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ARTICLE

Solving the Generalized Traveling Salesman Problem Using Sequential Constructive Crossover Operator in Genetic Algorithm

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ABSTRACT

The generalized travelling salesman problem (GTSP), a generalization of the well-known travelling salesman problem (TSP), is considered for our study. Since the GTSP is NP-hard and very complex, finding exact solutions is highly expensive, we will develop genetic algorithms (GAs) to obtain heuristic solutions to the problem. In GAs, as the crossover is a very important process, the crossover methods proposed for the traditional TSP could be adapted for the GTSP. The sequential constructive crossover (SCX) and three other operators are adapted to use in GAs to solve the GTSP. The effectiveness of GA using SCX is verified on some GTSP Library (GTSPLIB) instances first and then compared against GAs using the other crossover methods. The computational results show the success of the GA using SCX for this problem. Our proposed GA using SCX, and swap mutation could find average solutions whose average percentage of excesses from the best-known solutions is between 0.00 and 14.07 for our investigated instances.

KEYWORDS

Generalized travelling salesman problem; NP-hard; genetic algorithms; sequential constructive crossover; swap mutation

1 Introduction

The traditional travelling salesman problem (TSP) is an important combinatorial optimization problem (COP) in Operations Research and Computer Science. It can be represented as a network of n nodes (or cities) with a cost (or distance) matrix expressed by $C = [c_{ij}]$. The problem aims to find a least-cost Hamiltonian tour for the salesperson. Various exact and heuristic methods are present in the literature to solve this problem. However, in some real-life circumstances, additional restrictions have to be enforced on this usual TSP that creates some variant TSPs, for example, generalized TSP, multiple

TSP, TSP with backhauls, TSP with time windows, etc. Amongst different variants, the generalized TSP (GTSP) is considered to be studied here.

The GTSP was first presented in [1], an NP-hard problem [2]. In addition to the usual TSP, in this problem, the set of nodes (or cities), denoted by S, is grouped into m sets or clusters, suppose, S_1, S_2, ..., S_m with $S_1 \cup S_2 \cup \ldots \cup S_m = S$ and $S_i \cap S_j = \varnothing$ if $i \neq j$. The problem aims to find a least-cost Hamiltonian tour that contains exactly one node from every set S_i. Fig. 1 shows an example of a GTSP instance with 13 nodes and 7 sets.

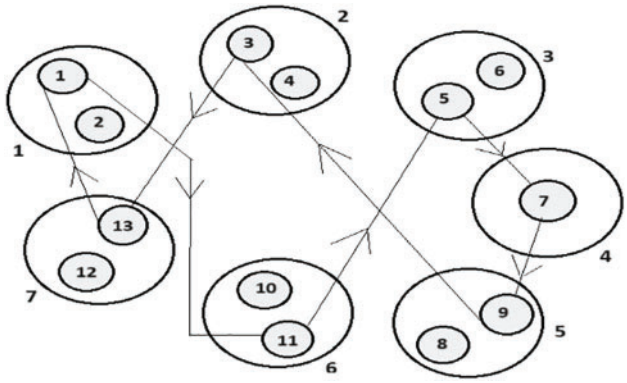


Figure 1: Graphical representation of a GTSP instance with n = 13 and m = 7

The GTSP turns into the usual TSP while the total number of cluster sets and the total number of nodes are same, and every set has only one node. If the associated cost matrix C is symmetric, then the GTSP is symmetric; otherwise, it is asymmetric. The clustered TSP is a variant of GTSP where all nodes in a cluster have to be visited successively before exiting the cluster [3]. The GTSP has many important real-life applications that include airplane routing, mail delivery [4], etc.

Heuristic/metaheuristic and exact procedures can be used to solve the problem. Branch and bound [5], dynamic programming [1], lexisearch algorithm [6], etc., are some well-known exact methods for finding exact solutions. However, finding exact solutions to the problem using these methods becomes impracticable even for moderate-sized problem instances. Therefore, many researchers used heuristic procedures to find heuristic solutions to the problem, and largely modern heuristic approaches to solve such COPs are named metaheuristic procedures. Genetic algorithms [7], ant colony optimization [8], variable neighborhood method [2], particle swarm optimization [9], etc., are some well-known metaheuristic methods for finding heuristic solutions. Generally, heuristic/metaheuristic approaches cannot make sure about exact optimal solutions but can find close to optimal solutions very quickly. Amongst metaheuristic approaches genetic algorithm (GA) is identified as a better metaheuristic technique for solving such COPs.

We are going to develop GA to find heuristic solutions to the problem. In GAs, as the crossover process performs a very crucial role, the crossover methods proposed for the traditional TSP could be adapted for the GTSP. The sequential constructive crossover (SCX) [10] and three other operators are adapted to use in GAs to solve the GTSP. The effectiveness of GA using SCX is verified on some GTSPLIB instances [11] first and then compared against GAs using the other crossover methods. The computational results show the success of the GA using SCX for this problem. Our proposed GA using SCX, and swap mutation could find average solutions whose average percentage of excesses from the best-known solutions is between 0.00 and 14.07 for our investigated instances. Furthermore, we showed a relative rank of the four crossover methods for the GTSP. In this study, we aim to test the

effectiveness of four crossover operators, we do not aim to achieve optimal (or best-known) solutions to the GTSP instances.

This paper is arranged as below: Section 2 briefly surveys the literature on the GTSP. Genetic algorithms using four crossover methods are developed in Section 3, while Section 4 reports the computational experiments of different GAs on asymmetric GTSPLIB instances. At the end, Section 5 describes some comments and closing remarks.

2 Literature Survey

The GTSP was introduced, and a dynamic programming technique was developed to solve the problem in [1], where the visited clusters identify a state. An integer linear programming technique was developed to find the solution to the problem and then applied to both non-Euclidean and Euclidean instances [12]. As reported, the technique solved the instance of size up to 50 cities and 10 clusters. A branch-and-bound technique was developed for solving the problem which was based on the minimal rooted tree [5]. As reported, the technique solved the instance of sizes up to 52 cities and 13 clusters. A branch-and-cut technique by combining branch-and-bound and cutting plane techniques was developed to solve the problem [11]. As reported, the technique solved the instance of sizes up to 442 cities and 89 clusters.

