A Deformation and a Break of Hanging Thin Film under Microgravity Conditions

A. Ovcharova[1] **and N.Stankous**[2]

Abstract: We consider a deformation of a thin film which is hanging between two solid flat walls under thermal load action. A two-dimensional model is applied to describe the motion of thin layers of viscous nonisothermal liquid under microgravity conditions. The model is based on the Navier-Stokes equations. A numerical analysis of the influence of thermal loads on the deformation and break of freely hanging thin films has been carried out. The mutual influence of capillary and thermo-capillary forces on thin film free surface position has been shown. The results of model problem solutions are presented.

Keyword: thin film, viscous non-isothermal liquid, free surface, capillary effect, Marangoni effect.

1 Introduction

Because of a large increase of industrial application of film coating, investigations of processes related to thin layers of viscous liquid are extremely important. There are an enormous number of recent publications from around the world devoted to this problem. In accordance with their purpose, these authors exploit different approaches to describe the processes occurring in thin layer of viscous liquid, and to solve the related problems. We focus our attention on the analysis of the mutual influence of capillary and thermo-capillary forces on the deformation and rupture of a freely hanging film under a thermal load applied to its free surface. Although we research the process under a micro-gravity effect, the same method can be used to solve similar problems with the effect of earth gravity.

Mathematical model. Consider a plane thin film with the density ρ, kinematical viscosity ν and coefficient of surface tension $\sigma(T)$ limited by two solid planes $x=0; x=L$ (fig.1). Here, $y=0$ is the plane of symmetry, and $f=f(t,x)$ is the free surface.

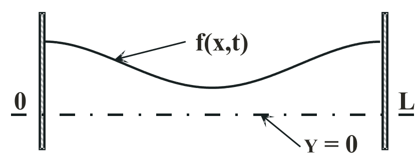

Figure 1:

The liquid film motion and heat transfer are described by Navier-Stokes equations which are written in terms of ψ, ω, and θ.

$$\frac{\partial\omega}{\partial t}+\frac{\partial}{\partial x}(\omega\frac{\partial\psi}{\partial y})-\frac{\partial}{\partial y}(\omega\frac{\partial\psi}{\partial x})=\Delta\omega; \qquad (1)$$

$$\Delta\psi+\omega=0; \qquad (2)$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial}{\partial x}(\theta\frac{\partial\psi}{\partial y})-\frac{\partial}{\partial y}(\theta\frac{\partial\psi}{\partial x})=\frac{1}{\mathrm{Pr}}\Delta\theta. \qquad (3)$$

The scales of the length, velocity and pressure are: h_0, ν/h_0, $\rho_0 v_0^2$ respectively, so that the Reynolds number is $\mathrm{Re}=1$. Pr is the Prandtl number; $\theta=(T-T_0)/\Delta T$ (T_0 is the characteristic temperature, and ΔT is a temperature drop).
The stream function is defined by the relations:

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}.$$

We assume that initial conditions reflect the fact that the film is plane and does not move.

[1] Lavrentyev Institute of Hydrodynamics of Siberian Division of Russian Academy of Sciences, RUSSIA. E-mail:ovcharova@hydro.nsc.ru
[2] National University, La Jolla, CA, USA. E-mail: nstankous@nu.edu

For $x = 0; \quad x = L$ the boundary conditions are non-slip ones:

$$\psi = 0, \qquad \frac{\partial \psi}{\partial n} = 0.$$

On the symmetry plane,

$$\psi = 0, \qquad \omega = 0.$$

Boundary condition for temperature (if necessary) can be given uniformly by the equation

$$\alpha\theta + \beta\frac{\partial \theta}{\partial n} = \gamma(t)$$

Then, for the plane of symmetry (for example) $\alpha = \gamma = 0; \beta = 1$.

On the free surface $f(x,t)$, the following conditions are set.

