# Magnetic Fluid Based Squeeze Film behavior between curved circular Plates and Surface Roughness Effect

**Nikhilkumar D. Abhangi[1] and G. M. Deheri[1]**

**Abstract:** Efforts have been directed to study and analyze the behavior of a magnetic-fluid-based squeeze film between curved rough circular plates when the curved upper plate (with surface determined by an exponential expression) approaches the stationary curved lower plate (with surface governed by a secant function). A magnetic fluid is used as the lubricant in the presence of an external magnetic field oblique to the radial axis. The bearing surfaces are assumed to be transversely rough and the related roughness is characterized via a stochastic random variable with non-zero mean variance and skewness. The associated Reynolds equation is averaged with respect to the random roughness parameter; then the related non-dimensional differential equation is solved with suitable boundary conditions in dimensionless form to obtain the pressure distribution, such a distribution being necessary for determining the expression of load carrying capacity and ensuing calculation of the response time. The results, presented graphically, indicate that the bearing system displays considerably improved performances as compared to bearing systems working with conventional lubricants. It is seen that the pressure, load carrying capacity and the response time increase with increasing the magnetization parameter. In particular, the load carrying capacity increases with respect to the upper plate's curvature parameter, while a symmetric distribution takes place with regard to the lower plate's curvature parameter. Even if the effect of transverse roughness is adverse in general, this investigation offers some indications for obtaining better performance in the case of negatively skewed roughness (by suitably choosing the curvature parameters of both the plates).

**Nomenclature**

$a$          Radius of the circular plate

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[1] Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar-388120, Gujarat, India

| | |
|---|---|
| $p$ | Lubricant pressure |
| $B$ | Curvature parameter of the upper plate |
| $C$ | Curvature parameter of the lower plate |
| $H$ | Magnitude of the magnetic field |
| $P$ | Dimensionless pressure $(P = -\frac{h_0^3 p}{\mu h_0 a^2})$ |
| $W$ | Load carrying capacity |
| $\overline{W}$ | Dimensionless load carrying capacity |
| $\Delta t$ | Response time |
| $\Delta T$ | Non-dimensional response time |
| $\alpha$ | Mean of the stochastic film thickness |
| $\sigma$ | Standard deviation of the stochastic film thickness |
| $\varepsilon$ | Measure of symmetry of the stochastic random variable |
| $\sigma^*$ | $\sigma/h_0$ |
| $\alpha^*$ | $\alpha/h_0$ |
| $\varepsilon^*$ | $\varepsilon/h_0^3$ |
| $\phi$ | Inclination angle |
| $\mu$ | Absolute viscosity of the lubricant |
| $\overline{\mu}$ | Magnetic susceptibility |
| $\mu_0$ | Permeability of the free space |
| $\mu*$ | Magnetization parameter $(\mu* = \frac{-\mu_0 \overline{\mu} k h^3}{\mu h_0})$ |

## 1   Introduction

The behavior of a squeeze film between various geometrical configurations of flat surfaces was analyzed by Archibald (1956). Murti (1974) discussed the performance of a squeeze film trapped between curved circular plates describing the film thickness by an exponential expression and it was established that the load carrying capacity increased sharply with curvature in the case of concave pads. Gupta and Vora (1980) investigated the corresponding problem considering the annular plates. Here the lower plate was taken to be flat. Ajwaliya (1984) studied this problem of squeeze film behavior taking the lower plate also to be curved. Wu (1970), (1972) dealt with the squeeze film performance when one of the surface was porous faced taking mainly two types of geometries namely, annular and rectangular. Various bearing configurations such as circular, annular, elliptical, rectangular and conical were investigated by Prakash and Vij (1973). They made a comparison between the squeeze film performances of different geometries of equivalent surface area. It was concluded that the circular plates registered highest transient load carrying capacity, other parameters remaining same.

The above studies made use of conventional lubricants. The application of a magnetic fluid as a lubricant was analyzed by Verma (1986). The magnetic fluid comprised of fine surfactant and magnetically passive solvent. Subsequently, the squeeze film behavior between porous annular disks in the presence of a magnetic fluid lubricant was presented by Bhat and Deheri (1991). It was established that the application of magnetic fluid lubricant enhanced the performance of the squeeze film. However, the plates were considered to be flat. But, in actual practice the flatness of the plate does not endure owing to elastic, thermal and uneven wear effects. With this end in view Bhat and Deheri (1993) studied the behavior of a magnetic fluid based squeeze film between cured circular plates. The magnetic fluid based squeeze film between curved plates lying along the surfaces determined by exponential, secant and hyperbolic function was analyzed by Patel and Deheri (2008), (2002.a), (2002.b). It was established that the application of magnetic fluid lubricant improved the performance of the squeeze film.

