

Lubrication of An Infinitely Long Bearing by A Magnetic Fluid

R. M. Patel¹, G. M. Deheri² and P. A. Vadher³

Abstract: An endeavor has been made to analyze the performance of an infinitely long magnetic fluid based hydrodynamic slider bearing in the presence of a magnetic fluid lubricant. The associated Reynolds' equation is solved with appropriate boundary conditions. Expressions for dimensionless pressure, load carrying capacity and friction are obtained. Computed values are displayed graphically. It is seen that the magnetic fluid lubricant improves the performance of the bearing system. The friction decreases at the moving plate while it increases nominally at the fixed plate due to magnetization. In order to extend the life period of this magnetic fluid based bearing system the outlet film thickness ratio plays a better role as compared to that of the aspect ratio.

Keywords: Magnetic fluid, Infinitely long bearing, Reynolds' equation, Pressure, Load carrying capacity.

Nomenclature

h	Fluid film thickness at any point (mm)
h_1	Maximum film thickness (mm)
h_2	Minimum film thickness (mm)
\vec{H}	Magnetic field
L	Length of the bearing (mm)
\bar{L}	$L/h_2 =$ Inverse of the ratio of the outlet film thickness to the length of the bearing
H	Magnitude of \vec{H}
k	Suitably chosen to suit the dimensions of both the sides of the strength of the

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	magnetic field.
m	Aspect ratio
u	Velocity in the X - direction
p	Lubricant pressure (N/mm ²)
P	Dimensionless pressure
w	Load carrying capacity (N)
W	Non-dimensional load carrying capacity
ϕ	Inclination of the magnetic field.
μ_0	Magnetic susceptibility
$\bar{\mu}$	Free space permeability
μ	Lubricant viscosity (N.s/mm ²)
μ^*	Dimensionless magnetization parameter
τ	Shear stress (N/mm ²)
$\bar{\tau}$	Dimensionless shear stress
F	Frictional force (N/mm ²)
\bar{F}	Dimensionless frictional force
F_0	Frictional force (at moving plate)
F_1	Frictional force (at fixed plate)

1 Introduction

The infinitely long slider bearing is the idealization of single sector shaped pad of a hydrodynamic thrust bearing. It consists of a fixed pad (stator) and a moving pad (slider) which is usually plane. It is encountered most often and is the simplest, in the sense that the expression for film thickness is simple and the boundary conditions are less complicated. In such type of bearings the film is non-diverging and continuous and so the problem of negative pressure does not arise. Such bearings are designed to support axial loads. These bearings are used in hydroelectric generators, gas turbines and other equipments.

In a landmark paper Lord Rayleigh (1918) analyzed the film profile for a one dimensional slider bearing carrying maximum load and concluded that it is step shaped with step height ratio being 1.866 and the lower step being 28.2% of the slider length. Analysis of a slider bearing problem is a classical one [Hamrock (1994), Majumdar (2008)]. The hydrodynamic lubrication of a non-porous slider bearing has been analyzed by Pinkus and Sternlicht (1961). Prakash and Vij (1973) investigated the performance of an inclined plane infinite slider bearing with an impermeable slider and a porous faced stator backed by a solid wall. Bhat (1978) extended the analysis of the above-mentioned paper to the porous composite slider bearing. Bhat and Patel (1981) carried out the analysis in another direction by

involving the squeeze film which caused increased load carrying capacity without changing the friction. Andharia, Gupta and Deheri (1998) analyzed the performance of a longitudinally rough hyperbolic slider bearing. The above-said investigation dealt with the conventional lubricant. Agrawal (1986) considered the configuration of Prakash and Vij (1973) with a magnetic fluid lubricant. He found that the performance was better than the one with conventional lubricant. Bhat and Deheri (1991) extended the analysis laid down in [Agrawal (1986)] by considering the magnetic fluid based porous composite slider bearing. They observed that magnetic fluid lubricant increased the load carrying capacity, unaltered the friction and shifted the centre of pressure towards the inlet. Gupta and Bhat (1979) presented the analysis for the performance of an inclined porous slider bearing with a transverse magnetic field. Deheri and Patel (2005) discussed the performance of a porous slider bearing with squeeze film formed by a magnetic fluid. Gertzos et al. (2008) derived the performance characteristics of a hydrodynamic journal bearing lubricated with a Bingham fluid by means of three dimensional computational fluid dynamics analysis. The theoretical study of hydrodynamic journal bearing lubricated with ferrofluids exhibiting non-Newtonian behavior was conducted by Osman et al. (2003). The discussion carried out by Urreta et al. (2009) summarizes essentially the work done towards the development of hydrodynamic lubricated journal bearing with magnetic fluids. Ochonski (2007) investigated the performance of sliding bearings lubricated with magnetic fluids.

