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Theoretical and Experimental Investigation of Water Flow through Porous Ceramic Clay Composite Water Filter

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Abstract: Water flow through point-of-use porous ceramic water treatment filters have been theoretically analyzed in this technical paper. Filters tested were manufactured by combining low cost materials namely, clay and sawdust. Three filters with distinct volume fractions of clay to sawdust (75:25, 65:35 and 50:50) were tested. Sintered clay filters casted in frustum shapes were structurally characterized using mercury intrusion porosimetry. A linear increase in porosity with volume fraction of sawdust was observed.
Flow experiments were carried out at constant room temperature and pressure. Potable tap water was used in these studies. Flows through filters occurring with drop in the head of water under gravity were statistically analyzed. Discharges through the filters were predicted with respect to independent variables of time for cumulative discharge and volume fraction of sawdust used for manufacturing the filters. The experimental data analysis predicts a multiplicative influence of time and volume fraction of sawdust respectively, on discharge from the filters. The results demonstrate a new theoretical approach for prediction of flow in similar types of heterogeneous porous media as discussed in this technical paper.

Keywords: Porous ceramic, volume fraction, multiplicative.

Nomenclature

$a, \bar{a}$	Model Coefficient
A	Internal Surface Area of the Filter Vessel
$b, \bar{b}$	Model Coefficient
G	Assumed Variable
h	Head of Water

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k	Intrinsic Permeability
p	Fluid Pressure Variable
Q	Discharge
t	Time
V	Flow Velocity
X_1, X_2	Assumed Variable Predictors
Y	Assumed Variable denoting Discharge
$\bar{Y}$	Assumed Variable for Mean Discharge

Greek Symbols

κ	Hydraulic Conductivity
μ	Viscosity
γ	Kinematic Viscosity
ρ	Density
σ	Standard Deviation of Lognormal Model
θ	Angle

1 Introduction

Water is a basic requirement to sustain life for human beings and animals around the world. There is a scarcity of potable water in developing countries. The people in these countries are not able to afford costly water purification technologies and devices at point-of-use (households) due to low per capita earnings [Islam (1996)]. Hence, there is a pressing need for low cost, locally produced, highly reliable and efficient water purification technologies and devices.

The requirements for filtration of water vary from region to region based on impurity characterizations such as, microbes [Wegmann, Michen and Graule (2008)], chemicals [Boujelben,Bouzid, Elouear, Feki, Jamoussi, and Montiel (2008); Genc-Fuhrman, Mikkelson and Ledin (2007)], metals [Lin, Wu and Lai (2008); Hatfield, Annable, Cjo, Rao and Klammler (2004); Ahmedna, Marshall, Hussieny, Rao and Goktepe (2004)], silt [Genc-Fuhrman, Mikkelson and Ledin (2007)] and organic matter [Lin, Wu and Lai (2008)]. Some of the point-of-use methods and devices for removing the above mentioned impurities are chlorination, solar disinfection, ceramic filters and bio-sand filters (specific in removal of metallic impurities) [Sobsey, Stauber, Casanova, Brown and Elliott (2008)].

Ceramic filters made of clay augmented with state-of-art technologies such as nano-filtration, can be used for providing cheap potable water in developing countries

[Hillie, Munasinghe, Hlope and Deraniyagala (2003)]. Past research and social work has helped millions of people across the globe to take advantage of clay composite ceramic filters for water purification [Sobsey, Stauber, Casanova, Brown and Elliott (2008); Hillie, Munasinghe, Hlope and Deraniyagala (2003)].

Similar type of clay ceramic water filters were encouraged and distributed for usage in developing countries by a nongovernmental organization named Potters for Peace [PFP (2006); Donachy (2004)]. The device was a low pressure water filter relying on gravity for its operation and was manufactured by combining locally available clay and sawdust (clay composite) and had a frustum shape. This filter has not been patented [PFP (2006)] and has been providing pure potable water in developing countries across Central America [Donachy (2004); Lee et al. (2001); Halem (2006)], West Africa [Halem (2006); Swanton (2008)] and South Asia [Sobsey, Stauber, Casanova, Brown and Elliott (2008); Swanton (2008); IRIN (2008); Brown and Sobsey (2006)]. A 99-100 % microbial removal was registered during several field tests [Franz (2005); Lantagne (2001)]. Health reports through 1993-2003 from Central America showed a 50% reduction in waterborne diseases after usage of these filters [Donachy (2004); Brown and Sobsey (2006)].

A general methodology for manufacturing this clay composite was followed and disseminated by Potters for Peace [PFP (2006); Donachy (2004)]. The composite filters were manufactured from moistened suspensions containing clay and sawdust in a 50:50 ratio by volume [Lee et al. (2001); Halem (2006)]. Due to the plasticity of moistened clay composite, it could mold under stress to any shape as required. The filters were casted in the shape of a frustum or flower pot [Donachy (2004)]. Sintering these filter molds to around 900°C introduces numerous pores in the mold serving its filtration capabilities [Lee et al. (2001); Halem (2006); Franz (2005); Hwang (2003); Dies (2003); Oyanedel-Craver and Smith (2008)].

Extensive literature is available on several empirical developments to analyze flow through these porous ceramic clay composite filters [Lee et al. (2001); Halem (2006); Lantagne (2001); Hwang (2003)].

Flow through a porous media is supposed to follow Darcy's law, as given by [Dagan (1989)]

$$Q = -\kappa A \Delta h / L \tag{1}$$

where Q is the discharge from the porous media, A is the surface area which holds the water, Δh is the change in head of water due to gravity flow, K is the hydraulic conductivity of the porous structure and L is the thickness of the porous media through which the water needs to percolate [Dagan (1989)].

