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A Novel Approach for Fracture-Network Morphology and Flow Simulation in Coalbed Methane Reservoirs

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ABSTRACT: Coalbed methane (CBM) reservoirs are typically characterized by ultra-low permeability, pronounced heterogeneity, and strong stress sensitivity. A primary challenge in horizontal-well hydraulic fracturing is to accurately characterize complex fracture morphology and efficiently simulate fracture-matrix flow, which is essential for coordinated optimization of production performance and economics. This study proposes the Fracture Connection Element Method (FCEM), which departs from conventional grid-based discretization by representing both matrix and fractures as nodes. Matrix nodes are generated via Poisson-disk sampling to achieve spatially uniform coverage. The hydraulic-fracture network is converted into a node-edge topology using Dijkstra's shortest-path algorithm, preserving both primary fractures and branching structures. On this connection-element system, an inter-nodal conductance (transmissibility) model is developed to characterize mass transfer among fractures, matrix, and their interactions. A semi-analytical formulation is further introduced for rapid production forecasting, enabling a favorable balance between numerical stability and computational efficiency. In a conceptual case under constant water-rate production, FCEM achieves an RMSE of 7.34% relative to the commercial simulator ECLIPSE while reducing runtime by approximately 80×. A dual-horizontal-well CBM field case is then validated against dynamic production data. History matching is performed using simultaneous perturbation stochastic approximation (SPSA), yielding match ratios of 84.87% and 80.31% for the two wells, respectively. Overall, FCEM reproduces pressure responses and fracture-network connectivity with high fidelity and offers improved efficiency, stability, and scalability compared with traditional grid-based approaches, providing a robust framework for fracturing-parameter design and production-schedule optimization in unconventional reservoirs.

KEYWORDS: Coalbed methane; naturally fractured reservoir; numerical reservoir simulation; connection element method

1 Introduction

With the gradual depletion of shallow coalbed methane (CBM) resources, the development of deep CBM has become an inevitable trend to adapt to the new development landscape. Compared with shallow CBM, deep CBM reservoirs exhibit significant differences in accumulation mechanisms, enrichment patterns, and occurrence states. Characterized by high *in-situ* stress and complex rock-mechanical conditions, deep reservoirs not only aggravate near-wellbore cracking and increase well-integrity risks [1,2]. The coupled high-stress and thermo-baric environment can also markedly influence fracture initiation and the resulting fracture-network morphology during hydraulic fracturing, thereby posing more severe engineering

challenges for development and production [3,4]. Consequently, conventional equivalent models based on idealized planar fractures are insufficient to effectively guide field operations [5,6].

In the development of unconventional oil and gas, hydraulic fracturing is a key stimulation technique that creates high-conductivity pathways in low-permeability formations, thereby significantly improving permeability and productivity. In deep unconventional reservoirs, particularly, the complex fracture networks formed during high-pressure injection govern the transport pathways between fractures and the matrix and the spatial distribution of productivity; their morphological characteristics have a decisive impact on production performance. Therefore, in-depth investigation of how fracturing parameters influence fracture propagation, and the construction of accurate multiphase-flow models based on fracture geometry, are prerequisites for optimizing stimulation strategies in unconventional reservoirs [7].

Research on hydraulic fracture propagation has primarily followed two routes: experimental approaches and numerical simulations. Experimental studies use uniaxial, triaxial, and true-triaxial loading systems to inject high-pressure fluid into rock specimens and infer fracture morphology—e.g., total fracture width—via monitoring methods such as ultrasonic pulse velocity. However, such methods struggle to realistically capture the heterogeneity and anisotropy of deep formations [8]. To overcome these limitations, numerical modeling of fracture propagation has become mainstream.

For complex hydraulic fractures, common numerical strategies can be categorized into continuum and discrete-medium models. The former include the equivalent continuum model [9] and dual-porosity/dual-permeability models [10,11], which treat fractures and matrix as elements of the same dimensionality. In discrete-medium models, fractures in higher-dimensional reservoirs are dimension-reduced (e.g., fractures represented as 1D line segments in a 2D domain). Building on these frameworks, many studies employ finite-difference (FDM) [12], finite-element (FEM) [13,14], and finite-volume methods (FVM) [15] for flow simulation. Continuum approaches are limited by a strong dependence of accuracy on mesh quality; overly coarse meshes lead to cumulative errors, while adequately resolving fractures and complex boundaries requires fine meshes that substantially increase computational cost. Hoteit and Firoozabadi (2006) [16] proposed a discrete-fracture model coupling mixed finite elements (MFE) with discontinuous Galerkin (DG), which performs well for flux computation among multiple intersecting fracture branches. Gordeliy and Peirce (2013) [17] developed an XFEM-based ND (Neumann → Dirichlet)/DN (Dirichlet → Neumann) mapping algorithm that matches reference solutions on non-conforming meshes and complex boundaries. Wang (2019) [18] introduced a global cohesive-zone model (CZM) approach to study interactions and propagation between hydraulic and natural fractures, enabling a unified description of turning, kinking, and branching. Nevertheless, the computational cost of these methods often grows exponentially with fracture density, limiting practical field applications. Huang et al. [19] used a 2D discrete-element model to elucidate the effects of stiffness-to-toughness ratios and interlayer stress contrasts on fracture-height growth; yet assumptions such as plane strain and no interfacial slip restrict extrapolation to 3D heterogeneous systems with slip.

Although discrete-fracture models reduce fracture dimensionality to ease complexity, generating high-quality conforming meshes remains difficult. To address this, the embedded discrete-fracture model (EDFM) has gained traction in recent years. EDFM uses orthogonal structured grids, requires no explicit meshing along fracture geometry, and avoids local refinement. However, conventional EDFM can incur significant pressure and rate errors for low-conductivity fractures and commonly assumes linear pressure distribution across fracture–matrix interfaces, leading to modeling bias. Building on Lee et al. (2000) [20], Losapio et al. proposed a local EDFM that relaxes the linear-pressure assumption and improves fracture–matrix coupling by employing local fine grids [21]. Tene et al. enhanced accuracy for low-conductivity fractures

by introducing additional non-neighboring connections (NNCs) [22]. While EDFM/DFM-based mesh-refinement schemes can now capture hydraulic-fracture geometry with good fidelity, limitations persist when handling fracture intersections, tips, and heterogeneous fracture–matrix coupling; moreover, mesh partitioning and NNC construction in complex networks substantially raise computational cost, hindering widespread field deployment.

To this end, Zhao et al. (2021) [23] proposed a lightning-breakdown-inspired mechanism for fracture growth and used it to invert post-fracture network morphology. This meshfree, fractal-inspired approach has few parameters and high efficiency, making it suitable for rapid morphology inversion and multi-scenario exploration. Building on this idea, Zhao et al. [24] introduced generalized finite-difference theory to establish a non-Euclidean physically connected-network equivalence and derived physically meaningful flow parameters that satisfy mass conservation, thereby proposing a new reservoir-simulation paradigm—the meshfree Connection Element Method (CEM). Based on the core concepts of CEM and by defining meshless nodal control volumes, Rao et al. [25] developed an extended finite-volume method, essentially a fully implicit scheme within the connection-element framework.