2 Analysis

The configuration of the bearing which is infinite in Z – direction is shown in Figure (i).

The non-porous slider moves with a uniform velocity \mathbf{u} in the X – direction. The length of the bearing is L . The magnetic field is oblique to the stator as in Agrawal (1986). Magnitude of applied magnetic field is taken be a function of x . We assume that the magnetic field has components of the form

$$\bar{H} = H(x)(\cos \phi, 0, \sin \phi); \quad \phi = \phi(x, z)$$

while H^2 satisfies the condition that it becomes zero at the interface of the bearing and the atmosphere i.e. $H^2(x) = 0$ at $x = 0$ and $x = L$. Hence $\nabla_x \bar{H} = 0$ in the present case leads to the equation for the inclination of magnetic field ϕ as

$$\cot \phi \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial z} = -\frac{1}{H} \frac{dH}{dx} \quad (1)$$

Following Prajapati (1995) we consider

$$H^2 = kL^2 \sin\left(\frac{\pi x}{L}\right)$$

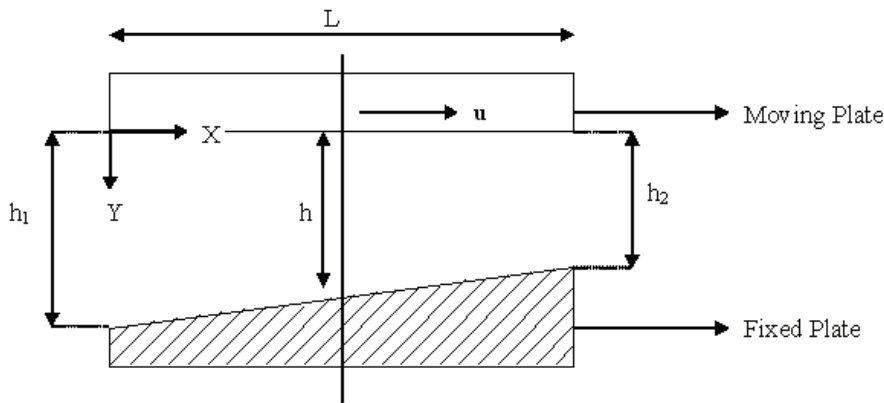


Figure i: Configuration of the bearing system

where L is the length of the bearing along $X -$ axis. Hence the direction ϕ of the magnetic field is given by the c eliminant

$$\operatorname{cosec}\left(\frac{\pi x}{L}\right) = c^2 \sin \phi$$

and

$$-c - \frac{\pi z}{L} = \int \frac{d\phi}{c \sin^2 \phi - 1} \tag{2}$$

With the usual assumptions of magnetohydrodynamic lubrication together with the assumption that the self field created by the magnetization of the fluid is neglected, the associated Reynolds' equation for the pressure distribution turns out to be [Pranjapati (1995), Basu et al. (2005)]

$$\frac{d}{dx} \left(p - \frac{\mu_0 \bar{\mu} H^2}{2} \right) = \frac{6\mu u \left\{ 1 + m \left(1 - \frac{x}{L} \right) \right\} - \lambda}{h_2^2 \left\{ 1 + m \left(1 - \frac{x}{L} \right) \right\}^3} \tag{3}$$

where $\lambda > 1$ and is given by $\frac{2(m+1)}{(m+2)}$, μ_0 is the magnetic susceptibility, $\bar{\mu}$ is free space permeability, μ is lubricant viscosity and m is the aspect ratio