Determine normal and tangent vectors to the free surface $f(x,t)$ in any its point as

$$\mathbf{n} = \left\{ \frac{-f_x}{\sqrt{1+f_x^2}}, \frac{1}{\sqrt{1+f_x^2}} \right\},$$
$$\mathbf{s} = \left\{ \frac{1}{\sqrt{1+f_x^2}}, \frac{f_x}{\sqrt{1+f_x^2}} \right\}$$

and assume that the coefficient of surface tension $\sigma(T)$ is a linear function of temperature:

$$\sigma(T) = \sigma_0(1 - \sigma_T(T - T_0)), \qquad \sigma_0 = \sigma(T_0),$$
$$\sigma_T = \frac{1}{\sigma_0}\frac{d\sigma}{dT}\Big|_{T=T_0}, \qquad \sigma_0, \sigma_T > 0.$$

To determine the free surface, the kinematical condition is utilized:

$$f_t + \sqrt{1+f_x^2}\frac{\partial \psi}{\partial s} = 0 \tag{4}$$

The boundary conditions for ψ and ω we can write now in explicit form:

$$\frac{\partial \psi}{\partial n} = v_s, \tag{5}$$

$$\omega = 2\left(\frac{v_s}{R} + \frac{\partial v_n}{\partial s}\right) + \mathrm{Mn}\frac{\partial \theta}{\partial s}, \tag{6}$$

where v_s is the solution to the equation

$$\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s} = 2\frac{\partial^2 v_s}{\partial s^2} + D; \tag{7}$$

$$D = \underbrace{-\frac{\partial \omega}{\partial n} + \mathrm{Ca}^{-1}\frac{\partial}{\partial s}\left[\frac{1}{R}\left(1 - \frac{\sigma_T \Delta T}{\sigma_0}\theta\right)\right]} \underbrace{-2\frac{\partial}{\partial s}\left(\frac{v_n}{R}\right) + v_n\omega + f_x\frac{v_n^2}{R}};$$

$$v_n = f_t/\sqrt{1+f_x^2}. \tag{8}$$

Here, v_s and v_n are the tangent and normal velocities of the free surface points; R is the radius of curvature of the surface $f(x,t)$; $1/R = f_{xx}/\sqrt{(1+f_x^2)^3}$; $\mathrm{Ca} = \rho_0 v_0 \nu/\sigma_0$ is the capillary number; $\mathrm{Mn} = \sigma_T \Delta T/(\rho_0 v_0 \nu)$ is the Marangoni number.

Note. In fact, $\sigma_T \Delta T \theta/\sigma_0 \ll 1$ for real liquids, and practically does not effect the position of the free surface, and will be omitted in further consideration.

We combine in D all the major and minor terms together. Two major terms show us which forces deform the film free surface. The first term $\partial\omega/\partial n$ presents the thermo-capillary forces, the second one $Ca^{-1}\partial(1/R)/\partial s$ is the capillary forces. The last three items are minor terms. Even more, if we are looking for a steady-state solution we can put the minor terms equal to zero.

To derive the formula (6) we utilized the expression for the continuity of the tangent stresses on the free surface. The equation (7) has been obtained as follows. The scalar product of the vector $\vec{s}(t,x)$ and the Navier-Stokes vector equation written in the natural variables $(\vec{v}, P)$ gives us a scalar equation for v_s. The derivative $\partial P/\partial s$ on the right-hand side of the resulting equation can be eliminated if to differentiate the equation for continuity of normal stresses on the surface with respect to s. Equation (7) plays a very important role. It helps to derive boundary conditions for the desired functions ψ and ω in explicit form, and in addition, as it will be shown below, this equation is an important tool for an investigation of the considered processes. Generally speaking, the equation (7) realizes the interchange of information between the flow inside the domain and its free surface. For the stationary case, a derivation of the equation (7) has been described in details by Ovcharova (1998).

