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Pendulum Thermal Vibrational Convection in a Liquid Layer with Internal Heat Generation

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Abstract: Thermal vibrational convection in a sector of a thin spherical liquid layer subjected to pendulum vibrations (spherical pendulum) is investigated theoretically and experimentally. Temperature non-uniformity inside the liquid is caused by uniformly distributed internal heat sources (one side of the layer is isothermal, the other one is adiabatic). Experiments are carried out under conditions of stable temperature stratification in the gravity field. Heat transfer and convective structure are investigated in a wide interval of governing dimensionless parameters. A critical increase of heat transfer is revealed as the vibrations intensity is increased, caused by average convection. It is shown that thermal convection is connected to the action of various thermo-vibrational mechanisms; the experimental threshold of convective stability is in good agreement with a theoretically determined one. Alongside with the thermal vibrational convection the occurrence of regular spatial structures which are not connected with temperature distribution is found and described.

keyword: Nonisothermal liquid, Pendulum vibrations, Thermal vibrational convection, Stability, Heat transfer.

1 Introduction

In practice, thermal vibrational convection is the "mean" convection that occurs in a nonisothermal incompressible liquid undergoing an oscillating force field (e.g., in a vibrating cavity). Convection is caused by mean mass forces generated in the liquid (which is subjected to oscillations) due to temperature (density) non-uniformity. In the case of high-frequency translational vibrations of a cavity along a definite direction the theoretical description of thermal vibrational convection has been done by the method of averaging (Zenkovskaya & Simonenko, 1966) and later has got wide application (see Gershuni & Lyubimov, 1998). In case of low or moderate frequencies the numerical study is used (Yan, Shevtsova & Saghir, 2006). Mean thermal vibrational effects are determined by a nonlinear interaction between the temperature field and the pulsational velocity component. As explained above, in case of translational cavity vibrations the oscillations are connected to non-uniformity of density (temperature), thus the mean vibrational effect is proportional to the square of the density non-uniformity $(\beta\Theta)^2$, it is characterized by the vibrational parameter $R_v = (b\Omega\beta\Theta L)^2/2\nu\chi$ (here b and Ω – amplitude and frequency of cavity translational vibration, L – characteristic length scale, β – thermal expansion coefficient, Θ – characteristic temperature difference, ν and χ – kinematic viscosity and thermal diffusivity coefficients).

Vibrational thermal convection undergoes qualitative modification in case of pendulum (combined translational – rotational) vibrations of a cavity (Kozlov, 1988). The presence of an isothermal pulsational velocity component, connected to the variation of cavity orientation in space, results in the occurrence of a new thermal vibrational mechanism which is proportional to the density non-uniformity $\beta\Theta$. This mechanism considerably surpasses the classical one (determined by the parameter R_v) in efficiency. A theoretical background for thermal vibrational convection in case of a spherical pendulum with relatively long handle was given by Kozlov (1989). The theoretical description of vibrational convection under vibrations of plane pendulum (Kozlov, 1988) was corroborated experimentally by Ivashkin & Kozlov (1987) and Ivanova & Kozlov (2003a).

The high efficiency of pendulum (combined translational – rotational) vibration is determined by a synchronous variation of isothermal pulsational velocity components (connected with rotary oscillation) and variation of the external force field – inertial one caused by translational component of vibration. In the absence of the translational vibration when the cavity makes only rotary oscillations (the handle of the pendulum is short) or liquid oscillations are connected mainly with the vibration

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of system elements or the cavity boundary, the linear on $\beta\Theta$ thermal vibrational mechanism remains (Lyubimov, 1995), however, in a weakened form. In this case the Schlichting mechanism of mean flows generation in Stokes boundary layers – "acoustic streaming" (Nyborg, W.L, 1965) takes place alongside with the thermo-vibrational mechanism (Ivanova & Kozlov, 2003b; Gershuni & Lyubimov, 1998). This mechanism of mean flows generation is not connected with temperature non-uniformity.

The results of theoretical analysis and experimental study of thermal vibrational convection in the sector of a relatively thin spherical layer of liquid with homogeneous internal thermal emission undergoing oscillations of a spherical pendulum are presented in this work. Experiments are carried out in a rectangular cavity (with one side much shorter than two others) fastened perpendicularly to the pendulum handle.

2 The theory of thermal vibrational convection in a cavity making oscillations of spherical pendulum

Following Kozlov (1988) consider the behavior of a non – isothermal liquid in a cavity, subjected to the oscillations of a spatial pendulum (Fig. 1).

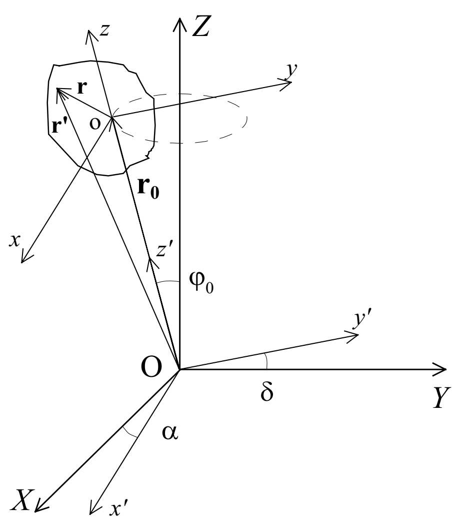

Figure 1 : Statement of problem: the cavity undergoes oscillations according to the law of a spherical pendulum.

Let's introduce the inertial system XYZ and non-inertial one $x'y'z'$ connected with the cavity. The origins of coordinates of both systems are in the center of pendulum; z' axis coincides with the pendulum handle and passes through the characteristic cavity point, for example through the center of mass. The pendulum handle (axis z') moves along a conical surface. Axis Z is directed along the main axis of the cone. The motion of axis z' in space (the cavity motion) is characterized by the variation of the angles between the axis Z and the projections of an axis z' on the main planes YZ and XZ: $\delta = \varphi_0 \cos\Omega t$, $\alpha = \varphi_0 \sin\Omega t$. Thus the axes of mobile system x' and y' perform harmonious oscillations, their average on time position coincides with the axes X and Y of the inertial system of coordinates.

