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Liquid Droplet Impact onto Flat and Rigid Surfaces: Initial Ejection Velocity of the Lamella

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Abstract: In this paper a theoretical approach is elaborated for modelling the impact and ensuing spreading behaviour of a liquid droplet after its collision with a flat and rigid surface. The major outcomes of such a study can be summarized as follows: 1) The propagating-shock-wave velocity associated with the droplet is not a constant value but depends on the impact velocity and the physical and geometrical properties of the droplet. 2) The initial radial ejection velocity of the lamella is proportional to the shock-wave velocity (a) and the impact velocity (u_0) according to the expression $(a \cdot u_0)^{1/2}$. 3) The deceleration behaviour of the initial lamella of fluid has an inverse root squared dependence on time ($t^{-1/2}$). Finally, for verification of the compatibility, the theoretical results are compared against experimental measurements.

Keywords: drop impact, lamella, ejection velocity, shock wave.

1 Introduction

The dynamics of drops and bubbles has attracted much attention over recent years, see, e.g., Jiménez et al. (2005), Lappa (2005a,b), Esmaeeli (2005), Cristini and Renardy (2006), Lowengrub et al., (2007), Sussman and Ohta (2007), Fedorchenko and Wang (2004), Haller et al., (2003), Kalantari and Tropea (2007), etc.

The highly inertial impact and spreading behavior of a liquid droplet on a rigid surface, in particular, is known to play an important role in different phenomena such as water hammer or flight of an airborne object through a rainy weather (Rein, 1993).

In practice, when the liquid motion inside a droplet suddenly comes to rest by sudden collision with a flat and rigid surface, a multi directional propagating shock wave near the droplet wall boundary is generated. The increase of the resulting pressure (about 10 bars) on the impacted surface becomes a source for erosion

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degradation. Presence of some possible foreign particles in emulsion form (and also the corresponding liquid acceleration inside the droplets) can further increase the resulting damages.

An analytical study based on a geometrical-acoustics model was adopted to analyze the initial expansion behavior of the contact line associated with these phenomena by Lesser (1981). The main driving assumption was that the propagating-shock-wave velocity inside the droplet is always constant.

Haller et al. (2003) showed that the initial velocity of the propagating individual wavelets is higher than the ambient speed of sound for higher impact velocities. They developed an additional analytical study accounting for the lateral liquid motion in the compressed area and compared with the numerical solution of the inviscid Eulerian flow equations.

In general, the differences between experimental and analytical results are believed to be related to the omission of factors such as the surface tension, size and the density of the liquid droplet, (which, in principle, should affect the velocity of the propagating shock waves).

Marengo et al. (1998) measured the initial spreading velocity of the lamella for water droplet impacting onto PVC surfaces. Their observations indicated that the initial ejection velocity of the lamella could reach velocities up to 21 times the impact velocity, greatly exceeding earlier estimations (8.3 times the impact velocity).

They also demonstrated that the time scale of the lamella ejection due to the shock wave is of the order of μs, one order larger than values reported in previous studies. This behaviour may be caused by the time delay necessary for formation of shock waves inside the droplet, as also speculated by Lesser (1981).

This study considers a new approach for estimating: a) the shock wave propagation velocity inside the liquid droplet and b) the initial ejection velocity of the lamella as a function of the impact velocity, drop size and physical properties of the liquid droplet. To analyze the results, a close comparison between the theoretical approach and experimental measurements is conducted.

2 Theory - maximum radial ejection velocity of the lamella

Consider a liquid droplet with a diameter of d_0 and an initial impact velocity of u_0 colliding with a flat and rigid surface. Immediately after the collision, a compressed liquid volume bounded by a spherical shock envelope (control volume, CV, shown in Fig.1) propagates in all directions inside the droplet. Based on the mass and momentum balance for the control volume, the pressure rise (dP) due to shock wave can be expressed as

$$dp = \rho \cdot a \cdot du \tag{1}$$

where ρ is the droplet liquid density, a is the velocity of the propagating shock waves inside the droplet and du is velocity change in the normal direction (equal to impact velocity u_0).