A heuristic method, by combining three techniques—initialization, insertion, and improvemen- was developed to solve the problem [13]. A new GA was proposed and improved by using 2-opt and swap techniques to solve the problem [14]. The Lin-Kernighan-Helsgaun technique was developed for finding solutions to the problem [15], and the technique could enhance the solution for some benchmark instances upon the best existing techniques. A memetic algorithm using breakout local search method was proposed to solve the problem [16]. Based bioinspired computing model, some parallel deoxyribonucleic acid (DNA) algorithms were developed to solve the problem [17]. By combining features of GA, a random-permutation based GA was developed to solve the problem [18]. By merging a core GA using a chromosome enhancement procedure and a shortest path algorithm with some local search techniques, a memetic algorithm was proposed to find the solution to the problem [19]. Using an adaptive neighborhood search technique, a heuristic technique was developed to find the solution to the problem [20]. Three new neighborhoods were illustrated and then integrated with an iterated local search technique to solve the problem [21]. Furthermore, cluster optimization was applied. A shortlist of algorithms for solving the problem is reported in Table 1.

Table 1: Algorithms for the GTSP

Algorithms	References
Dynamic programming	Henry-Labordere [1]
Integer linear programming	Laporte et al. [12]
Branch-and-bound method	Dimitrijevic et al. [5]
Branch-and-cut technique	Fischetti et al. [11]
A fast composite heuristic method	Renaud et al. [13]
Genetic algorithm	Silberholz et al. [14]
Lin-Kernighan-Helsgaun technique	Helsgaun [15]
Memetic algorithm	Bontoux et al. [16], Cosma et al. [19]
Parallel DNA algorithm	Ren et al. [17]

(Continued)

Table 1 (continued)

Algorithms	References
Random-permutation genetic algorithm	Khan et al. [18]
Neighborhood search technique	Smith et al. [20], Schmidt et al. [21]

3 Proposed Genetic Algorithm

Based on the survival-of-the-fittest theory among different populations in natural science, Goldberg proposed genetic algorithms (GAs) to find solutions to optimization problems. Since GAs are very flexible, and can encode the solutions easily as chromosomes, they are very popular. Several chromosomes are produced initially, termed together as the initial population, and then are processed through three major processes (operators), such as selection, crossover, and mutation, to achieve a suitable solution to the problem. Better chromosomes are picked probabilistically in the selection process. Two-parent chromosomes mate to produce suitable offspring chromosome(s) in the crossover process, while some genetic materials in the chromosomes are randomly altered in the mutation process. Amongst the operators, the crossover operator is the most powerful and important process to design and implement the GAs [22]. Therefore, the probability of using crossover is set very high, whereas the probability of mutation is set very low. The selection and the crossover operators are capable of speeding up the convergence of solutions.

3.1 Genetic Coding and the Initial Population

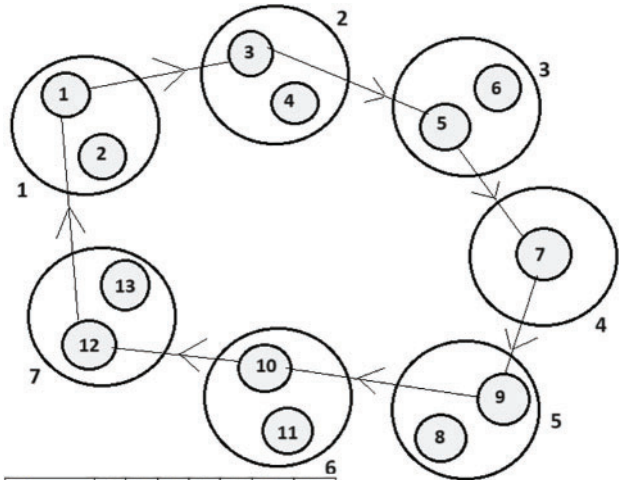
Using a GA to find the solution to a COP, at first, any chromosome representation method for a solution must be described. The most simple and common representation method is the path representation of the cities in a sequence. The bipartite chromosome is considered for the GTSP. It has two rows, where the first row shows the sequence of clusters, and the second row shows the city chosen from each corresponding cluster. The list of clusters (S_1, S_2, . . ., S_m) represents the Hamiltonian cycle {S_1−S_2−. . .−S_m−S_1}. Given a sequence of clusters (S_1, S_2, . . ., S_m), one of the salesman's tours may be as {v_1, v_2. . ., v_m), provided that the city $v_i \in S_i$, for all i ∈ {1, 2 . . ., m}. For example, suppose that n = 13 and m = 7, with the cluster sets: S_1 = {1, 2}, S_2 = {3, 4}, S_3 = {5, 6}, S_4 = {7}, S_5 = {8, 9}, S_6 = {10, 11} and S_7 = {12, 13}. If the sequence of cluster sets is (1, 2, 3, 4, 5, 6, 7, 1), then (1, 3, 5, 7, 9, 10, 12, 1) may be a tour, which is displayed in Table 2 and Fig. 2.

Table 2: A chromosome representation for the GTSP

Clusters	1	2	3	4	5	6	7
Cities	1	3	5	7	9	10	12

In Fig. 2, the second row represents a tour of the salesman whose objective cost is the total cost of the cities that exist in the path. Since the problem is minimization, the fitness cost, suppose f_i, of the i-th chromosome, is calculated as the inverse of the objective cost, o_i.

$$f_i = \frac{1}{1 + o_i} \tag{1}$$



Clusters	1	2	3	4	5	6	7
Cities	1	3	5	7	9	10	12

Figure 2: Representation of a tour by a chromosome

Thus, at the beginning, a population of pre-fixed size P_s is produced arbitrarily. Once the population is generated, the fitness of each chromosome is evaluated, and the population is passed to the next generation.

3.2 Selection Operator

In the selection process, fitter chromosomes are picked and passed to the following generation, and unproductive chromosomes are removed from the population. This operator upgrades the performance of GAs and can create diverse results all over generations. Furthermore, it assists in reliability between exploitation and exploration of the search space.

Various selection methods exist in literature. We consider the roulette wheel selection (RWS) [22] procedure that uses the fitness proportional law. The RWS is a very popular method that assigns every chromosome a portion of a roulette wheel, and the area of a portion is determined by the fitness of the corresponding chromosome. Next, the wheel is spun, and the chromosome that is ultimately pointed out by the pointer is chosen for the next generation of mating pools. Thus, chromosomes with higher fitness values have a bigger possibility of being chosen, and those with lower fitness values have a bigger possibility of being discarded. However, no one can guarantee that chromosomes with higher fitness values will always be selected. The selection probability pi is calculated for each chromosome in a population of size Ps as follows:

$$p_i = \frac{f_i}{\sum_{j=1}^{Ps} f_j}; i \in \{1, 2, \ldots, P_s\} \tag{2}$$

Then, the cumulative probability k_i, $i \in \{0, 1, .., P_s\}$ for each chromosome is calculated ($k_0 = 0$). After that, a random number, $r \in [0,1]$, is generated. If $k_{(i-1)} < r \leq k_i$, for $i \in \{1, 2, \cdots, P_s\}$, then the *i*-th chromosome is selected. The segment size and probability are kept fixed throughout this selection method. This is a very easy and simple method that assigns balanced distributed probabilities to all chromosomes and assigns the highest probability to the best chromosome.

