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A Hybrid Constitutive Model and Machine Learning Approach for Predicting the Dynamic Mechanical Behavior of SiCp/Al Composites

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ABSTRACT: SiCp/Al particle-reinforced aluminum matrix composites exhibit significant strain-rate sensitivity under high-speed impact and dynamic forming conditions, and their mechanical response is further affected by the volume fraction and particle size of the reinforcement phase. To describe these coupled effects, this study developed a hybrid constitutive modeling framework combining microstructure-based finite element simulation, the Johnson–Cook constitutive model, and machine learning. First, three-dimensional microstructural models of SiCp/7075Al composites were established using DIGIMAT FE and ABAQUS, and dynamic compression simulations were conducted under different strain rates, SiC volume fractions, and particle sizes. The simulated stress–strain curves were processed to construct the numerical database. A macroscopic Johnson–Cook model was subsequently calibrated using the low-volume-fraction composite data to provide a physically interpretable baseline response. On this basis, a neural network was employed to learn the stress discrepancy between the finite element simulation response and the Johnson–Cook baseline. The results demonstrate that, compared with the Johnson–Cook model alone, the hybrid model more accurately describes the dynamic stress–strain behavior of SiCp/Al composites under typical conditions. Additionally, strict condition-level holdout testing showed that the model retained a reasonable degree of numerical generalization capability for unseen strain rates, volume fractions, and particle sizes within or at the boundaries of the investigated simulation domain.

KEYWORDS: SiCp/Al composites; dynamic mechanical behavior; Johnson–Cook constitutive model; machine learning; hybrid modeling

1 Introduction

Particle-reinforced aluminum matrix composites have attracted considerable attention for lightweight structural applications because the incorporation of ceramic particles can improve the stiffness, strength, wear resistance, and thermal stability of aluminum alloys [1,2]. Among the various reinforcement phases, silicon carbide particles are commonly used in aluminum matrix composites because of their high hardness, high elastic modulus, and good thermal stability. SiC particle-reinforced aluminum matrix composites have been reported to exhibit high strength and wear resistance, and their plastic deformation and failure behavior are strongly affected by microstructural factors such as particle distribution, local strain localization, and particle–matrix interaction [3–5]. Therefore, an accurate description of the mechanical response of SiCp/Al composites is important for the structural design, simulation, and impact-resistance evaluation.

Under dynamic loading conditions, the stress–strain response of metal matrix composites is affected not only by plastic strain but also by the strain rate and microstructural parameters [6–8]. In SiCp/Al composites,

the volume fraction and particle size of SiC particles can influence the load transfer, local stress concentration, and deformation compatibility between the aluminum matrix and ceramic reinforcement. These factors make the dynamic mechanical behavior of SiCp/Al composites more complex than that of monolithic Al alloys. Consequently, a constitutive model used for such composites should reflect both the macroscopic strain rate-dependent response and the influence of reinforcement-related microstructural variables.

The Johnson–Cook constitutive model is one of the most widely used phenomenological models for describing the plastic flow stress of metals subjected to large strains, high strain rates, and high temperatures [9–11]. The original model was proposed by Johnson and Cook and has been extensively used in numerical simulations of impact, machining, forming, and other dynamic deformation processes. Its simple multiplicative form allows the effects of strain hardening, strain-rate sensitivity, and thermal softening to be represented using a limited number of parameters. However, the conventional Johnson–Cook model primarily describes the macroscopic responses of homogeneous materials. When applied to particle-reinforced aluminum matrix composites, it is difficult for the original formulation to explicitly capture the coupled effects of the SiC volume fraction, particle size, and particle–matrix interaction.

In recent years, data-driven and physics-informed machine learning methods have provided new possibilities for constitutive modeling [12–15]. Compared with traditional empirical models, machine learning models can approximate complex nonlinear relationships between input variables and material responses. However, purely data-driven models may suffer from limited interpretability and weak extrapolation ability when training data are insufficient or when the loading conditions change. To overcome these limitations, physics-informed and hybrid modeling strategies have been increasingly studied. For example, recent studies combined continuum-mechanics-based formulations with machine learning to construct constitutive models for strain rate-sensitive or history-dependent materials, showing that physical constraints can improve model consistency, generalization, and reliability [16–18].

Hybrid physics-based and data-driven modeling is particularly suitable for materials whose mechanical behaviors are controlled by both known physical mechanisms and complex nonlinear factors. Computational homogenization provides an effective framework for linking heterogeneous microscale responses to effective macroscopic material behavior in composite materials [19]. In multiscale solid mechanics, hybrid surrogate models have also been proposed by combining physics-based constitutive models with data-driven components; thus, the physical model provides an interpretable baseline, whereas machine learning enhances the expressive capability of the overall model. Similar ideas have also been used in heterogeneous and composite material modeling, where physically motivated model structures can help reduce the limitations of conventional black-box neural networks.

Therefore, this study establishes a hybrid framework for predicting the dynamic mechanical response of SiCp/Al composites by integrating microstructure-based finite element simulation, constitutive modeling, and machine learning. First, three-dimensional SiCp/7075Al microstructural models are constructed and dynamic compression simulations are performed under different strain rates, reinforcement volume fractions, and particle sizes to establish a numerical stress–strain database. A macroscopic Johnson–Cook model is then calibrated to provide the baseline constitutive response. Finally, a discrepancy-prediction MLP is introduced to learn the nonlinear discrepancy between the finite element response and the Johnson–Cook prediction. Strict condition-level holdout tests are further employed to evaluate the generalization capability of the proposed model under unseen material and loading conditions.

2 Data Sources and Dataset Construction

2.1 Microstructure-Based Finite Element Model

Three-dimensional microstructural models of SiCp/7075Al composites were constructed using the FE module of DIGIMAT. The 7075Al alloy was defined as the continuous matrix phase, whereas the SiC particles were treated as the discrete reinforcement phase. To simplify the microstructural geometry while retaining the principal reinforcement characteristics, the SiC particles were idealized as spherical inclusions and randomly distributed within the aluminum matrix. A series of microstructural models was generated by varying the SiC volume fraction and particle size while maintaining the same modeling procedure. The generated models were subsequently imported into ABAQUS for dynamic compression simulations.

In the microstructure-based finite-element model, the 7075Al matrix was described using the Johnson–Cook constitutive relation with $A = 473$ MPa, $B = 210$ MPa, $n = 0.3813$, and $C = 0.033$. Because the simulations were performed at room temperature, thermal softening was not considered. The SiC particles were treated as isotropic, fully elastic reinforcement, with a density of 4.21 g/cm^3 , an elastic modulus of 420 GPa , and a Poisson's ratio of 0.14 . The SiC/Al particle–matrix interface was represented by frictionless mechanical contact, and no interfacial reaction or formation of an additional phase was considered. The resulting stress–strain responses therefore reflect the combined effects of reinforcement content, particle size, and heterogeneous deformation within the composite. These simulated stress–strain curves were subsequently used to construct the numerical database for macroscopic Johnson–Cook reference calibration and machine-learning-based stress-discrepancy prediction.

The three-dimensional microstructural models were discretized using C3D10M elements and incorporated into a simplified SHPB numerical configuration. The projectile, incident bar, and transmitted bar were treated as rigid bodies. Frictionless surface-to-surface contact was prescribed between the projectile and incident bar, the incident bar and specimen, and the specimen and transmitted bar. The projectile length was 300 mm , while the incident and transmitted bars were each 1000 mm long with a diameter of 14 mm ; the specimen dimensions were $10 \text{ mm}^3 \times 10 \text{ mm}^3 \times 10 \text{ mm}^3$. Different nominal strain-rate conditions were generated by varying the projectile impact velocity from 10 to 100 m/s , corresponding to strain rates ranging from 1000 to $10,000 \text{ s}^{-1}$.

It should be emphasized that the Johnson–Cook relation used as the macroscopic baseline in the hybrid framework is distinct from the matrix-level Johnson–Cook model employed in the finite-element simulations. In the finite-element model, the Johnson–Cook relation describes the constitutive response of the 7075Al matrix phase. In contrast, the Johnson–Cook relation in the hybrid framework is recalibrated from the homogenized SiCp/Al stress–strain response and serves only as a composite-level reference for stress-discrepancy learning.

The data used in this study were derived from dynamic loading numerical simulations covering the stress–strain responses of SiCp/Al composites under a variety of typical conditions. These include datasets that span different strain rates, SiC volume fractions, and particle sizes, thus forming a representative multi-condition dataset (Fig. 1).

Specifically, the data includes the following categories: dynamic stress–strain curves for SiC volume fractions of 2% , 8% , 15% , and 30% under strain rates ranging from 1000 to $10,000 \text{ s}^{-1}$; stress–strain relationships at different SiC volume fractions (2% – 30%) under a fixed strain rate of 1000 s^{-1} ; and dynamic mechanical responses for different particle sizes (60 – $120 \text{ }\mu\text{m}$) under a SiC volume fraction of 8% and a strain rate of 1000 s^{-1} . In addition, dynamic stress–strain data for the 7075 aluminum alloy were included as a reference for the matrix material response, which helped analyze the dynamic mechanical behavior

differences between the matrix and composite materials. Importantly, the 7075 Al alloy data were not used in the training or error statistics of the current hybrid model.