The associated boundary conditions are

$$p = 0 \text{ at } x = 0 \text{ and } x = L$$

Solving equation (3) in view of the above boundary conditions we get the pressure distribution as

$$p = \frac{\mu_0 \bar{\mu} k L^2 \sin\left(\frac{\pi x}{L}\right)}{2} + \frac{6Lu\mu}{h_2^2 m} \left\{ -\frac{m+1}{(m+2) \left\{1+m\left(1-\frac{x}{L}\right)\right\}^2} + \frac{1}{\left\{1+m\left(1-\frac{x}{L}\right)\right\}} - \frac{1}{m+2} \right\} \quad (4)$$

Introducing the dimensionless quantities

$$m = \frac{h_1 - h_2}{h_2} \quad X = \frac{x}{L} \quad P = \frac{h_2^3 p}{\mu u L^2}$$

$$\mu^* = \frac{h_2^3 k \mu_0 \bar{\mu}}{\mu u} \quad \bar{L} = \frac{L}{h_2} \quad Y = \frac{y}{h}$$

one obtains the expression for pressure distribution in dimensionless form as

$$P = \frac{\mu^* \sin(\pi X)}{2} + \frac{6h_2}{mL} \left\{ -\frac{m+1}{(m+2) \{1+m(1-x)\}^2} + \frac{1}{\{1+m(1-x)\}} - \frac{1}{m+2} \right\} \quad (5)$$

The non-dimensional load carrying capacity per unit width is given by

$$W = \frac{h_2^3 w \pi}{\mu u L^4} = \pi \int_0^1 P dX \quad (6)$$

Therefore, the dimensionless load carrying capacity of the bearing is given by

$$W = \mu^* + \frac{6h_2 \pi}{L} \left[\frac{\ln(m+1)}{m^2} - \frac{2}{m(m+2)} \right] \quad (7)$$

The frictional force \bar{F} per unit width of the lower plane of the moving plate is obtained as

$$\bar{F} = \int_0^1 \bar{\tau} dX$$

where $\bar{\tau} = \left(\frac{h_2}{\mu u}\right) \tau$ is non-dimensional shearing stress. while

$$\tau = \frac{dp}{dx} \left(y - \frac{h}{2}\right) + \frac{\mu u}{h}$$

On simplifications this yields,

$$\bar{\tau} = \frac{dp}{dX} \bar{L} \{1 + m(1-x)\} \left(Y - \frac{1}{2} \right) + \frac{1}{\{1 + m(1-x)\}} \tag{8}$$

At $Y = 0$ (at moving plate), we find that

$$\bar{\tau} = -\frac{\bar{L}\mu^*\pi}{4} \{1 + m(1-x)\} \cos(\pi X) + \frac{6(m+1)}{(m+2)\{1 + m(1-x)\}^2} - \frac{2}{\{1 + m(1-x)\}} \tag{9}$$

Thus, in non-dimensional form the frictional force assumes the form

$$F_0 = -\frac{m\bar{L}\mu^*}{2\pi} - \frac{2\ln(m+1)}{m} + \frac{6}{(m+2)} \tag{10}$$

Further, at $Y = 1$ (at fixed plate), one obtains that

$$\bar{\tau} = \frac{\bar{L}\mu^*\pi}{4} \{1 + m(1-x)\} \cos(\pi X) - \frac{6(m+1)}{(m+2)\{1 + m(1-x)\}^2} + \frac{4}{\{1 + m(1-x)\}} \tag{11}$$

Lastly, in dimensionless form the frictional force comes out to be

$$F_1 = \frac{m\bar{L}\mu^*}{2\pi} + \frac{4\ln(m+1)}{m} - \frac{6}{(m+2)} \tag{12}$$

3 Results and discussion:

The expressions for the pressure distribution and load carrying capacity are presented in equations (5) and (7) respectively. Further, the frictional force at the bearing plate and the runner plate are determined by equations (10) and (12) respectively. A comparison with the conventional lubricant indicates that the non-dimensional pressure increases by

$$\frac{\mu^* \sin(\pi X)}{2}$$

and the load carrying capacity gets enhanced by μ^* . Furthermore, the friction at the moving plate gets reduced and at the fixed plate it increases by

$$\frac{m\bar{L}\mu^*}{2\pi}.$$

A close scrutiny of the results show that the non-dimensional load carrying capacity in the present case is approximately four times more than the case of the magnetic field wherein, the magnitude is taken as

$$H^2 = kx(L - x).$$

It is easy to see that setting the magnetization parameter to be zero one gets the study carried out in Basu et al. (2005).

