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On the Stability of the Hadley Flow under the Action of an Acoustic Wave

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Abstract: The effects of an acoustic wave on the instabilities occurring in a lateral differentially heated cavity are investigated numerically. Linear stability results show that the acoustic wave affects significantly the instability characteristics of such a Hadley flow. Indeed, the sound field is found to stabilize both two dimensional transverse stationary and three dimensional longitudinal oscillatory instabilities which are the most critical modes affecting the buoyant convection in the fluid layer. Nevertheless, when stabilized by an acoustic wave, the 2D modes turn from stationary to oscillatory, with the known consequences of such a change on mass and heat transfer, especially in horizontal Bridgman crystal growth process. The second feature to be mentioned is that the linear stability predicts a destabilization of 3D stationary modes that in the absence of acoustic waves are known to be excited at threshold values of the Grashof number highly above those of the two primary instabilities.

keyword: Instability, acoustic wave, thermally induced flow.

1 Introduction

"Acoustic streaming" is used to designate a stationary flow which occurs within a viscous fluid as a result of the presence of a sound wave or an oscillating body. Such a secondary flow can be of significant interest in some industrial applications, especially those involving heat and mass transfer. Indeed, forced convection resulting from acoustic streaming can be used to enhance heat transfer as proved by Vainstein et al. (1995) and Loh et al. (2002) who treated two different heat transfer problems. Vainstein et al. (1995) considered the heat transfer between two horizontal parallel plates kept at different constant temperatures. They proved theoretically, when neglecting natural convection appearing when such a Rayleigh-Bénard system become unstable, that the averaged Nusselt number is increased when an acoustic wave is propagated in the longitudinal direction, indicating an enhancement of the heat transfer between the two horizontal plates bounding the fluid layer. Loh et al. (2002) considered the problem of cooling a single aluminum plate. They proved experimentally and by means of numerical simulations that the cooling process is improved when the vibrating beam amplitude is increased for a vibration frequency of 28.4 kHz. For mass transfer problems, Suri et al. (2002) investigated the problem of mixing in closed containers. They pointed out the importance the position of the transducer producing the sound wave has on the mixing efficiency.

During the last decades, global flow control and especially the possible suppression of unsteady instabilities have become subjects of crucial importance because in principle they can lead to a better control of heat and mass transfer in several industrial applications such as crystal growth from the melt, e.g., Czochralski growth (CZ) (see, e.g., Tsukada and Kobayashi, 2005), floating-zone (FZ) (see e.g., Gelfgat et al., 2005) and Bridgman growth (see, e.g., Dold and Benz, 1997).

Recently, Lappa (2005a) has presented an overview of the two- and three-dimensional instabilities affecting low Prandtl fluids contained in open and confined cavities subject to different heating conditions and driving forces (thermocapillary and thermogravitational convection, for the latter case see also Melnikov and Shevtsova, 2005).

It is well known that transition from stationary to time dependent flow affects considerably the grown crystal structure and is generally responsible for the appearance of undesirable striations (see Hurle (1966), Haddad et al. (1999-2000)). In the last decades, different techniques have been used to control the flow and especially to delay the appearance of instabilities in order to preserve steady

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flow ensuring the growth of free striations crystals (see, e.g., the review of Lappa, 2005b and Amberg and Shiomi, 2005).

One of the most investigated ways of controlling and stabilizing the buoyant flow during Bridgman growth technique is the use of a static (see Hurle (1974), Kaddeche et al. (2003), Kaddeche et al. (2002)) or a rotating magnetic field (see BenHadid (2000), Dold and Benz (1995)-(1997)). The constant magnetic field (see Hurle (1974), Kaddeche et al. (2003) and Kaddeche et al. (2002)) is an efficient tool to damp the oscillations and stabilize the flow and consequently improve the grown crystal quality, especially when its direction is perpendicular to the fluid layer, see Kaddeche et al. (2003), Kaddeche et al. (2002).