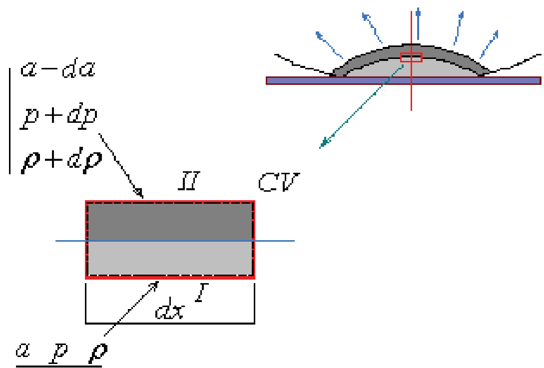

Figure 1: Element of a moving control volume (CV) around a propagating shock wave.

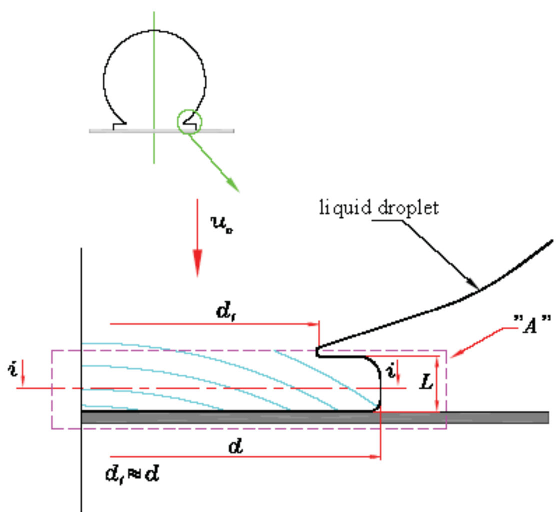

Figure 2: Bottom part of a liquid droplet near the rigid wall shortly after impact.

The dependency of the Velocity of the propagating shock waves inside the droplet on the droplet impact velocity is expressed as

$$a = a_0 + k \cdot u_0 \qquad (2)$$

where k is a constant value equal to 1.92 (see e.g. Haller et al. 2003 and Heymann 1969) and a_0 is speed of the acoustic waves inside the stationary liquid defined by

$$a_0 = \sqrt{B/\rho} \qquad (3)$$

Bulk modulus of the liquid droplet (B) shows that a change in the volume of fluid is expected as pressure changes; $B = dP/(dV/V_0)$, where dV/V_0 is relative compressibility of the liquid under the pressure change dP. However under the condition of $u_0/a_0 \ll 1$, the impact velocity has minor influence on the velocity of the propagating shock waves inside the liquid droplet according to Eq. 2.

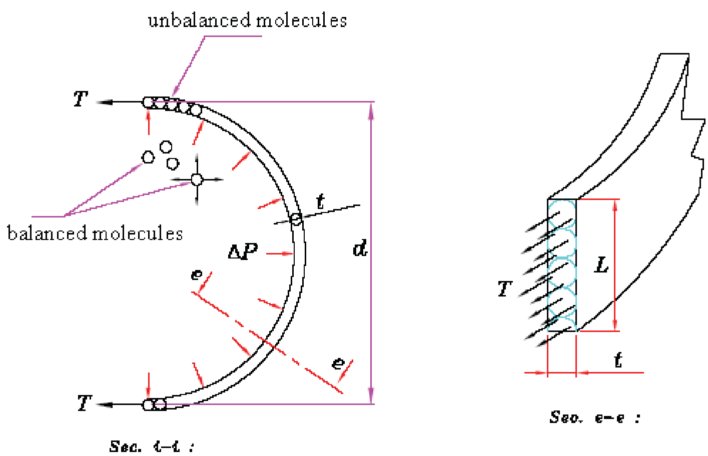

Figure 3: A unit long section of the cylindrical part of a lamella (Sec. i-i: in Fig.2).

