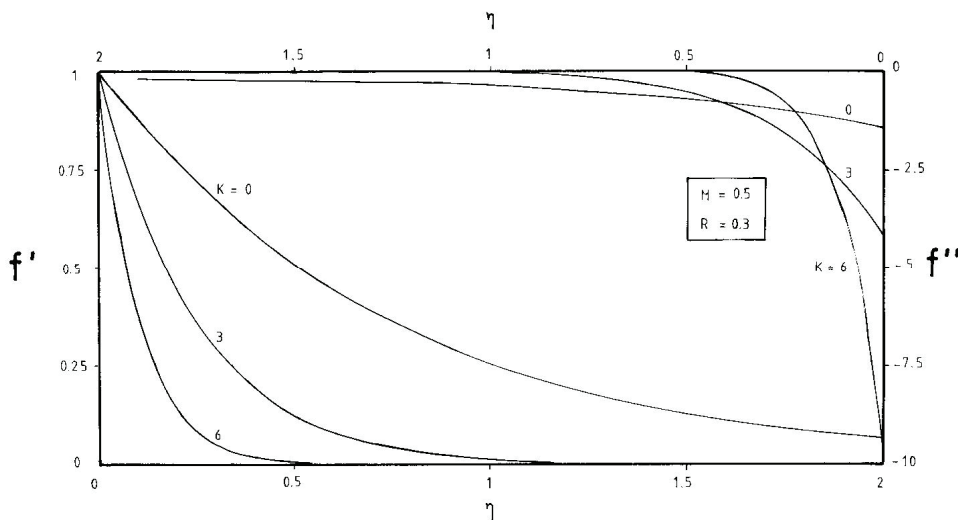


Figure 1: Velocity and velocity gradient profiles for selected values of  $K$  when  $M = 0.5$  and  $R = 0.3$  (suction).



Moreover, Fig. 2 depicts the changes in the  $f'(\eta)$  and  $f''(\eta)$  profiles at  $M = 0.5$  with changes in  $K$  in the case of blowing (i.e.,  $R = -0.3$ ). Note the effect's change of  $K$  in this case. It is observed that the velocity at any point near the surface slightly increases with increase in  $K$ ; but further away from the surface, the velocity decreases with increase in  $K$ . This new trend was nearly indistinguishable from graphical results obtained in Amir, Aliakbar, Farzad and Sadeghy (2009), and has already had detected for some non-Newtonian fluid flows (Cortell (2005a), Cortell (2006c)).

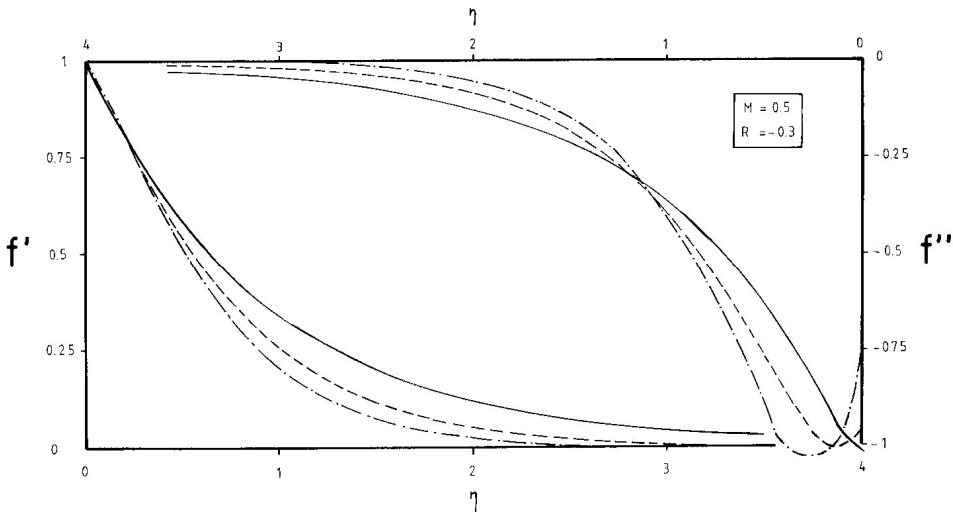


Figure 2: Velocity and velocity gradient profiles for selected values of  $K$  when  $M = 0.5$  and  $R = -0.3$  (blowing). [ $K=0$  (solid line);  $K=3$  (broken line);  $K=6$  (dashed-dotted line)].