In summary, under deep CBM conditions, existing approaches still struggle to simultaneously achieve (i) faithful representation of complex fracture-network morphology and connectivity, (ii) accurate fracture–matrix coupled transport, and (iii) computational efficiency required for optimization-oriented multi-scenario workflows. Equivalent-continuum and ideal planar-fracture assumptions weaken the description of realistic connectivity and interference pathways, whereas explicit discrete-fracture formulations and advanced coupled schemes, although accurate, remain constrained by conforming-mesh generation and high computational cost. EDFM alleviates meshing requirements, yet can still incur non-negligible bias for low-conductivity fractures and near intersections/tips, with additional overhead from grid partitioning and NNC construction. These limitations motivate the development of a fast flow-simulation framework that balances geometric expressiveness, physical consistency, and efficiency for productivity prediction and optimization in multi-fractured horizontal wells in deep CBM reservoirs.

Based on the meshfree CEM, this paper proposes a new Fracture Connection Element Method (FCEM) for the rapid simulation of fluid flow in unconventional reservoirs with multi-fractured horizontal wells featuring complex hydraulic-fracture morphologies. Comparisons with traditional numerical approaches validate the accuracy and efficiency of FCEM. Accordingly, this work provides a new pathway for flow simulation and optimization of hydraulic-fracture morphology in multi-fractured horizontal wells within unconventional reservoirs.

2 A New Flow-Simulation Method for Complex Fracture Morphology

In this method, the stimulated reservoir is represented as a set of discrete points according to the spatial distribution of the complex fracture geometry. Flow paths between nodes represent fluid exchange between the matrix and fractures. The inter-nodal connection transmissibility and connection volume (pore volume) are computed to characterize, respectively, the flow capacity and geometric attributes between nodes. Finally, treating each connection as a computational element, the pressure equation is solved to obtain the element flux, and the saturation equation is tracked to compute production at different perforation locations.

2.1 Fracture-Morphology Representation in FCEM

Unlike traditional grid-based models, the Connection Element Method (CEM) characterizes the reservoir with a set of discrete nodes. The method focuses on each node's domain of influence: all nodes within this domain are connected to the central node, forming multiple one-dimensional connection paths, as illustrated by the gray area in Fig. 1. For strongly heterogeneous reservoirs, the approach can still refine the

description of reservoir properties by adding nodes without injection/production rates at the corresponding locations. However, CEM is best suited to conventional reservoirs with regular well patterns and is difficult to apply directly to unconventional reservoirs with multi-fractured horizontal wells. On the one hand, if only the traditional well is treated as a node, the reservoir contains only a single node, making it impossible to establish mass-balance equations; a single well node cannot depict fluid flow among multiple media. On the other hand, the complex spatial distribution of fractures makes node placement challenging, and an excessive number of fracture nodes can markedly slow model construction.

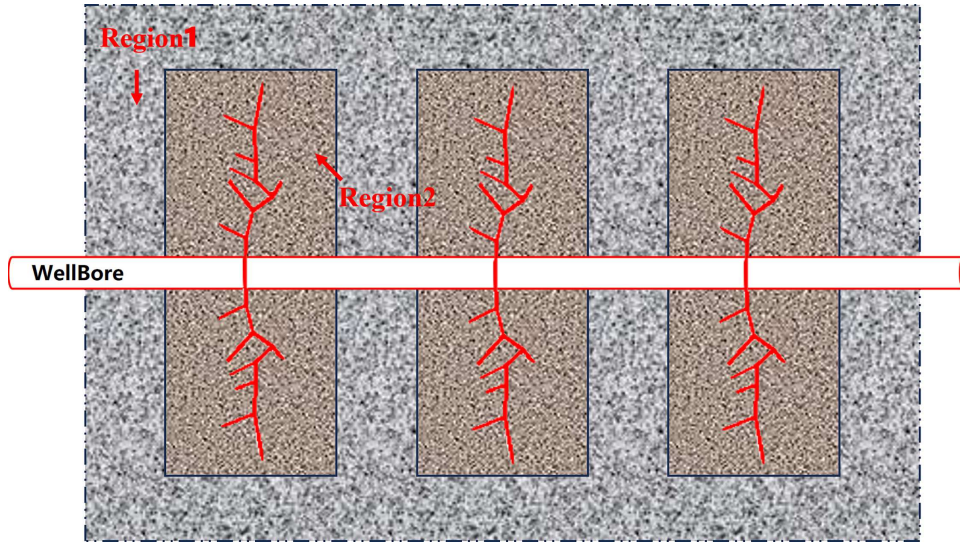


Figure 1: Structure of a fractured gas reservoir.

To represent the hydraulic-fracture morphology of multi-fractured horizontal wells, a nodal model is used to provide an equivalent description of the reservoir. The point-placement stage aims to generate representative fracture and matrix nodes as the basis for constructing subsequent connections. Fracture nodes are extracted from a binarized image of the original fracture map (i.e., a 0–1 Excel data file): cells with value 1 denote fracture locations and are directly converted to fracture-point coordinates, ensuring that the represented fracture geometry matches the source image. Matrix nodes are placed using Poisson-disk sampling to obtain a uniformly distributed point set with appropriate spacing, which yields a set of points uniformly distributed in space while maintaining appropriate spacing between each other. Its mathematical definition is:

$$\forall x_i, x_j \in S: \|x_i - x_j\| \geq r, S \sim P(r) \quad (1)$$

where x_i and x_j are any two matrix points in the set S , $\|\cdot\|$ denotes the Euclidean distance between two points, r is the prescribed minimum spacing between matrix points, and $S \sim P(r)$ indicates that the point set S is generated from a Poisson-disk process with radius r . The resulting discrete point cloud is shown in Fig. 2a.

To maximize computational speed, three types of connections are employed to describe two distinct reservoir regions. (1) Region 1 in Fig. 1 is the matrix zone, where no hydraulic fractures are present and the reservoir is relatively homogeneous; this zone is represented by a set of low-permeability nodes, and connection type ① in Fig. 2a describes matrix-to-matrix mass exchange within it. (2) Region 2 in Fig. 1 is the hydraulically stimulated zone, which contains high-permeability fracture nodes; connection type ②

in Fig. 2a describes fluid flow between fractures and matrix, while connection type ③ describes flow between fractures. The final computed connection transmissibilities are shown in Fig. 2b.

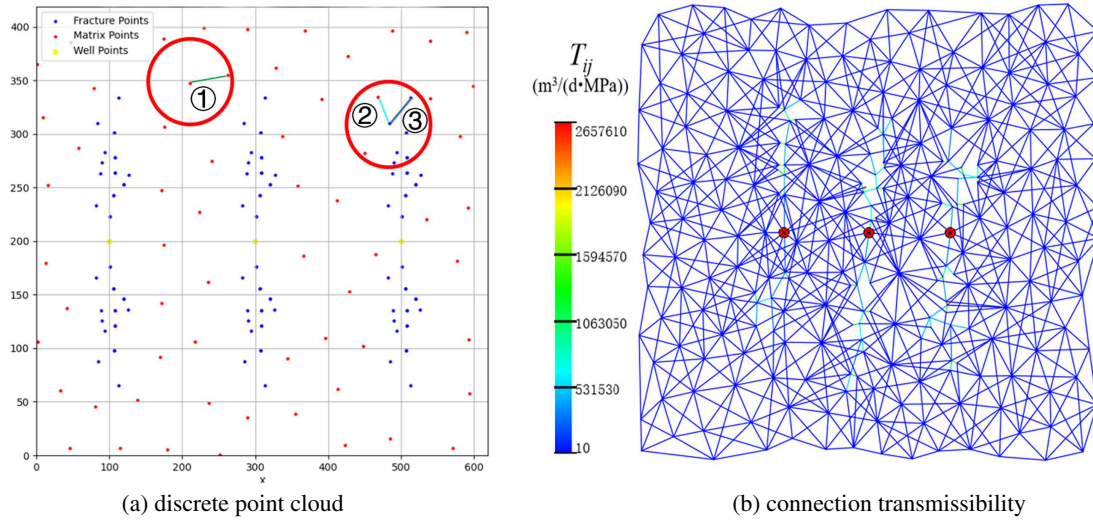


Figure 2: Construction of the connection network.