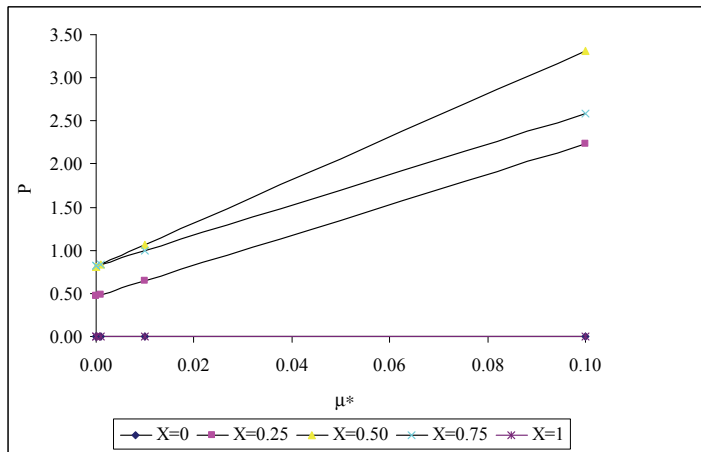


Figure 1: Variation of pressure with respect to μ^* and X ($m = 0.75$ & $h_2/L = 0.075$)

Figure (1), Figure (2) and Figure (3) present the non-dimensional pressure distribution with respect to the magnetization parameter for different vales of X, L/h_2 and the aspect ratio m respectively. It is clearly seen that the non-dimensional pressure increases sharply with increasing magnetization parameter, owing to the aspect ratio m and X, while, there is a marginal increase due to L/h_2 . Figure (4) describes the variation of non-dimensional pressure with respect to X for various values of L/h_2 . It is clearly seen that the pressure decreases with increasing values of L/h_2 . In Figure (5) one can find the dimensionless pressure distribution with respect to X for different values of aspect ratio m. It is observed that increasing values of m cause increased pressure.

In Figure (6) and Figure (7) we have the variation of load carrying capacity with respect to the magnetization parameter for different values of L/h_2 and the aspect ratio m. These figures suggest that the load carrying capacity increases considerably with respect to the magnetization parameter due to the aspect ratio m, while,

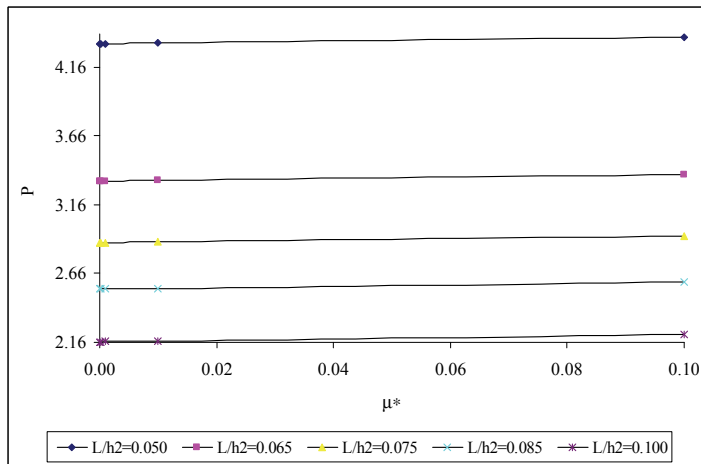


Figure 2: Variation of pressure with respect to μ^* and L/h_2 ($m = 0.75$ & $X = 0.5$)