Consider the case when the linear size of the cavity is small in comparison with the pendulum length and introduce the mobile coordinates system xyz connected with the cavity itself. The origin of coordinates will be chosen on the axis z, the axes x and y let's direct parallel to the axes x' and y'. The coordinates in mobile systems are presented in Fig. 1: $\mathbf{r}' = \mathbf{r} + \mathbf{r_0}$, where $\mathbf{r_0}$ – constant vector in the non-inertial system which is directed from the pendulum center to the beginning of coordinates of the system xyz.

The equations of vibrational convection of a non-isothermal liquid are obtained by the method of averaging, thus all the variables are presented as superposition of slowly varying (averaged) and oscillating components. The set of equations for slowly varying variables characterizes the averaged liquid behavior in the non-inertial coordinate system xyz connected with the cavity.

Omitting the detailed description of the procedure (see Kozlov, 1988**;** Kozlov, 1989), let's write out in a final dimensionless form the set of equations for slowly varying variables, describing the averaged thermal vibrational convection in a cavity subjected to high-frequency spatial oscillations induced by a spherical pendulum:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\mathrm{Pr}}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v} + (\mathrm{R_a}\gamma - 2\mathrm{R_v}\mathbf{k})T - \mathrm{R_k}T\nabla(\mathbf{w}_1\mathbf{i} + \mathbf{w}_2\mathbf{j}) \tag{1}$$

$$\mathrm{Pr}\frac{\partial T}{\partial t} + (\mathbf{v}\nabla)T = \Delta T \tag{2}$$

$$\mathrm{div}\mathbf{v} = 0 \tag{3}$$

$$\mathrm{div}\mathbf{w}_1 = 0, \quad \mathrm{rot}\mathbf{w}_1 = \nabla T \times \mathbf{i} - 2(\mathrm{R_k}/\mathrm{R_v})\mathbf{j} \tag{4}$$

$$\mathrm{div}\mathbf{w}_2 = 0, \quad \mathrm{rot}\mathbf{w}_2 = \nabla T \times \mathbf{j} + 2(\mathrm{R_k}/\mathrm{R_v})\mathbf{i} \tag{5}$$

The set of equations (1-6) differs from those presented in Kozlov (1989) only in the form. Here $\mathbf{k}$– unit vector related to the pendulum orientation $\mathbf{r}_0 = \mathrm{r}_0\mathbf{k}$, γ – the unit vector directed vertically upwards, other notations are conventional. As the units of distance, time, speed, pressure and temperature (variables $\mathbf{w}$) are chosen accordingly L, L^2/ν, χ/L, $\rho\nu\chi/L^2$ and characteristic temperature difference Θ.

The equations are written within the framework of the Boussinesq approximation $\beta\Theta << 1$, in the assumption of small angular oscillations $\varphi_0 << 1$ and relatively small cavity size $|\mathbf{r}| << |\mathbf{r_0}|$. The times of characteristic hydrodynamic and thermal processes are assumed to considerably surpass the period of oscillation, i.e. $\Omega L^2/\nu >> 1$ and $\Omega L^2/\chi >> 1$, where L is the reference scale (linear size of the cavity).

2.1 Boundary conditions, dimensionless parameters and additional variables

On borders S of the non-deformable cavity the conditions of heat exchange are set, the non-slip condition is satisfied $\mathbf{v}|_s = 0$ and the normal components of vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are absent; the last reflects the nonviscous character of high-frequency oscillations.

$$T|_s = T_s\,, \quad \mathbf{w}_1 \cdot \mathbf{n}|_s = \mathbf{w}_2 \cdot \mathbf{n}|_s = 0\,, \quad \mathbf{v}|_s = 0 \tag{6}$$

Here $\mathbf{n}$ is the unit vector normal to the boundary.

The set of equations contains the dimensionless parameters: Prandtl number Pr, gravitational Rayleigh number Ra, two vibrational parameters: R_v, which characterizes the action of classical thermo-vibrational mechanism, and $\mathrm{R_k}$, which is connected with the mean action of inertia forces – centrifugal force and Coriolis force:

$$\mathrm{Pr} = \frac{\nu}{\chi}, \quad \mathrm{R_a} = \frac{g\beta\Theta L^3}{\nu\chi},$$

$$\mathrm{R_v} = \frac{(\beta\Theta L\varphi_0 r_0\Omega)^2}{2\nu\chi}, \quad \mathrm{R_k} = \frac{(\varphi_0\Omega)^2 r_0\beta\Theta L^3}{2\nu\chi} \tag{7}$$

The additional solenoidal vector variables $\mathbf{w}_1$ and $\mathbf{w}_2$ represent the amplitudes of pulsational velocity components, caused by pendulum oscillations of the cavity in perpendicular planes – around the axes y and x correspondingly. It is worth noting that each of these variables has two parts: nonisothermal $\mathbf{w}_T$ connected to non-uniform liquid temperature and tangential acceleration, and isothermal $\mathbf{w_r}$ connected to the rotary cavity vibrations:

$$\mathbf{w}_1 = \mathbf{w}_{1T} + \mathbf{w}_{1\mathbf{r}},$$

$$\mathrm{rot}\mathbf{w}_{1T} = \nabla T \times \mathbf{i}, \quad \mathrm{rot}\mathbf{w}_{1_\mathbf{r}} = -\frac{2\mathrm{R_k}}{\mathrm{R}_v}\mathbf{j} \tag{8}$$

$$\mathbf{w}_2 = \mathbf{w}_{2T} + \mathbf{w}_{2\mathbf{r}},$$

$$\mathrm{rot}\mathbf{w}_{2T} = \nabla T \times \mathbf{j}, \quad \mathrm{rot}\mathbf{w}_{2\mathbf{r}} = \frac{2\mathrm{R_k}}{\mathrm{R}_v}\mathbf{i}$$

In case of our approximations (relatively long pendulum and small density non-uniformity) the isothermal and nonisothermal velocity components $\mathbf{w}_T$ and $\mathbf{w_r}$ are of the same order of magnitude. The limiting case $\mathrm{R_k} << \mathrm{R_v}$ corresponds to the action of translational vibrations of circular polarization (rotating force field).