2.2 Calculation of Flow-Characteristic Parameters

Define specific characteristic parameters to simulate flow through connection elements. These parameters represent flow behavior along each connection path. Determine transmissibility to quantify flow capacity between connected nodes.

Building on these foundational concepts, Sheng et al. developed a numerical simulation method for connection elements [26]. In their framework, nodes in the connection-element system lack defined control volumes. Introduce a control volume at the nodes to address source or sink terms in physical problems. Using mass conservation principles, define the nodal control volume as follows:

$$\begin{cases} \Omega_i \cap \Omega_j = \emptyset, \forall i \neq j \in \Lambda \\ \bigcup_{i \in \Lambda} \Omega_i = \Omega \\ \sum_{i \in \Lambda} V_i = V_\Omega \end{cases} \quad (2)$$

where Ω denotes the overall reservoir control domain; Ω_i represents the control domain of node i ; V_Ω is the total reservoir volume; and V_i is the control volume of node i .

Two rules govern nodal control volumes: they cannot overlap, and together they must cover the entire reservoir.

This derivation provides a way to calculate flow in fractured reservoirs. It uses the connection-element method and applies two-phase flow equations as an example.

2.2.1 Characterization of Matrix–Matrix Flow Parameters

The two-phase flow governing equations for a dual-medium matrix system are as follows:

$$\nabla \cdot \left(\frac{k_m}{B_g \mu_g} \nabla p_{g,m} \right) + q_{osi} \delta = \frac{\partial}{\partial t} \left(\frac{\phi}{B_g} \right) + \tau_{mf} \quad (3)$$

where k_m is the absolute permeability of the matrix system (mD); μ_g denotes the gas viscosity (mPa·s); B_g is the formation volume factor (dimensionless); ∇ is the nabla (gradient) operator; $p_{g,m}$ is the matrix pressure (MPa); q_{osi} is the volumetric flow rate at standard surface conditions (source/sink term, d^{-1}); δ is the Dirac delta function (dimensionless); t is time (d); and τ_{mf} is the mass-exchange term between matrix and fractures, representing the gas flux between the two systems, with units consistent with the source/sink term.

For Eq. (3), integrating the pressure-diffusion term over the nodal control volume yields Eq. (4).

$$\int_{V_i} \nabla \cdot \left(\frac{k_m}{B_g \mu_g} \nabla p_{g,m} \right) d\Omega = \sum_{\eta=1}^{n_j} \left(\frac{k_m}{B_g \mu_g} \right)_{\eta} T_{\eta}^j (P_{m,\eta} - P_{m,i}) \quad (4)$$

For the same connection element (i, j) , due to reservoir heterogeneity, the transmissibilities evaluated from the two sides are generally not identical. Denote the one-sided transmissibilities as:

$$\begin{cases} T_i^j = \bar{k}_{m,i} V_e^i e_i^j \\ T_j^i = \bar{k}_{m,j} V_e^j e_j^i \end{cases} \quad (5)$$

where $\bar{k}_{m,i}$ and $\bar{k}_{m,j}$ are the averaged matrix permeabilities in the control domains of nodes i and j , respectively; V_e^i and V_e^j are the corresponding volume contributions associated with the connection element; and e_i^j , e_j^i are geometric weights arising from the connection-element discretization. By mass conservation across the connection element, the geometric term satisfies.

$$V_e^i e_i^j = V_e^j e_j^i = G_{ij} \quad (6)$$

Taking a harmonic average to enforce symmetry $T_{ij} = T_{ji}$, we obtain:

$$T_{ij} = \frac{2T_i^j T_j^i}{T_i^j + T_j^i} = \frac{2\bar{k}_{m,i} \bar{k}_{m,j}}{\bar{k}_{m,i} + \bar{k}_{m,j}} G_{ij} \quad (7)$$

Accordingly, the harmonic-averaged equivalent permeability is defined as:

$$\bar{k}_{m,ij} = \frac{2\bar{k}_{m,i} \bar{k}_{m,j}}{\bar{k}_{m,i} + \bar{k}_{m,j}} \quad (8)$$

And thus:

$$T_{ij} = \bar{k}_{m,ij} G_{ij} \quad (9)$$

Finally, based on the connection-element mass-balance principle and the control-volume allocation constraint ($\sum_i V_i = V_{res}$), the geometric term G_{ij} can be assembled using the nodal control volume V_i and the neighborhood geometric weights $\{e_j^i\}_{j=1}^{n_i}$. This leads to the definition of the matrix–matrix transmissibility for the connection element (i, j) as:

$$T_{mm,ij} = \bar{k}_{m,ij} V_i \sum_{j=1}^{n_i} e_j^i \quad (10)$$

where n_i is the number of neighboring nodes of node i , and e_j^i denotes the geometric weight associated with the j -th neighbor.

2.2.2 Characterization of Fracture–Fracture Flow Parameters

The two-phase flow governing equation for a dual-medium fracture system is:

$$\nabla \cdot \left(\frac{k_f}{B_g \mu_g} \nabla p_{g,f} \right) + q_{g,well} = \frac{\partial}{\partial t} \left(\frac{\phi}{B_g} \right) - \tau_{mf} \quad (11)$$

where k_f is the absolute permeability of the fracture system (mD); μ_g is gas viscosity (mPa·s); B_g is the formation volume factor (dimensionless); ∇ is the gradient operator; $p_{g,sf}$ is the pressure of the fracture system (MPa); $q_{g,well}$ is the volumetric flow rate at standard surface conditions (source/sink term, d^{-1}); δ is the Dirac delta function (dimensionless); t is time (d); and τ_{mf} is the mass-exchange term between matrix and fractures.

Define the transmissibility between two fracture nodes as:

$$T_{f,ij} = \lambda \frac{\bar{k}_{f,ij} h \bar{w}_{f,ij}}{L_{f,ij}} \quad (12)$$

where $\bar{k}_{f,ij}$ is the harmonic-averaged permeability between fracture nodes i and j (mD); h is reservoir thickness; $\bar{w}_{f,ij}$ is the average fracture aperture between nodes i and j , $L_{f,ij}$ is the fracture length of the connection element (i, j).

2.2.3 Characterization of Two-Phase Flow Parameters between Matrix and Fractures

Because the connection-element framework does not prescribe a specific geometry for the nodal control volume, the interface area A between fractures and matrix inside a node's control region is represented by the product of the total fracture length $L_{f,i}$ within that region and the fracture height $h_{f,i}$. For a given central node i , $L_{f,i}$ is taken as one half of the sum of the lengths of all fracture connection elements having node i as an end point:

$$L_{f,i} = \frac{1}{2} \sum_{\zeta \in \Lambda} L_{f,i\zeta} \quad (13)$$

where Λ denotes the set of the other end nodes that form fracture connection elements with node i ; $L_{f,i\zeta}$ is the length of the corresponding fracture connection element (i, s) whose endpoints are nodes i and s (m).