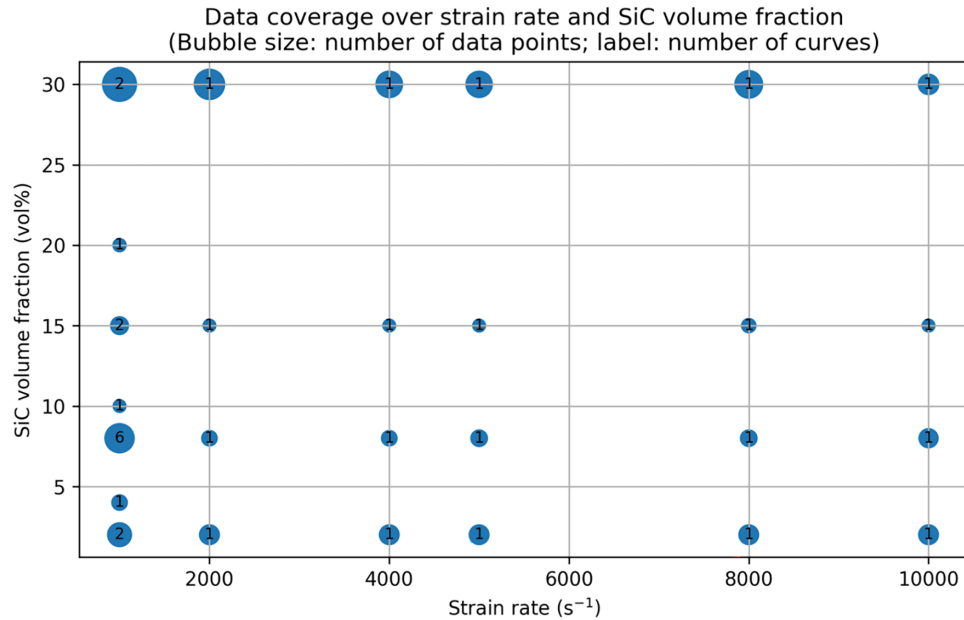


Figure 1: Schematic of data distribution under different strain rates, SiC volume fractions, and particle size conditions.

2.2 Structured Processing of Simulation Data and Dataset Construction

Numerical simulation results are typically presented as continuous stress–strain curves. Although this format offers intuitive advantages for material behavior analysis, it is not directly suitable for subsequent constitutive modeling and machine learning training. To address this issue, the dynamic-loading simulation results for the SiCp/Al composites were subjected to uniform data extraction and structural processing in this study. Through this labeling process, data from different sources and conditions were integrated into a unified format, enabling a systematic representation of the simulation data across a multidimensional parameter space. For data lacking clear particle size information, a missing-particle size marker was introduced, and reasonable fill values were applied to avoid misinterpreting the missing values as physically meaningful true particle sizes, thereby enhancing the consistency of the data representation.

Fig. 2 shows the dynamic stress–strain relationship curves for the 7075 aluminum alloy at different strain rates.

Figs. 3–5 show typical simulated stress–strain curves for different SiC volume fractions and strain rates. These curves were derived from numerical simulations that covered various reinforcement phase volume fractions, strain rates, and particle size conditions, forming the original input dataset for this study.

During data processing, the complete curves were not directly used for modeling. Instead, each stress–strain curve was discretized into point-level data. The curves were sampled point by point along the strain direction, with each data point corresponding to a specific strain and stress value. This method transforms continuous curves into a series of discrete sample points, significantly expanding the dataset available for modeling.

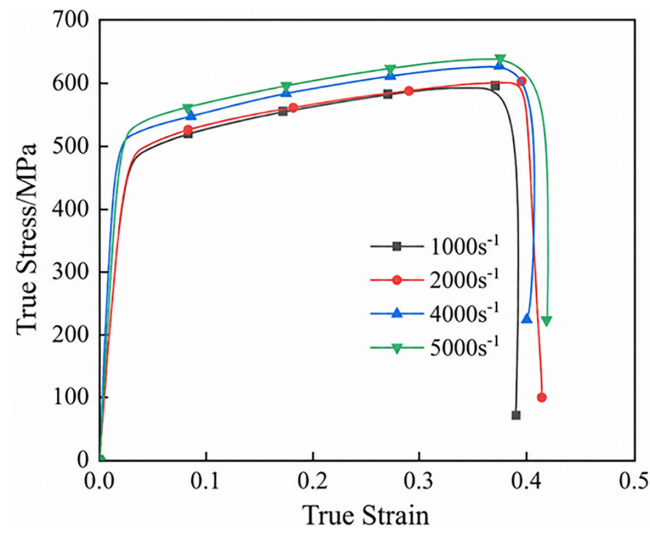


Figure 2: Dynamic stress–strain relationship curve for the 7075 aluminum alloy.

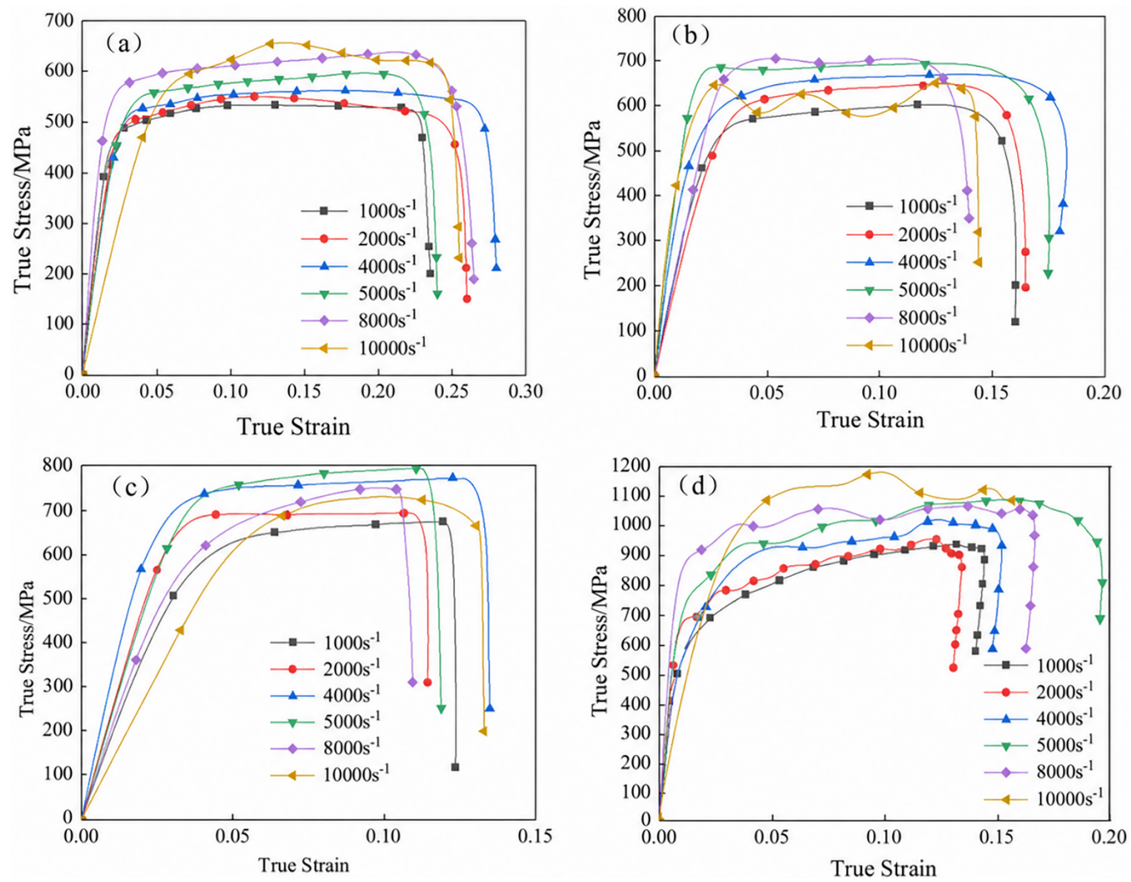


Figure 3: Dynamic stress–strain relationship curves for the composite material under different strain rates: (a) 2 vol%, (b) 8 vol%, (c) 15 vol%, and (d) 30 vol%.

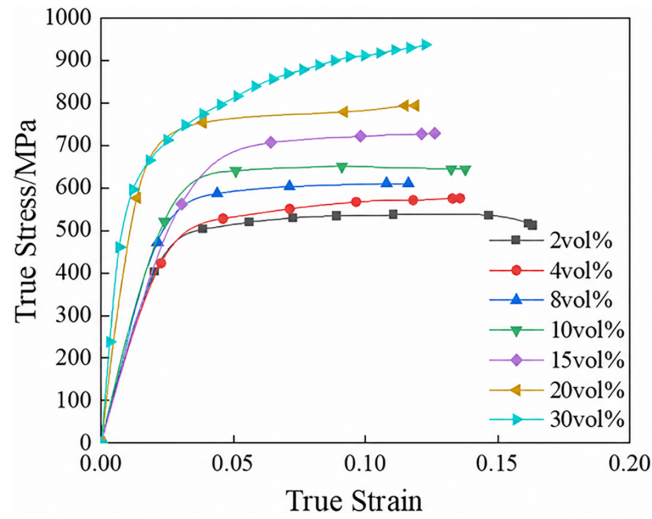


Figure 4: Stress–strain relationship curves for SiCp/Al composites with different SiC particle contents under a strain rate of 1000 s^{-1} .

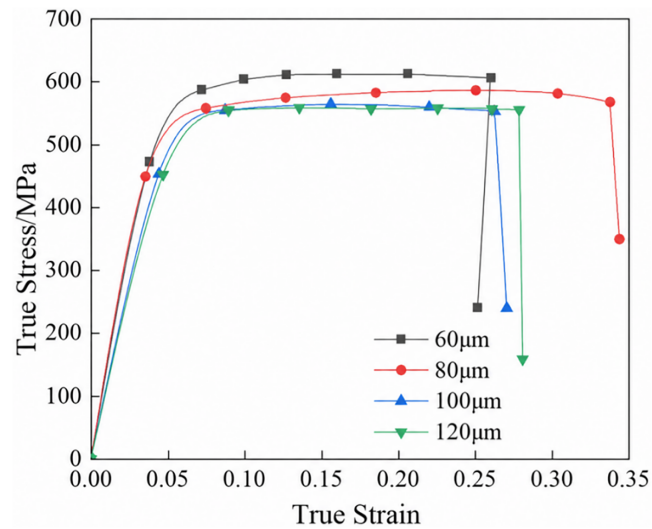


Figure 5: Dynamic stress–strain relationship curves for SiCp/Al composites with different particle sizes under a strain rate of 1000 s^{-1} .