Recently, for a Small-scale Floating-zone Silicon Growth process, Lan and Yeh (2005) have proven that a counter rotation speed less than 20 rpm, has only weak effects on both thermal field and fluid flow. Nevertheless, such a rotation has more significant consequences on the radial segregation for a transversal magnetic field than for an axial field case. Rotating magnetic field are also efficient to eliminate time dependent flow, see BenHadid (2000), Dold and Benz (1995)-(1997) and strength of the order of millitesla are sufficient when constant magnetic field need to have a strength of several hundred millitesla.

One can see that despite its considerable efficiency to stabilize the buoyant flow, the limitation of the magnetic field is probably its significant consumption of electrical energy.

Interestingly, Lappa (2005b) has also proposed the use of high-frequency axial vibrations (g-jitters) to damp unstable threedimensional convection in floating zones of silicon.

An alternative technique, more simple to implement and probably less expensive to use is a sound wave which will create an acoustic streaming flow. The resulting modification of the buoyant basic flow will in turn change the stability characteristics, namely, the critical thresholds and instability mechanisms, of the Hadley flow occurring in the melt

In this paper, we investigate the effects of a longitudinal sound wave amplitude and frequency on the stability of Hadley flow by means of a linear stability technique.

2 Linear stability

The stability of the basic flow (6-7) is investigated by means of numerical computations based on the linear stability theory. Such a method consists of following the evolution of an infinitesimal perturbation of velocity, pressure and temperature($\vec{v}$, p, θ)superimposed on the solution of the stationary problem$\left(\vec{V}_0, P_0, T_0\right)$ respectively. The evolution of these perturbations is governed by the linearized Navier-Stokes equations coupled to the energy and the continuity equations. The sketch of studied configuration is represented in Fig. 1, where H is the depth of the layer. The following boundary conditions are considered: No-slip, thermally insulating horizontal plates.

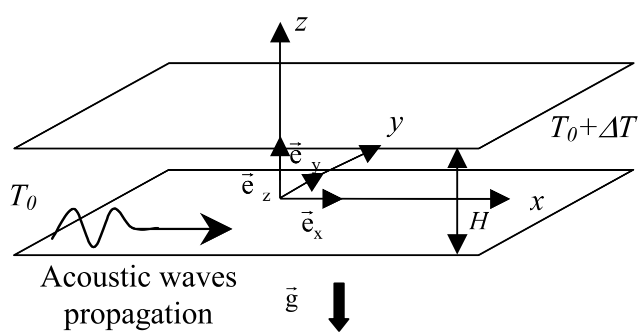

Figure 1 : Sketch of the studied configuration

If the variables are normalized using H, H^2/ν, ν/H and $\nabla\tilde{T}H$ ($\nabla\tilde{T}$is the constant horizontal temperature gradient) as scales for length, time, velocity and temperature respectively, this yields the following linearized system:

$$\vec{\nabla}\cdot\vec{v} = 0 \tag{1}$$

$$\frac{\partial\vec{v}}{\partial t} + \left(\vec{v}_0.\vec{\nabla}\right)\vec{v} + \left(\vec{v}.\vec{\nabla}\right)\vec{v}_0 = -\vec{\nabla}p + \nabla^2\vec{v} + Gr\theta\vec{e}_z \tag{2}$$

$$\frac{\partial\theta}{\partial t} + \vec{V}_0.\vec{\nabla}\theta + \vec{v}.\vec{\nabla}T_0 = \frac{1}{Pr}\nabla^2\theta \tag{3}$$

where the dimensionless numbers appearing in Eq.2 and Eq.3 are the Grashof and Prandtl numbers:

$$Gr = g\beta\nabla\tilde{T}H^4/\nu^2 \tag{4}$$

$$Pr = \nu/\kappa \tag{5}$$

β is the thermal expansion coefficient, T_0 is a reference value for the temperature, ν is the kinematic viscosity and