For the mass exchange between a matrix node and a fracture node, we write:

$$\tau_{mf,i} = T_{mf,i} (p_{m,i} - p_{f,i}) = \frac{2\lambda \bar{k}_{mf,i} L_{f,i} h_{f,i}}{d} (p_{m,i} - p_{f,i}) \quad (14)$$

Here we assume the fracture penetrates the entire reservoir thickness, i.e., the fracture height equals the reservoir thickness h . $\bar{k}_{mf,i}$ is the harmonic-averaged permeability for matrix–fracture crossflow within the control volume of node i (mD). d is the effective normal distance for matrix–fracture crossflow (m), which characterizes the equivalent seepage length. The effective height participating in matrix–fracture exchange is taken as $h_{f,i}$. The corresponding in-plane equivalent area of the control volume is:

$$S_i = \frac{V_i}{h_{f,i}} \quad (15)$$

We approximate this area by an equivalent square with side length:

$$l_i = \sqrt{S_i} = \sqrt{\frac{V_i}{h_{f,i}}} \quad (16)$$

Assuming that the fracture plane intersects the equivalent square at its center and that matrix–fracture exchange occurs symmetrically on both sides of the fracture, the one-sided normal distance from the fracture to the matrix domain spans $x \in [0, l_i/2]$. Under a uniform (area-averaged) approximation, the mean one-sided normal distance is:

$$d = \langle x \rangle = \frac{1}{l_i/2} \int_0^{l_i/2} x dx = \frac{l_i}{4} = \frac{1}{4} \sqrt{\frac{V_i}{h_{f,i}}} \quad (17)$$

Hence, the transmissibility between matrix and fracture is:

$$T_{mf,i} = \frac{\lambda \bar{k}_{mf,i} L_{f,i} h_{f,i}}{d} \quad (18)$$

2.3 Solution of the Flow-Governing Equations

Based on the local dual-medium formulation, the two-phase flow equations for fractured gas reservoirs are given by Eqs. (4) (matrix) and (11) (fractures). Using the nodal control volume, the discrete form of the flow equation is:

$$\sum_{j=1}^{n_i} \left[\frac{k_{ij}^{t+\Delta t}}{B_{g,ij}^{t+\Delta t} \mu_{g,ij}^{t+\Delta t}} T_{mm,ij} (p_{g,m,j}^{t+\Delta t} - p_{g,m,i}^{t+\Delta t}) \right] + Q_{g,well} = \frac{V_i}{\Delta t} \left[\left(\frac{\phi_i^{t+\Delta t}}{B_{g,i}^{t+\Delta t}} \right) - \left(\frac{\phi_i^t}{B_{g,i}^t} \right) \right] + \tau_{mf}^{t+\Delta t} \quad (19)$$

where i denotes the current matrix node and $j = 1, \dots, n_i$ denotes the neighboring matrix nodes connected to node i ; n_i is the number of matrix neighbors of node i ; superscripts t and $t + \Delta t$ denote the previous and current time levels, respectively, and Δt is the time-step size. $p_{g,m,i}^{t+\Delta t}$ and $p_{g,m,j}^{t+\Delta t}$ are the gas-phase pressures at matrix nodes i and j at time $t + \Delta t$. $k_{ij}^{t+\Delta t}$ is the equivalent absolute permeability on the matrix–matrix connection element between nodes i and j at time $t + \Delta t$. $B_{g,ij}^{t+\Delta t}$ and $\mu_{g,ij}^{t+\Delta t}$ are the gas formation volume factor (FVF) and gas viscosity on the connection element ij at time $t + \Delta t$, respectively. $T_{mm,ij}$ is the geometric transmissibility of the matrix–matrix connection ij , determined by the control-volume geometry and the inter-nodal distance. $Q_{g,well}$ is the gas-phase well source/sink term associated with the control volume of node i . V_i is the control volume assigned to node i . ϕ_i^t and $\phi_i^{t+\Delta t}$ are the porosities at node i at times t and $t + \Delta t$, respectively. $B_{g,i}^t$ and $B_{g,i}^{t+\Delta t}$ are the nodal gas FVFs at node i at times t and $t + \Delta t$, respectively. $\tau_{mf}^{t+\Delta t}$ represents the matrix–fracture inter-medium mass-exchange term (gas phase) within the control domain of node i , accounting for the transfer between the matrix and fracture systems.

Permeability uses a harmonic average, while viscosity and the formation volume factor use arithmetic averages:

$$\begin{cases} k_{ij}^{t+\Delta t} = \frac{2}{1/k_i^{t+\Delta t} + 1/k_j^{t+\Delta t}} \\ \mu_{g,ij}^{t+\Delta t} = \frac{\mu_{g,i}^{t+\Delta t} + \mu_{g,j}^{t+\Delta t}}{2} \end{cases} \quad (20)$$

2.4 Automatic History Matching

To improve parameter identifiability and prediction consistency for the field case, we employ an automatic history-matching strategy. Production time-series are used to construct an objective function, while connection-element parameters are treated as history-matching variables to calibrate the equivalent fracture-network model efficiently.

2.4.1 Objective Function

The model-parameter vector is:

$$m = [\dots, T_{ij}, \dots, V_i, \dots]^T \quad (21)$$

where T_{ij} is the equivalent transmissibility of the connection element between nodes i and j , and V_i is the control volume assigned to node i . Following a regularized formulation, the objective function consists of a prior-model term and a data-misfit term:

$$O(m) = \frac{1}{2} (m - m_{pr})^T C_M^{-1} (m - m_{pr}) + \frac{1}{2} (d_{obs} - g(m))^T C_D^{-1} (d_{obs} - g(m)) \quad (22)$$

where m_{pr} denotes the prior parameter vector; C_M is the parameter covariance matrix constraining deviations from the prior; d_{obs} is the observed production-data vector; $g(m)$ is the FCEM forward-simulation response corresponding to m ; and C_D is the data-error covariance matrix that controls data weighting and reliability. Minimizing $O(m)$ yields an automatically calibrated set of equivalent fracture-network parameters.

2.4.2 SPSA Optimization

Because the number of unknowns can be large and analytical gradients are unavailable, we use Simultaneous Perturbation Stochastic Approximation (SPSA). At iteration k , a random perturbation vector Δ_k is applied with perturbation magnitude c_k . A common two-sided SPSA gradient estimator is:

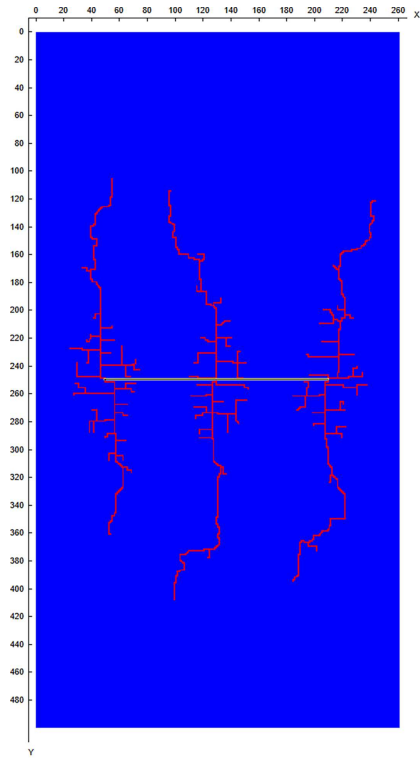
$$\hat{g}_k = \frac{O(m_k + c_k \Delta_k) - O(m_k - c_k \Delta_k)}{2c_k} \Delta_k^{-1} \quad (23)$$

where Δ_k^{-1} denotes componentwise inversion. The parameters are then updated with step size α_k :