It should be noted that the expression presented above for the speed of the acoustic waves (a_0) is with the assumption that the wave speed inside an infinite liquid is without any surrounding wall. This hypothesis may not be valid for the case of a liquid droplet. In this case the droplet-ambient interface acts as a strong copulating membrane resulting from the surface tension which creates complicated behaviour. With this assumption the value of the propagating shock wave velocity inside a droplet is expected to be different in comparison to Eq. 3.

To consider the effect of the surface tension and the droplet size on the propagating shock wave velocity inside a liquid droplet, first, assumption is that the bottom part of a liquid droplet near the rigid wall (shortly after the impact) has a cylindrical form denoted by "A" (see Fig.2). The total mass entering inside the cylindrical part ($\rho AL \cdot du/a$) at $t = L/a$ is equal to the amount of mass stored by an increase in the lamella diameter ($\rho L \cdot dA$); A being surface area of the ejecting lamella and ($LA \cdot d\rho$) is the mass stored due to the increase in density. Mass conservation yields

$$\rho AL \cdot du/a = \rho L \cdot dA + LA \cdot d\rho \tag{4}$$

Dividing Eq.4 by ρAL, and replacing du by Eq.1 and using definition of the bulk modulus (3), we arrive at

$$a = [\rho\,(1/B + dA/(A \cdot dP))]^{-0.5} \tag{5}$$

Figure 3 shows a unit long section of the cylindrical part of a lamella with an extremely thin wall thickness which relates $dA/(A \cdot dP)$ to the physical and geometrical properties of the liquid droplet. This equals to a differential CV subjected to an increase in internal pressure**.** The circumferential tensile force on the droplet wall (T), stress (S) and strain (ξ) can be expressed in static equilibrium condition as

$$\sum Fx = 0: \quad 2T - \Delta P \cdot (d \cdot 1) = 0$$

yielding

$$T = \Delta P \cdot d/2 \tag{6}$$

and

$$S = T/(t \cdot 1) = \Delta P \cdot d/2t \tag{7}$$

The hypothesis is that, the parameter σ/t in a liquid droplet is analogues with Young's modulus (E) in a solid material, i.e. $\sigma/t \sim E$ (or $\sigma/t = c_1 \cdot E$, c_1 is a constant coefficient). Therefore the strain (ξ) in the outer layer of a liquid droplet can be defined as $\xi = c_1 \cdot S/(\sigma/t)$.

Substituting S from Eq. 7 yields

$$\xi = c_1 \cdot \Delta P \cdot d/2\sigma \tag{8}$$

Since, the change in the cross sectional area of the lamella is related to the strain ($\xi = \Delta d/d$); therefore

$$\Delta A/A = 2(\Delta d/d) = 2\xi \tag{9}$$

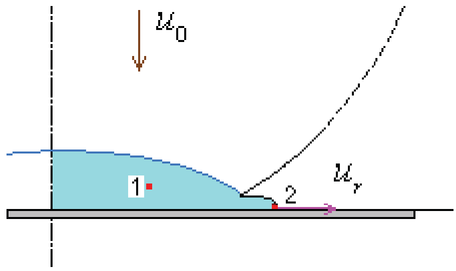

Figure 4: Initial phase of a liquid droplet impacting onto a rigid wall (ejection phase of the lamella).

Substituting (8) into Eq. (9) gives

$$\frac{\Delta A}{A \cdot \Delta P} = c_1 \cdot \frac{d}{\sigma} \tag{10}$$

Finally substituting (10) into Eq.5 and considering that the diameter of the lamella at a certain time is proportional to the droplet size, i.e., $d \propto d_0$ (or $d = c_2 \cdot d_0$), Eq.5 yields

$$a_0/u_0 = \left(\rho u_0^2/B + \varepsilon \cdot We\right)^{-0.5} \tag{11}$$

where We is the impact Weber number defined by $We = \rho u_0^2 d_0/\sigma$. The coefficient ε ($\varepsilon = c_1 \cdot c_2$) found to be approximately 5×10^{-8} based on the measurement data used in this study for different droplet liquids(eg: water, glycerol and silicon oil). Note that the coefficient ε may vary for different droplet liquids, but the given value satisfies the estimated value.