It is a well-established fact that the bearing surfaces tend to develop roughness after having some run-in and wear. The roughness appears to be random and disordered. The randomness and the multiple roughness scales both contribute to be complexity of the geometrical structure of the surfaces. Invariably, it is this complexity which contributes to most of the problems in studying friction and wear. The random character of the surface roughness was recognized by several investigators who resorted to a stochastic approach in order to mathematically model the roughness of the bearing surfaces (Tzeng and Seibel (1967), Christensen and Tonder (1969.a), (1969.b), (1970)). Tonder (1972) analyzed theoretically the transition between surface distributed waviness and random roughness. Tzeng and Seibel (1967) dealt with a beta probability density function for the random variable characterizing the roughness. This distribution is symmetrical in nature with zero mean and approximates the Gaussian distribution to a good degree of accuracy for certain special situations. Christensen and Tonder (1969.a), (1969.b), (1970) developed and modified this approach of Tzeng and Seibel (1967) in order to propose a comprehensive general analysis both for transverse as well as longitudinal surface roughness based on a general probability density function. The method adopted by Christensen and Tonder (1969.a), (1969.b), (1970) laid the frame work to analyze the effect of surface roughness on the performance of a bearing system in a number of investigations (Ting (1975), Prakash and Tiwari (1983), Prajapati (1991), Guha (1993), Gupta and Deheri (1996)). In most of these analyses the probability density function for the random variable characterizing the surface roughness was assumed to be symmetric with mean of the random variable equal to zero. However, in general this may only be true to the first approximation. In practice due to non-uniform rubbing of the surfaces the distribution of surface roughness may indeed be asym-

metrical. With this idea in view, Andharia, Gupta and Deheri (1997) discussed the effect of transverse surface roughness on the performance of a hydrodynamic squeeze film in a spherical bearing making use of general stochastic analysis. It was observed that the effect of transverse surface roughness on the performance of the bearing system turned out to be considerably adverse.

Here it has been proposed to study and analyze a magnetic fluid based squeeze film between curved transversely rough circular plates where in, the upper plate lies along the surface determined by an exponential expression while the lower plate is taken along a surface governed by secant function.

## 2 Analysis

Fig. 1 shows the configuration of the bearing system.



Figure 1: Bearing Configuration

The bearing surfaces are assumed to be transversely rough. The thickness $h(x)$ of the lubricant film is

$$h(x) = \overline{h}(x) + h_s \tag{1}$$

where $\overline{h}(x)$ is the mean film thickness while $h_s$ is the deviation form the mean film thickness characterizing the random roughness of the bearing surfaces. The deviation $h_s$ is considered to be stochastic in nature and governed by the probability density function

$$f(h_s), -c \leq h_s \leq c \tag{2}$$

where $c$ is the maximum deviation from the mean film thickness. The mean $\alpha$, the standard deviation $\sigma$ and the parameter $\varepsilon$ which is the measure of symmetry associated with random variable $h_s$ are governed by the relations

$$\alpha = E(h_s), \tag{3}$$

$$\sigma^2 = E[(h_s - \alpha)^2] \tag{4}$$

and

$$\varepsilon = E[(h_s - \alpha)^3] \tag{5}$$

where E denotes the expected value defined by

$$E(R) = \int_{-c}^{c} Rf(h_s)dh_s \tag{6}$$

It is taken into consideration that the upper plate lying along the surface determined by

$$Z_u = h_0 \left[\exp(-Br^2)\right]; \quad 0 \le r \le a \tag{7}$$

approaches with normal velocity $\dot{h}_0 = \frac{dh_0}{dt}$, to the lower plate lying along the surface

$$Z_l = h_0[\sec(-Cr^2) - 1]; \quad 0 \le r \le a \tag{8}$$

where $h_0$ the central distance between the plates and B and C are the curvature parameters of the corresponding plates. The central film thickness $h(r)$ then is defined by

$$h(r) = h_0 \left[\exp(-Br^2) - \sec(-Cr^2) + 1\right] \tag{9}$$

Axially symmetric flow of the magnetic fluid between the plates is taken into account under an oblique magnetic field