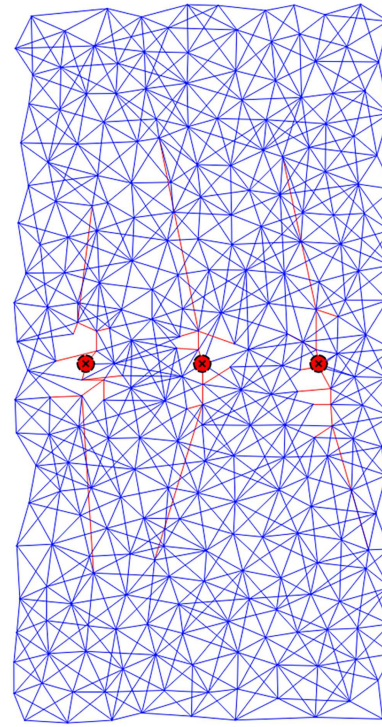
$$m_{k+1} = m_k - \alpha_k \hat{g}_k \quad (24)$$

2.5 Model Validation

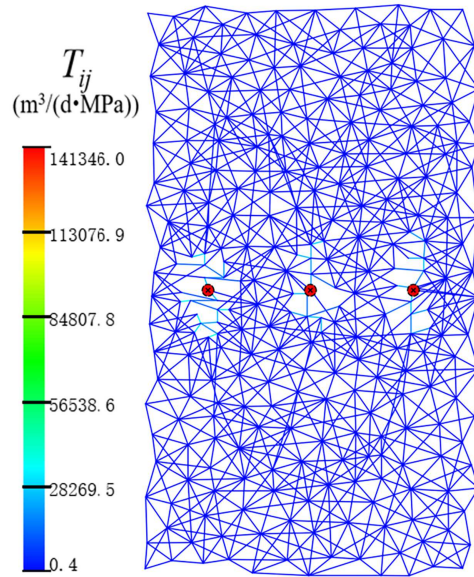
A fracture-network flow simulation is performed for the one-stage, three-cluster fracture-network morphology of the CBM fractured horizontal well shown in Fig. 3a. Based on the computed flow-characteristic parameters, the connection transmissibility and connection volume are obtained as shown in Fig. 3c,d. The comparative simulations were performed using the Eclipse E100 simulator. The reservoir was assumed to be a homogeneous two-dimensional model, and the medium properties were represented by prescribing the average permeabilities of the fracture and matrix, based on which the numerical simulations and comparisons were conducted. The basic model settings are listed in Table 1.



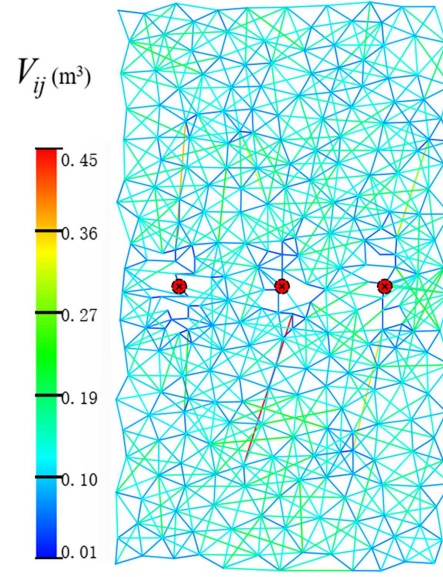
(a) Permeability distribution



(b) Connection generation



(c) Connection transmissibility



(d) Connection volume

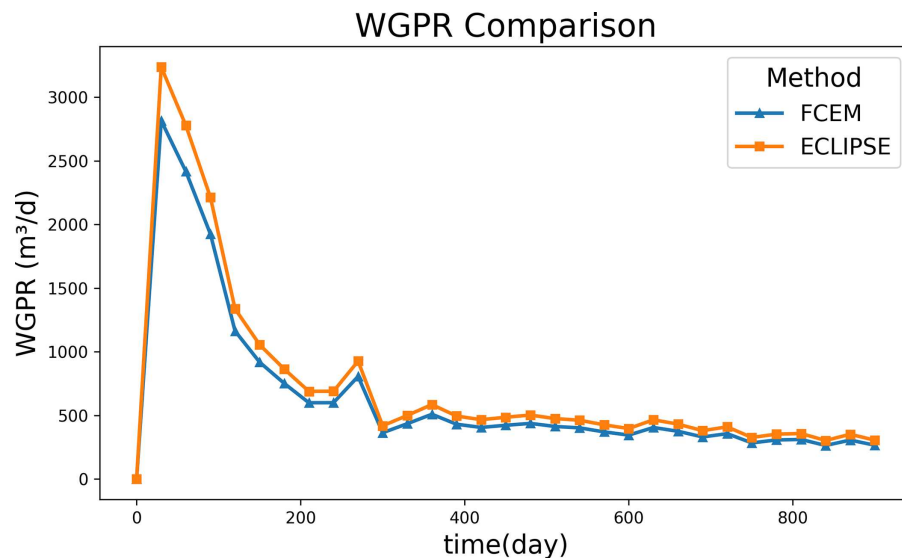
Figure 3: Distribution of reservoir petrophysical parameters and connection parameters.

Table 1: Conceptual case model parameters.

Parameter	Value	Parameter	Value
Number of production wells	1	Eclipse grid size/m	1*1*1
Total simulation time/day	900	Timestep/day	30
Grid length/m	500	Number of connection edges	2230
Grid width/m	261	Rock compressibility/MPa ⁻¹	0.00025
Number of connection nodes	269	Gas density/(kg/m ³)	0.79
Fracture permeability/mD	100	Matrix permeability/mD	0.1
Water saturation	0.3	Porosity	0.0175

To prevent an exponential increase in connections caused by excessive nodes during the initial discretization of complex fracture geometry, a more concise representation is adopted while keeping the overall fracture geometry essentially unchanged. Accordingly, the complex fracture network is simplified using morphological skeletonization combined with the Dijkstra shortest-path algorithm. This preserves the main-trunk topology of the fractures while markedly reducing the number of nodes, thereby improving the efficiency of subsequent flow simulation and computation (Fig. 3b).

Assuming this stage of fractures is simulated under a constant water-rate schedule for 900 days with a 30-day time step, gas production is first computed using the Eclipse numerical simulator. Using the transmissibilities and connection volumes obtained above, the proposed FCEM flow-simulation method is then applied to calculate gas production under the same water-rate schedule, and the results are compared with Eclipse, as shown in Fig. 4.

**Figure 4:** Comparison of single-well daily gas production by two methods.

Under identical conditions, the proposed method's simulation time is 15.2 s, vs. 1138.4 s for Eclipse. The RMSE between the two gas-rate curves is 7.34%, indicating that FCEM greatly improves computational efficiency while maintaining accuracy. Moreover, its nodal representation enables accurate depiction of fracture geometry and supports rapid, efficient fracture-network flow simulation.