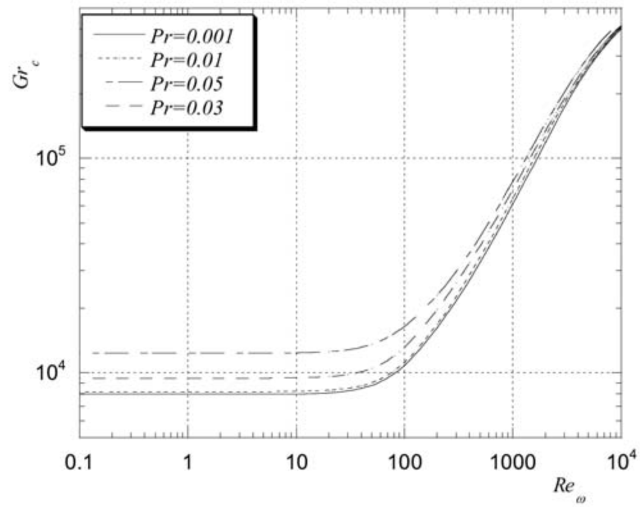

Figure 2 : Variation of Gr_c versus Re_ω for 2D modes and different values of Pr

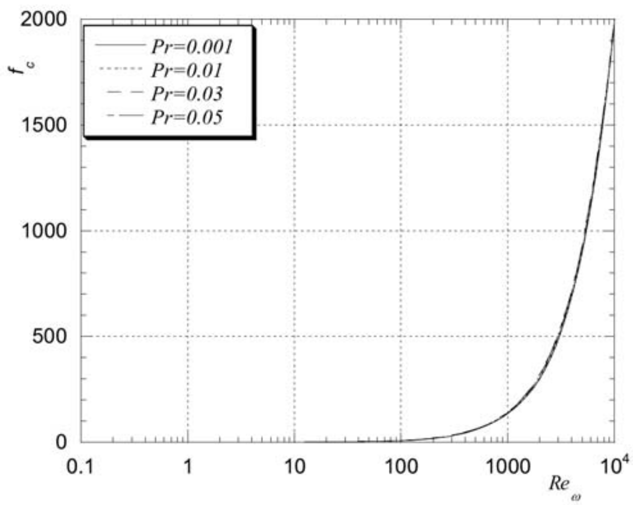

Figure 3 : Variation of the critical frequency f_c versus Re_ω for 2D modes and different values of Pr

κ is the thermal diffusivity. The basic flow $(\vec{v}_0, p_0, T_0)$ is obtained from the general equations of the stationary Hadley natural convection problem submitted to a longitudinal acoustic wave. The solution can be easily derived and one can find the following expressions:

$$\vec{v}_0(z) = \left(\frac{Gr}{6}\left(z^3 - \frac{z}{4}\right) + 2Re_\omega\left(6z^2 - 1\right)\right) \quad \vec{e}_x \tag{6}$$

$$T_0(x,z) = x + \frac{GrPr}{6}\left(\frac{z^5}{20} - \frac{z^3}{24} + c_1 z + c_2\right) + 2Re_\omega Pr\left(8z^4 - 4z^2 + 1\right) \tag{7}$$

Where z is the coordinate perpendicular to the fluid layer, $Re_\omega = \frac{u_\infty H}{\nu}$ is the acoustic Reynolds number, $u_\infty = \frac{3u_0^2}{8c_0}$ where u_0 is the amplitude of the sonic wave and c_0 is the speed of sound in the considered fluid. The constants c_1 and c_2 are given according to the thermal boundary conditions, namely: $c_1 = \frac{7}{960}$ and c_2=0 for thermally conducting boundaries, and $c_1 = \frac{1}{64}$ for thermally insulating boundaries. The constant c_2 is not involved in the linear stability calculations and consequently we do not need to have its exact value. The perturbation $(\vec{v}, p, \theta)$ is considered as a normal mode, so it can be written as follows:

$$(\vec{v}, p, \theta) = (\vec{v}(z), p(z), \theta(z))\, e^{i(hx+ky)+\omega t} \tag{8}$$

where h and k are the wave numbers in the x and y directions respectively and ω is a complex pulsation. Using Eq. 8, an eigenvalue problem is derived, namely: $LX = \omega MX$, where $x = (\vec{v}(z), p(z), \theta(z))$, L is a linear operator which depends on h, k, Gr, Pr and Re_ω, and M is a constant linear operator. Such an eigenvalue problem is solved using the spectral Tau Chebyshev collocation method. From the thresholds $Gr_0(Pr, Re_\omega, h, k)$, the critical Grashof number Gr_c is obtained after a minimization procedure with respect to h and k: $Gr_c = Inf_{(h,k)\in\mathbb{R}^2} Gr_0(Pr, Re_\omega, h, k)$