Hydraulic conductivity K is defined as [Dagan (1989)]

$$\kappa = \gamma k/\mu \tag{2}$$

where μ is the viscosity of water at a given temperature, γ is the specific gravity of water at that particular temperature and k is the intrinsic permeability defining the porosity and interconnectivity of the porous media [Dagan (1989)]. The flow velocity V through the medium can be written in the form

$$V = \frac{-k\left(\sum_{i=1}^{3}\frac{\delta p}{\delta x_i} - \gamma\sum_{i=1}^{3}\frac{\delta h}{\delta x_i}\right)}{\mu} \tag{3}$$

where $\sum_{i=1}^{3}\frac{\delta h}{\delta x_i}$ is the gradient function, p is the pressure and h is the variable head of water above the porous media [Chen, Huan and Ma (2006)]. Numerical models as well as image analysis has been used to study permeability [Arab, Semma, Pateyron, and El Ganaoui (2009); Chattopdhyay, Knight, Kapadia and Sarkar (1994); Chen, Huan and Ma (2006)].

Halem in 2006 found that, there is a change in flow rate with change in location of the manufacture of the filter. Experiments were carried out on filters from Cambodia, Ghana and Nicaragua and were made with clay to sawdust ratio of 50:50 [Halem (2006)]. From these observations it was envisaged that filters made with locally available clay and sawdust at these locations were influenced by their respective local material properties. The affected micro-structural properties such as porosity, for example filters from Cambodia, Ghana and Nicaragua have 43%, 39% and 34% porosity respectively [Halem (2006)]. The corresponding mean discharges from filters in the above mentioned countries were 0.76 l/hr, 2.41 l/hr and 0.85 l/hr respectively [Halem (2006)]. Similar empirical tests were conducted by researchers on samples from Mexico, Guatemala and Redart and concluded randomness in the behavior of similar volume fraction filters [Oyanedel-Craver and Smith (2008)].

In another study, it was concluded that porosity varies with increase in volume ratio of plant material [Lee et al. (2001)]. Porosity was predicted to be a linear polynomial fit with 99.45% accuracy as

$$y = 43.05 - 0.4865x + 0.0127x^2 \tag{4}$$

where y is the porosity of the filter sample and x is the volume ratio of plant material [Lee et al. (2001)]. A study analyzed influence of sintering temperature on porosity variations in clay ceramic filter containing Kaolin clays [Nandi, Uppaluri and

Purkait (2008)]. This study predicted a decrease in porosity with increase in sintering temperature in the range 850-1000°C [Nandi, Uppaluri and Purkait (2008)]. There would be a large influence on the properties of the final sintered product due to the shape, compositional materials used, processing environment and thickness [Bazant (1999); Chattopdhyay, Knight, Kapadia and Sarkar (1994); Worrall (1986)].

From the above discussion it is clear that a range of parameters influence the flow behavior through heterogeneous porous formations. Hence there is a requirement to identify the major parameters influencing discharge through non-uniform porous media. Earlier researchers have tried to use analytical models for the description of porous flows such as random walk models with Markovian chain based theories [Borgne, Dentz, and Carrera (2008)].

The present study identifies and discusses the major predictor variables that can influence the flow through clay composite ceramic filters. A statistical multi-parameter model elaborates the novel approach discussed in this paper, which would help us in understanding the material and transient effects on the discharge from frustum shaped filters and any other filter with a known geometrical structure.

2 Materials and Processing

2.1 Raw Materials

The composite filters were manufactured by combining specific volume ratio of materials namely, sawdust and clay. Sawdust was obtained from a local saw mill (Hamilton Supplies, Hamilton, NJ), containing 80% oak and 20% cedar as constituents. It was sieved manually using a 35-1000 micron mesh.

Clay supplied by Resco Products Inc. Pittsburgh, PA. USA, is a mix of illitic and kaolinitic clays.

2.2 Manufacturing Process

Volume fraction of clay and sawdust, in accordance with our requirements were mixed together in a commercial blender (Model A-200, The Hobart Manufacturing Company, Troy, OH). The mixture was blended for about 10-15minutes to accomplish uniform mixing. The blended composite was mixed with water in small volumes under continuous mixing. Clay to sawdust mixes of 75:25, 65:35 and 50:50, ratio by volume, required about 1.8-2 litres of water to be made into a 12 lb (5.443 kg) dough ball, enough to manufacture one filter. These ratios were chosen to study extremes in cumulative percolation time (basically from 5-10 hours to a week in similar filter variants).

Axial press forming is used to mold the dough to a shape. This can now be called the greenware in language of a potter [Cuff (1996)]. In this study, the greenware produced was frustum shaped. A 50 ton (45,359.237 kg) hydraulic press (TRD 55002, Torin Jacks Inc, Ontario Canada) was used for this purpose.

The press used a metallic frustum mold to form the greenware of requisite dimensions. The inner dimensions of the frustum were: length of axis, 26cm, lower base diameters, 20cm and upper base diameters, 23cm. The frustum wall and base had a thickness of 0.5cm and 1cm respectively.

For each of the volume fraction of sawdust discussed in this document, 6 filter greenwares with uniform geometric dimensions were manufactured. The greenwares were dried at room temperature. Time taken for drying the greenware varied (5-8days) for different constituent volume ratios.

After drying the greenwares were sintered in a gas and electric kiln (Ceramic Arts Department, Princeton University, Princeton, NJ). The filters were then pre-heated to 450-550°C for three hours in a gas kiln to burn off the saw dust [Cuff (1996); Maritan, Nodari, Mazzoli, Milano and Russo (2006)]. After slow cooling to room-temperature, the filters were re-heated to 955°C in an electric furnace. The initial heating rate of 50°C per hour was increased to about 100°C per hour beyond a furnace temperature of 200°C. The filters were sintered for 5 hours at the peak temperature of 955°C, then furnace cooled in air to room temperature.