3 Parameter Sensitivity on Runtime and Accuracy

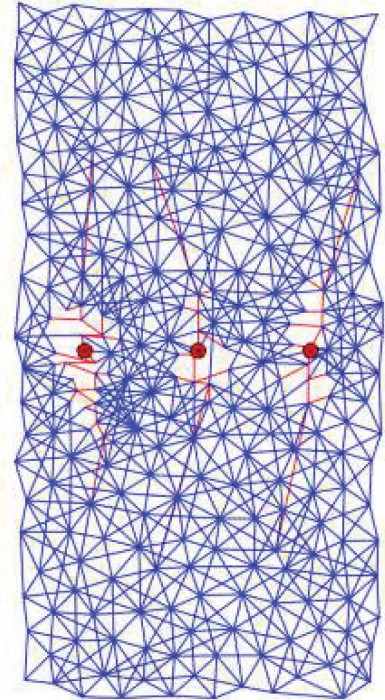
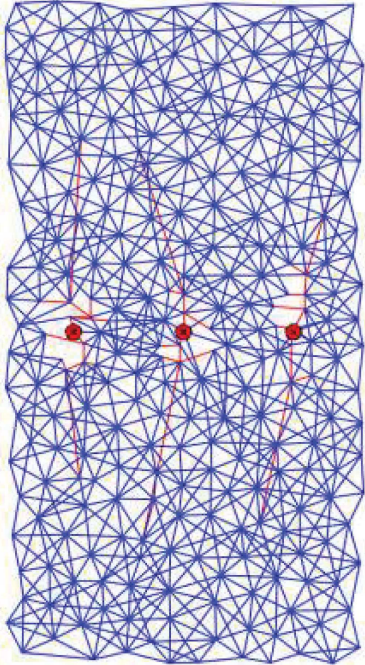
The computational efficiency and accuracy of the model mainly depend on the density of the matrix point cloud and the complexity of the fracture network. The Poisson-disk sampling used in this study and a skeletonization method fused with Dijkstra's shortest-path algorithm simplify the discrete representations of the matrix and fractures, respectively. While preserving the main-trunk topology of the fractures, this approach significantly reduces the number of nodes, thereby improving the efficiency of subsequent flow simulations without sacrificing accuracy. Therefore, this section conducts ablation studies on two key variables—fracture-morphology coarsening and matrix point-cloud density—to determine the optimal modeling standard.

3.1 Effect of Fracture Simplification

According to the Dijkstra algorithm, starting from the perforation point, one can enumerate all possible paths to the fracture tips. We extract all branches connected to the main trunk. Let the main-trunk length be L . Define a cutoff scale ℓ_c ; any branch with length shorter than ℓ_c is removed. We set $\ell_c = \{16, 13, 10, 5\}$ (in units consistent with the discretization) for sensitivity analysis while keeping the point-cloud density fixed. The evaluation metrics are the RMSE between the gas-rate curves computed by the proposed method and by Eclipse, as well as the runtimes of the two methods. The results are shown in [Table 2](#).

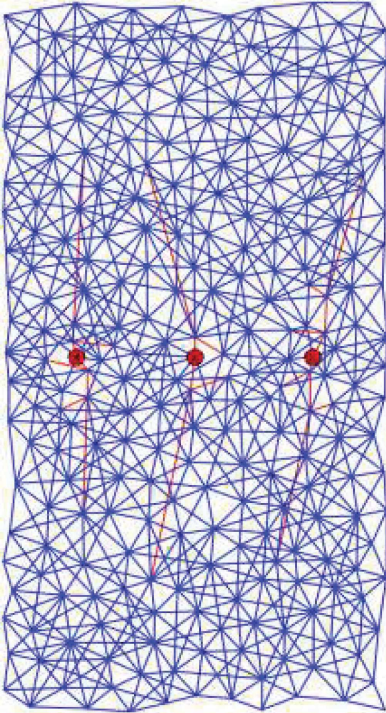
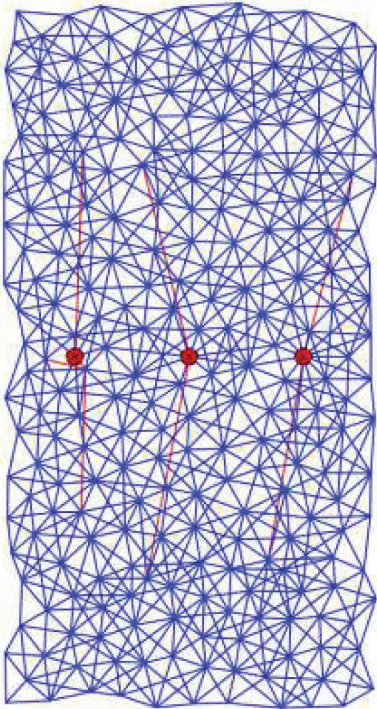
As shown in [Fig. 5](#), the comparisons under different coarsening scales indicate that while computational speed increases as fracture resolution decreases, the error relative to Eclipse grows markedly: at a scale of 16 the error is lowest (6.99%) but the runtime is longer (8.7 s); when the coarsening scale is reduced to 5, the computation is fastest (3.2 s) but the error is highest (19.28%), which is unacceptable for accurate simulation. By contrast, a coarsening scale of 13 achieves a better trade-off between error (7.34%) and runtime (5.4 s), substantially shortening computation time while maintaining high accuracy. Therefore, considering accuracy and efficiency, a coarsening scale of 13 is adopted for the final results.

Table 2: Effect of different coarsening scales on simulation accuracy and runtime.

Coarsening Scale ($\uparrow_c$)	Connection Model	RMSE	Runtime (s)
16		6.99%	8.7
13		7.34%	5.4

(Continued)

Table 2 (continued)

Coarsening Scale ($\Downarrow_c$)	Connection Model	RMSE	Runtime (s)
10		14.31%	4.5
5		19.28%	3.2

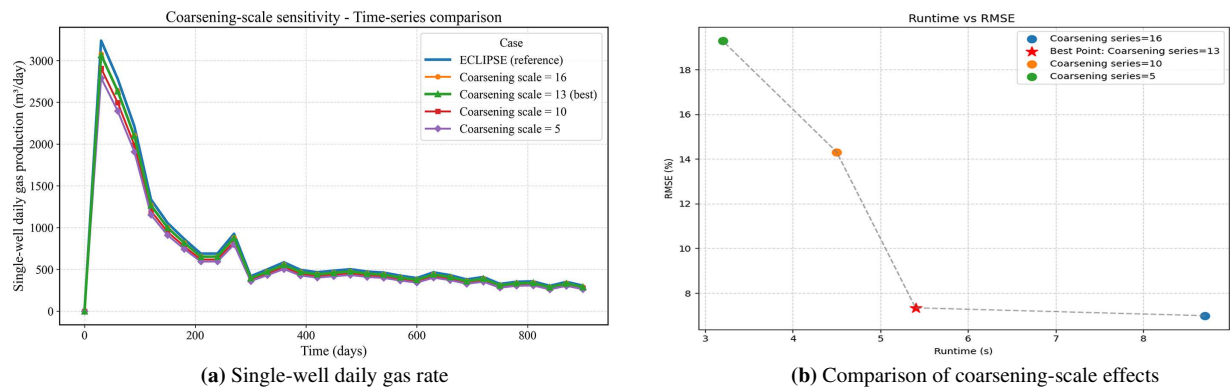
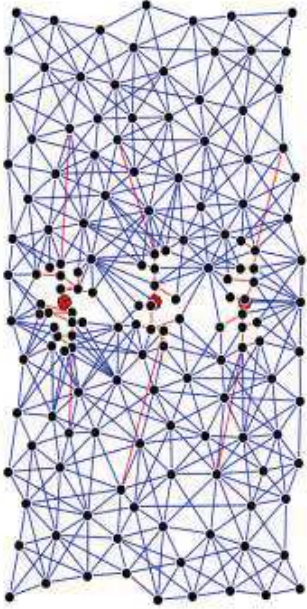


Figure 5: Impact of coarsening scale on gas-production prediction accuracy and computational efficiency.