3 Results

The Hadley flow (flow generated between infinite horizontal walls by a horizontal temperature gradient) is known to become unstable when the Grashof number Gr goes over a critical value Gr_c. For the situation considered in our study and relating to a confined cavity with thermally insulating boundaries, two major instabilities types are known. The first unstable modes which are two-dimensional and stationary appear for $Pr \leq 0.034$. For $Pr \geq 0.034$, three-dimensional oscillatory modes occur and become the most critical instabilities. A third kind of instabilities has to be mentioned, especially in the context of combined buoyancy-acoustic streaming driven convection, as it will be shown below. This last instability type consists of a stationary three-dimensional mode characterized by the values of critical thresholds which are highly above those of the two first primary instabilities. The aim of the following investigations is to study the effect of an acoustic wave on these three instability

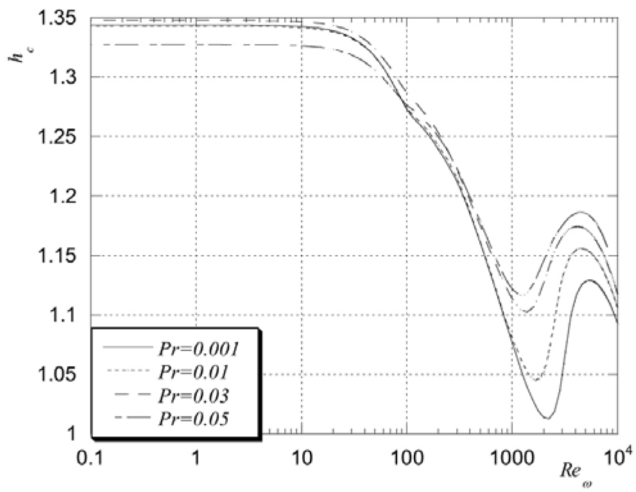

Figure 4 : Variation of the critical wave number h_c versus Re_ω for 2D modes and different values of Pr

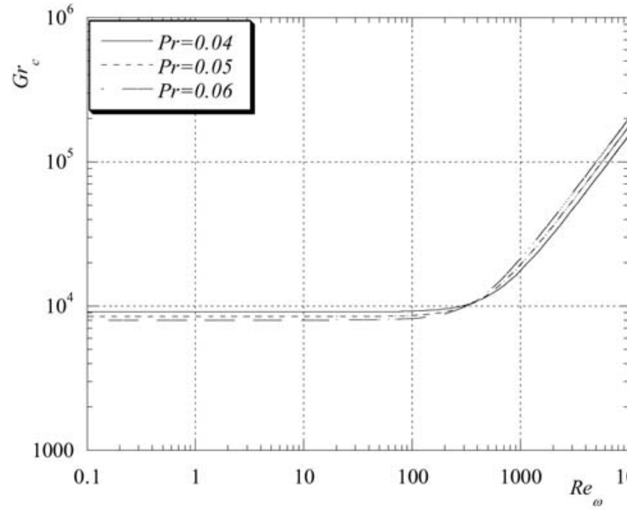

Figure 5 : Variation of Gr_c versus Re_ω for 3D instabilities and different values of Pr

types to discern whether such a wave will stabilize or destabilize the Hadley natural convection.