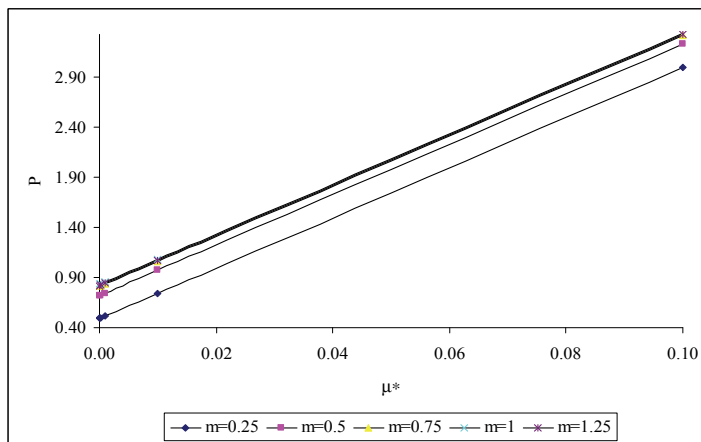


Figure 3: Variation of pressure with respect to μ^* and m ($h_2/L = 0.075$ & $\mu^* = 0.001$)

the ratio L/h_2 induces a marginal increase in the load carrying capacity. It is manifest that the load carrying capacity increases up to certain value of the aspect ratio and then tends to decrease marginally. Further, increasing values of the aspect ratio m and decreasing values of L/h_2 cause increased load carrying capacity (Figure (8)).

The variation of frictional force at the moving plate with respect to the magne-

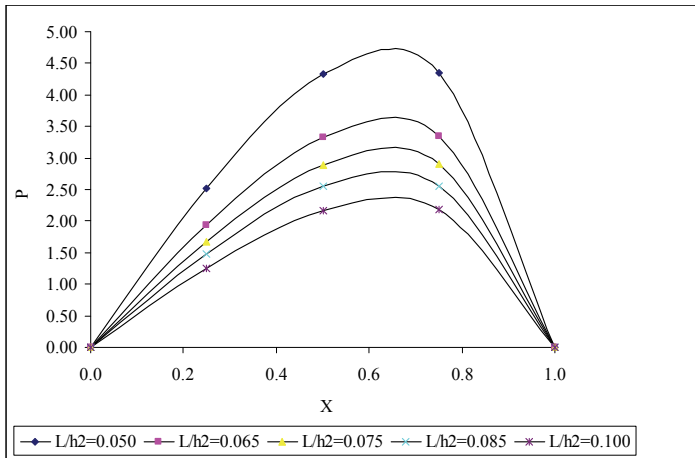


Figure 4: Variation of pressure with respect to X and L/h_2 ($m = 0.75$ & $\mu^* = 0.001$)

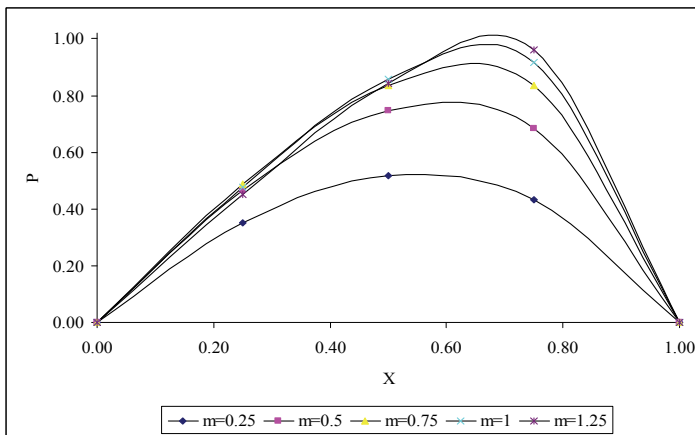


Figure 5: Variation of pressure with respect to X and m ($h_2/L = 0.075$ & $\mu^* = 0.001$)

tization parameter for different values of outlet film thickness ratio h_2/L and the aspect ratio m is presented in Figure (9) and Figure (10) respectively. It is clear that the friction decreases considerably with respect to the magnetization parameter μ^* . Furthermore, the effect of h_2/L with respect to the aspect ratio is almost negligible as can be seen from Figure (11). However, it is investigated that there is a nominal increase in the friction due to magnetization parameter at the fixed plate. The effect