3.3 Crossover Operators

As the crossover process performs a very crucial role, the crossover methods that were constructed for the traditional TSP could be adapted to apply to the GTSP. Here, partially mapped crossover (PMX) [23], ordered crossover (OX) [24], cycle crossover (CX) [25], and sequential constructive crossover (SCX) [10] operators are modified to apply to the GTSP. We plan to compare the working of these four crossover operators. For that, we plan to develop GAs using these crossover operators and run on asymmetric GTSPLIB instances of different sizes. Furthermore, we plan to show a relative rank of these crossover methods for the GTSP.

3.3.1 Partially Mapped Crossover Operator

The partially mapped crossover (PMX) technique was devised for the TSP that describes an exchange map substrings between two randomly selected points in parent chromosomes [23]. It is modified here and then employed on the GTSP as below. Suppose P1 and P2 are two parent chromosomes with costs 407 and 347, respectively, related to the cost matrix indicated in Table 3 with n = 13 and m = 7.

Table 3: Cost matrix

	Cluster	1		2		3		4	5		6		7	
Cluster	City	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0	41	31	86	25	57	7	13	21	19	41	47	41
	2	41	0	38	74	43	98	35	31	11	48	24	69	66
2	3	31	38	0	50	89	7	30	74	69	16	20	58	39
	4	86	74	50	0	89	92	34	9	69	13	44	79	61
3	5	25	43	89	89	0	56	28	35	68	86	82	83	19
	6	57	98	7	92	56	0	85	52	32	77	31	46	85
4	7	7	35	30	34	28	85	0	59	47	36	42	18	20
5	8	13	31	74	9	35	52	59	0	43	86	81	74	12
	9	21	11	69	69	68	32	47	43	0	50	16	95	19
6	10	19	48	16	13	86	77	36	86	50	0	29	8	40
	11	41	24	20	44	82	31	42	81	16	29	0	19	94
7	12	47	69	58	79	83	46	18	74	95	8	19	0	66
	13	41	66	39	61	19	85	20	12	19	40	94	66	0

These same parent chromosomes, as shown in Table 4, will be used to explain all crossover methods.

Table 4: The parent chromosomes P1 and P2

P1	2	5	4	6	7	1	3	P2	1	6	3	4	5	2	7
	3	8	7	10	13	2	5		1	11	5	7	9	3	13

The PMX operation example is presented in Figs. 3 and 4.

In P1 and P2, area between the cutoff points are shown by shading.

P1	2	5	4	6	7	1	3	P2	1	6	3	4	5	2	7
	3	8	7	10	13	2	5		1	11	5	7	9	3	13

Mapping segments of the clusters between them: 6⇔4 and 7⇔5.
These segments are copied to the offspring as follows.

O1	*	*	*	6	7	*	*	O2	*	*	*	4	5	*	*
				10	13							7	9		

Add other clusters with the cities from the alternative parent that do not result in illegal offspring

O1	1	*	3	6	7	2	*	O2	2	*	*	4	5	1	3
	1		5	10	13	3			3			7	9	2	5

The 1st * in O1 should be cluster 6, but is present in O1; thus, added cluster 4 with city 7 using the map 6⇔4.
The 2nd * in O1 should be cluster 7, but is present in O1; thus, added cluster 5 city 9 using the map 7⇔5. So, O1 is also created of cost 290.
Similarly, O2 is also created of cost 305.

O1	1	4	3	6	7	2	5	O2	2	7	6	4	5	1	3
	1	7	5	10	13	3	9		3	13	10	7	9	2	5

Figure 3: Steps of building the offspring O1 and O2 from parents P1 and P2 using PMX

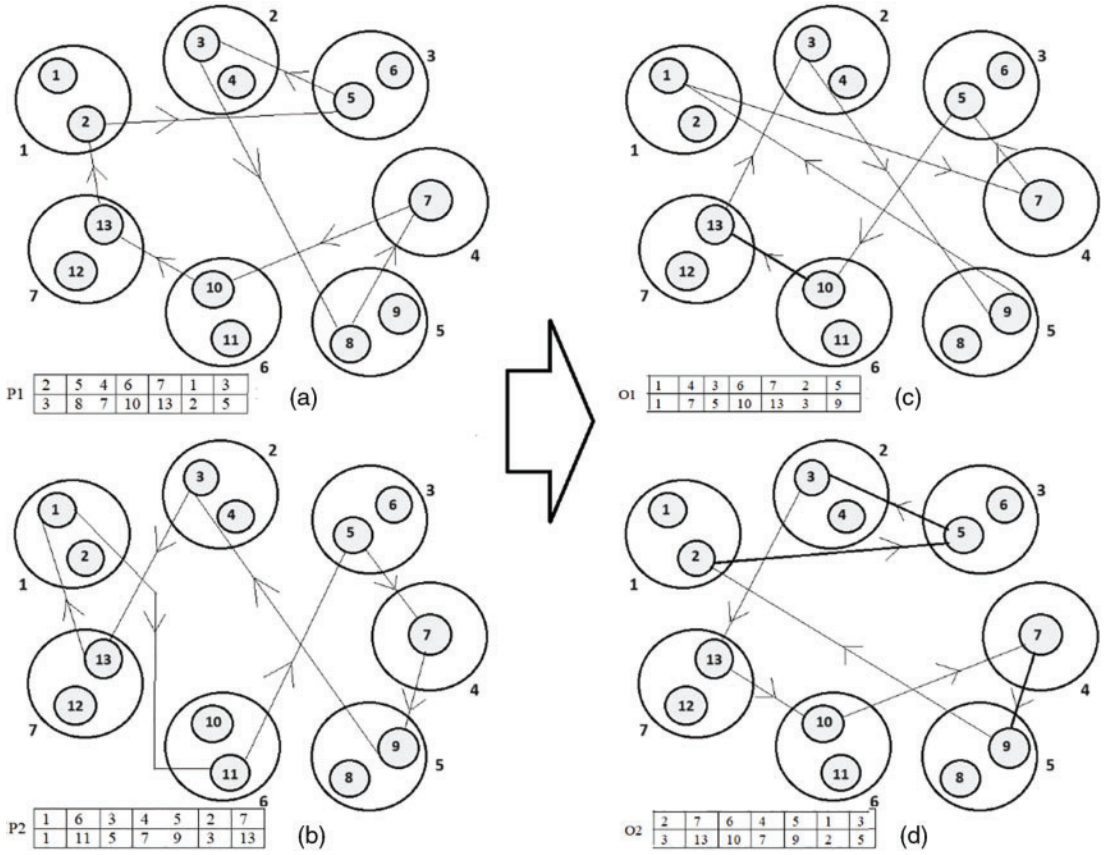


Figure 4: Illustration of the PMX. (a) The first parent chromosome P1. (b) The second parent chromosome P2. (c) The first offspring chromosome O1. (d) The second offspring chromosome O2

In GAs, generally, the crossover inherits the parents' characteristics. The crossover operator, which can preserve good characteristics of parents as offspring, is supposed to be a better operator. The parent chromosomes are shown in Fig. 4a,d. The bold edges shown in Fig. 4c,d are the edges that exist in either one of the parent chromosomes. In O1 and O2, only one and three edges respectively out of seven edges are chosen from either of the parent chromosomes. This means that 14.29% and 42.86% of the edges of the offspring chromosomes are chosen from the parent chromosomes.