Furthermore, to ensure comparability across data from different conditions within the same modeling framework, complete physical label information was added to each stress–strain data point. These labels include the strain rate, SiC volume fraction, particle size, and temperature. Through this labeling process, data from various sources and conditions were integrated into a unified format, allowing for the systematic representation of the simulation data in a multidimensional parameter space.

Fig. 6 illustrates the conversion of the stress–strain curves into point-level data and their labeled distributions. After structural processing, the original simulation results were transformed into a dataset suitable for constitutive model parameter calibration and machine learning training, thereby providing the foundation for subsequent hybrid constitutive modeling.

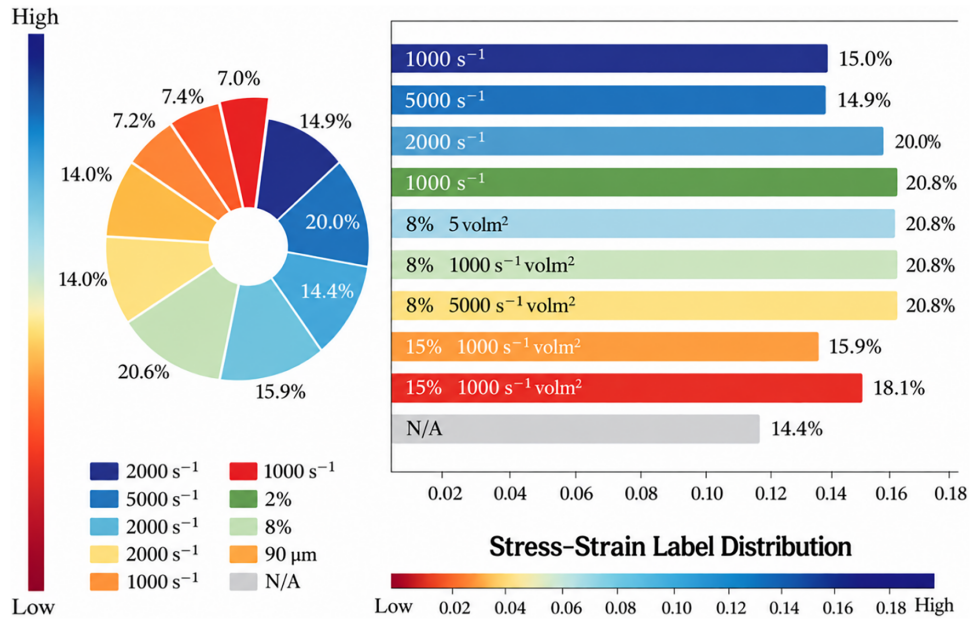


Figure 6: Conversion of stress–strain curves to point-level data and their labeled distribution.

3 Johnson–Cook Constitutive Model and Parameter Calibration

The Johnson–Cook constitutive model was employed in this study as a physics-based baseline model for predicting the dynamic flow stress of SiCp/Al composites. The main objective of this work was not to establish a new constitutive model through mechanical experiments but to develop a hybrid prediction framework by combining a conventional constitutive model with machine learning-based discrepancy prediction. Therefore, the Johnson–Cook model was first used to describe the primary strain-hardening and strain rate-dependent response, and the remaining prediction error was subsequently learned using a neural network.

The stress–strain data used in this study were obtained from the completed dynamic-loading numerical simulation curves described in Section 2. The dataset included the effects of strain, stress, strain rate, SiC volume fraction, particle size, and temperature. Because all the data correspond to room-temperature conditions, the thermal softening term in the original Johnson–Cook model was not considered in this study. The simplified Johnson–Cook constitutive equation is expressed as (1):

$$\sigma = (A + B\epsilon^n) (1 + C \ln \dot{\epsilon}^*) \quad (1)$$

where σ is the flow stress, ϵ is the plastic strain, $\dot{\epsilon}^*$ is the dimensionless strain rate, A is the initial flow stress at the reference strain rate, B is the strain hardening coefficient, n is the strain hardening exponent, and C is the strain-rate sensitivity coefficient.

The dimensionless strain rate is defined as:

$$\dot{\epsilon}^* = \dot{\epsilon} / \dot{\epsilon}_0 \quad (2)$$

where $\dot{\epsilon}$ is the strain rate and $\dot{\epsilon}_0$ is the reference strain rate. In this study, $\dot{\epsilon}_0$ was set to 1000 s⁻¹, corresponding to the lowest strain rate in the collected dynamic stress–strain dataset.

For the baseline Johnson–Cook parameter fitting, the stress–strain curves of the SiCp/Al composite with 2 vol% SiC under different strain rates were selected. The 2 vol% condition was not assumed to provide a globally optimal set of Johnson–Cook parameters for all reinforcement contents. Instead, it represents the lowest SiC volume fraction in the present dataset and was intentionally used as a low-reinforcement reference state. At relatively low reinforcement content, the macroscopic response is more strongly dominated by the aluminum matrix, whereas increasing particle content introduces progressively stronger reinforcement-related effects, including load transfer, local deformation heterogeneity, and particle–matrix interaction. Therefore, the Johnson–Cook model was calibrated from the 2 vol% reference condition to describe the primary strain-hardening and strain-rate-dependent response, whereas the effects associated with SiC volume fraction and particle size were retained in the stress discrepancy and learned by the neural network. It should also be noted that fitting a single conventional Johnson–Cook model to all reinforcement contents would produce an averaged effective parameter set because SiC volume fraction and particle size are not explicitly included in the classical formulation. In such a case, part of the reinforcement-related response would be implicitly absorbed into the fitted Johnson–Cook parameters, weakening the intended separation between the constitutive reference response and the data-driven discrepancy correction. Joint optimization of the Johnson–Cook and neural-network parameters was therefore not adopted, because the two additive components could compensate for each other and reduce the identifiability and physical interpretability of the baseline–correction decomposition.

In the initial unconstrained nonlinear fitting, nonphysical parameter combinations may occur, such as negative initial flow stress or excessively large hardening parameters. To avoid this problem, a constrained fitting strategy was adopted. The parameter A was determined from the reference stress–strain curve of the 2 vol% SiCp/Al composite at 1000 s^{-1} near the initial plastic deformation stage. The remaining parameters, B , n , and C , were obtained using bounded nonlinear least-squares fitting. The fitting was implemented using the `curve_fit` function in SciPy [20]. The fitted Johnson–Cook parameters are listed in Table 1.

Table 1: Johnson–Cook parameters fitted from simulation-based stress–strain data.

Parameter	Physical Meaning	Unit	Value
A	Initial flow stress at the reference strain rate	MPa	85.682
B	Strain hardening coefficient	MPa	530.918
n	Strain hardening exponent	–	0.1228
C	Strain-rate sensitivity coefficient	–	0.05733
$\dot{\epsilon}_0$	Reference strain rate	s^{-1}	1000

The fitting results show that the constrained Johnson–Cook model can provide a physically reasonable baseline prediction for 2 vol% SiCp/Al data. The fitting root mean square error (RMSE) and mean absolute error (MAE) were 95.016 and 67.592 MPa, respectively. These errors indicate that the simplified Johnson–Cook model can capture general strain-hardening and strain rate-dependent tendencies. However, it still has a limited ability to describe the complex effects caused by the SiC volume fraction, particle size, and particle–matrix interaction. Therefore, in the subsequent hybrid model, the Johnson–Cook prediction was used as the baseline stress, whereas the neural network was trained to learn the stress discrepancy between the baseline prediction and the numerical simulation data.

The main variables and value ranges of the dataset are summarized in Table 2.

Table 2: Main variables in the dataset and their value ranges.

Variable	Range/Description
Strain, ϵ	0.000342–0.3439
Stress, σ	29.145–1180.758 MPa
Strain rate, $\dot{\epsilon}$	1000–10,000 s ⁻¹
SiC volume fraction, VSiC	2–30 vol%
SiC particle size, dSiC	60–120 μm for particle-size-dependent curves; set as 0 when particle size was not explicitly available
Temperature, T	293 K, fixed at room temperature
Missing particle-size indicator, m_d	0 or 1

Based on the fitted Johnson–Cook model, the baseline stress for each data point was calculated as follows:

$$\sigma_{JC} = (A + B\epsilon^n)(1 + C \ln(\dot{\epsilon}/\dot{\epsilon}_0)) \quad (3)$$

where the parameters A , B , n , C , and $\dot{\epsilon}_0$ are those listed in Table 1. The stress discrepancy used for machine learning was then defined as:

$$\Delta\sigma = \sigma - \sigma_{JC} \quad (4)$$

where σ is the stress value obtained from the numerical simulation data, σ_{JC} is the Johnson–Cook baseline prediction, and $\Delta\sigma$ is the stress discrepancy to be learned by the neural network. This discrepancy-learning strategy allows the Johnson–Cook model to retain its physical interpretability, whereas the neural network compensates for nonlinear deviations caused by the reinforcement content, particle size, and other coupled factors.

4 Hybrid Physics-Guided Neural Network Framework

4.1 Overall Framework of the Proposed Method

Fig. 7 shows the overall framework of the proposed hybrid constitutive prediction method. The framework consists of three parts: input feature preprocessing, Johnson–Cook baseline prediction, and neural network-based discrepancy prediction. The Johnson–Cook model is used to provide the physics-based baseline stress, whereas the neural network learns the stress discrepancy, which cannot be fully described by the constitutive equation.