2 Computation Method

Under such decoupled boundary conditions on the free surface, the problem (1) - (8) can be solved by any method used for heat and mass transfer problems in terms of the variables ψ, ω, θ in closed domains. Note that in the frame work of the model, equations (1) - (3) are of the same type , and therefore the same computational procedure can be applied to solve them. In the present paper, we utilized the method of solution for regular domain. The domain occupied by the liquid is mapped onto rectangular with sides $0 \le \xi \le L$, $0 \le \eta \le 1$ by using transformation

$$x = \xi, \quad y = \eta(f(x,t)).$$

In this case, all boundaries of the domain including the free surface lie on the coordinate lines. Every equation (1)-(3) can be written in the form

$$\frac{\partial \Phi}{\partial t} = \frac{1}{Bf}\left[\frac{\partial}{\partial \xi}\left(B_{11}\frac{\partial \Phi}{\partial \xi} + B_{12}\frac{\partial \Phi}{\partial \eta} - A\Phi\frac{\partial \psi}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(B_{12}\frac{\partial \Phi}{\partial \xi} + B_{22}\frac{\partial \Phi}{\partial \eta} + A\Phi\frac{\partial \psi}{\partial \xi}\right)\right] + R_1\frac{\partial \Phi}{\partial \eta} + R_2. \quad (9)$$

Here

$$B_{11} = f(\xi), \quad B_{12} = -f_\xi \eta, \quad B_{22} = \frac{1 + {B_{12}}^2}{f(\xi)} \quad (10)$$

For $\Phi = \omega$:

$$B = 1, \quad A = 1, \quad R_1 = \frac{f_t}{f}\eta, \quad R_2 = 0,$$

For $\Phi = \theta$:

$$B = Pr, \quad A = Pr, \quad R_1 = \frac{f_t}{f}\eta, \quad R_2 = 0,$$

For $\Phi = \psi$:

$$B = \frac{1}{\lambda}, \quad A = 0, \quad R_1 = 0, \quad R_2 = \lambda\omega,$$

where λ is the iteration parameter to solve Puasson's equation for ψ.

If we take new designations:

$$U(\Phi) = B_{11}\frac{\partial \Phi}{\partial \xi} + B_{12}\frac{\partial \Phi}{\partial \eta} - A\Phi\frac{\partial \psi}{\partial \eta},$$

$$V(\Phi) = B_{12}\frac{\partial \Phi}{\partial \xi} + B_{22}\frac{\partial \Phi}{\partial \eta} + A\Phi\frac{\partial \psi}{\partial \xi}.$$

then equation (9) will be

$$\frac{\partial \Phi}{\partial t} = \frac{1}{Bf}\left[U_\xi(\Phi) + V_\eta(\Phi)\right] + R_1\frac{\partial \Phi}{\partial \eta} + R_2. \quad (11)$$

The set of boundary conditions on the free surface ($\eta = 1$) can be written as:

$$\frac{\partial f}{\partial t} + \frac{\partial \psi}{\partial \xi} = 0, \quad (12)$$

$$\frac{\partial \psi}{\partial \eta} = \frac{(v_s + B_{12}v_n)\sqrt{1 + B_{12}^2}}{B_{22}}, \quad (13)$$

$$\omega = 2\left(\frac{v_s}{R} + \frac{1}{\sqrt{1 + B_{12}^2}}\frac{\partial v_n}{\partial \xi}\right) + \frac{\text{Mn}}{\sqrt{1 + B_{12}^2}}\frac{\partial \theta}{\partial \xi}, \quad (14)$$

where v_s is the solution to the equation

$$\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial \xi} = 2\frac{\partial}{\partial \xi}\left(\frac{1}{\sqrt{1 + B_{12}^2}}\frac{\partial v_s}{\partial \xi}\right) + D; \quad (15)$$

$$D = -\left(B_{12}\frac{\partial \omega}{\partial \xi} + B_{22}\frac{\partial \omega}{\partial \eta}\right) + \text{Ca}^{-1}\frac{\partial}{\partial \xi}\left(\frac{1}{R}\right) - 2\frac{\partial}{\partial \xi}\left(\frac{v_n}{R}\right) + v_n\omega - B_{12}\frac{v_n^2}{R};$$

$$v_n = f_t/\sqrt{1 + B_{12}^2}. \quad (16)$$

Note that equation (11) is presented in the divergent form, and the kinematical condition (12) expresses the mass conservation law.