3.1 *Effect of an acoustic wave on the two-dimensional stationary instabilities*

Initially, we begin our investigations by considering the effect of an acoustic wave on the two-dimensional instabilities which are dominant for $Pr \leq 0.034$ in the pure thermal case (when no acoustic wave is applied). Our aim is to determine the influence of such a wave characteristics on the critical thresholds of the two-dimensional stationary modes. The conducted numerical computations allow us to conclude that the acoustic wave stabilize the 2D modes by increasing the critical Grashof number values Gr_c. Indeed, Fig. 2, illustrating the variation of Gr_c versus Re_ω, shows an increase of that critical parameter as Re_ω goes from 0 to 10^4. It is also worth noting that this stabilization of the flow seems to occur in three different stages, depending on the values of Re_ω. For $Re_\omega <$ 10, the critical Grashof number Gr_c varies very slightly and remains almost equal to its value for Re_ω=0 (when no acoustic wave is applied), namely: $Gr_c \sim Gr_c(Re_\omega{=}0)$. For $Re_\omega > 100$, or even $Re_\omega > 200$ and $Pr > 0.01$, the critical Grashof number Gr_c is found to vary linearly with Re_ω. Consequently, for $Re_\omega > 200$, the stabilization law can be written as follows: $Gr_c \sim Re_\omega$. For the high values of Re_ω, and more precisely for $Re_\omega > 5000$, the acoustic wave is seen to be less efficient in stabilizing the flow. Indeed, the increase of the thresholds Gr_c has been characterized by fitted law which scales as: $Gr_c \sim Re_\omega^{0.5}$. The Fig. 3 illustrates a fundamental change affecting the nature of the two-dimensional instabilities when an acoustic wave is applied. One can notice that such 2D modes which where stationary for Re_ω = 0 (when no acoustic wave is applied), become oscillatory when an acoustic wave is applied ($Re_\omega > 0$). Moreover, the increase of the critical frequency f_c which remains moderate for $Re_\omega < 100$, becomes extremely sharp for $Re_\omega > 1000$. For the critical wave number h_c, its variation with $Re_\omega$is more complex compared to that of Gr_c, in particular for $Re_\omega > 1000$ where the function $h_c = f(Re_\omega)$ is no more monotonous as illustrated in Fig. 4. Nevertheless, for $Re_\omega < 1000$, one can note, that globally, the size of the marginal cells increases. For $Re_\omega > 1000$, after a stage where the marginal cells shrink, they start to lengthen again. Finally, we can notice in Fig. 8, that the stabilization process seems to be almost the same regardless of the value of the Prandtl number Pr.

3.2 *Effect of an acoustic wave on the three-dimensional oscillatory instabilities*

The acoustic wave has also a stabilizing effect on the oscillatory longitudinal three-dimensional instabilities, but contrary to the case of the two-dimensional modes, the oscillatory nature of 3D instabilities is preserved. Fig. 5, where we report the variation of Gr_c versus Re_ω, shows that Gr_c increases with Re_ω. In Fig. 5, two types of behaviors can be noted depending on the values of Re_ω.

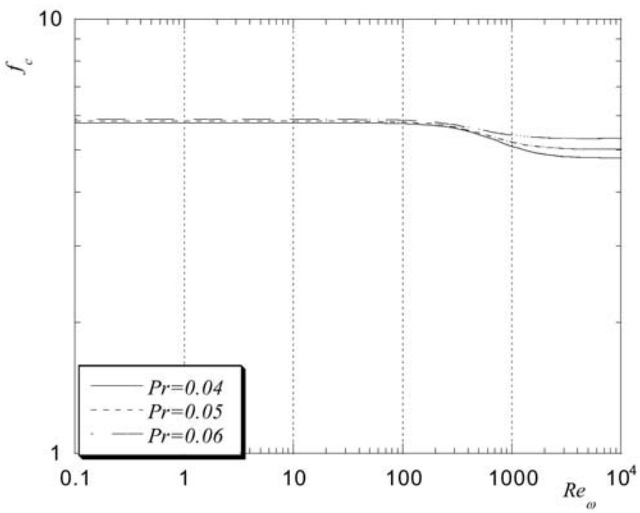

Figure 6 : Variation of the critical frequency f_c versus Re_ω for 3D instabilities and different values of Pr