Once the firing is completed the filters are left in the furnace for 5-10 hours to cool down. Greenwares were transformed into structurally rigid red colored composite ceramic filters, ready for use in experiments.

3 Experimental procedure

3.1 Material Analysis

The material formed after sintering the greenware should be structurally distinct owing to the variations in its basic constituents. Mercury intrusion porosimetry is used to measure the porosity of the ceramic composite structure of the filter.

Small material samples with a dimension of approximately $\sim$3mm x 3mm x 3mm each were cut from each of the ceramic composite filters used here. Each of these three compositionally different samples was separately poured into the experimental chamber (the penetrometer) of the Micrometrics Autopore III: 9400 digital analyzer used to calculate material porosity. Generally, the samples occupied about one-half of the penetrometer bulb volume. The analyzer follows a two step pressure analysis test on the ceramic porous sample, taking advantage of the non-wetting behavior of Mercury.

The main aim of the experiment is to identify the effect of the volume ratio of the basic material constituents on porosity of the sintered clay ceramic material. Micrometric Autopore III: 9400 digital analyzer plots a pore size distribution curve and calculates the average porosity of the material.

3.2 Flow Experiments

Many researchers have conducted filtration tests focusing on the long term effects on the flow behavior [Donachy (2004); Halem (2006); Swanton (2008); Brown and Sobsey (2006); Lantagne (2001)]. It is necessary to understand the flow behavior at the microscopic time scale to establish variability due to material modifications and surface interactions taking place during the transient flow through filter media.

The sintered filters of each volume fraction were fully saturated with water by dipping them in a water bath containing purified water (Barnstead/Thermolyne, EASYpure uv/uf, Model D8611) for about 12 hours. This is done to simulate the actual discharge from the ceramic composite filter in a fully working condition.

The experimental setup as shown in Fig. 1 consists of three major components. First, a ceramic composite filter filled with water. Secondly, a vessel for collection of discharged filtrate from the filter.

The experimental setup was covered by plastic wrap to prevent evaporation as well as external influences and impurities. Finally, the filtrate collection vessel was placed on a load cell (Model LSC 7000-50, Omega Engineering Incorporated, Stamford, CT).

The load cell was connected to a LabView card (NI PCI-6259, National Instruments, Austin, TX) via a 68-Pin Digital and Trigger I/O Terminal Block (CB-68LP, National Instruments, Austin, TX) to a LabView software (Version 8.0, National Instruments, Austin, TX).

The data acquisition (DAQ) system can be used to calculate the amount of filtrate water collected at a resolution of one millisecond.

The data are collected in a format to assist with the study of the flow analysis and recorded as follows: The filtrate volume accumulated in the measuring vessel is recorded every second for the first 20 minutes of the start of filtration. Secondly the readings are accessed every minute for the next 3 hours. After this, data are collected every 3 minutes until the last drop of water within the filter.

The complete draining of water under gravity from within a fully filled composite ceramic filter is termed as one experiment. These experiments are conducted consecutively three times, one after the other, on each of the 6 filters manufactured for the different clay and sawdust configurations (75:25, 65:35 and 50:50 by volume).

A C-S (Volume of Clay in %–Volume of Sawdust in %) and C-S-E (Volume of Clay

Figure 1: Model experimental setup for flow experiments conducted for different filter variants.

in %–Volume of Sawdust in %-Experiment Number) notation has been used in this document to describe the filters and their corresponding experiments.

It is important to note that time has been a factor in constant and variable pressure filtration models [Johnston (1995)]. Similarly it has also been found that permeability has been a factor in the models for percolation through porous media [Hatfield, Annable, Cjo, Rao and Klammler (2004); Dagan (1989); Johnston (1995)].

The objective is to establish the basic variables and their mutual interactions influencing the flow rate of the filtrate from the ceramic composite filters. Thus the experiment focuses on the material constituents and their qualitative interactive effects on the flow through the heterogeneous system used in this study.

4 Results

4.1 Porosity Variations

The porous structures for ceramic composite filter materials with three compositions (75:25, 65:35 and 50:50 ratios by volume) of clay and sawdust were analyzed. The pore sizes in all the samples showed a wide range distribution within 0.001μm to 100μm as plotted in Fig. 2a, 2b, and 2c.

The density of number of pores with sizes within 0.001μm to 1μm was observed to be very high in all the samples tested irrespective of its constituent quantities.

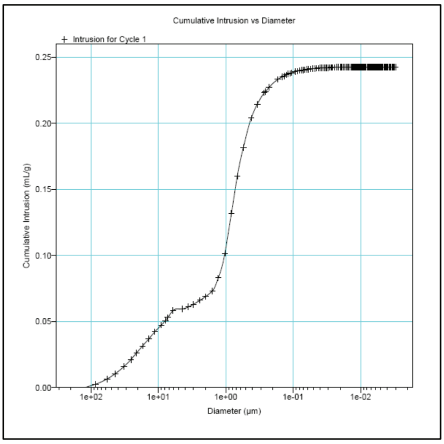

Figure 2a: Pore size distribution for the sample from the circular base of the 75-25 frustum filter

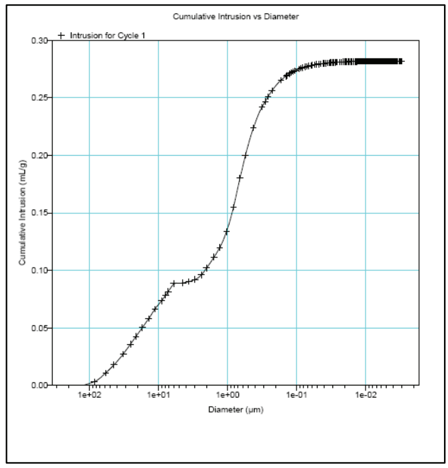

Figure 2b: Pore size distribution for the sample from the circular base of the 65-35 frustum filter.