The presence of a rotary vibrational component (in addition to translational one) as follows from (1) results in, firstly – reformation of mean force field (gravity), secondly – generation of the additional vibrational volumetric force proportional to $\beta\Theta$.

Pendulum thermal vibrational convection exhibits several specific features, and the shape of the cavity plays an important role. Let's consider the convection in a sector of spherical layer of thickness h (the plane layer fastened perpendicularly to the pendulum shoulder). In the approximation $h/r_0 << 1$ the isothermal pulsational velocity components far from the lateral borders of the layer are of a specific kind: $\mathbf{w}_{1\mathrm{r}} = \mathrm{R_k}\,/\,\mathrm{R}_v(1-2z)\mathbf{i}$ and $\mathbf{w}_{2\mathrm{r}} = \mathrm{R_k}\,/\,\mathrm{R}_v(1-2z)\mathbf{j}$ (the origin of coordinates of non-inertial system is on the inner boundary of the layer, the length unit is h). The total action of the vibrational mechanism determined by parameter R_k in this case results in a renormalization of the steady (gravity) field (Ivanova & Kozlov, 2003a), it is possible to exclude variables $\mathbf{w}_{1\mathrm{r}}$ and $\mathbf{w}_{2\mathrm{r}}$ from consideration and rewrite the set of equations in the form:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\mathrm{Pr}}(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v} + (\mathrm{R_a}\gamma + 2\mathrm{R_k}\mathbf{k})T - \mathrm{R_v}T\nabla(\mathbf{w}_1\mathbf{i} + \mathbf{w}_2\mathbf{j}) \tag{9}$$

$$\Pr\frac{\partial T}{\partial t}+(\mathbf{v}\nabla)T=\Delta T \tag{10}$$

$$\mathrm{div}\mathbf{v}=0 \tag{11}$$

$$\mathrm{div}\mathbf{w}_1=0,\quad \mathrm{rot}\mathbf{w}_1=\nabla T\times\mathbf{i} \tag{12}$$

$$\mathrm{div}\mathbf{w}_2=0,\quad \mathrm{rot}\mathbf{w}_2=\nabla T\times\mathbf{j} \tag{13}$$

In such form the set of equations is equivalent to the one of the classical thermal vibrational convection under translational vibrations of circular polarization. The important difference consists of the modification of the steady (gravity) field due to rotary vibrations. It is worth mentioning that the sign of the term with parameter R_k in (9) changes in comparison with (1).

Thus, in the thin plane layer filled with nonisothermal liquid and fastened perpendicularly to the pendulum handle the total convective action of the centrifugal field and Coriolis force (averaged thermo-vibrational effect) is similar to the action of some effective force field directed to the center of pendulum and similar in form to the gravity force. In the frame of made approximations about relatively small linear sizes of the cavity the field is considered homogeneous. The direction of this effective force field is opposite to the direction of centrifugal force (the vector $\mathbf{k}$ is directed along the pendulum shoulder).

Accordingly, the nature of the vibrational mechanism R_k in plane layers allows simulating the presence of a static force field in weightlessness or to change and even completely exclude the action of gravity field (i.e. its effects on the thermal convection) in Earth conditions. It is true at $\mathrm{Ra}\gamma+2\mathrm{R}_k\mathbf{k}=0$, i.e. when $\mathrm{Ra}=2\mathrm{R}_k$, and the unit vectors γ and $\mathbf{k}$ are of opposite direction (average position of the pendulum – vertical, of the layer – horizontal, a point of pendulum hanging is above the cavity). Certainly, the action of the classical convective mechanism R_v (connected with translational vibrational component) remains. It allows, in particular, using the pendulum vibrations for experimental study of classical vibrational convection in plane layers in Earth conditions in a wide range of Rayleigh numbers, including the weightlessness conditions.

3 Vibrational convection in a plane layer of liquid with uniform internal heat release

Consider the vibrational convection in the horizontal plane layer undergoing oscillations induced by a spherical pendulum (Fig. 2). The borders of the layer are perpendicular to the pendulum shoulder, the average position of the shoulder – vertical. The distance from the point of pendulum hanging, located below the layer to the center of the layer remains constant and equal to r_0.

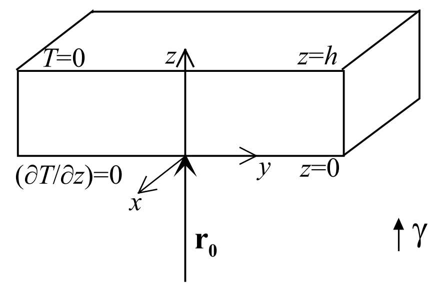

Figure 2 : Geometry of the problem and the coordinate system.

The bottom layer boundary is adiabatic (i.e. the normal component of heat flux is zero), the top one – isothermal, non-uniformity of liquid temperature is connected with internal heat release (Q – volumetric density of internal heat sources – the amount of heat released into the unit volume per unit time).