Equation (11) for θ (temperature), ω (vortex) and ψ (stream function) has been solved by the stabilizing-correction finite-difference scheme (see Jim Douglas, jr. and H.H.Rachford, jr. (1956) and N.N. Yanenko (1967,1971)) written in the form

$$\frac{\Phi^{k+1/2} - \Phi^k}{\tau} = \frac{1}{Bf}\left[U_\xi^k(\Phi) + V_\eta^{k+1/2}(\Phi)\right] + R_1\Phi_\eta^{k+1/2} + R_2$$

$$\frac{\Phi^{k+1}-\Phi^{k+1/2}}{\tau}=\frac{1}{Bf}\left[U_{\xi}^{k+1}(\Phi)-U_{\xi}^{k}(\Phi)\right] \quad (17)$$

$$V^{k+1/2}(\Phi)=B_{12}\Phi_{\xi}^{k}+B_{22}\Phi_{\eta}^{k+1/2}+A\Phi^{k+1/2}\frac{\partial\psi}{\partial\xi}$$
$$U^{k+1}(\Phi)=B_{11}\Phi_{\xi}^{k+1}+B_{12}\Phi_{\eta}^{k+1/2}-A\Phi^{k+1}\frac{\partial\psi}{\partial\eta} \quad (18)$$

This scheme is one of the efficient finite-difference schemes using fractional steps. The first fractional step gives the full approximation of the equation, and the second step is a correctional one and serves the purpose of stability improvement.

For implementing the scheme (17), (18) within the rectangle reflecting the transformed domain the calculational grid is built using the standard procedure:

$$\xi_n=(n-1)\Delta\xi, \quad \Delta\xi=L/NB,$$
$$n=1,\ldots,NN, \quad NN=NB+1,$$
$$\eta_m=(m-1)\Delta\eta, \quad \Delta\eta=1/MB,$$
$$m=1,\ldots,MM, \quad MM=MB+1.$$

For differential expressions like $(a_{11}\Phi_{\xi})_{\xi}$, $(a_{22}\Phi_{\eta})_{\eta}$, $(a_{1}\Phi)_{\xi}$, $(a_{2}\Phi)_{\eta}$, $(a_{12}\Phi_{\xi})_{\eta}$, $(a_{12}\Phi_{\eta})_{\xi}$, we use approximations of the second order accuracy by finite-difference analogs Λ_{11}, Λ_{22}, Λ_{1}, Λ_{2}, Λ_{12}, Λ_{21}, that have traditional representations. After replacement the derivatives in (17) by corresponding finite differences and substitution $V_{\eta}(\Phi)$ and $U_{\xi}(\Phi)$ by finite-difference analogs from (18) for every half-step in time for all interior points $((n=2,\ldots,NB;\ m=2,\ldots,MB))$ we will get the system of linear finite-difference equations for the function $\Phi(\xi_n,\eta_m)$. The system has three-diagonal structure with prevailing diagonal elements of the matrix, and can be solved effectively by the double-sweep method with specific boundary conditions.

The equation (15) is the Burgers-Hopf type equation with a right-hand side, and it requires the high accuracy for its solution because the boundary conditions for the functions ψ and ω depend on the solution. To get the solution, the high accuracy conservative scheme has been utilized. The high order of accuracy is required for the reason that numerical viscosity typical for the first order approximation schemes would not suppress the viscosity of the equation. Thus, the implicit scheme of the second order accuracy in time and space has been used to approach left-hand parte of the equation (15). However, in this case, steep gradients occur, and the solution is close-to-discontinuous. As Ostapenko (1987, 2000) had shown, that more accurate flow parameters in steep-gradient regions are produces by conservative schemes, i.e. flux terms must be approximated in divergent form. For that reason, we rewrite the equation (15)

$$\frac{\partial v_s}{\partial t}+\frac{1}{2}\frac{\partial}{\partial\xi}(v_s)^2=2\frac{\partial}{\partial\xi}\left(\frac{1}{\sqrt{1+B_{12}^2}}\frac{\partial v_s}{\partial\xi}\right)+D$$

and approximate the second term in the left-hand side as following :

$$\frac{1}{2}\frac{\partial}{\partial\xi}(v_s)^2\rightarrow\frac{1}{2}\frac{(v_s^2)_{n+1}^{k+1}-(v_s^2)_{n+1}^{k+1}}{2\Delta\xi}$$
$$\rightarrow\frac{1}{2}\frac{(v_s)_{n+1}^{k}(v_s)_{n+1}^{k+1}-(v_s)_{n-1}^{k}(v_s)_{n-1}^{k+1}}{2\Delta\xi}.$$