Porosity variations within a particular filter were distinct, for example, the porosity values on the walls of the filter vessel were not similar to those at the base [Halem (2006)]. These random pore size distributions may introduce non -uniform variation in porous media flow behavior. This variability throughout the filter would suggest development of probabilistic models for predicting flow through the ceramic composite filter.

The variations in average porosity were observed with the changes in the volume ratio of the filter constituents. These average porosity values in percentage were an output calculated by the Autopore III:9400 digital analyzer.

The graph in Fig. 3 implies that a linear relationship may be used to fit the increase in average porosity with increase in sawdust content by volume. This also supports the explanation of influence of carbonate content on increasing porosity [Cultrone, Sebastian, Elert, Torre, Cazalla and Rodriguez-Navarro (2004)].

Porosity variations with location as shown in Fig. 4 may be due to natural modifications in the raw material type and its configuration with respect to the local geographical environment.

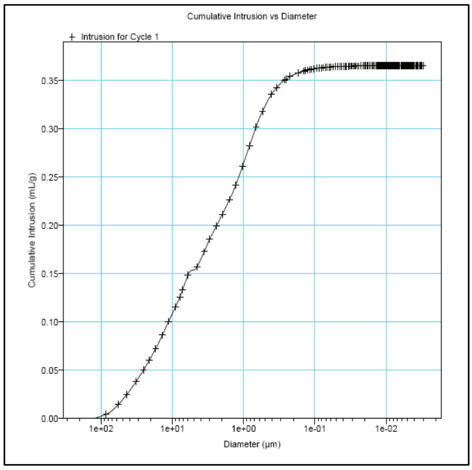

Figure 2c: Pore size distribution for the sample from the circular base of the 50-50 frustum filter

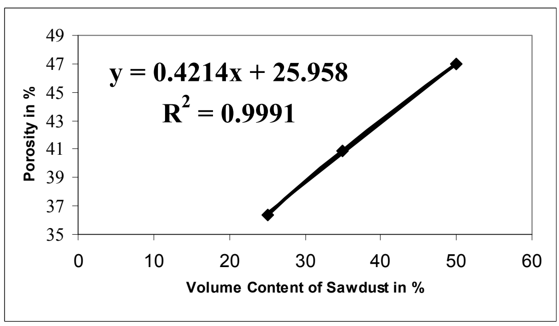

Figure 3: Linear equation fit for an increase in porosity with increase in sawdust for filters with 25%, 35% and 50% of sawdust by volume.

4.2 Statistical Modeling of Discharge through Ceramic Filters

From the series of filtration experiments simulating water percolation through ceramic composite filters the initial filtrate volume measurements from all the three distinct filters 50-50, 65-35 and 75-25 has been plotted in the Fig. 5 to illustrate their behavior.

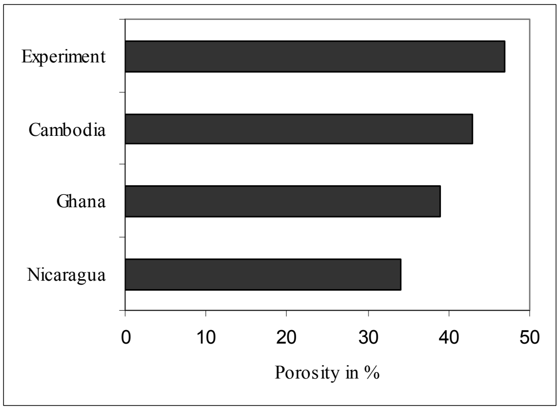

Figure 4: Comparative study on porosity variations (of the circular base of the 50-50 frustum filter at different locations and experiments conducted in this study [Lee et al. (2001); Halem (2006); Lantagne (2001)].

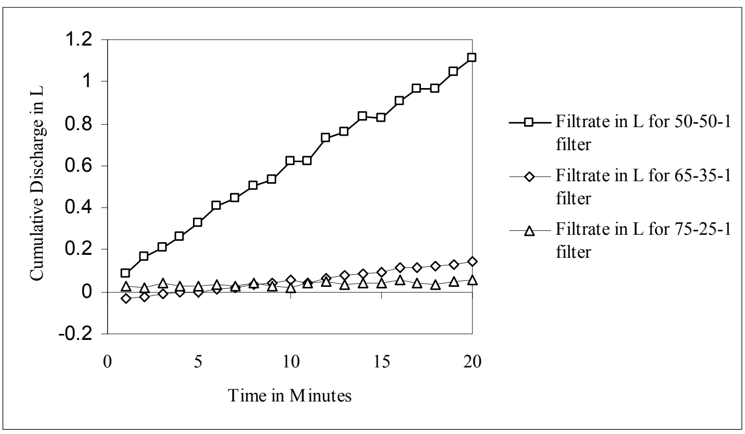

Figure 5: Behavior of cumulative discharge from the filters at the initiation of water percolation.