The heat transfer equation differs from (10) as the internal heat sources must be added. Its dimensionless form is:

$$\Pr\frac{\partial T}{\partial t}+(\mathbf{v}\nabla)T=\Delta T+1 \tag{14}$$

The other dimensionless equations, (9) and (11–13), do not change if the unit of length is thickness of layer h and the unit of temperature is $Qh^2/\rho\, c_p$, where c_p – specific heat coefficient.

Dimensionless parameters take the form:

$$\mathrm{Ra}=\frac{g\beta Qh^5}{\rho c_p\nu\chi^2},$$

$$\mathrm{R}_v=\frac{(\varphi_0\Omega\beta r_0Qh^3)^2}{2\nu\chi^3(\rho c_p)^2},\quad \mathrm{R}_k=\frac{(\varphi_0\Omega)^2\beta r_0Qh^5}{2\nu\chi^2\rho c_p} \tag{15}$$

Under the chosen boundary conditions (the nondimensional temperature of the isothermal border is assumed to be $T = 0$)

$$z = 0: \quad \partial T/\partial z = 0, \quad z = 1: \quad T = 0 \tag{16}$$

the quasi-equilibrium distributions of temperature and pulsational velocity components read:

$$T_0 = \frac{1}{2}\left(1 - z^2\right)$$

$$\mathbf{w}_{01} = \frac{1}{6}(1 - 3z^2)\,\mathbf{i} \tag{17}$$

$$\mathbf{w}_{02} = \frac{1}{6}(1 - 3z^2)\mathbf{j}$$

Vectors $\mathbf{w}_{01}$ and $\mathbf{w}_{02}$ are equal and mutually perpendicular. Under the action of vibrations the liquid layers, which are normal to the pendulum shoulder, make plane-parallel circular oscillations in a horizontal plane. The amplitude of oscillations depends on the coordinate z and is determined by the temperature distribution.

3.1 Convective stability of a quasi-equilibrium state

Consider the problem of quasi-equilibrium stability of a plane liquid layer. Let us introduce the normal perturbations of temperature T', pressure p', average velocity $\mathbf{v}'$ and pulsational velocity components $\mathbf{w}'_1$ and $\mathbf{w}'_2$. Substituting the perturbations into the basic set (9, 11–14) and linearizing it, one obtains the set of equations for the perturbations:

$$\frac{\partial \mathbf{v}'}{\partial t} = -\nabla p' + \Delta \mathbf{v}' + T'\left(\mathrm{R_a}\gamma + 2\mathrm{R_k}\mathbf{j}\right) - \mathrm{R_v}\nabla(\mathbf{w}_{01}\mathbf{i} + \mathbf{w}_{02}\mathbf{j})) - \mathrm{R_v}\nabla(\mathbf{w}'_1\mathbf{i} + \mathbf{w}'_2\mathbf{j})T_0 \tag{18}$$

$$\mathrm{Pr}\frac{\partial T'}{\partial t} + \mathbf{v}'\nabla T_0 = \Delta T'$$

$$\mathrm{rot}\,\mathbf{w}'_1 = \nabla T' \times \mathbf{i}, \quad \mathrm{rot}\,\mathbf{w}'_2 = \nabla T' \times \mathbf{j}$$

$$\mathrm{div}\,\mathbf{v}' = 0, \quad \mathrm{div}\,\mathbf{w}'_1 = 0, \quad \mathrm{div}\,\mathbf{w}'_2 = 0$$

Let's consider the normal perturbations

$$\left(\mathbf{v}',\, T',\, \mathbf{w}'_1,\, \mathbf{w}'_2\right) = (\mathbf{v},\, T,\, \mathbf{w}_1,\, \mathbf{w}_2)\exp\left(ik_1x + ik_2y - \lambda t\right)$$

We shall limit ourselves with the case of monotonous perturbations which threshold is defined by the condition $\lambda = 0$. After the usual transformations connected to elimination of the pressure perturbations and the horizontal components of $\mathbf{v}$, $\mathbf{w}_1$ and $\mathbf{w}_2$, and replacement $w = -(ik_1w_{1z} + ik_2w_{2z})/k^2$ one obtains the set of homogeneous equations for the amplitudes:

$$\Delta\Delta v - (\mathrm{Ra} + 2\mathrm{R}_k)k^2 T + \mathrm{R}_v k^2 \dot{T}_0 (T + \dot{w}) = 0$$

$$\Delta T - \dot{T}_0 v = 0 \tag{19}$$

$$\dot{T} + \Delta w = 0$$

Here Δ stands for the operator $\Delta \equiv (\partial^2/\partial z^2) - k^2$; $\dot{} \equiv \partial/\partial z$ – differentiation with respect to the transversal coordinate, v – z-component of velocity perturbation.

Normal components of the mean and pulsational velocities are to be zero at the solid boundaries of the layer, as well as $\partial v / \partial z$. The temperature perturbation vanishes at the isothermal boundary, normal component of heat flux is equal to zero at the adiabatic boundary. Thus the amplitudes v, w and the temperature T satisfy the conditions:

$$z = 0: \quad v = \dot{v} = w = \dot{T} = 0$$

$$z = 1: \quad v = \dot{v} = w = T = 0$$

The problem (19), (20) coincides with the problem of quasi-equilibrium stability of a plane layer (2D case) under linear translational vibrations (Gershuni et al., 1989). The combined action of gravity and thermo-vibrational mechanism is determined by the complex $(\mathrm{Ra} + 2\mathrm{R}_k)$. The basic difference consists of the degeneration of the form of the perturbations. The amplitude problem does not contain wave-numbers k_1 and k_2 separately, only the square of the wave vector $k^2 = k_1^2 + k_2^2$ is the parameter of the problem. Thus the ratio between k_1 and k_2 is not determined and wide variety of perturbations (rectangular, square, hexagonal …) correspond to the threshold. The linear stability theory does not answer the question of their competition, the non-linear analysis is needed. This situation is similar to classical Rayleigh-Bènard stability problem.