Then, we approximate the first term in the right-hand side by the finite-difference operator Λ_{11} on the upper time step $(k+1)$, and the rest of the differential operators have traditional presentations. For the derivative $\partial\omega/\partial\eta$ on the free surface, we use one-side approximation of the second order, and the value $\partial\omega/\partial n$ is taken from the previous time step. In this case, we will get a system of finite-difference equations with three-diagonal structure, which has been solved by the double-sweep method. Boundary conditions for the equation (15) can be found from the boundary conditions for ψ given on the side sites of the domain as well as from the physical conditions of the problem.

A general algorithm for solving the problem is the following. Let's say that at a time moment $t_k=k\tau$ position of the free surface $f(x,t_k)$, distribution of temperature θ, vortex ω and stream function ψ are known for the whole domain. Using (12), we will find a new position of the free surface $f(x,t_{k+1})$, and using (10), we determine the coefficient matrix $B_{11},\ B_{12},\ B_{22}$ for the equation (11). Finally, if we solve that equation with new coefficients for temperature θ, vortex ω and stream function ψ respectively, we will get a new distribution of those functions for the next time moment $t_{k+1}=(k+1)\tau$ and for the iteration $s=0$. Iterations for every time step keep going until the condition

$$\max_n \frac{|(\Phi(\xi_n))^{s+1}|-|(\Phi(\xi_n))^{s}|}{|(\Phi(\xi_n))^{s+1}|}<\varepsilon$$

has been met for all the functions $\omega,\ \theta,\ \psi$. Here, s is a number of iterations, and ε is the required accuracy. This is a pattern for one step in time. The rest of the steps are similar. If we seek a steady-state solution of the problem the following condition should be met

$$\max_n \frac{|(f(\xi_n))^{k+K}|-|(f(\xi_n))^{k}|}{|(f(\xi_n))^{k+K}|}<\varepsilon$$

where k is the number of the time step, K is some given number of steps, and ε is the required accuracy.

As one can see from the description above, the calculation process at first, does not break the conservation laws, and at the second, the approximation operator saves its elliptic type for every time step and every iteration. All those properties provide the convergence of the iteration process in total.

3 Numerical results and discussion

We consider the deformation and the rupture of a freely hanging film under the action of thermal load. V.V. Pukhnachev and S.B. Dubinkina (2006) investigated a model of the deformation of a free weightless liquid film with rims fixed at a plane contour to the action of thermocapillary forces within the framework of long-wave approximation. The plane and axisymmetric cases were studied in detail. The equilibrium forms of a freely hanging non-isothermal film were calculated. From our point of view, the most interesting case is when capillary number Ca and Maragoni number Mn are small, but the thermal load is not yet great enough to break the film. Numerical results for $Ca^{-1}=1500$, $Mn=0,5$, $Pr=1$ are presented on fig. 2. In this calculation the ratio of the film length to its half-thickness equal to 100. The dimensionless half-thickness of the film equal to 1. At the initial moment of time, the thin film is in the state of rest and the temperature $\theta(x,y,t)$ of the film is equal 0.

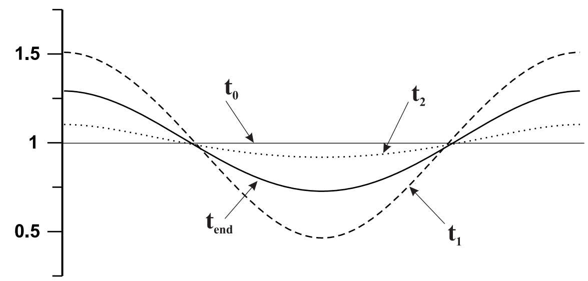

Figure 2: A position of the free surface for different time moments: $t_0<t_1<t_2<t_{end}$; $t_0=0$ corresponds to initial position of the free surface; $t_1=16,875$ is the half-time of vibrations; $t_2=33,75$ is the time of one period of the oscillations. That process repeats again, and oscillation amplitude is decreasing slowly, but oscillation frequency keeps to be constant; $f(x,t_{end})$ is the result of steady-state solution; $t_{end}=437,5$. Calculation was prepared for the parameters: $Pr=1$; $Ca^{-1}=1500$; $Mn=0,5$.