The main idea of this framework is not to replace the constitutive model with a black-box neural network but to combine the interpretability of the Johnson–Cook model with the nonlinear fitting capability of machine learning. The Johnson–Cook model captures the primary effects of strain hardening and strain-rate sensitivity, whereas the neural network compensates for the nonlinear deviations associated with the SiC volume fraction, particle size, and particle–matrix interaction.

This design is inspired by the physics-guided learning concept of Deep Material Networks (DMNs) [21,22]. In DMNs, the micromechanical relations are embedded into the network structure through homogenization building blocks such that the network learns material interaction mechanisms rather than only empirical input–output correlations. Similarly, the present work embeds the Johnson–Cook constitutive relation into the prediction process and uses a neural network only to learn the predicted stress discrepancy.

Therefore, the proposed method can be considered as a simplified physics-guided Johnson-Cook-guided discrepancy-learning framework.

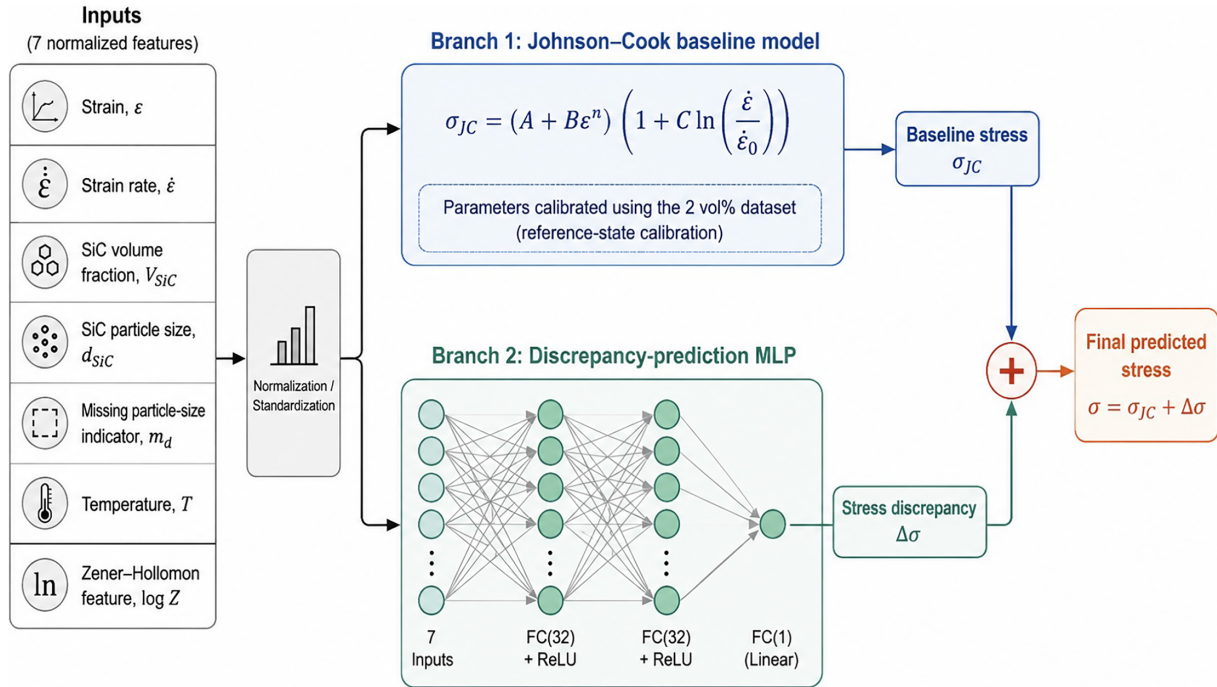


Figure 7: Schematic of the proposed Johnson-Cook-guided discrepancy-prediction MLP framework.

The final stress prediction is expressed as:

$$\sigma_{pred,i} = \sigma_{JC,i} + \Delta\sigma_{NN,i} \quad (5)$$

where $\sigma_{pred,i}$ is the predicted stress of the i th data point, $\sigma_{JC,i}$ is the Johnson-Cook baseline stress, and $\Delta\sigma_{NN,i}$ is the predicted stress discrepancy predicted by the neural network.

4.2 Input Feature Representation and Preprocessing

The input features were selected based on the loading conditions, material composition, and available microstructural information in the dataset. For each data point, the input vector is defined as:

$$\mathbf{x}_i = [\epsilon_i, \dot{\epsilon}_i, V_{SiC,i}, d_{SiC,i}, m_{d,i}, T_i, \log Z_i]^T \quad (6)$$

where ϵ_i is the strain, $\dot{\epsilon}_i$ is the strain rate, $V_{SiC,i}$ is the SiC volume fraction, $d_{SiC,i}$ is the SiC particle size, T_i is the temperature, and $\log Z_i$ is a strain rate-temperature coupled feature, where $m_{d,i}$ represents the missing-particle-size indicator. For curves without explicit particle-size information, $m_{d,i} = 1$ and $d_{SiC,i} = 0$ were assigned; otherwise, $m_{d,i} = 0$.

The strain rate-temperature coupled feature is calculated as:

$$\log Z_i = \ln(\dot{\epsilon}_i) + \frac{Q}{RT_i} \quad (7)$$

where Q is the apparent activation energy, R is the gas constant, and T_i is the absolute temperature. Because the temperature in this study was fixed at room temperature, this feature mainly enhanced the representation of the strain rate-dependent behavior.

For curves without explicit particle-size information, d_{SiC} was set to 0. This value does not represent the real particle size but indicates that the particle size was not available for the corresponding curve. As real particle-size-dependent data are within 60–120 μm , the value 0 can be clearly distinguished from valid particle-size values.

Before training, all input features were standardized to reduce the influence of different numerical scales [23]:

$$\mathbf{x}'_i = \frac{\mathbf{x}_i - \boldsymbol{\mu}_x}{\mathbf{s}_x} \quad (8)$$

where $\mathbf{x}'_i$ is the standardized input vector, $\boldsymbol{\mu}_x$ is the mean vector, and $\mathbf{s}_x$ is the standard deviation vector of the input features.

4.3 Johnson–Cook Baseline Prediction

In the proposed framework, the Johnson–Cook model is used as a physics-based baseline rather than as the final prediction model. For the i th data point, the baseline stress is calculated as:

$$\sigma_{JC,i} = (A + B\varepsilon_i^n) \left[1 + C \ln \left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_0} \right) \right] \quad (9)$$

where A , B , n , and C are the Johnson–Cook parameters obtained in Section 3, and $\dot{\varepsilon}_0$ is the reference strain rate. In this study, $A = 85.682$ MPa, $B = 530.918$ MPa, $n = 0.1228$, $C = 0.05733$, and $\dot{\varepsilon}_0 = 1000 \text{ s}^{-1}$.

The Johnson–Cook model provides a physically meaningful estimate of the stress response related to the strain hardening and strain-rate sensitivity. However, it does not explicitly include the SiC volume fraction or particle size. Therefore, its prediction error contains useful information regarding reinforcement-related nonlinear effects. This error is further learned by the neural network.

4.4 Stress Discrepancy Learning Formulation

The stress discrepancy is defined as the difference between the numerical simulation stress and the Johnson–Cook baseline prediction:

$$\Delta\sigma_i = \sigma_{data,i} - \sigma_{JC,i} \quad (10)$$

where $\sigma_{data,i}$ is the stress value obtained from the numerical, and $\Delta\sigma_i$ is the target stress discrepancy.

The neural network is trained to approximate this stress discrepancy:

$$\Delta\sigma_{NN,i} = f_{NN}(\mathbf{x}'_i; \boldsymbol{\theta}) \quad (11)$$

where f_{NN} represents the nonlinear mapping learned by the neural network, and $\boldsymbol{\theta}$ denotes the trainable parameters, including weights and biases.

This discrepancy-learning strategy reduces the burden on the neural network because the network does not need to learn the entire stress response from zero [24]. Instead, it focuses on the part of the material response that is not captured by the Johnson–Cook model, such as the effects of the reinforcement content, particle size, and composite heterogeneity.

4.5 Neural Network Architecture

The discrepancy prediction model was implemented using a feedforward multilayer perceptron [25,26]. The conceptual structure is shown in Fig. 7. In the present implementation, the network takes the standardized input vector as the input and outputs the predicted stress discrepancy.

The forward propagation of the neural network can be written as:

$$\begin{aligned}\mathbf{z}^{(l)} &= \mathbf{W}^{(l)} \mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \\ \mathbf{a}^{(l)} &= \text{ReLU}(\mathbf{z}^{(l)}) = \max(0, \mathbf{z}^{(l)})\end{aligned}\quad (12)$$

where $\mathbf{W}^{(l)}$ and $\mathbf{b}^{(l)}$ are the weight matrix and bias vector of the l th layer, $\mathbf{z}^{(l)}$ is the linear transformation result, and $\mathbf{a}^{(l)}$ is the activated output. A rectified linear unit function [27] was used because it can introduce a nonlinear representation while avoiding excessive computational complexity.

In the present study, the discrepancy-prediction model was implemented using a compact feedforward MLP consisting of two hidden layers with 32 neurons each. This architecture was not assumed to be universally optimal; rather, it was selected to balance nonlinear representation capability and model complexity considering the relatively limited number of independent curve-level conditions in the present database. Feedforward neural networks possess strong nonlinear function-approximation capability, and they have been successfully employed in data-driven constitutive modeling, including cases with limited material data. In the present hybrid framework, the MLP does not learn the complete stress-strain response from scratch. Instead, the Johnson-Cook model provides the primary constitutive response, and the MLP is used only to approximate the remaining stress discrepancy. Consequently, a compact MLP was considered sufficient for the present low-dimensional discrepancy-regression task while avoiding unnecessary model capacity. The output layer provides predicted stress discrepancy, where l denotes the output layer. Therefore, the figure should be understood as a conceptual illustration of the hybrid discrepancy prediction framework, rather than a strict layer-by-layer representation of the code.