The stability boundary is presented in Fig. 3. The signs on vibrational parameter R_k and gravitational Rayleigh number are identical, if the direction of the pendulum

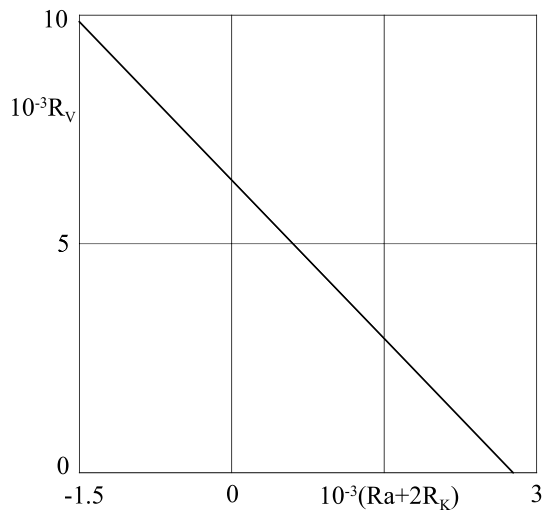

Figure 3 : The border of vibroconvective quasi-equilibrium stability in the plane liquid layer undergoing the high-frequency oscillations induced by a spherical pendulum: average position of the layer – horizontal; average position of the pendulum – vertical.

shoulder and the gravity are opposite ($\gamma \cdot \mathbf{k} = 1$). The negative value of gravitational Rayleigh number corresponds to the case when the adiabatic boundary is above.

It's worth mentioning, that the results are valid in the limiting case of high frequencies when the thickness of boundary Stokes layers is negligibly small.

4 Experimental study of thermal vibrational convection in a plane liquid layer with internal thermal emission

Experimental setup includes the mechanical vibrator, the cavity and a measuring part. The oscillations of a spherical pendulum 1 (Fig. 4) with the cavity 2 fastened on it are produced by the disk 3 rotating at some definite frequency with eccentric finger 4 on it. The finger is connected with the pendulum platform 5 by the spherical bearing 6. The special mechanism 7 is used for pendulum hanging in order to prevent the pendulum rotation around the vertical axis. The length of pendulum handle - from the point of hanging up to the cavity center - is $r_0 \sim 50$ cm. The angular amplitude of pendulum oscillations could be varied in the range $\varphi_0 = 0 - 0.1$ radian by changing the distance between the finger 4 and an axis of rotating disk 3. Rotation of disk with constant frequency in an interval $f \equiv \Omega/2\pi = 0.2 - 11$ Hz is set by the engine of a direct current 8. The vibrator is collected on a massive metal frame 9.

The frequency of vibrations is measured by means of an electronic tachometer with accuracy 0.01 Hz. The amplitude of angular pendulum oscillations is measured using the track of the laser beam on the screen (diode laser is fastened on the pendulum handle); with an accuracy not lower than 10^{-3}radian.

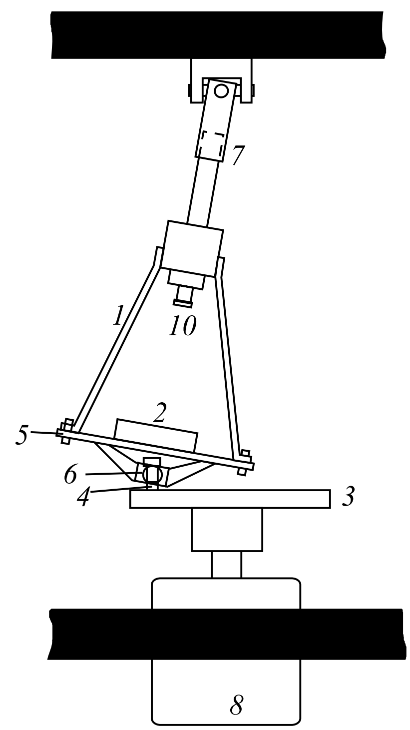

Figure 4 : The sketch of the vibrator (spherical pendulum).

The plane layer is formed by aluminum heat-exchanger 1 (Fig. 5), plexiglas walls 2 and the glass plate 3. The heat-exchanger has the channels inside it for circulation of thermostatic liquid; its internal surface (isothermal boundary of the layer) is electrically insulated by thin plastic film, thickness 0.1 mm. Copper electrodes 4 are mounted on the opposite face walls of the layer. Glass plate (the top border of the cavity) is covered with thermo-insulating material 5. The cavity size varies: $8.25 \times 5.73 \times 0.44$, $7.70 \times 5.70 \times 0.32$ or $7.40 \times 5.78 \times 0.21 cm^3$.

The temperature of the metal border is set constant; the temperature of the adiabatic one is determined by the intensity of internal heat sources. The uniform internal thermal emission is provided by electric current of industrial frequency, the voltage on the electrodes is adjusted by means of a transformer. Water with the small contents of copper salt (3-5 %) is used in experiment. The electric current through the liquid and the voltage are measured by digital voltmeters.

For temperature measurements the TERMODAT-15M1 (accuracy 0.1 k) is used. Copper resistance thermometers are used. A thermal sensor 9 is mounted into the heat-exchanger body and takes its temperature; another integrating thermometer is pasted on the internal side of the glass plate (adiabatic boundary).

Observation and video registration of convective flows are carried out through the transparent cavity glass border (in this case the external heat insulator is absent) by means of a miniature video camera 10 (Fig. 4). The camera is fastened on the pendulum and performs vibrations together with it.