3.3.2 Ordered Crossover Operator

The ordered crossover (OX) technique was devised for the traditional TSP that creates offsprings by picking a subchromosome from a chromosome and saving the relative cluster sequence with subsequent cities from another [24]. It is modified here and then applied to the GTSP as in Figs. 5 and 6.

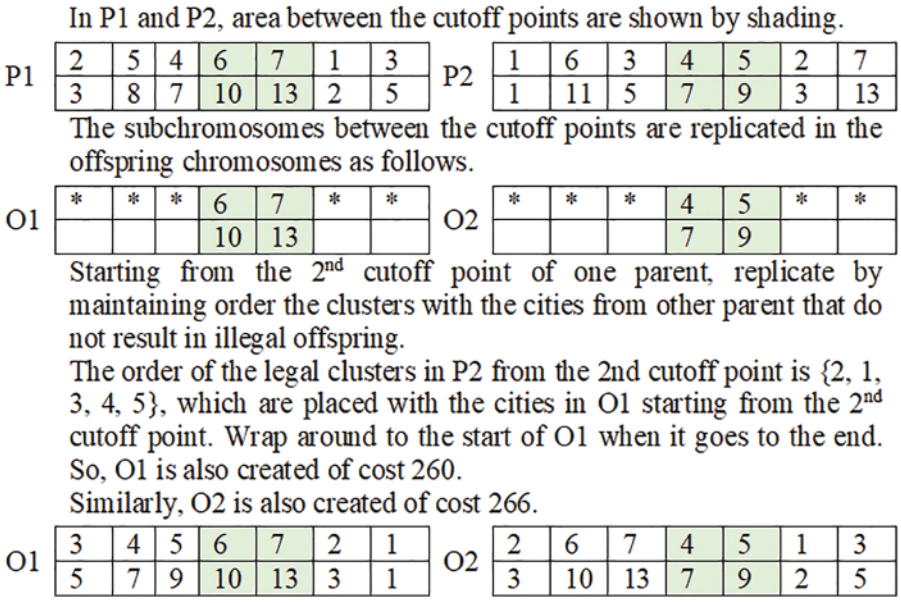


Figure 5: Steps of building the offspring O1 and O2 from parents P1 and P2 using OX

The parent chromosomes are shown in Fig. 6a,b. The bold edges shown in Fig. 6c,d are the edges that exist in either one of the parent chromosomes. In O1 and O2, only three and four edges respectively out of seven edges are chosen from either of the parent chromosomes. This means that 42.86% and 57.14% of the edges of the offspring chromosomes are chosen from the parent chromosomes.

3.3.3 Cycle Crossover Operator

The cycle crossover (CX) technique was devised for the traditional TSP that creates offspring so that each cluster (and its position) and the subsequent city start from one of the chromosomes [25]. It is modified here and then applied to the GTSP as in Figs. 7 and 8.

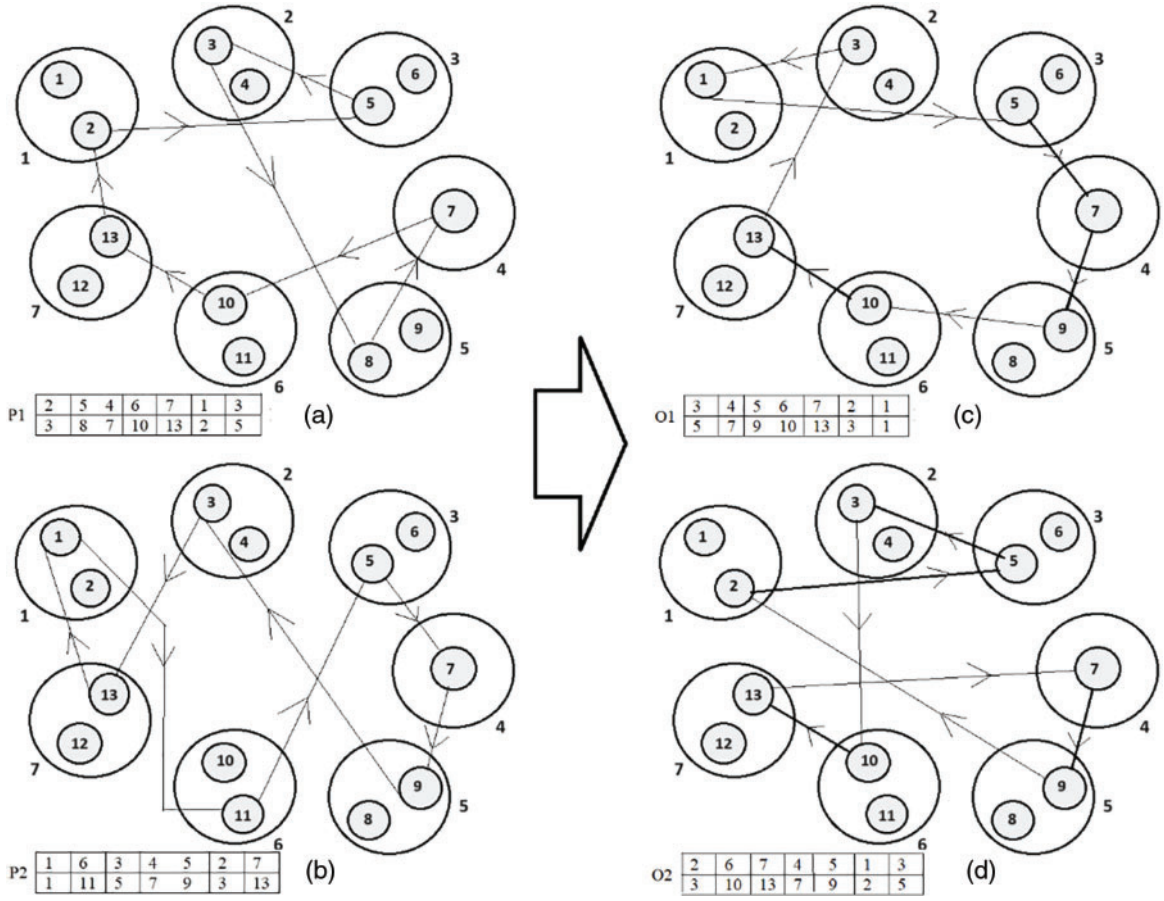


Figure 6: Illustration of the OX. (a) The first parent chromosome P1. (b) The second parent chromosome P2. (c) The first offspring chromosome O1. (d) The second offspring chromosome O2

P1

2	5	4	6	7	1	3
3	8	7	10	13	2	5

P2

1	6	3	4	5	2	7
1	11	5	7	9	3	13

The O1 and O2 are generated by considering the 1st clusters and their corresponding cities from P1 and P2, respectively.