$$\overline{H} = (H(r)\cos\phi(r,z),\ 0,\ H(r)\sin\phi(r,z)) \tag{10}$$

whose magnitude $H$ vanishes at $r = a$; for instance; $H^2 = ka(a - r)$, $0 \le r \le a$ where $k$ is a suitably chosen constant so as to have a magnetic field of required strength, which suits the dimensions of both the sides. The direction of the magnetic field plays a pivotal role since $\overline{H}$ has to satisfy the equation

$$\nabla \overline{H} = 0, \quad \nabla \times \overline{H} = 0. \tag{11}$$

Therefore, $\overline{H}$ arises out of a potential function and the inclination angle $\phi$ of the magnetic field $\overline{H}$ with the lower plate is determined by the first order partial differential equation

$$\cot\phi \frac{\partial\phi}{\partial r} + \frac{\partial\phi}{\partial z} = \frac{1}{2(a - r)} \tag{12}$$

whose solution is determined from the equations

$$c_1^2 \cos ec^2 \phi = a - r,$$

$$z = -2c_1 \sqrt{(a - c_1^2 - r)} \tag{13}$$

where $c_1$ is a constant of integration.

The modified Reynolds equation governing the film pressure $p$ then can be obtained as [(1996), (2002.a), (2002.b)]

$$\frac{1}{r}\frac{d}{dr}\left[ rg(h)\frac{d}{dr}\left( p - 0.5\mu_0\overline{\mu}H^2 \right) \right] = 12\mu\dot{h}_0 \tag{14}$$

where

$$g(h) = h^3 + 3\sigma^2 h + 3h^2\alpha + 3h\alpha^2 + 3\sigma^2\alpha + \alpha^3 + \varepsilon \tag{15}$$

Introducing the non-dimensional quantities

$$\overline{h} = h/h_0,\ R = r/a,\ \mu* = \frac{-\mu_0\overline{\mu}kh^3}{\mu\dot{h}_0},\ P = -\frac{h_0^3 p}{\mu a^2\dot{h}_0} \tag{16}$$

$$\sigma = \sigma/h_0,\ \varepsilon = \varepsilon/h_0^3,\ B = Ba,\ C = Ca^2$$

and solving the concerned Reynolds equation with the associated boundary conditions

$$P(1) = 0, \quad \frac{dP}{dR} = -\frac{\mu*}{2}\ \text{at } R = 0 \tag{17}$$

one can avail the non-dimensional pressure distribution as

$$P = \frac{\mu*}{2}(1 - R) + 6\int_R^1 \frac{R}{G(h)}dR \tag{18}$$

where in

$$G(h) = \overline{h}^3 + 3\overline{h}^2\alpha* + 3\sigma*^2\overline{h} + 3\overline{h}\alpha*^2 + \varepsilon* + 3\sigma*^2\alpha* + \alpha*^3$$

The dimensionless load carrying capacity is given by

$$\overline{W} = -\frac{Wh_0^3}{2\pi\mu a^4\dot{h}_0} = \frac{\mu*}{12} + 3\int_0^1 \frac{R^3}{G(h)}dR \tag{19}$$

where the load carrying capacity $W$ is obtained from the relation

$$W = 2\pi \int_0^a r p(r) dr \tag{20}$$

Lastly, the response time in dimensionless form is determined from the relation

$$\Delta T = \frac{\Delta t W h_0^2}{\pi \mu a^4} = \overline{W} \int_{\overline{h}_1}^{\overline{h}_2} \frac{1}{G(h)} d\overline{h} \tag{21}$$

where

$$\overline{h}_1 = \frac{h_1}{h_0}, \quad \overline{h}_2 = \frac{h_2}{h_0} \tag{22}$$

## 3 Results and discussions

Equations (18), (19) and (21) represent the expressions for non-dimensional pressure P, load carrying capacity $\overline{W}$ and response time $\Delta T$. It is evident that these performance characteristics depend on various parameters such as $\mu*$, $\sigma*$, $\alpha*$, $\varepsilon*$, $B$ and $C$. These parameters describe respectively, the effect of magnetic fluid lubricant, standard deviation of roughness, variance associated with roughness, measure of symmetry, the upper plate's curvature parameter and the lower plate's curvature parameter.

The equation (19) tends to suggest that the load carrying capacity of the bearing increases by 0.083 $\mu*$. Setting the roughness parameters $\sigma*$, $\alpha*$ and $\varepsilon*$ to be zero one gets the performance of a magnetic fluid based squeeze film trapped between curved circular plates lying along the surfaces determined by exponential function and secant function. Furthermore, taking the magnetization parameter to be zero the present study reduces to the performance of squeeze film behavior between curved circular plates.