3.2 Effect of Matrix Point-Cloud Density

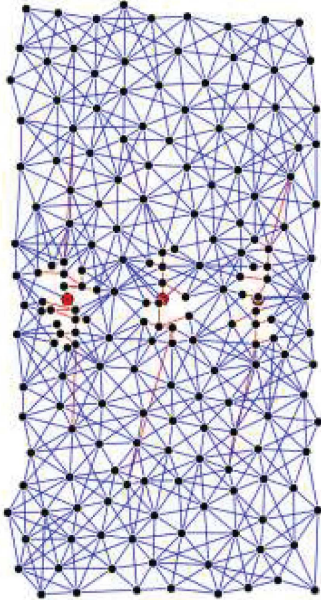
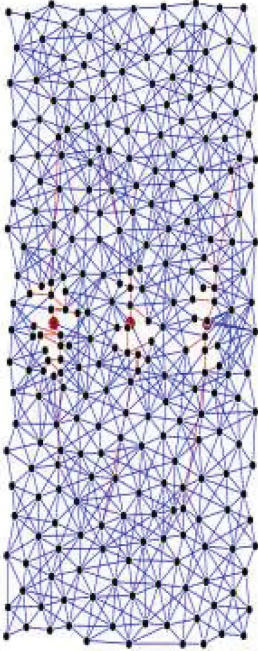
Because matrix points are generated via Poisson-disk sampling, the density is controlled by the minimum allowed distance between points: a candidate point is retained only if its Euclidean distance to all other matrix points exceeds this threshold. Based on the actual fracture morphology in Fig. 3a, we choose minimum interpoint distances of 30, 25, 20, and 15, respectively, and perform comparisons under the same fracture geometry and maximum connection radius. The evaluation metrics are the RMSE of the gas-rate curve relative to Eclipse and the runtime. The results are summarized in Table 3.

Table 3: Effect of different point-cloud radius on simulation accuracy and runtime.

Point-Cloud Radius (m)	Connection Model	RMSE	Runtime (s)
30		26.23%	2.9

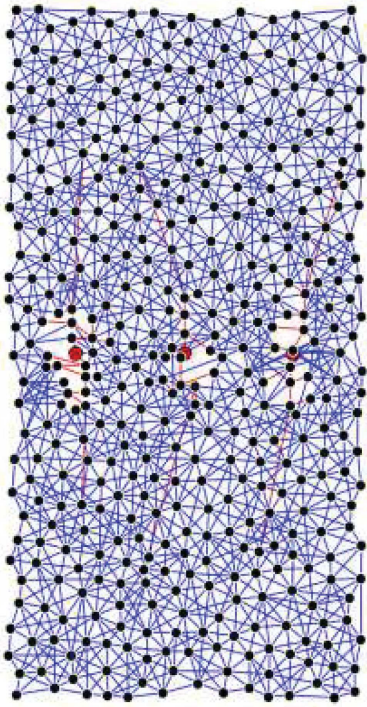
(Continued)

Table 3 (continued)

Point-Cloud Radius (m)	Connection Model	RMSE	Runtime (s)
25		15.68%	4.2
20		7.34%	5.4

(Continued)

Table 3 (continued)

Point-Cloud Radius (m)	Connection Model	RMSE	Runtime (s)
15		5.73%	15.2

As shown in Fig. 6, the comparisons under different point-cloud radii indicate that although computational speed improves as fracture resolution decreases, the error relative to Eclipse increases markedly: with a radius of 15, the error is the lowest (5.73%) but the runtime is the longest (15.28 s); when the point-cloud radius is 30, the computation is the fastest (2.9 s) but accompanied by the highest error (26.23%), which is insufficient to ensure simulation accuracy. By contrast, a radius of 20 achieves a better balance between error (7.34%) and runtime (5.4 s), substantially shortening computation time while maintaining high accuracy. Therefore, considering accuracy and efficiency, a point-cloud radius of 20 is adopted for the final results.

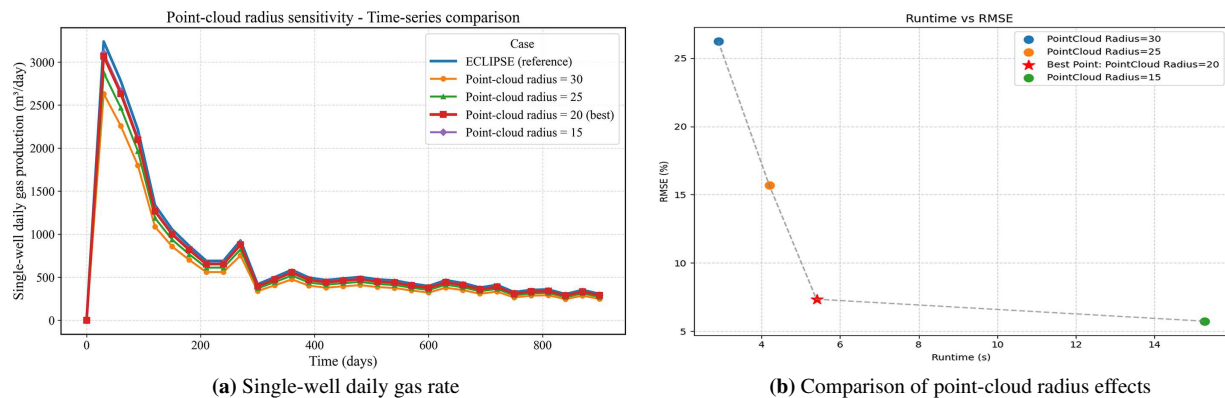


Figure 6: Impact of point-cloud radius on gas-production prediction accuracy and computational efficiency.

4 Field Case Study

To further verify the model's stability, numerical tests under identical operating conditions are conducted based on the actual CBM dual-horizontal-well reservoir model shown in Fig. 7a.

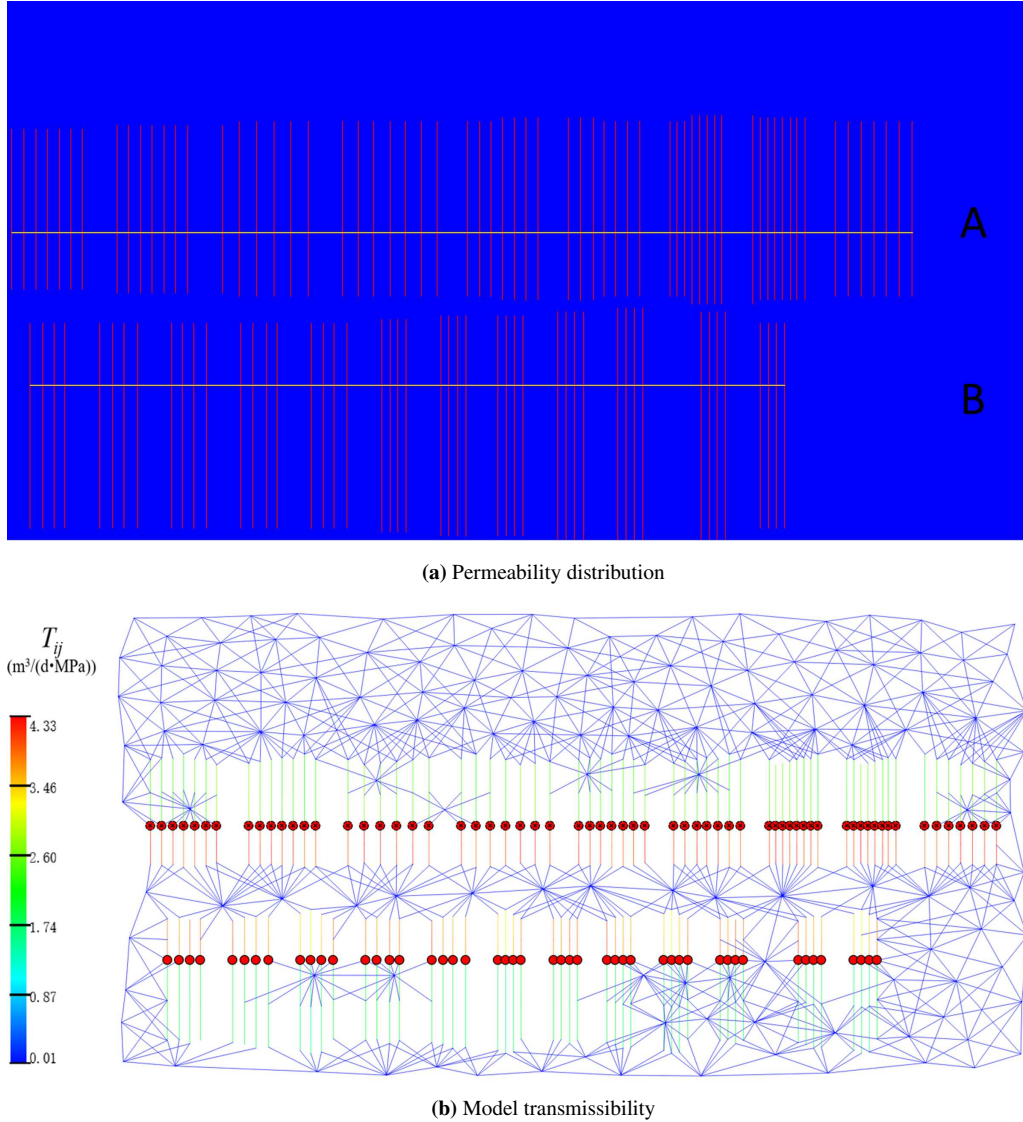
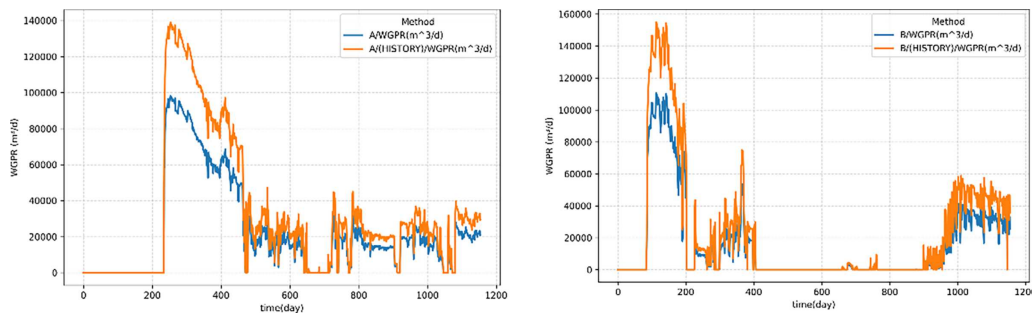