O1

2	*	*	*	*	*	*
3						

O2

1	*	*	*	*	*	*
1						

Every cluster in the offspring must be included from one of its parents (from the same place).

In O1, the present cluster 2 is in 1st position. In P2, the cluster in the 1st position is 1, which is in the 6th position in P1. So, this cluster 1 with city 2 is copied and placed in 6th position in O1.

Similarly, in O2, the present cluster 1 is in 1st position. In P1, the cluster in the 1st position is 2, which is in the 6th position in P2. So, this cluster 2 with city 3 is copied and placed in 6th position in O2.

Figure 7: (Continued)

O1	2	*	*	*	*	1	*	O2	1	*	*	*	*	2	*
	3					2			1					3	

In O1, the present cluster 1 is in 6th position. In P2, the cluster in the 6th position is 2, which is already present in O1. So, a cycle is completed.
Similarly, in O2, the present cluster 2 is in 6th position. In P1, the cluster in the 6th position is 1, which is already present in O2. So, a cycle is completed.
Thus, choose the remaining clusters with their corresponding cities from the other parent.
Hence, complete offsprings are created of costs 293 and 301, respectively.

O1	2	6	3	4	5	1	7	O2	1	5	4	6	7	2	3
	3	11	5	7	9	2	13		1	8	7	10	13	3	5

Figure 7: Steps of building the offspring O1 and O2 from parents P1 and P2 using CX

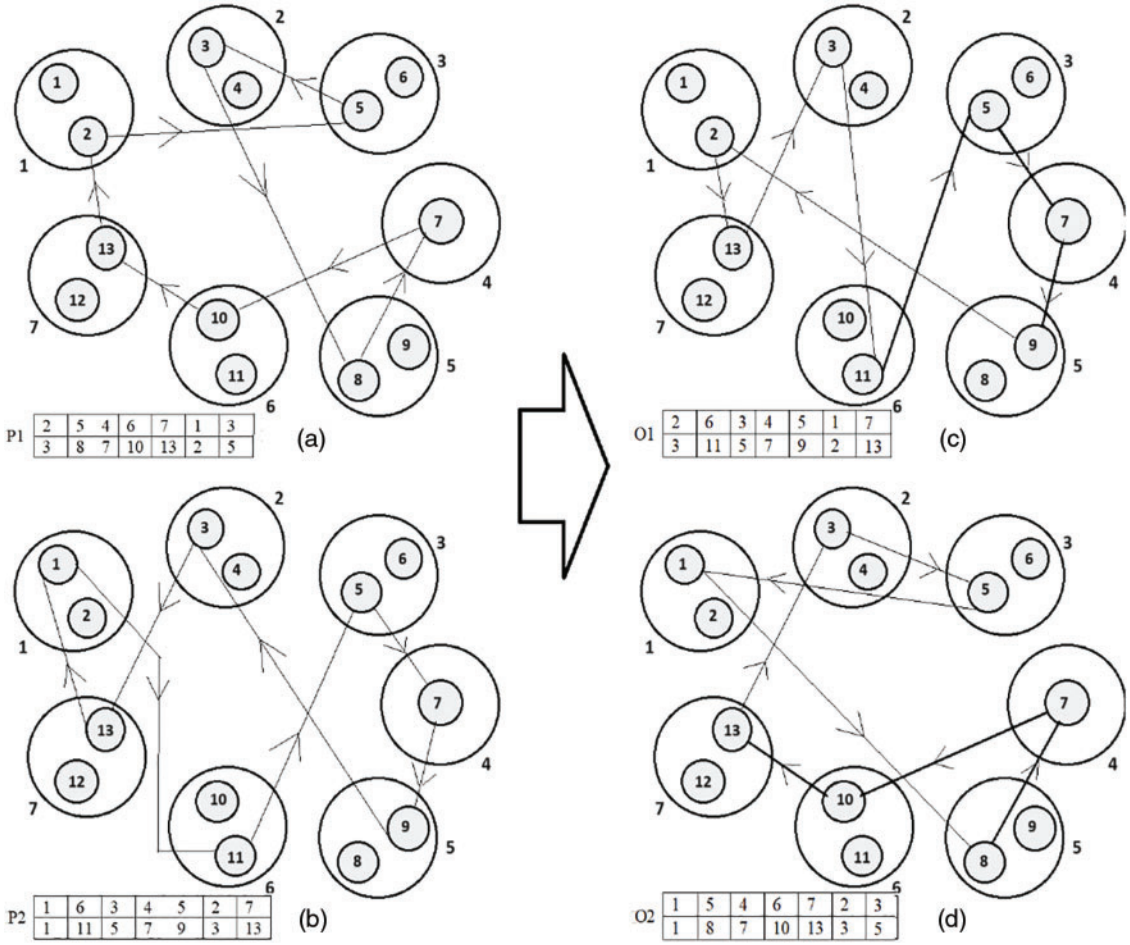


Figure 8: Illustration of the CX. (a) The first parent chromosome P1. (b) The second parent chromosome P2. (c) The first offspring chromosome O1. (d) The second offspring chromosome O2

The parent chromosomes are shown in Fig. 8a,b. The bold edges shown in Fig. 8c,d are the edges that exist in either one of the parent chromosomes. In both O1 and O2, only three edges out of seven edges are chosen from either of the parent chromosomes. This means that 42.86% of the edges of the offspring chromosomes are chosen from the parent chromosomes.

3.3.4 Sequential Constructive Crossover Operator

The sequential constructive crossover (SCX) technique was devised for the traditional TSP which produces one offspring only that checks sequentially both parents and picks the 1st untouched (legitimate) gene from everyone that is seen after the current one. If a legitimate gene is not detected in a parent, it wraps around the parent and chooses the first legitimate gene. Then the cost of each legitimate gene from the current gene is computed, and the cheaper one is augmented to the offspring [10]. This technique was applied successfully to other combinatorial optimization problems ([26–28]). We adapted this technique to the GTSP and reported in Algorithm 1 [7].