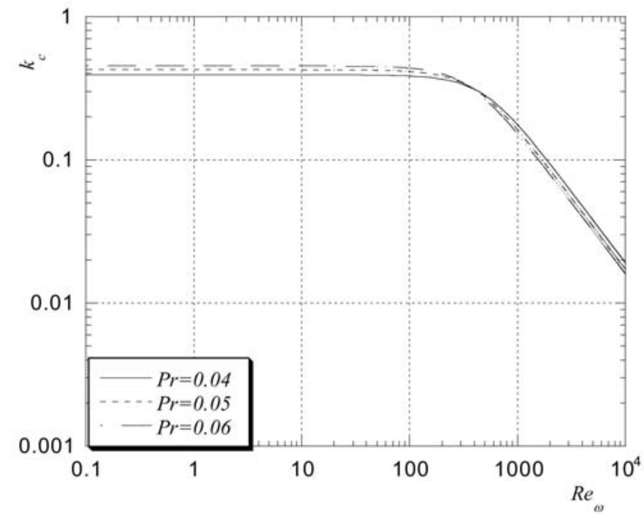

Figure 7 : Variation of the critical wave number k_c versus Re_ω for 3D instabilities and different values of Pr

For $Re_\omega < 100$, the critical Grashof number Gr_c varies very slightly and remains almost equal to its value for $Re_\omega = 0$ (when no acoustic wave is applied), namely: $Gr_c \sim Gr_c(Re_\omega = 0)$. For $Re_\omega > 500$ and $Pr > 0.04$, the critical Grashof number Gr_c is found to vary linearly with Re_ω : $Gr_c \sim Re_\omega$. The critical frequency f_c illustrated on Fig. 6 is seen to vary very slightly with the acoustic Reynolds number Re_ω. For the considered range of Re_ω, one can note that the critical frequency takes two values which are almost constant: for $Re_\omega < 100$ we have $f_c \sim 6$, and for $Re_\omega > 1000$ the critical frequency has another constant value, namely: $f_c \sim 5$. For the critical wave number k_c, its variation with Re_ω can be divided in two stages as illustrated on Fig. 7: during the first stage corresponding to $Re_\omega < 100$, the values of k_c are almost constant, and then k_c begins to decrease according to the scale law $k_c \sim Re_\omega^{-1}$, which indicates a lengthening of the marginal cells size. In addition, and contrary to what was noted for the two-dimensional instabilities, one can remark from Fig. 8, that the stabilization is more efficient when Pr increases. Finally, we can clearly note, when observing Fig. 8, that the values of Gr_c for $Re_\omega = 0$ and $Re_\omega = 5000$, are very close when $Pr \to 0$, confirming the thermal origin of these instabilities.

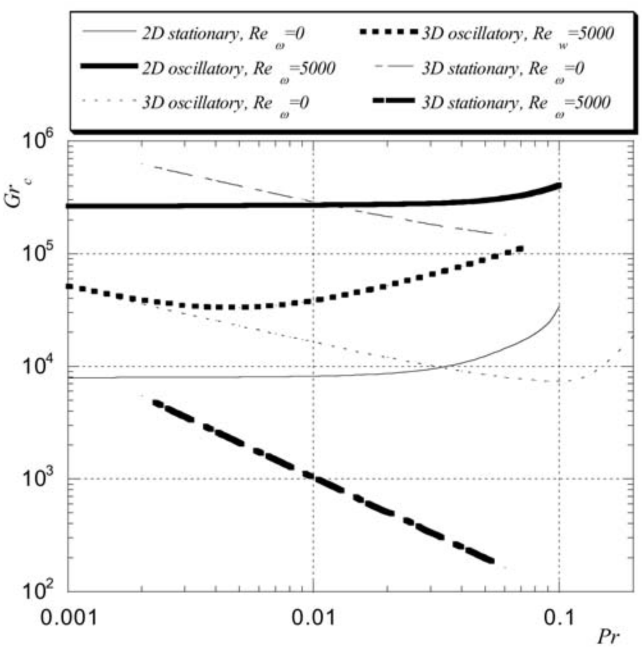

Figure 8 : Variation of Gr_c versus Pr for different values of Re_ω

3.3 *Effect of an acoustic wave on the three-dimensional stationary instabilities*