4.2.1 Theoretical Development

Nonlinearity is also detected by visual inspection of the rising curves in Fig. 5. The filtrate volume or cumulative discharge Y_i is a random function of time X_i for i = 0, 1, 2,. . . ..n. The total filtrate volumeY_n, accumulated and collected, is a result of a finite discharge from a filter in a specific interval of time. The two successive discharges vary each other by a random ratio or a transfer function. This can be written as

$$\frac{Y_i}{Y_{i-1}} = X_i^{b_i}, \text{ for } i = 1, 2 \ldots\ldots n \tag{5}$$

Eqs. 5 is a stochastic model, describing the relationship between the cumulative discharge Y_i at step i, from the previous cumulative discharge at step $i-1$ and transfer function X_i^{bi}. This transfer function X_i^{bi} contains the effect of the predictor variable X_i at step i of the stochastic model [Soboyejo (1965)]. Since there are n predictor variables in the generalized multi-parameter stochastic model; Eqs. 5 represents only one of the steps for i = 1, 2,. . . . . . n. The generalization has been elaborated in Appendix 1. It can be inferred from Fig. 5 that filtrate volume Y_i at each and every time step is incremented independently.

There are wide deviations in the discharge during the three consecutive experiments on each of the filters discussed here, as illustrated in Figs. 6, 8 and 10 for filters manufactured with 50-50, 65-35 and 75-25 clay to sawdust by volume ratios respectively.

Linear transformations can be used to remove the nonlinearity, thus providing a new response variable [Soboyejo (1965); Benjamin and Cornell (1970)].

The curves in Figs. 6, 8 and 10 may be characterized using a discrete time stochastic birth process [Soboyejo (1965)]. This was confirmed by performing regression data analysis on the basis of principles in stochastic methods [Benjamin and Cornell (1970); Ang and Tang (2007); Cox and Miller (1967)].

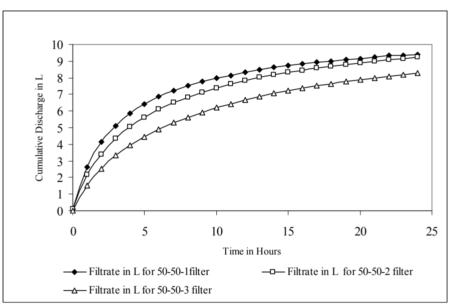

Figure 6: Discharge from three consecutive experimental runs on a 50-50 filter.

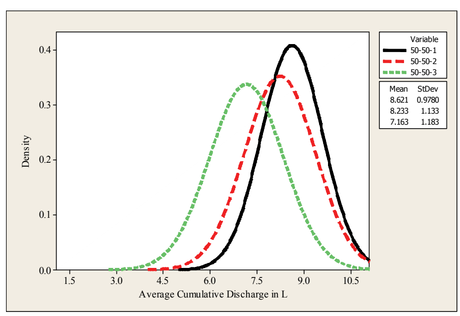

Figure 7: Histograms and normal distribution of cumulative discharge for consecutive experimental runs on the 50-50 filter

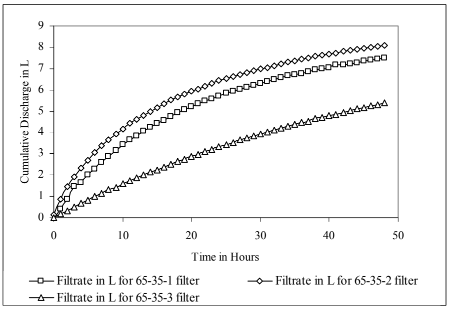

Figure 8: Cumulative discharge from three consecutive experimental runs on a 65-35 filter.

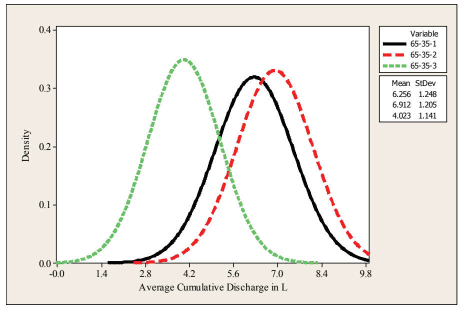

Figure 9: Histograms and normal distribution of discharge for consecutive experimental runs on the 65-35 filter.

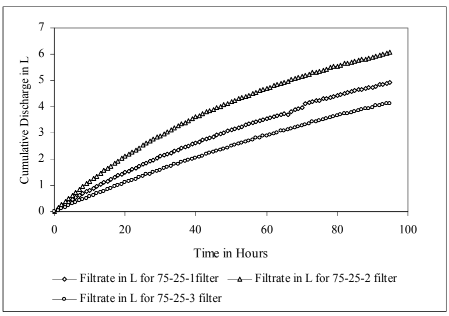

Figure 10: Discharge from three consecutive experimental runs on a 75-25 filter.

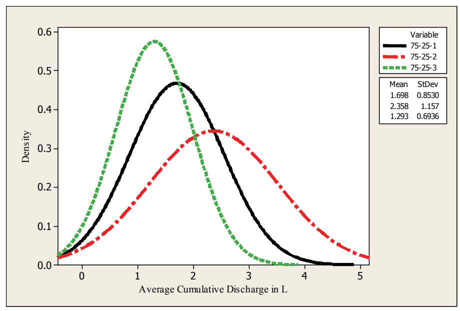

Figure 11: Histograms and normal distribution of discharge for consecutive experimental runs on the 75-25 filter. See above comment.