4.6 Loss Function and Model Training

The training target of the neural network is the stress discrepancy $\Delta\sigma_i$. Therefore, the loss function is defined as the mean squared error between the target stress discrepancy and predicted stress discrepancy:

$$\mathcal{L}_{res}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N (\Delta\sigma_i - \Delta\sigma_{NN,i})^2 \quad (13)$$

where N is the number of training samples.

Similarly, the loss function can be expressed in terms of the final stress prediction as:

$$\mathcal{L}_{\sigma}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N [\sigma_{data,i} - (\sigma_{JC,i} + f_{NN}(\mathbf{x}'_i; \boldsymbol{\theta}))]^2 \quad (14)$$

The network parameters were optimized using the Adam algorithm [28]. In the implementation, the maximum number of training iterations was set to 2500. Both the input features and target stress discrepancy were standardized before training to improve the convergence stability.

The training procedure is summarized in Table 3.

Table 3: Training procedure of the proposed hybrid constitutive model.

Step	Operation
1	Construct the stress–strain dataset from the finite element simulation curves.
2	Calibrate the macroscopic Johnson–Cook reference baseline using the 2 vol% SiCp/Al simulation data.
3	Calculate the Johnson–Cook baseline stress $\sigma_{JC,i}$.
4	Calculate the stress discrepancy $\Delta\sigma_i = \sigma_{data,i} - \sigma_{JC,i}$.
5	Construct and standardize the input feature vector.
6	Train the neural network to predict $\Delta\sigma_{NN,i}$.
7	Obtain the final prediction using $\sigma_{pred,i} = \sigma_{JC,i} + \Delta\sigma_{NN,i}$.
8	Evaluate the model using holdout tests and error metrics.

4.7 Validation Strategy

Strict condition-level holdout tests were conducted to evaluate the generalization capability of the proposed hybrid framework. Rather than randomly dividing individual stress–strain points, all data belonging to a selected material or loading condition were excluded from model development and used exclusively for testing. Six holdout conditions were considered, including three intermediate conditions (20 vol% SiC, 5000 s^{−1}, and 100 μm) and three upper-boundary conditions (30 vol% SiC, 10,000 s^{−1}, and 120 μm), as summarized in Table 4. The intermediate conditions were used to assess interpolation within the investigated parameter domain, whereas the upper-boundary conditions were used to assess extrapolation relative to the remaining training range.

Table 4: Condition-level holdout cases used for interpolation and upper-boundary extrapolation evaluation.

Holdout Condition	Evaluation Type	Purpose
20 vol% SiC	Interpolation	Prediction at an unseen intermediate reinforcement content
5000 s ^{−1}	Interpolation	Prediction at an unseen intermediate strain rate
100 μm	Interpolation	Prediction at an unseen intermediate particle size
30 vol% SiC	Upper-boundary extrapolation	Prediction at the upper reinforcement-content boundary
10,000 s ^{−1}	Upper-boundary extrapolation	Prediction at the upper strain-rate boundary
120 μm	Upper-boundary extrapolation	Prediction at the upper particle-size boundary

In each holdout test, the Johnson–Cook baseline was calibrated exclusively using the available 2 vol% reference subset within the corresponding training partition. For strain-rate holdout tests, the held-out strain-rate condition was additionally excluded from the 2 vol% calibration subset. Thus, no data associated with the held-out condition were used either for Johnson–Cook calibration or for neural-network training.

This whole-condition holdout strategy is more stringent than random point-level splitting because all discretized points belonging to the same held-out material or loading condition remain exclusively in the test set. The prediction accuracy was evaluated using RMSE, MAE, and MAPE.

These holdout tests are more rigorous than random data splitting because the entire material or loading conditions are excluded from the training set [29]. Therefore, they can better assess whether the model learns generalizable relationships rather than interpolating only nearby data points.

The prediction accuracy was evaluated using the RMSE, MAE, and mean absolute percentage error (MAPE) [30–32]. These metrics were used to compare the Johnson–Cook baseline model and the final hybrid model, thereby quantifying the improvement introduced by discrepancy learning.

Although discretization of the stress–strain curves produces a relatively large number of point-level samples, these samples are not fully independent because multiple points originate from the same material/loading curve. Therefore, the effective diversity of the dataset is more appropriately characterized by the number of independent curve-level conditions. To reduce the risk of overly optimistic evaluation, no random point-level split was used for generalization assessment. Instead, all points associated with each held-out condition were reserved exclusively for testing. The relatively compact 32×32 MLP architecture was also adopted to avoid unnecessary model capacity.

4.8 Summary of the Hybrid Learning Mechanism

The proposed model combines constitutive modeling and machine learning in a complementary manner. The Johnson–Cook model provides a physically interpretable baseline for strain hardening and strain-rate sensitivity, whereas the neural network learns the discrepancy prediction associated with reinforcement-related nonlinear effects. Compared with the pure Johnson–Cook model, the hybrid model has a stronger ability to describe the influence of the SiC volume fraction and particle size. Compared with a pure neural network, the proposed model has better physical interpretability because the main stress response is constrained by a constitutive equation.

Therefore, the proposed Johnson–Cook-guided discrepancy-prediction MLP provides a suitable framework for predicting the dynamic mechanical behavior of SiCp/Al composites under different strain rates, reinforcement contents, and particle sizes.

5 Results and Discussion

5.1 Curve Prediction Results under Typical Conditions

Under typical test conditions, the microstructure-based finite element simulation results, Johnson–Cook model predictions, and hybrid model predictions were compared. The Johnson–Cook model accurately reflects the overall stress–strain trend of a material under dynamic loading. However, when the volume fraction increases or the microstructural conditions change, the model exhibits clear systematic deviations in predicting the stress levels and hardening behaviors. By contrast, the hybrid model based on discrepancy learning provided better adherence to the numerical simulation curves across the entire strain range, particularly under mid-to-high volume fraction conditions. The improvement in curve fitting was most apparent in the peak and plateau stages. This suggests that combining a physical constitutive model with machine-learning-based discrepancy predictions enhances the ability of the model to describe the coupled dynamic mechanical behavior influenced by multiple factors, as shown in [Figs. 8–11](#).

The results indicate that the hybrid model can track the simulation curve closely across the entire strain range, with particular advantages under high volume fraction conditions. Notably, when numerical oscillations are present in certain stress–strain curves, the hybrid model may still inherit some data fluctuations at specific intervals.

5.2 Generalization Prediction Results under Strict Holdout Conditions

In contrast to the typical condition curve comparisons, this section further evaluates the generalizability of the model using strict holdout tests. In each holdout test, all data associated with the held-out condition were excluded before model development. The Johnson–Cook baseline remained calibrated exclusively using the 2 vol% reference subset available within the corresponding training partition. For strain-rate holdout tests, the held-out strain-rate condition was additionally excluded from the 2 vol% calibration

subset. Therefore, no data from the held-out condition were used either for Johnson–Cook calibration or for neural-network training.

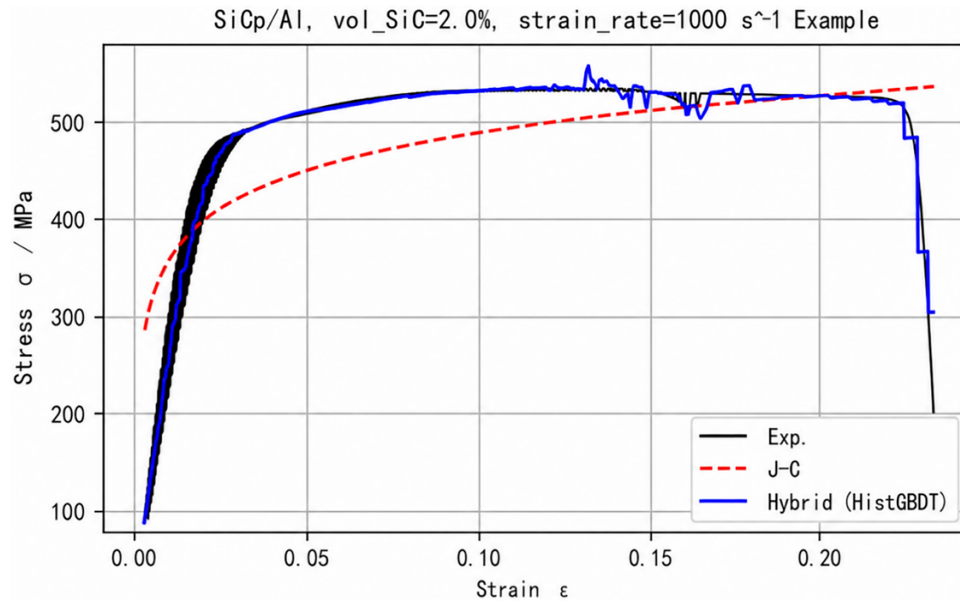


Figure 8: Comparison of simulation results, Johnson–Cook model, and hybrid model for SiCp/Al (2 vol%, 1000 s⁻¹).