Furthermore, the influences of the magnetic parameter  $M$  on velocity profiles have also been investigated. In the case of suction (i.e.,  $R = 0.3$ ), Figure 3 demonstrates graphically that, for fixed  $K$ , the momentum boundary-layer becomes thinner as the magnetic field parameter  $M$  increases. This effect diminishes with increase in  $K$ . It is further obvious that large elasticity and magnetic field numbers provide a lower influence of  $M$  on velocity profiles.

In the case of blowing (i.e.,  $R = -0.3$ ), Figure 4 illustrates graphically that the same  $M$  influences on velocity profiles are reached. It is concluded that our numerical results confirm the general conclusion made in Hayat, Abbas and Sajid (2006), Amir, Aliakbar, Farzad and Sadeghy (2009). That is, for all of the calculations, we have observed that the elasticity or magnetic field parameter influences are to decrease the boundary-layer thickness with its increases.

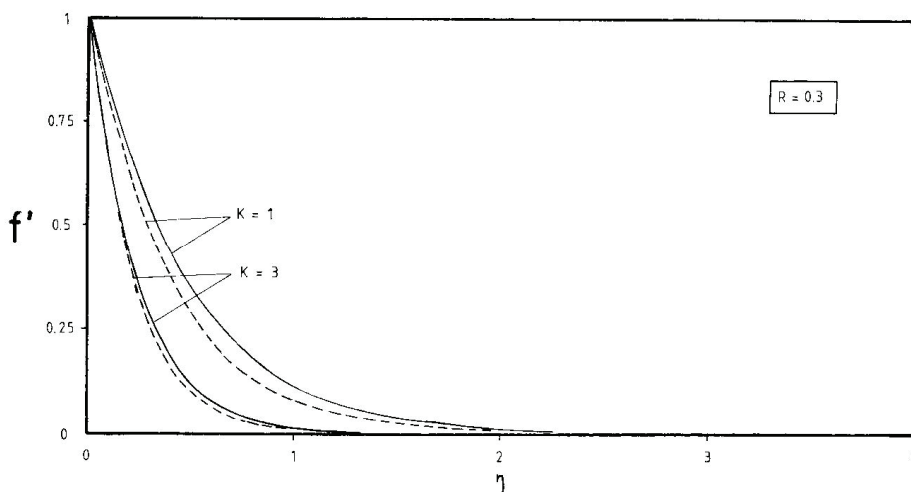


Figure 3: Velocity profiles for two selected values of  $K$  and  $M$  in the case of suction ( $R = 0.3$ ) [ $M = 0.5$  (solid line);  $M = 2$  (broken line)].