The consideration of such three-dimensional stationary instability would present only little interest if the considered Hadley flow was not submitted to an acoustic wave. Indeed, when no acoustic wave is applied ($Re_\omega = 0$), these stationary three-dimensional modes are characterized by thresholds critical values which are highly above those of the two first instabilities (2D stationary and 3D oscillatory). When an acoustic wave is applied, such 3D stationary modes are destabilized and become the most dangerous mode for certain values of the Prandtl number Pr and the acoustic Reynolds number Re_ω. This result is well illustrated in Fig. 8 where we notice a significant decrease of Gr_c when Re_ω reaches the value of 5000. One can remark that in such conditions, the 3D stationary modes become the most critical instabilities for $0.002 \leq Pr \leq 0.06$. It is also worth pointing out that such instabilities preserve their stationary character when an acoustic wave is applied ($f_c = 0, \forall\, Re_\omega \geq 0$), and that for a given Pr their size increases with Re_ω. It's also worthwhile to notice, as illustrated in Fig. 9, that the destabilization efficiency of the acoustic wave on these modes is very significant for $Re_\omega \leq 500$ and become relatively moderate for $Re_\omega \geq 1000$.

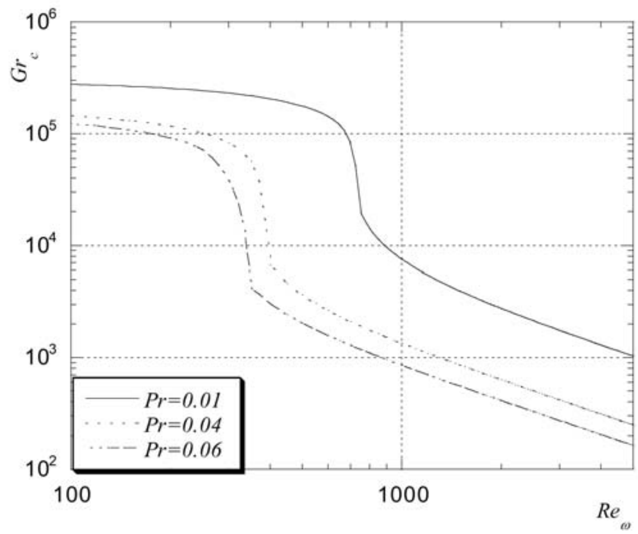

Figure 9 : Variation of Gr_c versus Re_ω for 3D stationary instabilities and different values of Pr

4 Discussion

We have characterized the effect of an acoustic wave on the stability of the Hadley natural convection through a linear stability analysis. The conducted numerical computations allow us to prove that applying an acoustic wave will stabilize both the two-dimensional stationary and three-dimensional oscillatory modes. These two stabilization processes are quite different. The acoustic wave preserve the oscillatory character of the 3D modes, but it fundamentally changes the 2D stationary modes which become oscillatory. This capital result can have harmful consequences on the crystal growth process where it is known that oscillations can produce undesirable striations in the grown crystal. A second result in connection with the influence of the Prandtl number on the stabilization process has to mentioned: even though Pr does almost have no impact on the 2D modes stabilization, it appears to be a significant parameter in influencing the stabilization of the 3D oscillatory modes.

Finally, another fundamental result has to be pointed out, it is related to the behavior of the three-dimensional stationary instabilities with respect to an acoustic wave. Unlike the two first instabilities (2D modes and 3D oscillatory modes) which are stabilized, this last instability (3D stationary mode) is destabilized under the action of an acoustic wave and becomes the most dangerous mode for a certain range of the Prandtl number Pr and the acoustic Reynolds number Re_ω.

References

Amberg, G.; Shiomi, J. (2005) : Thermocapillary flow and phase change in some widespread materials processes. *FDMP*, vol 1, pp. 81-95.

Ben Hadid, H. (2000) : MHD damped buoyancy driven flows. In: H. Aref and J.W. Philips (ed), *Mech. New Mellenium, Proc. ICTAM, Kluwer,* pp. 289.