If the variables are transformed in the following manner,

$$G_i = \frac{X_i}{Y_i}$$

The random variable G_i jointly characterizes the random variable X_i andY_i. The relationship between the transformed random variable andX_i, is expressed as a straight line as given below [Soboyejo (1965); Soboyejo (1973)].

$$Y_i = \frac{X_i}{a_i + b_i X_i}. \tag{6}$$

where X_i for i= 0, 1, 2,.....n and for i=1 denotes time t. Here a_i and b_i are model parameters and are found to characterize individualistic material behavior and time respectively (elaborated in the Appendix 1) [Bulmer (1979); Soboyejo (1965); Soboyejo, Ozkan, Papritan and Soboyejo (2001)]. The transformation can be used model the behavior of the flow through the ceramic composite filter with improved values for the coefficient of determination, R^2 [Soboyejo (2006)]. This would help in determining a better empirical relationship for the flow through each of the individual filters.

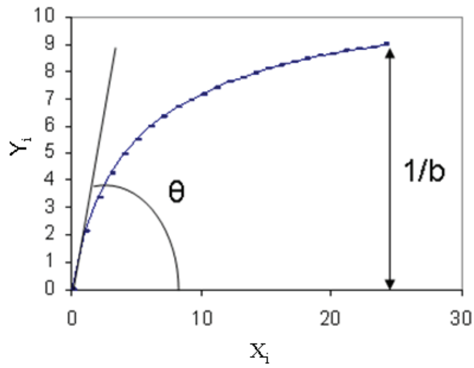

Figure 12: Description of the specific birth process model proposed in Eqs. 6 for *i*=1 [Soboyejo (2006)].

Rate of change of $Y_i$can be written as the first derivative of Eqs. 6 as [Soboyejo (2006)],

$$\frac{dY_i}{dX_i} = \frac{a_i}{(a_i + b_i X_i)^2} \tag{7}$$

From Fig. 12, at X_i= 0, for i= 1, we have

$$\frac{dY_i}{dX_i} = \frac{1}{a} = \tan\theta$$

where the derivative is also given as the arc tangent of the angle θ formed by each of the rising curves plotted in Fig 6, 8 and 10 respectively and as illustrated in Fig. 12. Eqs. 6 may again be expressed as

$$Y_i = \frac{1}{\frac{a_i}{X_i} + b_i} \tag{8}$$

From Fig. 12, as X_i tends to infinity, the Eqs. 8 reduces to the form,

$$Y_i = \frac{1}{b_i}. \tag{9}$$

This value of discharge may define the extrapolated value of the asymptote of each of the curves in Figs. 6, 8 and 10 respectively at requisite time durations.

4.2.2 Flow Variation in Each Filter within Discrete Experiments

Figs. 7, 9 and 11, compare the flow behavior of 50-50 filters with those of the 65-35 and 75-25 filters. The mean value of the filtrate volume decreases with number of experiments for all the 6 50-50 filters tested. This is shown from Fig. 7 as

$$\overline{Y}_{50-50-1} > \overline{Y}_{50-50-2} > \overline{Y}_{50-50-3}.$$

In this regard, the mean discharge values for 65-35 and 75-25 filters follow a different behavior as plotted in Figs. 9 and 11 respectively, and is explained below as,

$$\overline{Y}_{65-35-1} < \overline{Y}_{65-35-2} > \overline{Y}_{65-35-3}, \quad \overline{Y}_{75-25-1} < \overline{Y}_{75-25-2} > \overline{Y}_{75-25-3}.$$

Apart from these variabilities, the discharges from each of the experiments were characterized by discrete time stochastic processes. The discharge from the experiments done on the 50-50 filters can be expressed as

$$Y = \frac{X_1}{0.432 + 0.0943X_1} \tag{10}$$

where the model parameters *a* and *b* are 0.432 and 0.0943 respectively. The statistical coefficient of determination for the model (R^2) was found to be 99.9% based on the statistical regression analysis performed in Minitab Statistical Software (Version 15, Minitab inc, State College, PA).

Similarly a birth process model predicts the discharge through the 65-35 filters with coefficient of determination of 99.7 % as

$$Y = \frac{X_1}{2.25 + 0.098X_1} \tag{11}$$

where the model parameters *a* and *b* are 2.25 and 0.0943 respectively. The discharge from the 75-25 filter specimens has been found to follow a birth process with a coefficient of determination of 99% as shown below

$$Y = \frac{X_1}{10.9 + 0.0855X_1} \tag{12}$$

where the model parameters *a* and *b* are 10.9 and 0.0885 respectively.

4.2.3 Discharge Variability due Multiple Parameters

From Figs. 5 and 13, it is very clear that each of the filters behave differently. The average discharge for three filters with different manufacturing material compositions is shown in Fig. 13. There are 6 filters for each of the three filter variants with different manufacturing material composition. All the curves follow a rising trend with decreasing slopes with increase in time irrespective of their compositional variability.

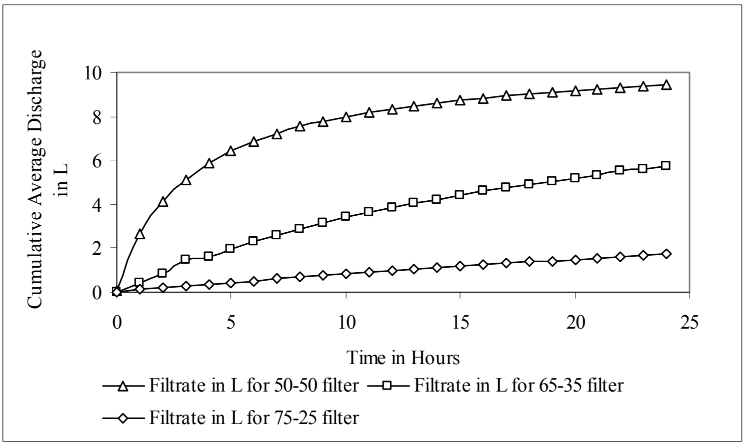

Figure 13: Measured discharge from three distinct filters for a duration of 24 hours.