Algorithm 1 : Sequential constructive crossover algorithm

Input: Cost matrix C, Crossover probability P_c, Parent chromosomes.
Output: Offspring chromosome.
Generate a random number $r \in [0,1]$.
if $(r \leq P_c)$ then **do**
  Set t = first cluster, p = first city from first parent.
  The offspring includes 'cluster t' with 'city p'.
  for i = 2 **to** m **do**
    In both chromosomes, take the first 'legitimate cluster' which is found after the 'cluster t'.
    if no 'legitimate cluster' is present in any chromosome, **then**
      Search from start of that chromosome (wrap around) and take the first 'legitimate cluster' that is found after the 'cluster t'.
    end if
    Suppose 'cluster x' with 'city α' and 'cluster y' with 'city β' are found in the first and second chromosomes, respectively.
    if $(c_{p\alpha} < c_{p\beta})$ then **do**
      The 'cluster x' with 'city α' is attached to the offspring.
    Else
      The 'cluster y' with 'city β' is attached to the offspring.
    end if
    Rename the current cluster as the 'cluster t' and the current city as the 'city p', and continue.
  end for
end if
Return the offspring chromosome

The SCX operation example is presented in Fig. 9 and 10. The chromosomes P1 and P2 in Fig. 10a,b, respectively, are the parent chromosomes, while O in Fig. 10c is their offspring. The bold edges shown in Fig. 10c are the edges that exist in either one of the parents. Five edges of seven edges are chosen from either of the parents. This means that 71.44% of the edges of the offspring are chosen from the parents. Out of five edges, there is no common edge selected from either of the parents; only one edge (7, 10) is chosen from P1, while four edges (13, 1), (5, 70), (9, 3) and (3, 13) are chosen from P2. The edges (1, 5) and (10, 9) are new edges that have smaller costs. Thus, SCX can preserve better characteristics of the parents in their offspring. Furthermore, SCX can create various offspring.

P1

2	5	4	6	7	1	3
3	8	7	10	13	2	5

P2

1	6	3	4	5	2	7
1	11	5	7	9	3	13

The offspring O is created by choosing the 1st cluster 2 and its corresponding city 3 from P1.

The legitimate clusters after cluster 2 in P1 and P2 are cluster 5 with city 8 and cluster 7 with city 13, respectively. The costs from city 3 to city 8 and from city 3 to city 13 are 74 and 39, respectively. Since cluster 7 with city 13 is cheaper, it is added to the offspring.

O

2	*	*	*	*	*	*
3						

=>

2	7	*	*	*	*	*
3	13					

The legitimate clusters after cluster 7 in P1 and P2 are cluster 1 with city 2 and cluster 1 (wrapped around) with city 1, respectively. The costs from city 13 to city 2 and from city 13 to city 1 are 66 and 41, respectively. Since cluster 1 with city 1 is cheaper, it is added to the offspring.

The legitimate clusters after cluster 1 in P1 and P2 are cluster 3 with city 5 and cluster 6 with city 11, respectively. The costs from city 1 to city 5 and from city 1 to city 11 are 19 and 94, respectively. Since cluster 3 with city 5 is cheaper, it is added to the offspring.

O

2	7	1	*	*	*	*
3	13	1				

=>

2	7	1	3	*	*	*
3	13	1	5			

The legitimate clusters after cluster 3 in P1 and P2 are cluster 5 (wrapped around) with city 8 and cluster 4 with city 7, respectively. The costs from city 5 to city 8 and from city 5 to city 7 are 35 and 28, respectively. Since cluster 4 with city 7 is cheaper, it is added to the offspring. Finally, complete offspring O of cost 288 is created.

O

2	7	1	3	4	*	*
3	13	1	5	7		

=>

2	7	1	3	4	6	5
3	13	1	5	7	10	9

Figure 9: Steps of building the offspring O from parents P1 and P2 using SCX

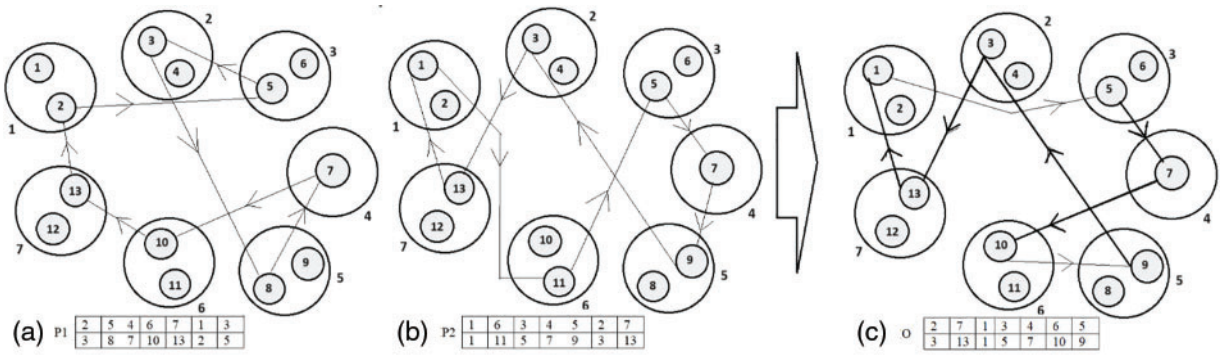


Figure 10: Illustration of the SCX. (a) The first parent chromosome P1. (b) The second parent chromosome P2. (c) The only offspring chromosome O

3.4 Mutation Operator

Mutation operators can increase the variety in the population by initiating random variations in the GA population. It randomly chooses some gene(s) in a parent chromosome and then changes the

genes(s); thus, changing the information. We apply the swap mutation (SM) technique with probability, P_{mt}, that randomly picks two genes from a chromosome and then swaps them [29].

Suppose P is a parent chromosome, and the 3rd and 6th positions are randomly picked. Then cluster 1 with city 1 and cluster 6 with city 10 are exchanged in their positions to create the mutated chromosome E as follows.

P

2	7	1	3	4	6	5
3	13	1	5	7	10	9

=> E

2	7	6	3	4	1	5
3	13	10	5	7	1	9

3.5 Simple Genetic Algorithms

We have developed simple GAs using the traditional GA methods and GA procedures. They start with an arbitrary initial population, choose stronger chromosomes by using the roulette wheel selection technique, apply a selected crossover technique, and swap mutation technique. A simple GA is defined in Algorithm 2.