Dold, P.; Benz, W. (1995) : Convective temperature fluctuations in liquid gallium in dependance on static and rotating magnetic fields. C*rystal Res. Technol.* Vol 30, pp. 8.

Dold, P.; Benz, W. (1997) : Modification of Fluid Flow and Heat Transport in Vertical Bridgman Configurations by Rotating Magnetic Fields. C*rystal Res. Technol.* Vol 32, pp. 51.

Gelfgat A.Yu., Rubinov A., Bar-Yoseph P.Z. and Solan A., (2005), "On the Three-Dimensional Instability of Thermocapillary Convection in Arbitrarily Heated Floating Zones in Microgravity Environment" , FDMP, Vol. 1, no.1, pp. 21-32

Haddad, F.Z.; Garandet, J.P.; Henry, D.; Ben Hadid, H. (1999) : Analysis of the unsteady segregation in crystal growth from a melt: I. Fluctuating interface velocity. *J. Crystal Growth*, vol 204, pp. 213-223.

Haddad, F.Z.; Garandet, J.P.; Henry, D.; Ben Hadid, H. (2000) : Analysis of the unsteady segregation in crystal growth from a melt Part II: Fluctuating convection velocity. *J. Crystal Growth*, vol 220, pp. 166-175.

Hurle, D.T.J. (1966) : Temperature oscillations in molten metals and their relationship to growth striae in melt-grown crystals. *Phil. Mag.* Vol 13, pp. 305.

Hurle, D.T.J. (1974): Convective temperature oscillations in molten gallium. *J. Fluid Mech.* Vol 64, pp. 565-576.

Kaddeche, S.; Henry, D.; Ben Hadid, H. (2003) : Magnetic stabilization of the buoyant convection between infinite horizontal walls with a horizontal temperature gradient. *J. Fluid Mech.* Vol 480, pp. 185-216.

Kaddeche, S.; Henry, D.; Putelat, T.; Ben Hadid, H. (2002) :] Instabilities in liquid metals controlled by constant magnetic field—Part I: vertical magnetic field. *J. Crystal Growth,* vol 242, pp. 491-500.

Kaddeche, S.; Henry, D.; Putelat, T.; Ben Hadid, H. (2002) : Instabilities in liquid metals controlled by constant magnetic field—Part II: horizontal magnetic field. *J. Crystal Growth,* vol 242, pp. 501-510.

Lan, C.W.; Yeh, B.C. (2005) : Effects of rotation on heat flow, segregation, and zone shape in a small-scale floating-zone silicon growth under axial and transversal magnetic fields. *FDMP*, vol 1, no.1, pp. 33-43.

Lappa, M. (2005a): On the nature and structure of possible three-dimensional steady flows in closed and open parallelepipedic and cubical containers under different heating conditions and driving forces. *FDMP*, vol 1, no 1, pp. 1-19.

Lappa M. **(2005b)**: Review: Possible strategies for the control and stabilization of Marangoni flow in laterally heated floating zones, *FDMP*, vol.1, no.2, pp. 171-188.

Loh, B.K.; Hyun, S.; Ro, P. I.; Kleinstreur, C. (2002): Aoustic streaming induced by ultrasonic flexural vibrations and associated enhancement of convective heat transfer. *J. Acoust. Soc,* Am, vol 111, pp. 875-883.

Melnikov D. E.; Shevtsova V. M. (2005): Liquid Particles tracing in three-dimensional buoyancy-driven flows, *FDMP,* vol.1, no.2, pp. 189-199.

Suri, C. ; Takenaka, K. ; Yanagida, H. ; Kojima, Y. ; Koyama, K. (2002) : Chaotic mixing generated by acoustic streaming. *Ultrasonics,* vol 40, pp. 393-396.

Tsukada, T.; Kobayashi, M.; Jing, C.J.; Imaishi, N. (2005): Numerical simulation of CZ crystal growth of oxide. *FDMP*, vol 1, no.1, pp. 45-62.

Vainstein, P.; Fichman, M.; Gutfinger, C. (1995): Acoustic enhancement of heat transfer between two parallel plates. *Int. J. Heat Mass Transfer,* vol 38, pp. 1893-1899.