Let's put the thermal load on the free surface of the film

$$\theta(x,t)=A\sin(\pi x/L),\quad 0\le x\le L,\quad A=10.$$

where L is the length of the film. For $x=0$ and $x=L$, $\theta=0$; on the symmetry plane $\partial\theta/\partial n=0$. Under the action of the thermal load, the free surface of the film starts oscillations in relation to some equilibrium position. The oscillation amplitude is decreasing slowly, but the oscillation frequency is constant. We can explain that fact using the equation (7). When the film is in the state of rest the right-hand side of the equation (7) is equal to 0. As soon as the thermal load starts acting the right-hand side of the equation (7) is changing because of the term $\partial\omega/\partial n$, where ω is defined by (6). The derivative $\partial\omega/\partial n$ has different signs on two sides from the middle of the film because the temperature gradients have opposite directions in those areas. For that reason, tangent velocity also has opposite signs and directions from the middle of the film to its ends. It looks like thermo-capillary forces stretch the film into opposite directions. At the same time, capillary forces are increasing because the curvature of the film is increasing. Capillary forces try to level the film and come back it to initial position. This process is lasting until capillary and thermo-capillary forces make some compromise. The compromise is the steady-state solution. Solid line on fig. 2 shows the position of the free surface corresponding to the steady-state solution. Note that there are two node points in the position where the thin film has constant thickness equal to 1. Oscillations of the free surface are similar to the oscillations of the thin membrane held on the distance of $L/4$ from film ends.

Let's increase the thermal load and consider three different boundary conditions for the temperature on the free surface.

Case (a): the same as above, only A=15.

$$\theta(x,t) = Asin(\pi x/L), \quad 0 \le x \le L, \quad A = 15.$$

Case (b):

$$\theta(x,t) = A, \quad L/2 - h_0 \le x \le L/2 + h_0,$$

at the rest of the film surface $\partial\theta/\partial n = 0$; h_0 is the half of film thickness.

Case (c):

$$\theta(x,t) = A, \quad L/2 - h_0 \le x \le L/2 + h_0,$$

at the rest of the film surface $\theta = 0$; h_0 is the half of film thickness.

The position of the film free surface for all three cases (a),(b) and (c) for the same moment of the time t* is shown on fig.3. As we can see, the film has the smallest thickness in the case (a). Now, if we continue our calculations, then, in the case (b), the thickness of the film will be decreasing until the film is broken. For the cases (a) and (c), the film free surface will be making oscillations. The solid line on the fig.3 presents the steady state solution for these cases. The isolines of the stream function and the temperature are presented on fig.4 and fig.5. If we will increase the thermal load some more, then in the case (a) the thin film will be broken without oscillations, and in the case (c) the film thickness will decrease very slightly. M.El-Gammal and J.M. Floryan (2006) considered interface deformation and thermocapillary rupture in a cavity with free upper surface to concentrated heating. The results determined for large Biot and zero Marangoni numbers show the existence of limit points beyond which steady, continuous interface cannot exist and processes leading to the interface rupture develop.