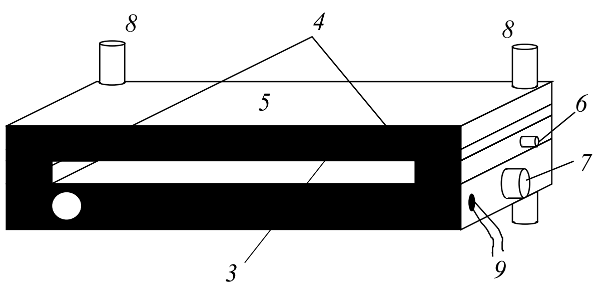

Figure 5 : The scheme of the cavity (vertical cross-section view): 1 – heat exchanger, 2 and 3 – lateral and upper heat-insulated borders, 4 – electrodes, 5 – heat-insulating cover, 6 – pipes for filling the cavity with liquid, 7 – the entrance for thermostatic liquid, 8 – fixing elements, 9 – temperature sensor.

The experimental technique is the following. At some definite intensity of internal thermal emission Q and amplitude of pendulum oscillations the temperature of the adiabatic border T_a is measured depending on vibrations frequency f. The temperature measurement is carried out in the established mode. The intensity of thermal emission, amplitude of vibrations and thickness of the layer are varied in experiments.

The experiments correspond to the stable stratification of liquid in gravity field; gravitational Rayleigh number Ra has negative value. The excitation of thermal convection is possible only due to vibrational mechanisms determined by dimensionless parameters R_v and R_k (the last one has the positive value). The convection excitation due to quasi-equilibrium instability results in increase of heat flux through the layer and in decrease of the temperature of the adiabatic boundary.

5 Experimental results

The temperature of the adiabatic border T_a varies (Fig. 6) with increase of vibrations frequency at some definite amplitude of vibration φ_0 and thermal emission Q intensity.

One can see three characteristic sites on the temperature curves $T_a(f)$. At the first (site *I*, Fig. 6) the temperature does not depend on the frequency of vibrations f, that corresponds to a condition of mechanical quasi-equilibrium. The temperature T_a is in agreement with the corresponding theoretical values for purely thermal diffusive conditions within several percent, testifying the absence of convective flows in the layer.

With increase of the vibration frequency the temperature of the adiabatic boundary at first goes down in a threshold way (it testifies the increase of heat transfer and convection occurrence) then goes up near to the initial value. The area of local change (increase) of heat transfer is marked as site *II* in Fig. 6. A further increase of frequency results in the next critical growth of heat transfer: the temperature of the adiabatic boundary quickly and monotonously reduces (site *III*).

The temperature of the adiabatic boundary and the depth of the second site increase with Q. A variation of the layer thickness and the amplitude of vibrations results in a deformation of the temperature curves, however, the curves keep their shape (three sites still exist on the curves).

6 Discussion of results

The excitation of convective flow in a layer of liquid steadily stratified in gravity field ($\mathrm{Ra} < 0$) under a pendulum vibration is connected to the action of vibroconvective mechanisms and is accompanied by an increase of heat transfer through the layer. The Nusselt number Nu as a function of the vibrational parameter R_k is shown in the Fig. 7. The parameter Nu is defined as the ratio of the adiabatic boundary temperature in the absence of convection and the temperature, measured during the vibrational experiment. At small values of R_k the Nus-

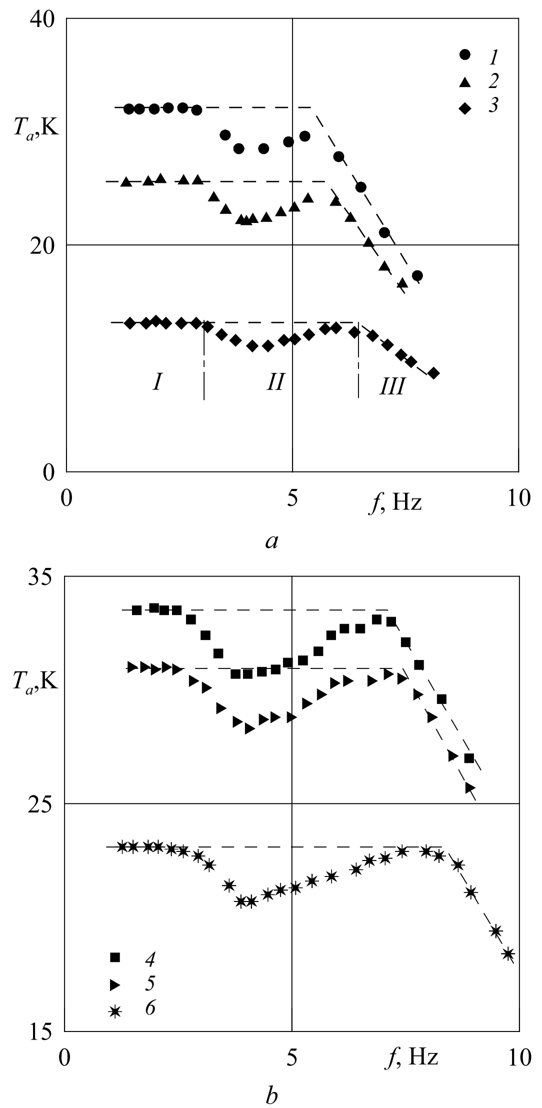

Figure 6 : Temperature of the adiabatic border T_a versus the frequency of vibration f (the layer thickness $h = 0.44$ cm) at amplitude of vibrations $\varphi_0 = 0.097$ (a) and 0.069 rad (b), $Q = 2.06$, 1.63, 0.82, 2.17, 1.99, 1.44 W/cm^3 (marks 1–6).

selt number is about the unit. This site corresponds to a quasi-equilibrium condition (the absence of mean convection in the layer). With increase of R_k the parameter Nu grows up to some value and after it smoothly goes down (the dashed line corresponds to $Nu = 1$). With a further increase of R_k a monotonous growth of the Nusselt number takes place, demonstrating the development of thermal convection. The curves $Nu(R_k)$ for the layers of different thickness $h = 0.32$ and 0.21cm have a similar form.