The obtained expression (11) is derived for spreading (depositing) droplets on a flat and rigid target, where the phenomenon of the lamella ejection is evident. The Deposition occurs at the higher impact Weber numbers We>5, when portions of the spreading droplet lose their kinetic energy due to the dissipation in the boundary layer of the spreading droplet, see e.g., Pasandideh-Fard et al. (1996), and Kalantari and Tropea (2007). The upper limit of the impact Weber number for the deposition of a droplet without splashing depends also on some additional parameters such as surface roughness; see e.g., Kalantari and Tropea (2006), Mundo et al. (1998), and Cossali et al. (1997). Expression (11) indicates a dependency of the propagating shock wave velocity on both the impact Weber number (We) and on the Bulk modulus of the liquid droplet (B). Considering the definition of the impact Weber number, Eq. 11 shows that the propagation velocity of the shock waves inside a liquid droplet is larger for smaller droplets and also true for higher surface tensions. This is in consistent with the experimental measurements illustrated in Fig. 5.

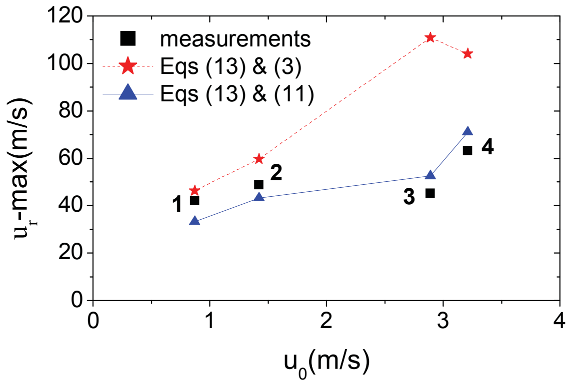

Figure 5: Maximum initial ejection velocity of the lamella for different liquid droplets impacting onto a glass surface. Single points: experimental data obtained by Rioboo et al. (2002): 1) Isopropanol (u_0 =0.87 m/s, d_0 =3.3 mm), 2) Isopropanol (u_0 =1.42 m/s, d_0 =3.29 mm), 3) Glycerine (u_0 =2.89 m/s, d_0 =2.49 mm), 4) Water (u_0 =3.21 m/s, d_0 =2.76 mm)). Points along the solid line: computations according to Eqs. (13) & (11); and points along the dashed line: computations according to Eqs. (13) & (3).

The maximum value of a rising pressure inside a liquid droplet (given by Eq.1) occurs immediately after the impaction and exactly above the rigid wall attached to the contact line. Therefore immediately after the collision, contact line of the droplet rapidly is expected to be ejected outward. For estimating the maximum initial ejection velocity of the lamella, the energy balance between points 1 and 2 is considered (see Fig.4), yielding

$$\frac{p_1}{\rho_1} + \frac{V_{1x}^2}{2} = \frac{p_2}{\rho_2} + \frac{u_r^2}{2} \tag{12}$$

where P_1 is the maximum hammer pressure inside the liquid droplet occurring immediately after the impact ($P_1 = \rho \cdot a \cdot u_0$), V_{1x} is a radial component associated with the propagating shock wave velocity, P_2 is the ambient pressure and u_r is the initial radial ejection velocity of the lamella. Assuming that the shock waves are to be propagating vertically near the rigid wall and ambient pressure is negligible; the energy balance (12) gives the maximum radial ejection velocity of the lamella as