Figure 7: Fracture morphology and connection model for the field well case.

Based on the model parameters in Table 4, the proposed Fracture Connection Element Method (FCEM) is used to model and simulate the reservoir; the modeling result is shown in Fig. 7b. Under a constant water-rate scheme, the single-well daily gas and water rates for Wells A and B are simulated as in Fig. 4. Using a 1-day time step over 1154 days, the computation finishes in 114 s, yielding the simulated production curves. The initial agreement between the simulated and actual daily gas-rate curves is 70.59% for Well A and 71.28% for Well B, as shown in Fig. 8.

Table 4: Field case model parameters.

Parameter	Value	Parameter	Value
Number of production wells	1	Eclipse grid size/m	$1 \times 1 \times 1$
Total simulation time/day	1154	Timestep/day	1
Grid length/m	1300	Number of connection edges	3020
Grid width/m	1000	Rock compressibility/ MPa^{-1}	0.00025
Number of connection nodes	704	Gas density/ (kg/m^3)	0.79
Fracture permeability/mD	100	Matrix permeability/mD	0.1
Water saturation	0.1	Porosity	0.0185

**Figure 8:** Single-well daily gas-rate curves.

As Fig. 8 shows, it is worth noting that the relatively modest pre-history-matching accuracy mainly stems from two factors. First, the reservoir properties are constructed from a limited set of well-log/interpretation data and obtained via interpolation, which inevitably cannot fully capture the true fine-scale heterogeneity and localized preferential flow pathways. Second, to improve computational efficiency, FCEM employs an equivalent connection-element/control-volume discretization to reduce and simplify the complex fracture-network representation, which introduces additional approximation errors. The combined effect of input-data uncertainty and equivalent discretization therefore, leads to noticeable deviations from the measured daily gas-rate curves in certain periods, motivating the subsequent automatic history-matching procedure to calibrate key parameters and improve agreement with field observations. We introduce the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm for automatic history matching to address this. The objective function is constructed from production history (daily gas rate, bottom-hole pressure, etc.), while each connection element's transmissibility and control volume serve as the history-matching variables. SPSA parameters are set to: 200 iterations, step size 0.4, 3 perturbations per iteration, and perturbation step size 0.2. The matched results are shown in Fig. 9.

Building on connection parameters computed from petrophysical properties, FCEM's simulated and history-matched single-well gas-rate curves align closely with the measured trends, achieving 84.87% and 80.31% agreement for Wells A and B, respectively. Compared with conventional ECLIPSE simulations, which require 1,300,000 grid cells for a 1300 m $\times$ 1000 m reservoir, the FCEM model uses only 704 nodes and 3020 connections in a mesh-free representation. While maintaining accuracy, the discretization size is reduced by 1850 $\times$, and runtime drops from hours to minutes, achieving significant acceleration and scalability.

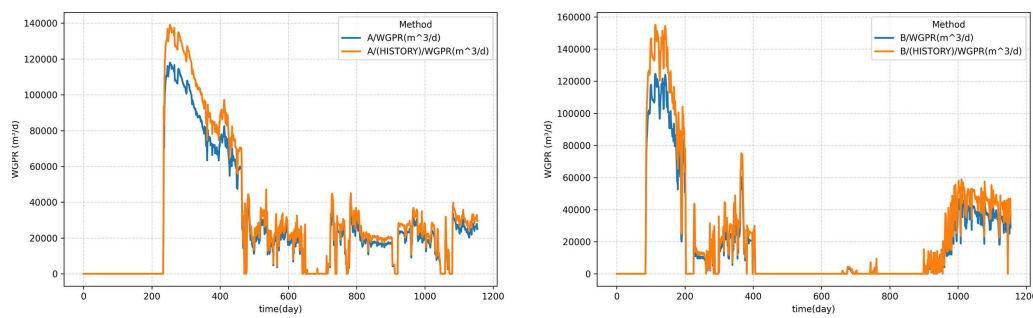


Figure 9: Single-well daily gas-rate curves after history matching.

5 Conclusions

This study addresses the engineering challenges of deep coalbed methane (CBM) by introducing a mesh-free approach to fracture-matrix modeling and flow simulation, based on a connection framework called the Fracture Connection Element Method (FCEM). Initially, Poisson-disk sampling is employed to generate a uniform matrix point cloud, which prevents mesh distortion and resolution issues commonly encountered near complex fractures in traditional grid-based discretization. The complex fracture network is then simplified using morphological skeletonization combined with Dijkstra's shortest-path algorithm, which retains the main trunk and branch structures while discretizing the fractures into a point cloud. Finally, within the FCEM framework, a node-to-node equivalent transmissibility model is developed to represent multiphase flow between the fractures and matrix, as well as within each domain.

In a conceptual CBM example and a field case involving dual horizontal wells, the FCEM method accurately simulates fracture connectivity and greatly improves computational efficiency. However, the current method assumes that fractures extend across the entire thickness of the target sublayer, reducing the 3D problem to a 2D in-plane simulation of fracture-network connectivity and flow. Future work will expand the FCEM method to explicitly account for 3D fracture systems and integrate more complex geological conditions and multiphysics processes, aiming to enhance its applicability and decision-support capabilities for a wider range of engineering scenarios.

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