Algorithm 2: Simple genetic algorithm

Input: *n, m, C,* P_s, P_c, P_{mt}.
Output: *Best chromosome along with its cost.*
Create an initial population of pre-fixed size P_s *randomly.*
Evaluate the population.
Generation $= 0$.
While *the stopping criterion is not achieved.*
Generation $=$ *Generation* $+ 1$.
Select better chromosomes using a selection technique.
Perform a crossover technique with probability P_c.
Perform swap mutation technique with probability P_{mt}.
Evaluate the population.
end while
Return *the best chromosome along with its cost.*

4 Computational Experiments

Four simple GAs with the crossover methods–PMX, OX, CX, and SC-with swap mutation (SM) are encoded in Visual C++ and tested on asymmetric GTSPLIB problem instances [11] executed on a laptop with the specification as Intel(R) Core(TM) i7-1065G7 CPU@1.30 GHz and 8.00 GB RAM under MS Windows 11. Every GA is run 50 times for every instance. For each instance, we report the best-known solution (BKS) reported in [30], the best-obtained solution (BOS), the average obtained solution (AOS), and the average computational time (ACT) (in seconds) to notice the BOS for the first time within 50 executions, by all algorithms. Since GAs are dominated by the parameters- population size (P_s), crossover probability (P_c), mutation probability (P_{mt}), and a stopping condition.

First, we check the behavior of the crossover techniques on the asymmetric instances. We set $P_s = 100$, $P_c = 1.0$, and at most 10n generations for stopping the GAs. Table 5 shows a comparative study among different GAs using different crossover methods with no mutation technique. We indicated the best results by **boldfaces**.

Table 5: Results by GAs using crossover methods without a mutation method for some instances

Instance	BKS	PMX			OX			CX			SCX		
		BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT
4br17	31	**31**	**31.00**	0.00	**31**	**31.00**	0.00	**31**	**31.00**	0.00	**31**	**31.00**	0.00
7ftv33	476	**476**	534.30	0.00	**476**	501.00	0.00	**476**	543.75	0.00	**476**	**480.85**	0.00
8ftv35	525	545	582.25	0.00	**525**	561.25	0.00	579	645.25	0.00	**525**	**556.55**	0.00
8ftv38	511	554	599.10	0.00	**511**	535.75	0.00	569	628.30	0.00	**511**	**516.55**	0.00
9p43	5563	**5563**	5570.65	0.00	**5563**	**5564.60**	0.01	5564	5572.30	0.00	**5563**	5565.65	0.00
9ftv44	510	**510**	562.40	0.00	**510**	**552.45**	0.00	567	678.10	0.00	**510**	522.80	0.00
10ftv47	569	594	672.75	0.00	589	607.90	0.00	697	783.05	0.00	**569**	**606.90**	0.00
10ry48p	6284	6547	6905.90	0.00	6339	6583.05	0.00	6987	7468.35	0.00	6320	**6481.95**	0.00
11ft53	2648	2711	2914.75	0.00	2654	2886.50	0.00	2850	3318.90	0.00	2656	**2712.80**	0.00
12ftv55	689	751	784.80	0.00	**689**	714.75	0.00	817	953.55	0.00	**689**	**690.20**	0.00
13ftv64	708	793	867.05	0.00	763	821.75	0.00	896	1132.20	0.00	735	**777.05**	0.00
14ft70	7707	8202	8721.15	0.02	7978	**8260.50**	0.01	8819	9508.30	0.00	8074	8271.10	0.00
15ftv70	594	687	759.85	0.00	625	726.05	0.00	947	1157.05	0.00	613	**652.10**	0.00
20kro124p	11203	13808	15975.80	0.01	13549	14963.10	0.01	21667	23278.80	0.00	11999	**12753.25**	0.00
35ftv170	1205	1515	1773.75	0.03	1532	1743.30	0.06	3377	2772.60	0.01	1359	**1463.35**	0.01
65rbg323	471	673	765.25	0.13	637	702.55	1.05	950	1014.80	0.02	548	**578.65**	0.07
72rbg358	693	1033	1090.90	0.10	860	935.20	1.79	1105	1215.85	0.02	803	**849.55**	0.09
81rbg403	1170	1235	1567.25	0.14	1244	1302.40	3.04	1491	1531.90	0.05	1290	**1319.10**	0.12
89rbg443	632	920	1187.35	0.34	976	1038.75	3.80	1356	1443.60	0.05	748	**799.70**	0.20

Observing the boldfaces in Table 5, the GA using SCX is shown to be the topmost one. Observing the best-obtained solution, GAs using PMX, OX, CX, and SCX, could obtain the BKS (minimum once in 50 runs) for four, seven, two, and eight instances respectively. So, we can say that SCX, OX and PMX are the topmost, second topmost and third topmost crossover methods, while CX is the most inferior one. Furthermore, observing the average obtained solution in the table, one can draw the same conclusion. Fig. 11 shows the percentage of the excess of the average obtained solution over BKS, which is calculated by the formula: Average Excess (%) $= 100 \times (\text{AOS/BKS} - 1)$. Looking at the figure, we have the same observation. So, this figure also validates the success of GA using SCX.

We further conducted a two-tailed Student *t*-test with a 5% significance level between GA using SCX and GAs using other crossover operators for the above instances to verify our above interpretations. We reported the results in Table 6. As presumed, SCX is the best-ranked crossover, OX is the second-best, PMX is the third-ranked, and CX is the most inferior.

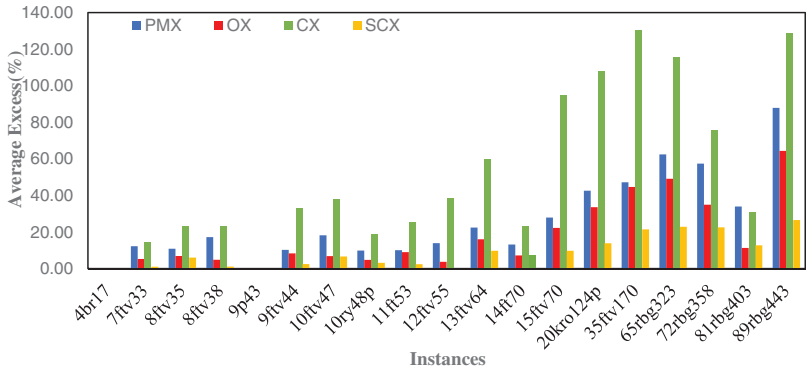


Figure 11: Average Excess (%) by crossover methods without a mutation for some instances

Table 6: Results of the *t*-test on antisymmetric GTSPLIB instances

Crossover	Inferior crossovers
SCX	PMX, OX, CX
OX	PMX, CX
PMX	CX

We now run the GAs using crossovers and the swap mutation operator. Naturally, setting the same mutation probability for every instance is a very difficult job. After so many trials, we set $P_c = 0.90$, $P_{mt} = 0.25$, and at most 20n generations for stopping the GAs. We reported a comparative study amongst the GAs using various crossover techniques and SM in Table 7.