The Eqs.6 is extended to a multi-parameter model to predict filter discharge on the basis of time and compositional variability. Here the stochastic process occurring within the filter system is represented in Fig. 14.

From Fig. 14, the multi-parameter model may be written as

$$G_i = a\prod_{i=1}^{k} X_i^{b_i} \tag{13}$$

where the model constants of a, *bi* for *i*= 1.2. . . . . . k, are derived by regression analysis on the experimental data. It should be noted that derived transformation of multiplicative variables produces multiplicative variables [Benjamin and Cornell (1970)].

The model indicates the lognormal influences of naturally occurring materials such as sawdust and water on the process [Limpert, Stahel and Markus (2001)]. The inherent non-linearity resulting from the influence of the natural variables is removed by log transformation on both sides of Eqs. 13. This can be expressed as

$$\ln G_i = \ln a + \sum_{i=1}^{k} b_i \ln X_i \tag{14}$$

The Eqs. 14 can be used for estimating model constants a_i and b_i. Multi-parameter regression analysis is carried out with two major random variables to predict percolation through the ceramic composite filter.

The mean value of $G_i$is expressed as [Ang and Tang (1975); Benjamin and Cornell (1970)]

$$E(G) = a \prod_{i=1}^{k} E(X_i^{b_i}) \tag{15}$$

The second central moment of the lognormal distribution representing $G_i$may be expressed as [Ang and Tang (1975); Benjamin and Cornell (1970)]

$$Var(G) = \sum_{i=1}^{k} b_i^2 \left[\frac{\sigma_{X_i}}{X_i}\right]^2 G_i^2 \tag{16}$$

It is found that there is no correlation between the random variables considered in this multi-parameter model hence the covariance term Cov (X_i,X_j) tends to zero, from the predictor variables analyzed with Minitab statistical software (Version 15, Minitab Inc., State College, PA). Also, the R^2 value is the combined influence of the independent parameters in this model [Haldhar and Mahadevan (2000)].

In this model,$G_i = \frac{X_1}{Y_i}$ and the predictor variables are X_1 (time) and X_2 (volume ratio of sawdust).

The variation of G_i was examined and G_i was modeled to a linear lognormal equation in Eqs. 13 with a square of the multiple-correlation coefficient, R^2 of 95.1%.

$$G = 11.4 \times X_1^{0.32} X_2^{-3.02} \tag{17}$$

Eqs. 17 may, therefore, be used to characterize the percolation through the filter manufactured with a known volume ratio of material constituent, at a specific time.

5 Discussion

The average cumulative discharge Y_i for the three different filters is seen to increase with an increase in the amount of saw dust X_2, as shown in Fig. 13. The stochastic birth process behavior shown by the discharge from these porous clay ceramic filters is theoretically elaborated in Appendix 1.

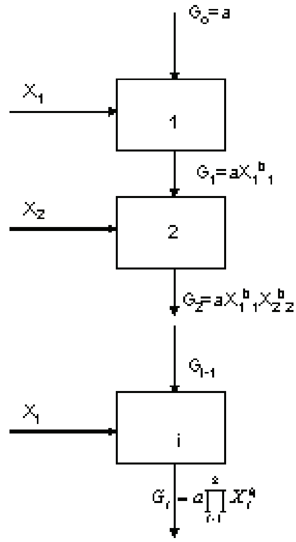

Figure 14: Diagrammatic representation of a stochastic process influenced by multiple parameters [Soboyejo (2006)]

The difference in percolation behavior within consecutive experimental trials was more noteworthy for the 50-50 filters with respect to the other filters. This distinctiveness may be attributed to the individual influences of clay and sawdust content on the structure of the filter material with time X_1 for the cumulative discharge process in each experimental trial.

The variability for cumulative discharge from each filter of a particular type (50-50, 65-35 or 75-25) with respect to X_1 is well established. It is known that there is negligible statistical correlation between the two variables X_1 and X_2. In view of this, the effect of variability between these predictor variables is extremely low and insignificant.

The variability in discharge can be attributed to variability in material constituents, as well as the variability arising from the manufacturing or production processes.

From Eqs. 17, it can be seen that the coefficient b_1 of the predictor variable X_2 is much larger when compared to the coefficient b_2 of predictor variable X_2. This clarifies the greater influence of predictor variable X_1 compared to X_2 on flow behavior in the clay ceramic porous media filter system with a specific volume fraction of sawdust [Bulmer (1979); Soboyejo (1965)].

The variability attributed to material constituents is much greater than the possible variability arising from the effect of time. Therefore, the variance or error of prediction of cumulative discharge with respect to time is much smaller than the error due to volume fraction of material constituents in this study.

An analysis of variance is carried out to elaborate this notion. From Eqs. 16 and 17, it can be further stated that,

$$Var(G) \approx (b_1)^2 Var(X_1) + (b_2)^2 Var(X_2) \tag{18}$$

Var (G) is the error in the prediction of the new transformed variable and Var (X_1) and Var (X_2) are variance or error expected in the individual predictor variables X_1 and X_2respectively. Discussions on Eqs. 18 are given elsewhere and will therefore not be given in this article (Bennett (2006); Soboyejo (1968); Soboyejo, Ozkan, Papritan and Soboyejo (2001)).

From empirical data analysis and calculations Eqs. 18 can be solved as

$$0.848 \approx (0.32)^2 [0.6703] + (-3.02)^2 [0.0812] \tag{19}$$

$$0.848 \approx 0.0686 + 0.7405$$

It is important to note from Eqs. 19 that material composition variability $((b_2)^2 Var(X_2))$ is much greater than the variability arising from the influence of time X_1 for cumulative discharge $((b_1)^2 Var(X_1))$.