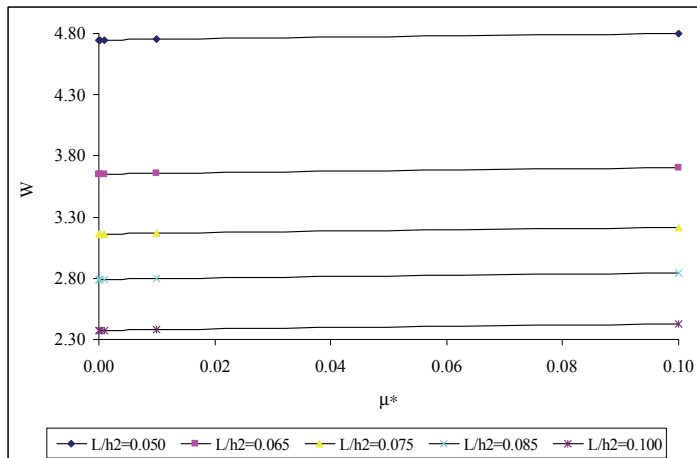


Figure 6: Variation of load carrying capacity with respect to μ^* and L/h_2 ($m=0.75$)

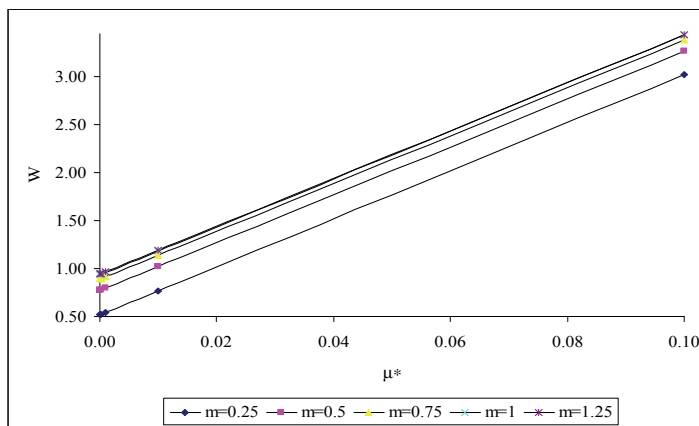


Figure 7: Variation of load carrying capacity with respect to μ^* and m ($h_2/L = 0.075$)

of h_2/L with respect to the aspect ratio is almost similar to the corresponding case at the bearing plate (Figure (12)).

4 Conclusion

Thus, the magnetic fluid lubricant improves the performance of the bearing system. However, this study makes it clear that even if, there is a magnetic fluid lubricant,

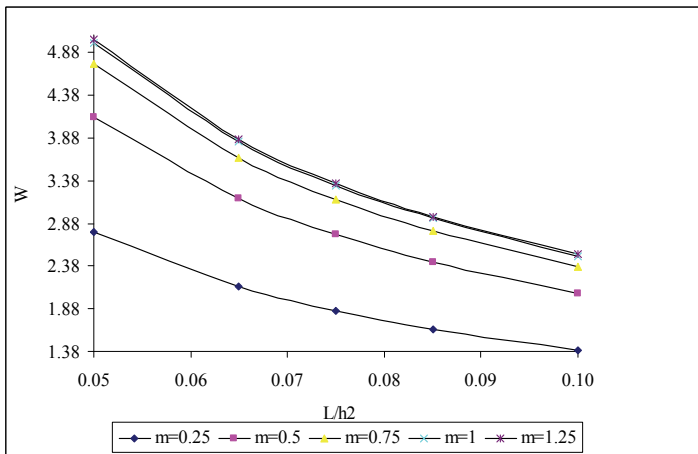


Figure 8: Variation of load carrying capacity with respect to L/h_2 and m ($\mu^* = 0.001$)

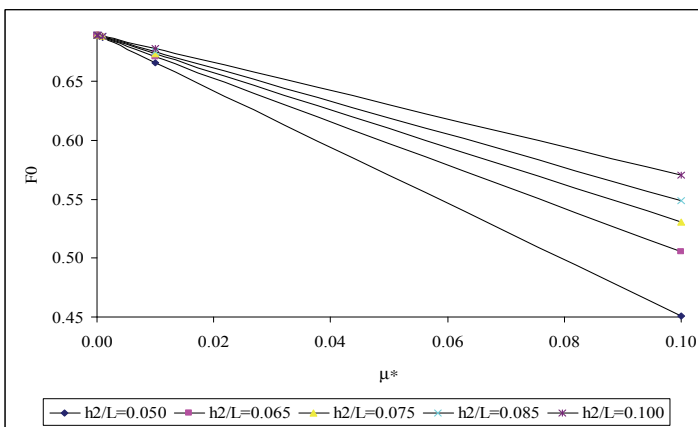


Figure 9: Variation of frictional force with respect to μ^* and h_2/L (at moving plate) ($m = 0.75$)