One can find the threshold values of vibrational parameters R_v and R_k at definite negative values of Ra using the critical frequencies of convection excitation (Fig. 6).

The increase of heat transfer in the areas *II* and *III* is caused by vibrational mechanisms of a different nature.

Let's consider the area *III* in which the critical growth of heat transfer with increase of vibration intensity is caused by the development of pendulum thermal convection (see the theoretical parts 2–3). The threshold of excitation of thermal convection can be found by crossing the dashed lines on Fig. 6. The experimental points corresponding to various conditions of internal heating and the different cavity size h, are in good agreement one with another on the plane of theoretically predicted dimensionless complexes $Ra + 2R_k$, R_v (Fig. 8).

Let's note that the complex $R_a + 2R_k$ is determined by the sum of two large parameters of opposite signs (convection takes place in the liquid steadily stratified in gravity field at large negative values of gravitational Rayleigh number). The deviation of the experimental critical value R_k from theoretically predicted at given R_a and R_v values is about 25 % (Fig. 9).

The experimental stability threshold is below the theoretically predicted one. The nature of this phenomenon could be connected to the difference of experimental conditions and theoretical assumptions. The surface of the cold heat-exchanger is covered with thin plastic film for the purpose of electrical isolation. The thickness of the cover together with gluing substance is about 0.3 mm. Thus the assumption of a high heat conductivity of the boundary is not provided. This could be important for the perturbations. In Gershuni & Zhukhovitsky (1972) it was shown, that thermal perturbations penetrate into boundaries of a low heat conductivity that results in a significant reduction of the threshold of thermal convection excitation. The well known critical value of Rayleigh number $R_{acr} = 1708$ (onset of convection in horizontal layer with borders of different temperature) corresponds to the case of high heat conductivity of the boundaries. The temperature perturbations vanish at the boundaries in this case. If the borders have the same heat conductivity as the liquid the stability threshold is one and a half time lower compared with the case of high heat conductivity of borders. It could be the reason why the theoretical curve in Fig. 8 corresponding to the case of boundary

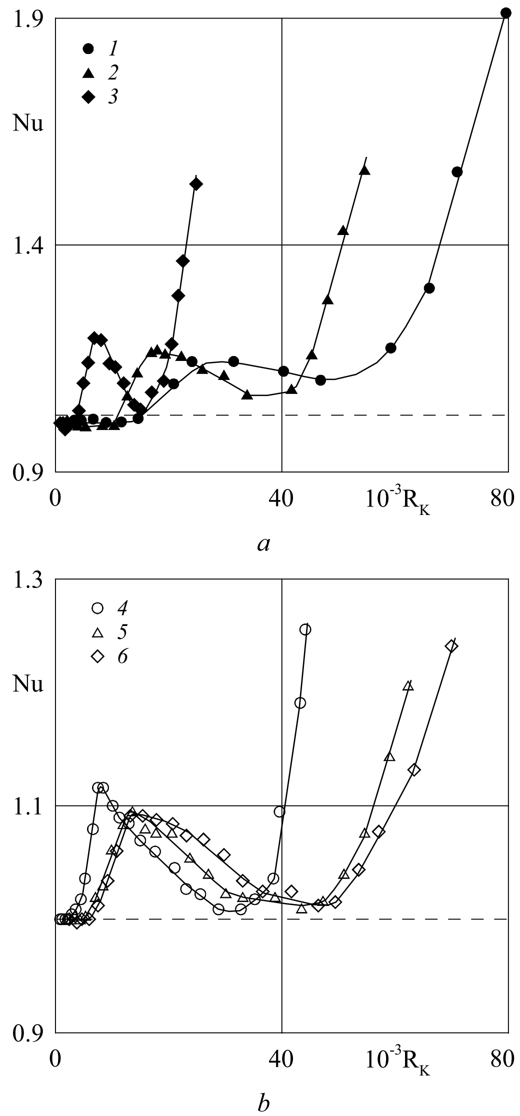

Figure 7 : The Nusselt numbers Nu as a function of the vibrational parameter R_k for the layer $h = 0.44$cm; the experimental data correspond to Figure 6.

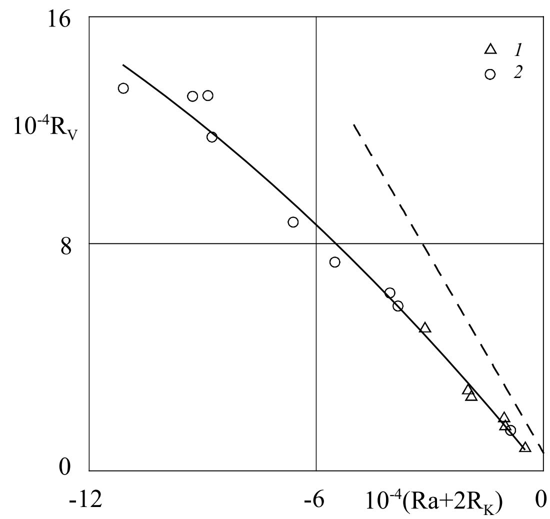

Figure 8 : The threshold value of vibrational parameter R_v versus the complex $Ra + 2R_k$ for $h = 0.32$ (marks 1) and 0.44 cm (2), dashed line – the theoretical stability border (Figure 3).