Table 7: Results by GAs using different crossover methods and SM for some instances

Instance	BKS	PMX			OX			CX			SCX		
		BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT
4br17	31	**31**	**31.00**	0.00	**31**	**31.00**	0.00	**31**	**31.00**	0.00	**31**	**31.00**	0.00
7ftv33	476	**476**	488.12	0.00	**476**	486.20	0.00	**476**	511.50	0.00	**476**	**476.00**	0.00
8ftv35	525	**525**	560.30	0.00	**525**	554.20	0.00	**525**	565.30	0.00	**525**	**534.80**	0.00
8ftv38	511	513	531.00	0.00	**511**	518.60	0.00	516	541.00	0.00	**511**	**512.45**	0.00
9p43	5563	**5563**	5563.80	0.00	**5563**	5564.65	0.00	**5563**	5563.90	0.00	**5563**	**5563.00**	0.00
9ftv44	510	**510**	539.20	0.00	**510**	534.20	0.00	543	550.65	0.00	**510**	**517.10**	0.00
10ftv47	569	578	589.20	0.00	573	583.70	0.00	575	602.25	0.00	572	**580.60**	0.00
10ry48p	6284	6319	6484.95	0.01	6309	6435.45	0.01	6309	6450.75	0.00	**6284**	**6309.95**	0.00
11ft53	2648	2683	2780.75	0.01	2673	2729.55	0.01	2674	2734.35	0.01	2651	**2668.65**	0.00
12ftv55	689	**689**	698.55	0.00	**689**	693.55	0.00	**689**	715.05	0.01	**689**	**690.13**	0.00
13ftv64	708	756	820.20	0.01	735	800.20	0.01	751	829.65	0.01	**708**	**733.45**	0.00
14ft70	7707	7980	8283.20	0.01	7960	8233.20	0.01	7965	8226.40	0.01	7922	**8047.25**	0.01
15ftv70	594	646	693.55	0.01	636	643.55	0.01	647	676.30	0.01	**594**	**604.40**	0.00
20kro124p	11203	13564	15867.85	0.04	13564	14767.85	0.04	13572	14493.50	0.05	12108	**12779.50**	0.01
35ftv170	1205	1502	1595.32	0.31	1414	1485.65	0.31	1636	1807.85	0.33	1281	**1370.55**	0.38

(Continued)

Table 7 (continued)

Instance	BKS	PMX			OX			CX			SCX		
		BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT	BOS	AOS	ACT
65rbg323	471	695	756.70	1.03	625	683.32	1.03	636	678.10	1.88	500	**531.10**	3.78
72rbg358	693	987	1037.55	1.77	878	908.63	1.59	869	900.60	2.65	708	**749.80**	7.11
81rbg403	1170	1357	1388.40	2.25	1267	1298.40	2.15	1317	1349.55	3.77	1271	1300.40	9.34
89rbg443	632	893	1134.75	2.79	897	986.24	2.79	910	987.85	3.89	**632**	688.60	6.48

Observing the boldfaces in Table 7, the GA using SCX and SM is proven to be the topmost GA. Observing the best-obtained solution, the GA using PMX and SM, the GA using OX and SM, the GA using CX and SM, and the GA using SCX and SM, could obtain the BKS (minimum once in 50 runs) for six, seven, five, and eleven instances respectively. From this examination, we can confirm that the GA using SCX and SM is the topmost GA, the GA using OX and SM is the second topmost, the GA using PMX and SM is the third topmost, and the GA using CX and SM is the most inferior. Furthermore, observing the average obtained solution in the table, we can draw the same conclusion.

Looking at the average obtained solutions in Table 7, we can have a conclusion. GA using SCX and SM is the best one, GA using OX and SM is the second best, GA using CX and SM is the third best, and GA using PMX and SM is the worst. The results shown in Fig. 12 also prove the success of GA using SCX and SM.

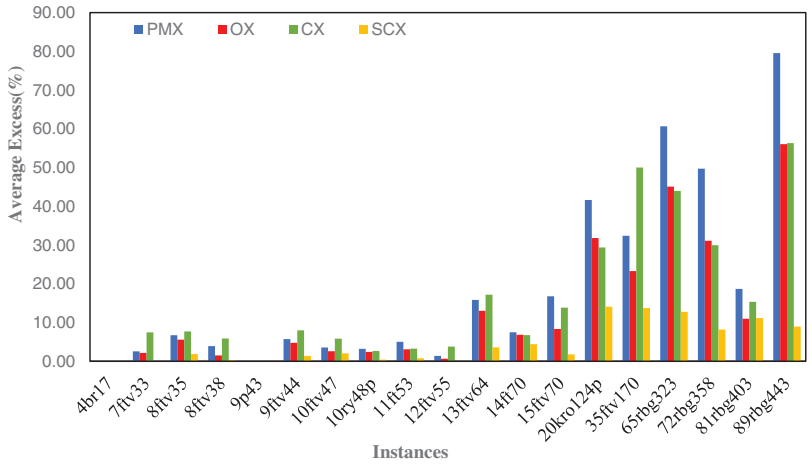


Figure 12: Average Excess (%) by crossover methods with SM operator for some instances

5 Conclusions and Future Work

We have modified ordered crossover (OX), sequential constructive crossover (SCX), partially mapped crossover (PMX), and cycle crossover (CX) for genetic algorithms (GAs) to find solutions to the generalized traveling salesman problem (GTSP). We then illustrated the working of the crossover techniques using an example and encoded the GAs using these crossover methods in Visual C++. Then a comparative study among PMX, OX, CX, and SCX for some asymmetric GTSPLIB instances. On

the basis of solution quality, it is followed that crossover PMX is better than crossover CX, crossover OX is better than crossover PMX, and crossover SCX is the topmost. Furthermore, GAs using four crossover techniques and the swap mutation (SM) were compared and found that GA using PMX and SM is the most inferior and GA using SCX and SM is the topmost. Thus, SCX with or without a mutation technique is the topmost crossover technique. Our GA using SCX and SM could find average solutions whose average percentage of excesses from the best-known solutions is between 0.00 and 14.07 for our investigated instances.

In this current study, we aimed to compare the solutions by four crossover techniques to find solutions to the GTSP. Our aim was not to increase the solution quality by a GA using any crossover technique merged with a mutation technique. So, we did not utilize any local search method in GAs to increase the solution quality, and so, we could not find optimal/best-known solutions for many problem instances. However, the integration of a better local search method into the algorithm might solve those instances optimally that is under our future investigation. Furthermore, our SCX can also be modified to apply to other combinatorial optimization problems, which is also under our next investigation.

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Availability of Data and Materials: The datasets generated and/or analyzed during the current study are available in the GTSPLIB repository, https://www.cs.rhul.ac.uk/home/zvero/GTSPLIB/ (accessed on 25/02/2024).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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