$$u_r = \sqrt{2au_0} \tag{13}$$

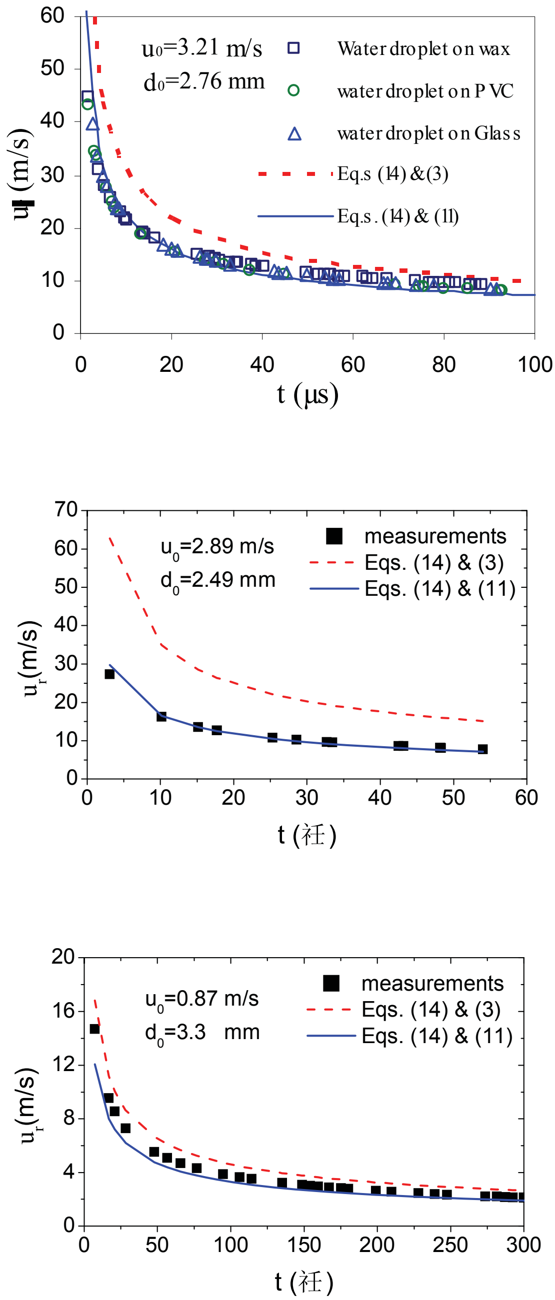

Figure 6: Time history of initially ejection velocity of the lamella: a) water droplet on different surfaces, b) Glycerine droplet on glass, and c) Isopropanol droplet on glass; experimental measurements by Rioboo et al. (2002).

Based on the experimental data shown in Fig. 6, the deceleration associated with the maximum radial ejection velocity of the lamella with time is obtained in the form of

$$u_r(t) = \sqrt{2au_0} \cdot (t/\tau)^{-0.5} \tag{14}$$

where t is the time after the impact (in μs) and τ is the instant of the lamella ejection related to the time delay necessary for the formation of the shock waves inside the liquid droplet. The value of $\tau = 1\mu s$ satisfies the experimental observations used in this study and also dimensional request of (14).

3 Results

Using Eqs. (11) and (13), a comparison between experimental data and the theoretical maximum initial ejection velocity of the lamella has been carried out for different liquid droplets. It is illustrated in Fig. 5.

The results presented in this figure indicate that computing the shock-wave propagation velocity with (3) overestimates the initial maximum ejection velocity of the lamella for glycerine, where the liquid droplet has lower surface tension but higher density. The deviation in the speed of the initial maximum ejection velocity of the lamella for water droplet is also evident in this figure. In contrast the computed initial maximum ejection velocity of the lamella (Eq.11) gives better results for both liquid droplets and indeed the theoretical results are in a good agreement with the experimental data used in this study, see Fig. 5.

The time dependence behaviour of the initial ejection velocity of the lamella for different liquid droplets (eq. (14)) is illustrated in Figs. 6a, b, and c indicating a good agreement with the experimental data.

4 Conclusions

It has been shown that the propagation velocity of the shock waves inside a liquid droplet impacting with a solid surface depends on the physical and geometrical properties of the liquid droplet.

According to the derived theoretical expressions: a) the initial radial ejection velocity of the lamella is much smaller than the computed shock wave velocity inside the liquid droplet; b) the ejection velocity is much greater than the impact velocity.

Comparison of the experimental and theoretical results show that the time associated with the initial ejection of the lamella is independent of the wetability of the surfaces.

Estimations of the initial velocity of the radial lamella ejection based on experiments are in a good agreement with the presented theoretical data.

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