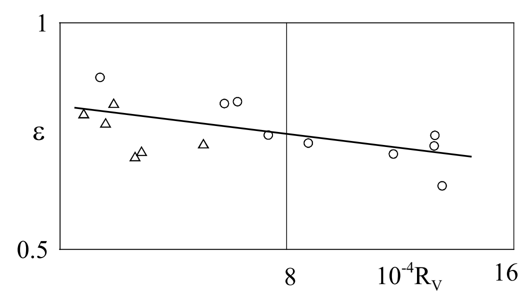

Figure 9 : The ratio $\varepsilon \equiv R_k(exp)/R_k(theor)$ versus the R_v; the data correspond to Figure 8.

of high heat conductivity goes above the experimental points.

Another theoretical position, which is not always valid in the experiment, is the approximation of high frequencies (dimensionless frequency of vibrations $\omega \equiv \Omega h^2/\nu >> 1$). The Stokes boundary layers are supposed to be negligibly thin. The conditions of the present experiment correspond to moderate frequencies $\omega \approx 500 - 1500$.

We shall discuss the nature of local heat-transfer increase in the area *II* (Fig. 6a) only briefly, as it is not connected with thermal convection and leaves the frameworks of the present work. As follows from observations the growth of heat-transfer in area *II* is caused by the occurrence of regular spatial vortical structures (Fig. 10). The lattice is observed in a limited area of frequencies. The form and the spatial period of structures is identical in both isothermal and nonisothermal cases.

In case of thermal convection the liquid flow was visualized with a suspension of aluminum powder, in isothermal liquid – plastic particles of practically neutral buoyancy of diameter ~ 0.06 mm. The amount of visualizing matter was insignificant; in the absence of vibrations the layer of particles on the cavity bottom did not exceed one caliber.

The dimensionless periods of the structures in experi-

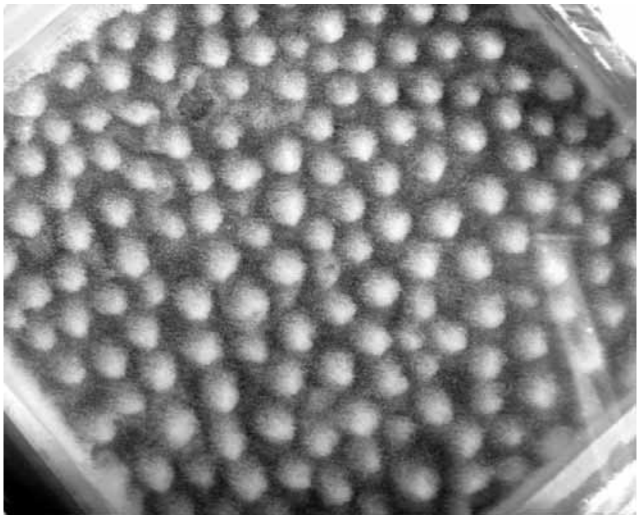

Figure 10 : Photo of vibrational vortexes near the cavity bottom, the top view (isothermal case, water, rectangular cavity $9.2 \times 9.2 \times 1.05 \text{cm}^3$, $\varphi_0 = 0.097$rad, $f = 5.12$Hz).

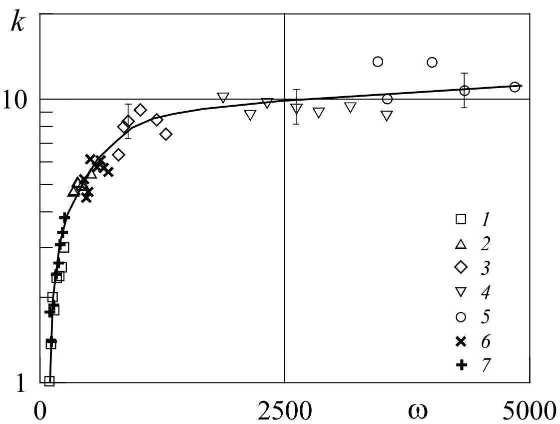

Figure 11 : Wave number $k = 2\pi h/\lambda$ (λ – the distance between the neighboring vortexes) as a function of dimensionless frequency (water; amplitude of vibration $\varphi_0 = 0.097$rad). Points 1–5 correspond to experiments with isothermal liquid and layer thickness $h = 0.21$, 0.32, 0.52, 0.85 and 1.05 cm; 6 and 7 – to non-isothermal case: $h = 0.44$cm, $\text{Ra} = -75 \cdot 10^3$; $h = 0.21$cm, $\text{Ra} = -6.2 \cdot 10^3$.

ments with layers of various thickness are in good agreement among themselves on the plane ω, k (Fig. 11). The wave number (the layer thickness plays the role of the length unit) monotonously grows with increase of the dimensionless frequency. The typical discrepancy of wavenumber is shown by error bars. At $\omega > 10^3$ the growth rate decreases. It's worth noting that the wavelength varies in a wide interval, at low ω it is few times larger than the layer thickness, at high frequencies – smaller. The formation of such structures is specific for pendulum (combined translational – rotary) vibrations and do not manifest itself in the absence of one vibrational component, in case of pure translational vibrations of circular polarization or pure rotational oscillations (Kozlov & Selin, 2005).

7 Concluding remarks

Thermal vibrational convection in a plane layer undergoing high-frequency oscillations induced by a spherical pendulum has been investigated.

It has been theoretically shown, that the average action of oscillating Coriolis and centrifugal forces on the non-isothermal liquid (in the specific case of a sector of a thin spherical layer) is equivalent to the action of a static force field directed to the center of the spherical pendulum. Experimental results confirm this conclusion. In case of experiments carried out in the gravity field the vibrational convection in the plane layer is determined by the parameter R_v and the complex one $\text{R}_\text{a} + 2\text{R}_\text{k}$.

Besides the threshold excitation of thermal pendulum vibrational convection a new kind of instability has been observed experimentally, i.e. the occurrence of regular spatial vortical structures. It has been shown that the formation of such structures has not a thermal nature – it is observed in isothermal liquids too.

The pendulum (combined rotary-translational) type of vibration has been found to play the most important role.

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