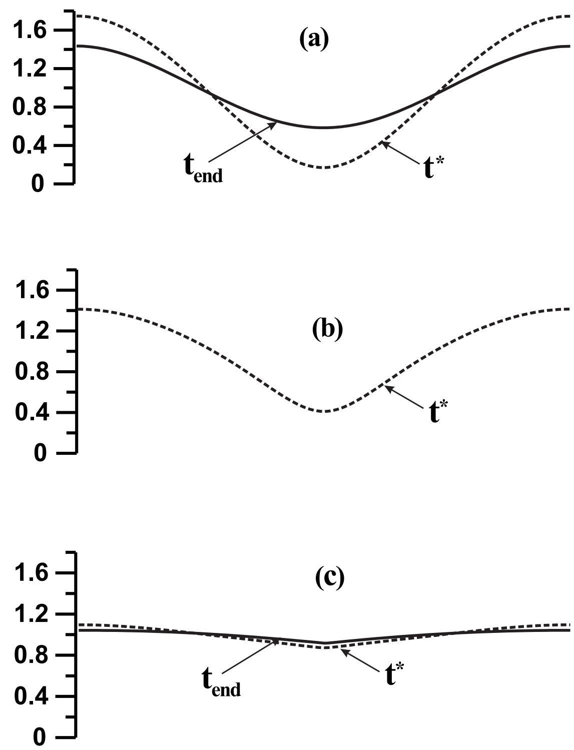

Figure 3: The position of free surface for different cases of thermal load (a-c) on free surface. Stroke lines show the position of free surface for all 3 cases (a-c) in the same time moment $t^* = 17,75$. This time is the half-time of vibrations for case a). Solid lines show the result steady- state solution for cases a) and c). Calculations are prepared for parameters: $Pr = 1; Ca^{-1} = 1500; Mn = 0,5$.

Now, let's consider the case (c) for different parameters. We increase the Marangoni number in three times, capillary number in thirty times, and

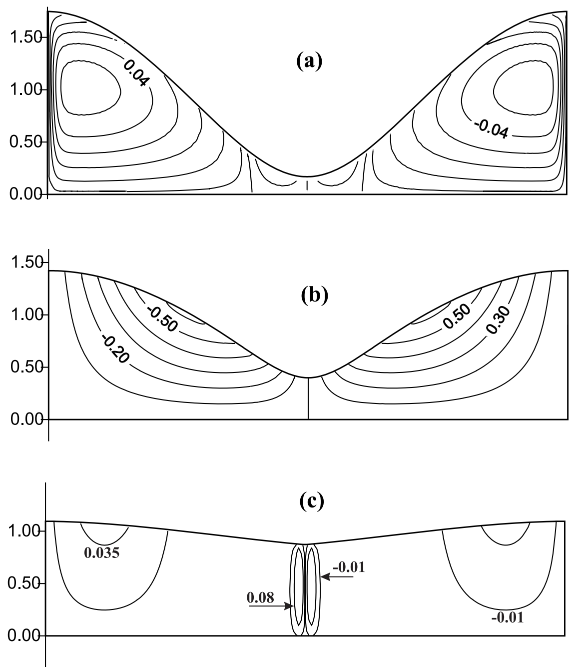

Figure 4: The distribution of stream function for all cases 3a-3c in the same time moment $t^* = 17,75$. This time is the half-time of the vibrations for case a). Calculations are prepared for the parameters: $Pr = 1$; $Ca^{-1} = 1500$; $Mn = 0,5$.

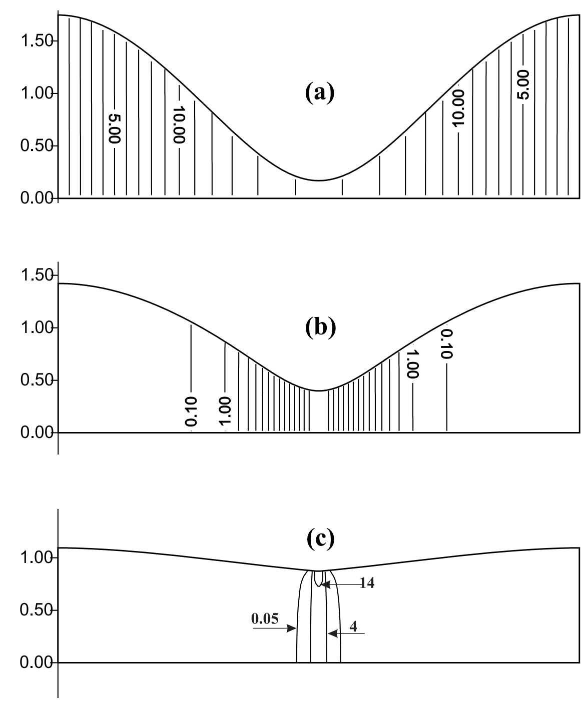

Figure 5: The isotherms for all cases 3a-3c in the same time moment $t^* = 17,75$. This time is the half-time of vibrations for case a). Calculations are prepared for the parameters: $Pr = 1$; $Ca^{-1} = 1500$; $Mn = 0,5$.

we decrease the thermo-load in three times. An example of numerical calculation for $Ca^{-1} = 50$, $Mn = 1,5$, $Pr = 1$, $\theta = 5$ is given on fig.6. In this case, the film thickness is vanishing very fast, and the break of the film happens so fast that disturbances of the free surface won't reach the solid walls.