The variation of load carrying capacity $\overline{W}$ with respect to the magnetization parameter $\mu*$ is presented for various values of roughness parameters $\sigma*$, $\alpha*$, $\varepsilon*$ and the curvature parameters $B$ and $C$ respectively in Fig. 2-6.

It is indicated from these figures that the load carrying capacity rises sharply with respect to the magnetization parameter although, the effect of $\mu*$ is almost negligible up to the value 0.01 as shown in Fig. 7-11.

Besides, among the roughness parameters the combined effect of the magnetization parameter and skewness is more pronounced.
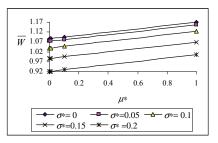
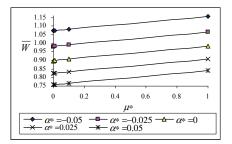Figure 2: Variation of load carrying capacity with respect to $\mu*$ and $\sigma*$



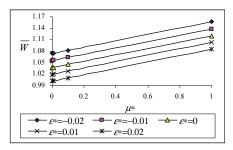Figure 3: Variation of load carrying capacity with respect to $\mu*$ and $\alpha*$



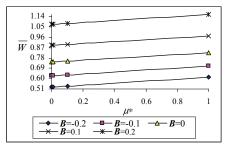Figure 4: Variation of load carrying capacity with respect to $\mu*$ and $\varepsilon*$



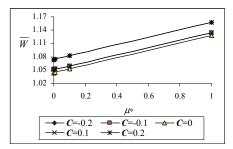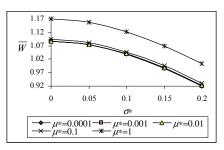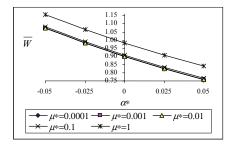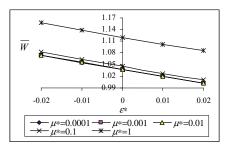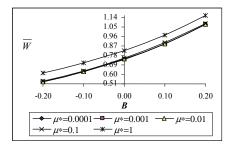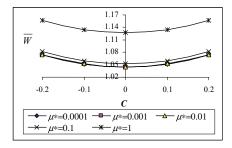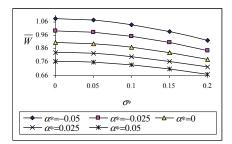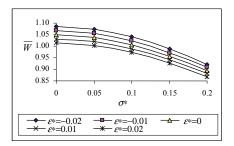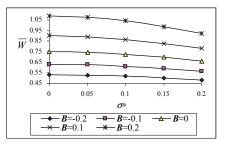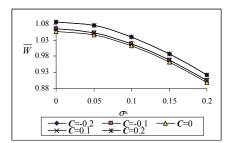Figure 5: Variation of load carrying capacity with respect to $\mu*$ and $B$



Figure 6: Variation of load carrying capacity with respect to $\mu*$ and $C$



Figure 7: Variation of load carrying capacity with respect to $\sigma*$ and $\mu*$

Figure 8: Variation of load carrying capacity with respect to $\alpha*$ and $\mu*$



Figure 9: Variation of load carrying capacity with respect to $\varepsilon*$ and $\mu*$



Figure 10: Variation of load carrying capacity with respect to $B$ and $\mu*$



Figure 11: Variation of load carrying capacity with respect to $C$ and $\mu*$



Figure 12: Variation of load carrying capacity with respect to $\sigma*$ and $\alpha*$



Figure 13: Variation of load carrying capacity with respect to $\sigma*$ and $\varepsilon*$

Figure 14: Variation of load carrying capacity with respect to $\sigma*$ and $B$



Figure 15: Variation of load carrying capacity with respect to $\sigma*$ and $C$

Fig. 12-15 describes the effect of the standard deviation associated with roughness on the distribution of the load carrying capacity.

It can be easily seen from these figures that the effect of the standard deviation is considerably adverse, in the sense that the load carrying capacity decreases substantially, although the effect of standard deviation is negligible upto 0.05 as can be seen from Fig. 16-19.
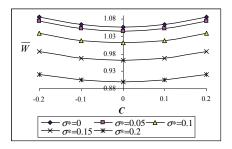


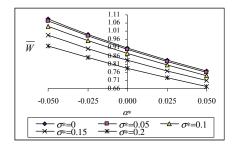Figure 16: Variation of load carrying capacity with respect to $C$ and $\sigma*$



Figure 17: Variation of load carrying capacity with respect to $\alpha*$ and $\sigma*$

The negative effect of $\sigma*$ is a little bit less with respect to the measure of symmetry as compared to that of variance associated with roughness.