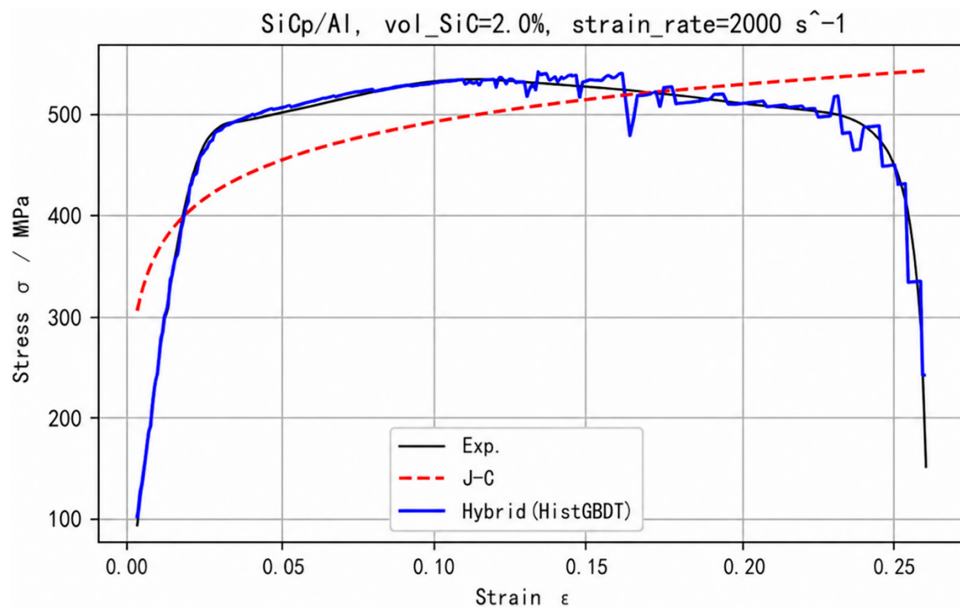


Figure 9: Comparison of simulation results, Johnson–Cook model, and hybrid model for SiCp/Al (2 vol%, 2000 s⁻¹).

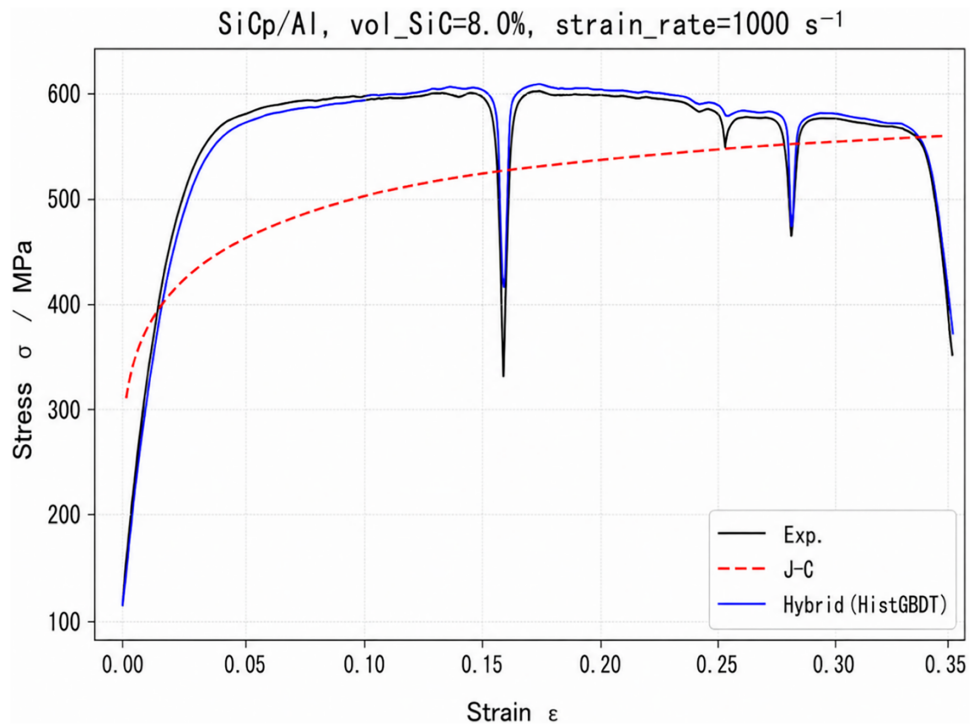


Figure 10: Comparison of simulation results, Johnson–Cook model, and hybrid model for SiCp/Al (8 vol%, 1000 s^{-1}).

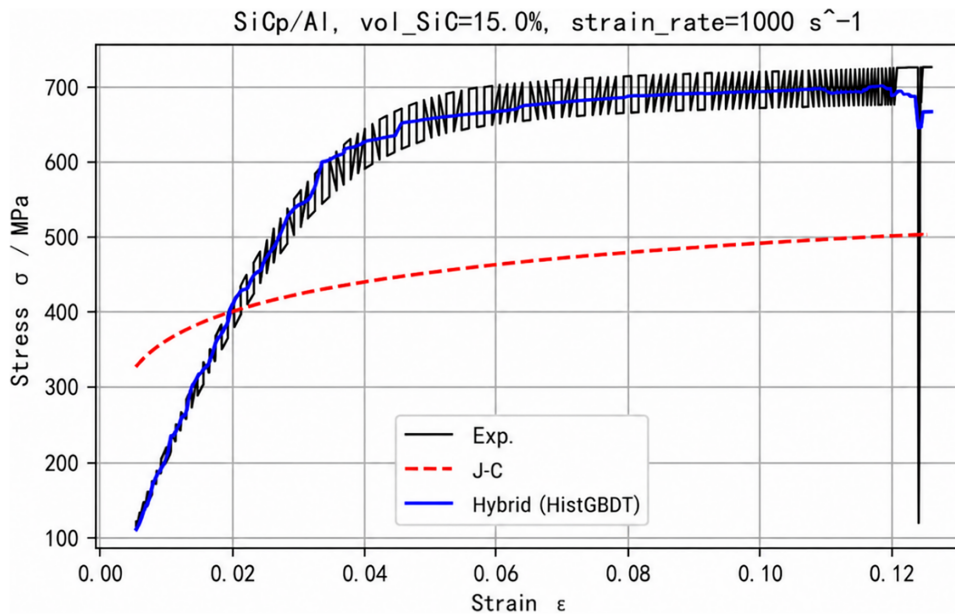


Figure 11: Comparison of simulation results, Johnson–Cook model, and hybrid model for SiCp/Al (15 vol%, 1000 s^{-1}).

5.2.1 Holdout Strain Rate Condition ($sr = 10,000 \text{ s}^{-1}$)

In the strict strain rate holdout test, the curves for different volume fractions under a strain rate of $10,000 \text{ s}^{-1}$ were used as the test set. As shown in Fig. 12, the Johnson–Cook model generally underestimated

the stress levels under different volume fraction conditions, whereas the hybrid model effectively corrected this bias in most cases. For the 15 and 30 vol% conditions, the hybrid model exhibited a significant improvement in the overall stress level and shape of the early-to-middle segment of the curve. However, for the 2 and 8 vol% conditions, although the hybrid model outperformed the Johnson–Cook model in some regions, local fluctuations and deviations remained in the latter half of the curve. This demonstrates that under strict extrapolation conditions, the model exhibits some generalization capability for strain-rate effects, but its stability still depends on the volume fraction.

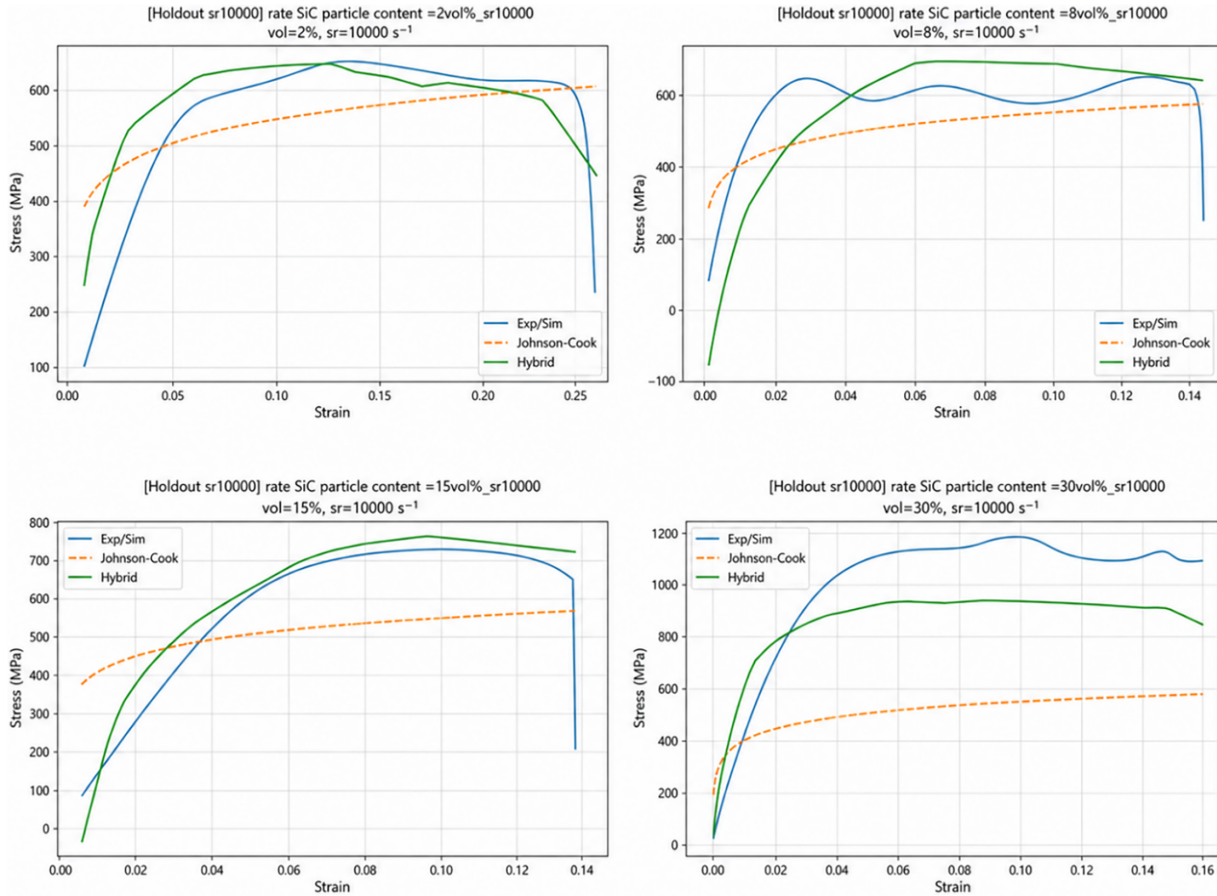


Figure 12: Comparison of simulation results, Johnson–Cook model, and hybrid model for holdout strain rate condition ($sr = 10,000 \text{ s}^{-1}$) with different volume fractions.