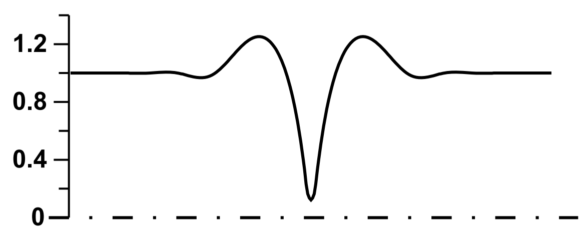

Figure 6: A deformation of the free surface and quick rupture of the thin film. Parameters of the problem are:$Pr = 1$;$Ca^{-1} = 50$;$Mn = 1,5$. The thermal load on the free surface is $\theta = 5$ if $L/2 - h_0 \leq x \leq L/2 + h_0$, at the rest of film surface $\theta = 0$. At the solid walls $\theta = 0$.

4 Conclusions

The processes occurring in freely hanging film under a thermal load influence have been investigated. A mathematical model describing a motion of the viscous non-isothermal liquid thin layers under micro-gravity has been developed. The model is based on Navier-Stokes equations. The numerical analysis of the deformation and the break of freely hanging film under effect of a thermal load has been carried out. In the equation for the tangent velocity on the free surface points, the terms responsible for the deformation and break of the film under the thermal load have been found. Mutual influence of capillary and thermo-capillary forces on the free surface position has been shown. Note: strengthen the effect of the thermo-capillary forces in test problems the thermal load has been chosen in the form of temperature on the film free surface.

Acknowledgement: This work was supported by the Russian Foundation for Basic Research, project no. 04-01-00355 and the Program for State Support of Leading Scientific Schools, project no. 5873.2006.1

References

Douglas, Jim, jr.; Rachford, H.H., jr. (1956): On the numerical solution of heat conduction problems in two and three space variables. *Trans. of the Amer. Math. Soc.*, vol.82, no. 2, pp. 421-439.

El-Gammal, M.; Floryan, J.M. (2006): Thermocapillary Effects in Systems with Variable Liquid Mass Exposed to Concentrated Heating. *FDMP: Fluid Dynamics & Materials Processing*, vol.2, no.1, pp.17-26.

Ostapenko, V.V. (1987): Method for Theoretical Estimation of Disbalances of Nonconservative Difference Schemes on Shock Waves. *Dokl. Akad. Nauk SSSR.*, vol.295, no.2, pp.292-297.

Ostapenko, V.V. (2000): Construction of High Order Accurate Shock - Capturing Finite-Difference Schemes for Unsteady Shock Waves. *Comput. Math. Math. Phys.*, vol.40, no. 12, pp. 1784-1800.

Ovcharova, A.S. (1998): Method of Calculating Steady-State Flows of a Viscous Fluid with Free Boundary in Vortex-Stream Function Variables. *Journal of Applied Mechanics and Technical Physics*, vol.39, no. 2, pp. 211-219.

Pukhnachev, V.V.; Dubinkina, S.B. (2006): A Model of Film Deformation and Rupture under the Action of Thermocapillary Forces. *Fluid Dynamics*, vol. 41, no. 5, pp. 755-771.

Yanenko, N.N. (1967): The Fractional Step Method for Multidimensional Problems in Mathematical Physics. *Nauka, Novosibirsk* [in Russian].

Yanenko, N.N. (1971): *The Method of Fractional Steps (The Solution of Problems of Mathematical Physics in Several Variables)*. Engl. transl. eg. by M. Holt. - Berlin etc.: Springer-Verl.