the role of aspect ratio and the outlet film thickness ratio are crucial especially, from life period point of view because the friction decreases at one plate where as it reverses at the other plate.

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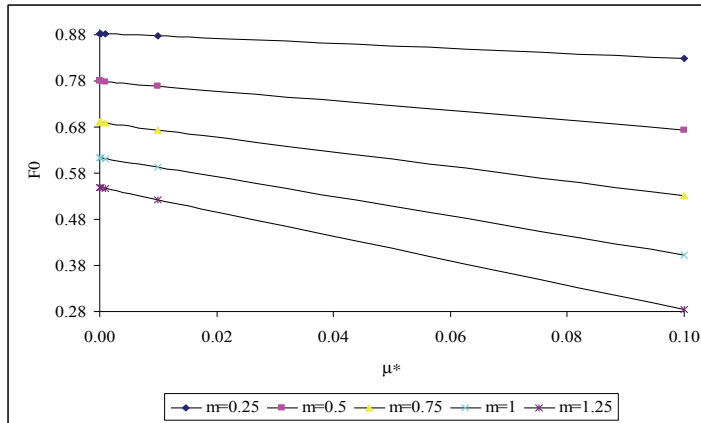


Figure 10: Variation of frictional force with respect to μ^* and m (at moving plate) ($h_2/L = 0.075$)

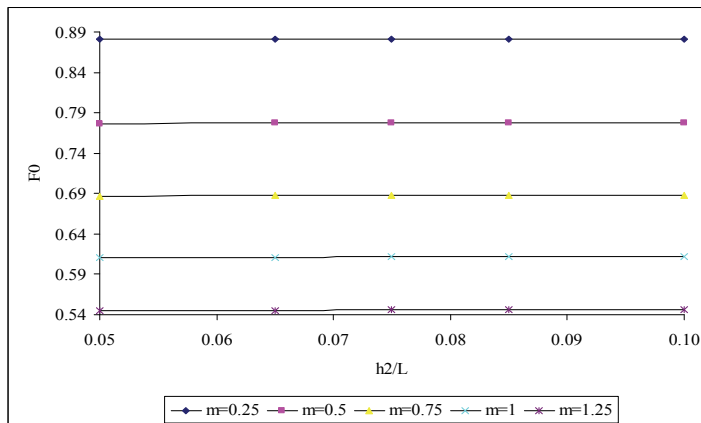


Figure 11: Variation of frictional force with respect to h_2/L and m (at moving plate) ($\mu^* = 0.001$)

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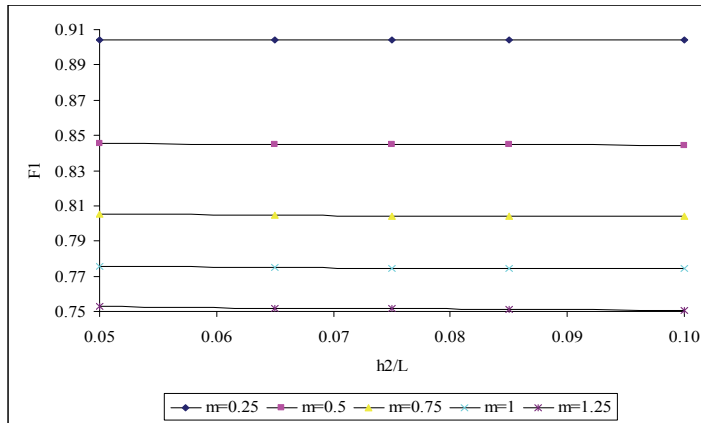


Figure 12: Variation of frictional force with respect to h_2/L and m (at fixed plate) ($\mu^* = 0.001$)

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