5.2.2 Holdout Volume Fraction Condition ($vol = 30\%$)

In the volume fraction holdout test, all curves under the 30 vol% condition were excluded, ensuring that the model did not encounter high volume fraction conditions during the training phase. As shown in Fig. 13, the Johnson–Cook model significantly underestimates the overall stress level for high volume fraction materials, whereas the hybrid model better identifies the strengthening trend, especially under low to moderate strain-rate conditions, where its predictions in the early to middle strain regions are notably superior to those of the Johnson–Cook model. However, as the strain rate increases from 1000 to $10,000 \text{ s}^{-1}$, the hybrid model gradually shows an early drop in the later stages, resulting in insufficient ability to maintain the plateau stress level. This indicates that the current model can capture the enhanced strengthening trend of high volume fraction materials relative to low volume fraction ones but still struggles to fully characterize

the complex nonlinear strengthening mechanisms under the coupled conditions of high volume fraction and high strain rate.

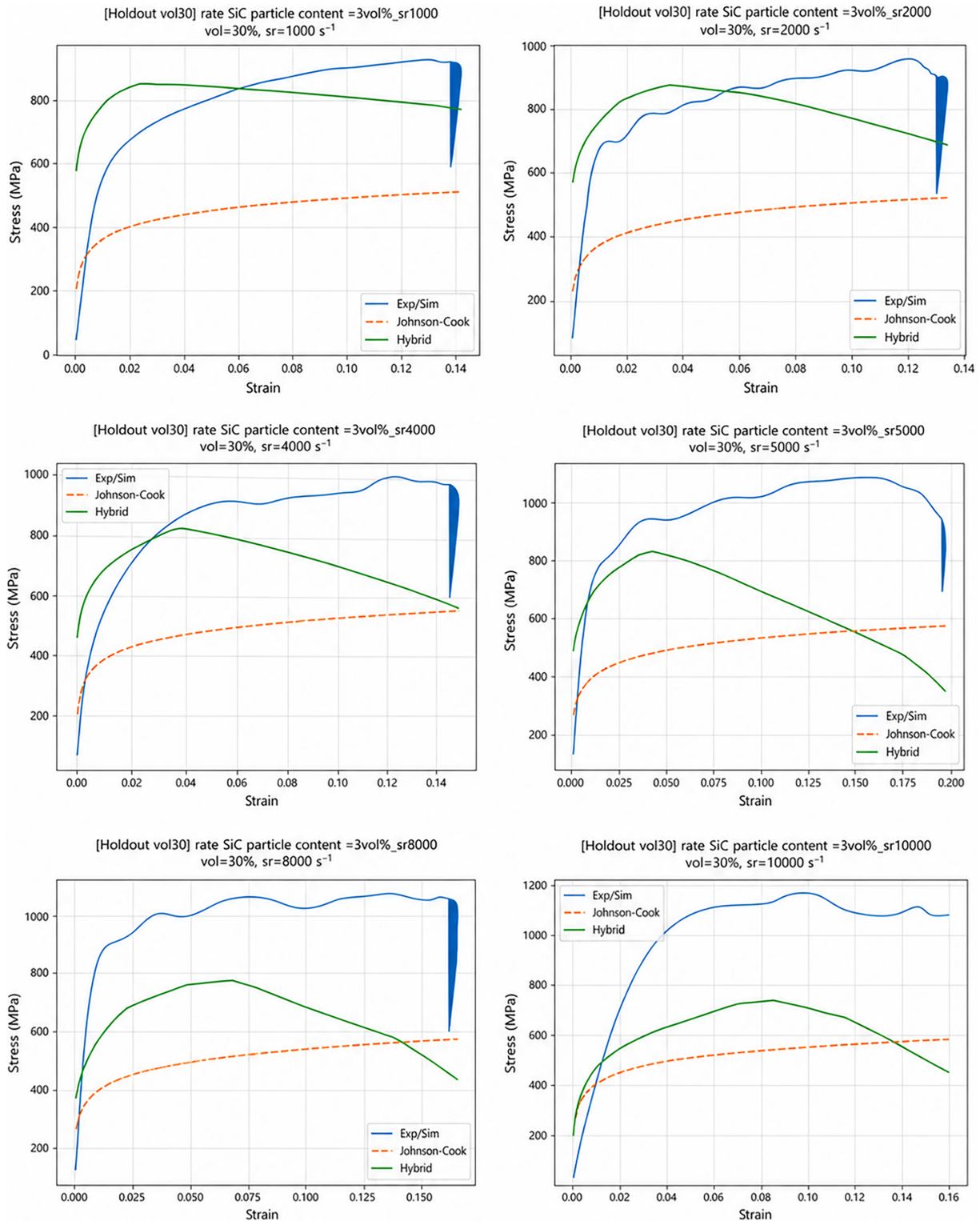


Figure 13: Comparison of simulation results, Johnson–Cook model predictions, and hybrid model predictions under different conditions for the volume fraction of vol = 30%.

5.3 Original Point-Level Error Statistics

Three error metrics were used to quantitatively evaluate the prediction accuracy of the Johnson–Cook and hybrid models: RMSE, MAE, and MAPE.

The RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\sigma_{pred,i} - \sigma_{data,i})^2} \quad (15)$$

The MAE is defined as:

$$MAE = \frac{1}{N} \sum_{i=1}^N |\sigma_{pred,i} - \sigma_{data,i}| \quad (16)$$

The MAPE is defined as:

$$MAPE = \frac{100\%}{N} \sum_{i=1}^N \left| \frac{\sigma_{pred,i} - \sigma_{data,i}}{\sigma_{data,i}} \right| \quad (17)$$

where N is the number of data points, $\sigma_{pred,i}$ is the predicted stress, and $\sigma_{data,i}$ is the collected stress value of the i -th data point. The RMSE emphasizes large deviations, MAE reflects the average absolute error, and $MAPE$ describes the relative prediction error. Using these metrics together provides a more complete evaluation of the prediction accuracy.

To ensure a consistent comparison across the three types of holdout tests, the point-level RMSE for both the training and test sets was first computed. The results are summarized in Table 5 and Fig. 14. In all three tests, the hybrid model exhibited a significantly lower error in the training set than the Johnson–Cook model, indicating that the discrepancy-prediction MLP effectively learned the systematic discrepancy of the Johnson–Cook baseline model.

Table 5: Comparison of point-level RMSE between the Johnson–Cook model and the hybrid model under three types of holdout tests.

Test Type	Train RMSE (JC)/MPa	Train RMSE (Hybrid)/MPa	Test RMSE (JC)/MPa	Test RMSE (Hybrid)/MPa	Train Points	Test Points	Train Curves	Test Curves
Holdout vol = 30%	130.37	28.78	393.39	299.21	5726	5884	28	7
Holdout sr = 10,000 s ⁻¹	292.51	56.42	314.02	137.17	10,495	1115	31	4
Holdout d = 120 μm	297.18	55.55	75.13	38.65	11,399	211	34	1

More importantly, the hybrid model outperformed the Johnson–Cook model in the test set across the board. For the volume fraction holdout test, the test set RMSE decreased from 393.39 to 299.21 MPa (Fig. 15); for the strain rate holdout test, it dropped from 314.02 to 137.17 MPa; and for the particle size holdout test, it decreased from 75.13 to 38.65 MPa. These results demonstrate that the hybrid model not only improves the curve-fitting accuracy for the observed conditions but also exhibits some degree of generalization ability for unseen conditions.

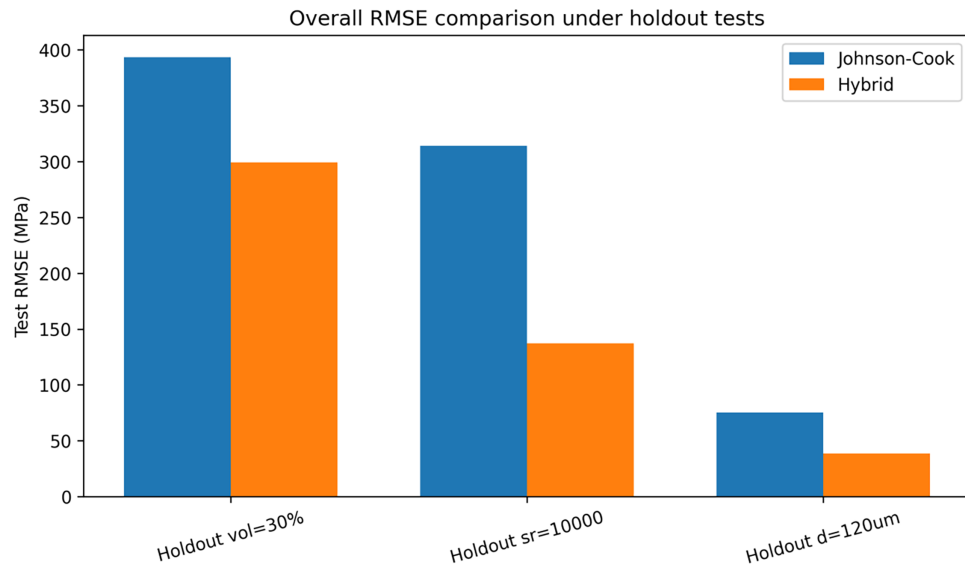


Figure 14: Comparison of overall RMSE between the Johnson–Cook model and hybrid model under three types of holdout test conditions.