This in turn confirms that error of prediction of cumulative discharge with time is much smaller than the error due to the amount of manufacturing material constituents of the filter used in this study.

Hence, investigations need to be performed to assess the variability in average cumulative discharge between individual clay ceramic filter variants and also within each individual variant filter.

From Figs. 7, 9 and 11, it is found that each of the three types of filters tested have cumulative discharges following the central limit theorem and have approximately equal standard deviations which tend to be the basic assumptions of tests for analysis of variance[Walpole and Myers (1993)].

The basic hypothesis was that mean average cumulative discharge for all the filter variants in this problem are equal to each other. This investigation was carried out with one- way analysis of variance with a 95% confidence interval using Minitab software (Version 15, Minitab Inc., State College, PA). The results are given in Tab.1.

From Tab.1, the value of F-statistic indicates that there is a large difference in mean average cumulative discharge values between the three filter variants studied in this problem, and not much variations within a particular filter variant.

Table 1: ANOVA results.

Average cumulative discharge vs. volume fraction of sawdust (VF) in the porous clay ceramic filter					
Sources of Variation	Sum of Squares	DOF	Mean Squares	F-Statistic	p value
Between VF	449.10	2	224.55	118.16	0.00
Within VF	131.12	69	1.90		
Total	580.22	71			

This rejects our hypothesis and confirms with the discussion on Eqs. 19. It is also seen that p value is smaller than 0.05, proving that volume fraction of sawdust is a significant contributor to the efficient percolation behavior of the filter.

The results of the ANOVA and Eqs. 19 are supported by the multi-parameter regression analysis done on dimensionless variables as shown in the Tab. 2.

Table 2: Lognormal model for porous ceramic clay composite water filter discharge (Y) using X_1 – time and X_2 – volume fraction of sawdust, using $Y = a\prod_{i=1}^{\kappa=2} X_i^b$.

Predictor Variables\Model Coefficients	$\overline{a}$	$\overline{b_1}$	$\overline{b_2}$	R^2
X_1	-0.423	0.992		0.147
X_2	-29.5	0.992	8.16	0.98

The values of the coefficients of predictor variables X_1 and X_2 are provided in Tab.2. Model coefficient $\overline{a}$ takes a negligibly small value indicating very low initial percolation rates, which has been observed from Figs. 6, 8 and 10 respectively. In comparison to the predictor parameter time, the volume fraction of sawdust used in the manufacture of the porous ceramic clay composite water filter influences the discharge Y the most. This is true for a family of filters with different compositional mixes of clay and sawdust by volume.

The results of this study are applicable only to the material configurations, porous vessel size and shape tested. With any change in material configurations, shape or size, the new technological principles developed in this study, can be applied to the new characteristics to obtain the new model. It is intended to extend the present technology to include new configurations, shapes and sizes in future investigation.

6 Conclusion

1. The micro-structural properties are substantially influenced by the quantity of the material used in the manufacture of the ceramic composite filter. A linear trend in the variation of porosity is seen with change in volume ratio of the material constituents used to manufacture the filter.

2. A single parameter stochastic birth process model for estimating the flow through a porous clay ceramic vessel with a specific material composition has been developed. Discharge follows a specific behavior depending on the amount of material constituent used in the manufacture of that filter.

3. A lognormal multi-parameter model has been proposed for predicting the flow behavior through a heterogeneous system with varying volume ratio of materials.

4. The contribution of time, volume ratio of the manufacturing materials as well as type of process play an important role in efficiently defining hydrodynamic behavior of a non uniform porous media.

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Appendix I

Assuming the cumulative discharge from a filter, Y at any time t, where t>0 and Y_i is a function of random variable X_i; i= 1, 2,...,n with an initial value Y_0 at time t = 0. Y_i, the cumulative discharge from a particular porous media filter, is obtained from the step–by-step contributions of n random variables as is also clear from Fig. 4. The cumulative discharge through a given filter is a stochastic birth process, with a well defined asymptotic value Y and is given as [Soboyejo (1965)]

$$Y = \frac{X}{a+bX}$$

Y_i is given by Eqs.6 as shown below

$$Y_i = \frac{X_i}{a_i + b_i X_i} \text{ for } i = 1,2,\ldots,n.$$

The new response variable considered is given by

$$G_i = \frac{X_i}{Y_i}$$

Considering the step-by-step contributions of n random variables it can be shown that

$$G_i = \frac{X_i}{Y_i} = G_{i-1} + b_i X_i,$$

then for i = 1 we have

$G_1 = G_o + b_1X_1$

When i = 2

$G_2 = G_1 + b_2X_2$

$G_2 = G_0 + b_1X_1 + b_2X_2$

$\ldots$

$G_n = G_0 + b_1X_1 + b_2X_2 + \ldots\ldots + b_nX_n$

Here G_0= initial value ofG_i, not related to any of the predictors, namely X_i for i= 1, 2, . . . ,n.

$G_0 = a$, is only related to the material structural properties [Soboyejo (1965)]. The coefficient of the predictor variables X_i for i= 1, 2, 3,. . . ,n are b_1, b_2, . . . , b_n which provides us information about the importance of the influence of each of the predictor variables on response G_i in this case [Bulmer (1979); Soboyejo (1965)]. These coefficients also represent the variability of G for a unit change in X_i for i= 1, 2, 3,. . . ,n [Haldhar and Mahadevan (2000)]. It should be noted that with the increase in the number of predictor variables, there is decrease in the deviation or error of prediction by the model [Haldhar and Mahadevan (2000)].