In Fig. 20-22, one can have the effect of variance on the variation of load carrying capacity.

These figures make it clear that $\alpha*$(+ve) decreases the load carrying capacity while $\alpha*$(-ve) increases the load carrying capacity. Furthermore, it is indicated that the combined effect of the upper plate's curvature parameter and the negative variance tends to be significantly positive. The effect of the measure of symmetry on the
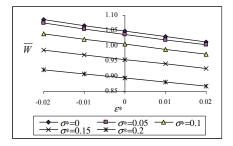
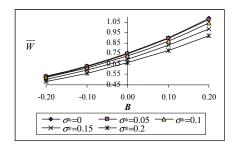Figure 18: Variation of load carrying capacity with respect to $\varepsilon*$ and $\sigma*$



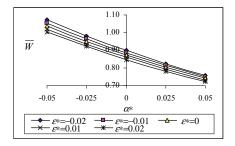Figure 19: Variation of load carrying capacity with respect to $B$ and $\sigma*$



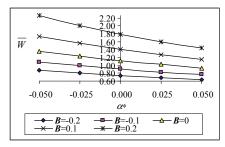Figure 20: Variation of load carrying capacity with respect to $\alpha*$ and $\varepsilon*$



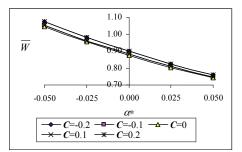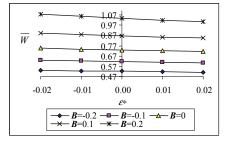Figure 21: Variation of load carrying capacity with respect to $\alpha*$ and $B$



Figure 22: Variation of load carrying capacity with respect to $\alpha*$ and $C$

distribution of load carrying capacity is depicted in Fig. 23-24.



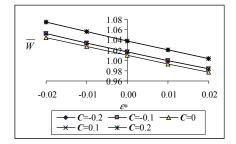Figure 23: Variation of load carrying capacity with respect to $\varepsilon*$ and $B$



Figure 24: Variation of load carrying capacity with respect to $\varepsilon*$ and $C$

As in the case of variance here also the load carrying capacity decreases due to positively skewed roughness, while the negatively skewed roughness increases the load carrying capacity. In addition, there is the symmetric distribution of the load carrying capacity with respect to the lower plate's curvature parameter which can be seen from Fig. 25-28.

In addition, the combined positive effect of the negatively skewed roughness and the upper plate's curvature parameter dominates the positive effect of the negatively skewed roughness and the lower plate's curvature parameter. Interestingly, it is noticed that the rate of increase in load carrying capacity with respect to the magnetization parameter is more with respect to the lower plate's curvature parameter as compared to that of upper plate's curvature parameter. Besides, it is revealed that the positive effect induced by negatively skewed roughness gets further enhanced owing to negative variance resulting in the fact that the combined effect of negatively skewed roughness and negative variance is significantly positive. Lastly, from the expression (21) it is found that the trends of response time $\Delta T$ are identical with those of load carrying capacity. A comparison of this investigation with the study of Patel and Deheri [12] suggests that the increase in load carrying capacity is substantially more considerable here.

## 4   Conclusion

Albeit, in general, the effect of transverse roughness is adverse, this article reveals that by properly choosing the curvature parameter of both the plates and the magnetization parameter the performance of the bearing system can be improved considerably in the case of negatively skewed roughness. This investigation offers some indications even for extending the life period of the bearing system.
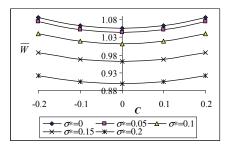
Figure 25: Variation of load carrying capacity with respect to $C$ and $\sigma*$



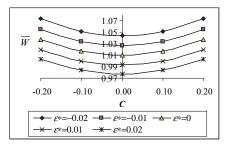Figure 26: Variation of load carrying capacity with respect to $C$ and $\alpha*$



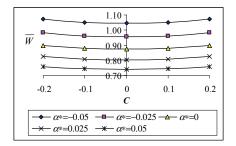Figure 27: Variation of load carrying capacity with respect to $C$ and $\varepsilon*$



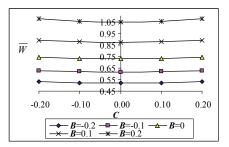Figure 28: Variation of load carrying capacity with respect to $C$ and $\varepsilon*$

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