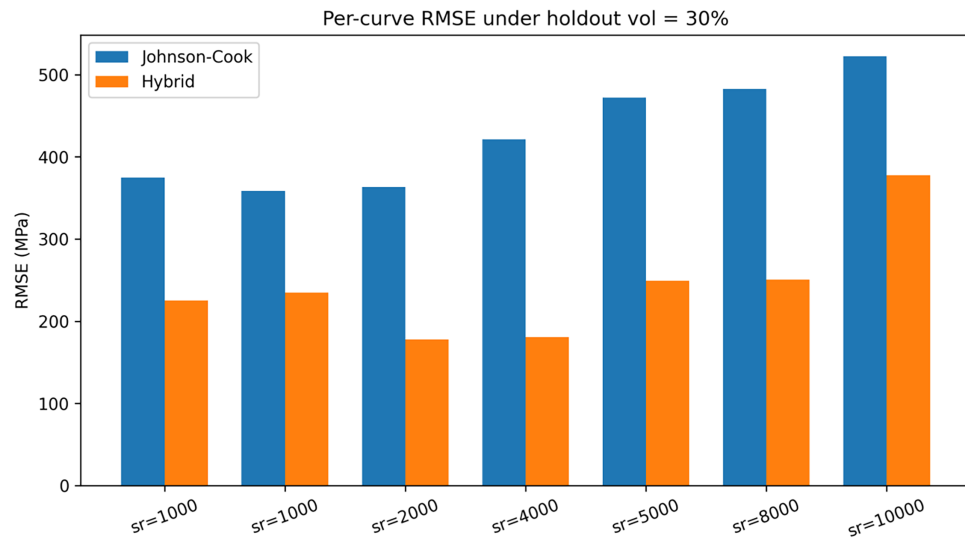


Figure 15: Comparison of RMSE for different strain rate curves under the volume fraction condition of 30 vol%.

From the comparison between the different test types, the most noticeable error improvement is observed in the strain-rate holdout test, suggesting that the model is better at correcting for unseen high-strain-rate conditions. The particle size holdout test also shows good correction ability, indicating that the model can recognize the effects of microstructural factors when the particle size and its missing marker are included. However, the improvement was relatively limited in the high-volume-fraction holdout test, suggesting that the model still faces challenges in capturing the complex behavior of high-volume-fraction materials.

5.4 Strict Error Evaluation Using 30 vol% as a Representative Condition

This study adopted a stratified evaluation strategy for result analysis. In the three types of holdout tests, the first step was to use the original point-level RMSE for a horizontal comparison. Building on this, a more rigorous evaluation was conducted using the 30 vol% holdout test as a representative condition by applying a post-peak failure segment truncation strategy to evaluate the RMSE, MAE, and MAPE for a valid deformation range.

Because dynamic stress–strain curves often experience phenomena such as local instability, damage evolution, or numerical oscillations after the peak stress [33–35], directly applying error statistics to all points along the full curve can amplify the impact of failure tail segments [36] on the model evaluation. Therefore, a more reasonable approach is to truncate the failure tail and evaluate only the error during the valid deformation stage. This provides a more accurate reflection of the ability of the model to describe the behavior of the material during the effective deformation phase.

As presented in Table 6, for the 30 vol% holdout test, the overall RMSE for the Johnson–Cook model is 412.73 MPa, whereas that of the hybrid model is reduced to 229.97 MPa. The MAE of the Johnson–Cook model is 394.80 MPa, whereas that of the hybrid model achieves 178.09 MPa. The MAPE also decreases from 49.90% to 42.64%. These results indicate that when the failure tail is excluded, the hybrid model provides a more accurate prediction of the stress levels in the effective deformation phase, providing a better reflection of the reinforcement behavior.

Table 6: Comparison of overall errors after truncating the failure segment for the 30 vol% holdout test.

Metric	Johnson–Cook	Hybrid
RMSE/MPa	412.73	229.97
MAE/MPa	394.80	178.09
MAPE/%	49.90	42.64

When analyzing the results curve by curve, the hybrid model shows the most significant reduction in RMSE and MAE at strain rates of 2000, 4000, and 5000 s^{-1} , indicating a strong ability to correct stress discrepancies for high-volume fraction materials under medium strain rates. At 8000 s^{-1} , the hybrid model still outperforms the Johnson–Cook model, but the improvement becomes less pronounced. At the extreme strain rate of 10,000 s^{-1} , although the hybrid model alleviates the overall underestimation of stress levels, it still fails to maintain the plateau stress in the latter stages of the curve, suggesting that the model has not fully captured the complex strengthening mechanisms of high volume fraction composites under extreme strain rates.

5.5 Strict Condition-Level Holdout Validation and Model Generalization

To further evaluate numerical generalization beyond the three representative boundary-condition analyses discussed above, an expanded set of strict whole-condition holdout tests was conducted. Three intermediate conditions (20 vol% SiC, 5000 s^{-1} , and 100 μm) were completely excluded from model development to evaluate interpolation, while three upper-boundary conditions (30 vol% SiC, 10,000 s^{-1} , and 120 μm) were withheld to evaluate extrapolation relative to the remaining training range. Because all stress–strain points associated with each selected condition were excluded simultaneously, these tests evaluate prediction under genuinely unseen simulation conditions rather than random interpolation among points from the same curves.

As summarized in Table 7, the hybrid framework consistently outperformed the conventional Johnson–Cook model for all six unseen conditions. The RMSE reduction ranged from 21.81% to 85.69%, with an average reduction of approximately 58.6%. Particularly, the largest improvement was observed for the strain-rate interpolation case (5000 s^{-1}), where the hybrid model reduced RMSE from 331.37 to 47.41 MPa.

Table 7: Performance comparison between the Johnson–Cook baseline and hybrid model under strict intermediate-condition holdout validation.

Holdout Condition	Type	JC RMSE (MPa)	Hybrid RMSE (MPa)	Improvement
20 vol%	Interpolation	260.78	131.30	49.65%
5000 s^{-1}	Interpolation	331.37	47.41	85.69%
100 μm	Interpolation	83.90	15.16	81.93%

An important limitation of the present study is that both model development and validation are based on stress–strain responses generated by the same finite-element modeling framework. Therefore, the reported prediction errors quantify agreement with the numerical reference responses rather than direct prediction errors for experimentally tested SiCp/7075Al composites. Any assumptions or systematic discrepancy of the JC baseline in the underlying finite-element model may consequently be inherited by the learned model. Independent SHPB or other high-strain-rate experimental validation will therefore be required in future work before the proposed framework can be regarded as an experimentally validated constitutive model for real SiCp/7075Al composites.

6 Conclusion

Based on the microstructure-based finite element simulation database established for SiCp/Al particle-reinforced aluminum matrix composites, this study developed a hybrid modeling framework combining a macroscopic Johnson–Cook baseline with neural-network discrepancy prediction. This approach addresses the dynamic mechanical behavior prediction of composites under different strain rates, SiC volume fractions, and particle sizes. The major conclusions of this study are as follows:

- (1) The dynamic stress–strain curves obtained from the finite element simulations were discretized into point-level samples and labeled according to strain rate, SiC volume fraction, particle size, and temperature, thereby establishing a structured dataset for constitutive modeling and machine-learning training.
- (2) The Johnson–Cook model was calibrated using multi-strain rate data from SiCp/Al composites with a 2 vol% SiC volume fraction as the basis. The results show that the Johnson–Cook model adequately reflects the overall deformation trend of low-volume-fraction composites under dynamic loading conditions and can serve as a baseline constitutive relationship with a physical meaning.
- (3) Building on the Johnson–Cook baseline model, a multilayer feedforward neural network was used to learn the stress discrepancies between the numerical simulation stress and baseline prediction, leading to a “baseline constitutive + discrepancy prediction” hybrid model. This method enhances the ability to describe the dynamic stress–strain response under multifactor coupling without sacrificing the physical interpretability of traditional constitutive models. The results under typical conditions show that the hybrid model tracks the overall trend of the curves better than the Johnson–Cook model, with more significant correction effects for mid- to high-volume-fraction conditions.
- (4) To evaluate model generalization more rigorously, strict condition-level holdout tests were conducted. For each test, the held-out condition was excluded from model development, while the Johnson–Cook

baseline was calibrated only from the available 2 vol% reference subset within the corresponding training partition. The hybrid model achieved lower prediction errors than the stand-alone Johnson–Cook baseline under the investigated unseen simulation conditions.

- (5) In a comparison of the three holdout types, the hybrid model showed the most substantial improvement in the strain rate holdout test and performed well in correcting for particle size effects. However, in the high-volume-fraction holdout test, the improvement was limited, indicating that capturing the complex behavior of high-volume-fraction materials remains a challenge.
- (6) A more rigorous error evaluation was performed using a 30 vol% volume fraction holdout test, in which the failure segment after the peak stress was truncated. After this truncation, the hybrid model showed significant improvements in RMSE, MAE, and MAPE compared with the Johnson–Cook model, suggesting a better prediction accuracy during the effective deformation phase for high-volume-fraction materials. However, at extreme strain rates ($10,000 \text{ s}^{-1}$), the hybrid model still struggles to capture the full complexity of high volume fraction materials' behavior under such conditions.

In summary, within the investigated numerical parameter domain, the proposed hybrid modeling approach substantially improves agreement with the microstructure-based finite-element reference responses compared with the stand-alone Johnson–Cook model. The framework therefore provides a promising numerical strategy for combining constitutive modeling with machine-learning-based discrepancy correction. Further validation using independent high-strain-rate experimental data is required to assess its predictive capability for real SiCp/Al composites.

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Ethics Approval: Not applicable. This study did not involve human participants or animals.

Conflicts of Interest: The authors declare no conflicts of interest.

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