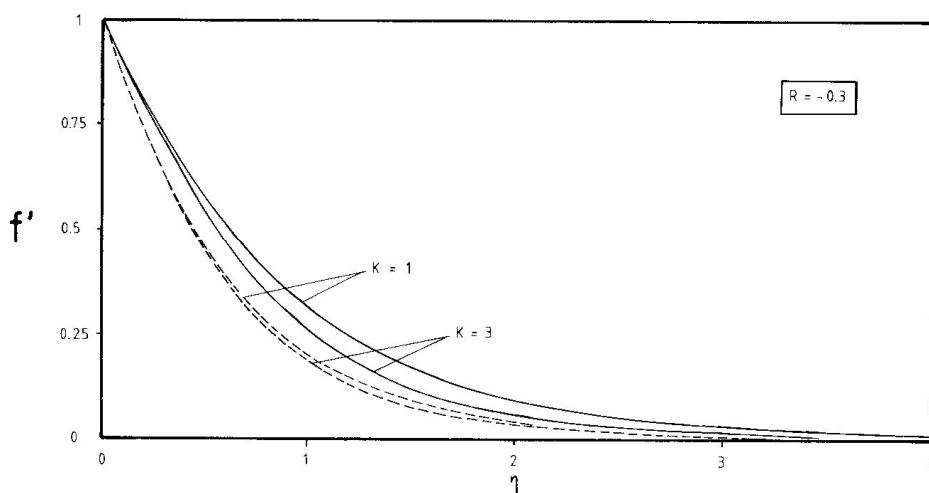


Figure 4: Velocity profiles for two selected values of  $K$  and  $M$  in the case of injection ( $R = -0.3$ ) [ $M = 0.5$  (solid line);  $M = 2$  (broken line)].

### 3 Discussions and conclusions

In this note we analyze magnetohydrodynamic boundary-layer flow in an electrically conducting viscoelastic UCM fluid over a sheet, which is stretched in a linear

fashion under the effects of suction and blowing at the surface. The fluid is at rest and the motion is created by the sheet.

The numerical values of the skin-friction parameter  $f''(0)$ , the characteristic non-dimensional quantities  $\eta_\delta$  and  $\eta_\infty$ , as well as the entrainment velocity of the fluid  $f_\infty$  (from which the amount of fluid dragged by the stretching sheet can be analyzed) are shown in tabular form. It seems that no investigation has been made which provides the appropriate results for parameters as  $f_\infty$ . Thus, it seems appropriate to communicate such results and, at the same time, to compare with the existing analytical HAM results in the literature for the problem under investigation. Our numerical results confirm the existing general trends in the open literature which is very scarce as far as numerical/HAM comparisons are concerned.

To this end, the governing partial momentum equation is transformed to ordinary one by exploiting the similarity procedure and the resulting equation system is solved numerically using a 4<sup>th</sup> order Runge-Kutta algorithm along with a shooting method from which we are capable to encounter, for each set of parameters entering the problem, the corresponding  $\eta_\infty$  and  $f_\infty$  results along with the additional condition at infinity  $f''(\infty) \rightarrow 0$ . The latter has been accounted throughout our numerical procedure. It is also worth while to underline that our iterative procedure provides, for each numerical solution, the integration domain (i.e.,  $\eta_\infty$ ) as a natural part of the approach, and there is no necessity to select the  $\eta_\infty$  value before calculations. If one enforces the far field conditions by using an inadequate (small) finite value  $\eta = \eta_\infty$  the accuracy of the numerical results could be severely contaminated (see also Cortell. (2008), El-Mistikawy (2009)). Our straightforward approach finds best values of the missed velocity gradient at the wall  $f''(0)$ , and the results for related problems have already been used for comparison in the development of several later investigations: Ishak, Nazar, Pop (2009). Graphs and tables show the influences of the parameters entering into the problem. From our numerical results the following conclusion may be drawn:

1. An augment of the fluid's elasticity yields a diminution of the amount of fluid dragged by the stretching surface (the increase of the parameter  $K$  leads to the decrease of  $f_\infty$ ), and this is true for both the suction and blowing cases.
2. The blowing broadens the values of  $\eta_\delta$  and  $\eta_\infty$ , whereas an opposite trend can be seen for the case of suction.
3. The momentum boundary-layer thickness becomes thinner as the elastic parameter  $K$  increases.
4. A larger elasticity and magnetic field numbers provides a lower influence of the magnetic parameter  $M$  on velocity profiles.

In summary, the article underlines that accurate numerical solutions to strongly nonlinear problems can be achieved and we can apply our approach to a variety of nonlinear problems in the science and engineering. In this manner, a suitable helpfulness for researchers to analyze highly nonlinear problems can be reached (Cortell (2005c), Bataller (2008b), (2008c)).

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