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ARTICLE

An Adaptive Hybrid Metaheuristic for Solving the Vehicle Routing Problem with Time Windows under Uncertainty

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ABSTRACT: The Vehicle Routing Problem with Time Windows (VRPTW) presents a significant challenge in combinatorial optimization, especially under real-world uncertainties such as variable travel times, service durations, and dynamic customer demands. These uncertainties make traditional deterministic models inadequate, often leading to suboptimal or infeasible solutions. To address these challenges, this work proposes an adaptive hybrid metaheuristic that integrates Genetic Algorithms (GA) with Local Search (LS), while incorporating stochastic uncertainty modeling through probabilistic travel times. The proposed algorithm dynamically adjusts parameters—such as mutation rate and local search probability—based on real-time search performance. This adaptivity enhances the algorithm's ability to balance exploration and exploitation during the optimization process. Travel time uncertainties are modeled using Gaussian noise, and solution robustness is evaluated through scenario-based simulations. We test our method on a set of benchmark problems from Solomon's instance suite, comparing its performance under deterministic and stochastic conditions. Results show that the proposed hybrid approach achieves up to a 9% reduction in expected total travel time and a 40% reduction in time window violations compared to baseline methods, including classical GA and non-adaptive hybrids. Additionally, the algorithm demonstrates strong robustness, with lower solution variance across uncertainty scenarios, and converges faster than competing approaches. These findings highlight the method's suitability for practical logistics applications such as last-mile delivery and real-time transportation planning, where uncertainty and service-level constraints are critical. The flexibility and effectiveness of the proposed framework make it a promising candidate for deployment in dynamic, uncertainty-aware supply chain environments.

KEYWORDS: Vehicle routing problem with time windows (VRPTW); hybrid metaheuristic; genetic algorithm; local search; uncertainty modeling; stochastic optimization; adaptive algorithms; combinatorial optimization; transportation and logistics; robust scheduling

1 Introduction

Efficient logistics and transportation are essential to modern supply chain management, with direct implications for operational costs, environmental sustainability, and customer satisfaction. One of the most prominent challenges in this field is the *Vehicle Routing Problem with Time Windows (VRPTW)*, which involves determining optimal routes for a fleet of vehicles to serve customers within predefined time intervals [1]. By introducing time window constraints, the VRPTW significantly increases the complexity of the classical Vehicle Routing Problem (VRP) [2].

In practice, routing performance is often influenced by uncertainties such as fluctuating travel times, variable service durations, and changing customer demands. Traditional deterministic models struggle to accommodate such variability, frequently resulting in suboptimal or infeasible solutions [3]. To address these challenges, stochastic and robust optimization methods have been developed to enable more reliable and resilient routing under uncertainty [4].

Metaheuristic algorithms—particularly hybrid approaches—have demonstrated strong performance in solving VRPTW and its variants. By combining global exploration with local refinement, these methods effectively balance the search process across large and complex solution spaces [5]. Early work in this area combined Genetic Algorithms (GA) with Local Search (LS) to enhance both solution quality and computational efficiency [6]. More recent studies have introduced advanced hybridizations incorporating modern heuristics and adaptive strategies. For example, a study proposed an Improved Genetic Ant Colony Optimization (IGA-ACO) algorithm that integrates Solomon's insertion heuristic for population initialization, achieving faster convergence and better route planning under time and capacity constraints [7]. Similarly, a self-adaptive combination of Intelligent Water Drops and Simulated Annealing has been used in green logistics to reduce emissions while maintaining adaptability [8]. Other work has shown that coupling GA with Record-to-Record Travel (GA-RR) improves both route robustness and efficiency in capacitated VRP scenarios [9].

Despite these advances, many existing metaheuristics still rely on static parameters or fixed algorithmic structures, limiting their responsiveness to changing problem landscapes or uncertain inputs. Adaptive mechanisms that dynamically tune algorithmic parameters based on performance feedback can further enhance solution quality, robustness, and convergence speed [10].

In this study, we propose an Adaptive Hybrid Metaheuristic (AHM) that integrates Genetic Algorithms with Local Search while incorporating stochastic modeling of travel-time uncertainty to address the VRPTW under realistic operating conditions. The primary contributions of this work are as follows:

1. **Development of an Adaptive Hybrid Metaheuristic:** A novel algorithm that adjusts key parameters dynamically to effectively balance exploration and exploitation during the optimization process.
2. **Integration of Uncertainty Modeling:** A probabilistic framework that captures the variability of real-world travel times through scenario-based simulations.
3. **Comprehensive Performance Evaluation:** Extensive testing on benchmark instances from Solomon's suite, demonstrating the algorithm's superiority in robustness, feasibility, and computational efficiency.

The remainder of this paper is organized as follows: Section 2 reviews related work in VRPTW and hybrid metaheuristics; Section 3 presents the problem formulation; Section 4 details the proposed methodology; Section 5 presents the experimental setup; Section 6 presents the results and a discussion; and Section 7 concludes with insights and future research directions.

2 Related Work

The Vehicle Routing Problem with Time Windows (VRPTW) has been extensively studied due to its critical role in logistics and transportation. This section reviews significant contributions, focusing on recent advancements in exact methods, metaheuristic approaches, uncertainty modeling, and green logistics.

2.1 Exact Methods for VRPTW

Early research on VRPTW primarily employed exact algorithms, such as branch-and-bound and dynamic programming, to find optimal solutions. However, these methods often faced scalability issues with larger problem instances. Recent studies have aimed to enhance the efficiency of exact methods. For instance, Baldacci et al. [11] provided a comprehensive review of exact algorithms for VRPTW, discussing their applicability and limitations in large-scale scenarios.

2.2 Metaheuristic Approaches

To address the limitations of exact methods, metaheuristic algorithms have been widely adopted. Techniques like GAs, Ant Colony Optimization (ACO), and Particle Swarm Optimization (PSO) have been applied to VRPTW with notable success. Marinakis et al. [12] introduced a multi-adaptive PSO for VRPTW, demonstrating improved performance over traditional PSO variants. Furthermore, Zahedi and Khalilzadeh [13] proposed a hybrid metaheuristic algorithm combining the Nondominated Sorting Genetic Algorithm II (NSGA-II) with Teaching–Learning-Based Optimization (TLBO) to solve a bi-objective capacitated electric vehicle routing problem with time windows and partial recharging. Their method efficiently balances routing cost and environmental impact, illustrating the growing interest in sustainable and multi-objective VRP solutions.

2.3 Handling Uncertainty in VRPTW

Real-world applications of VRPTW often involve uncertainties such as stochastic customer demands and variable travel times. Traditional deterministic models may not adequately capture these uncertainties, leading to suboptimal solutions. Recent research has focused on stochastic and robust optimization methods to better handle these challenges. Florio et al. [14] discussed Bayesian learning techniques for correlated demands in stochastic VRP, offering insights into handling demand uncertainty effectively. Additionally, Indrianti et al. [15] developed a Green Vehicle Routing Problem (GVRP) model that integrates complex real-world constraints—including emission reduction and delivery limits—using GAs.

2.4 Green Logistics and Sustainability

With the growing emphasis on sustainability, recent studies have integrated environmental objectives into VRPTW. Zahedi and Khalilzadeh [13] offered a hybrid metaheuristic algorithm combining NSGA-II and teaching–learning-based optimization for a bi-objective capacitated electric vehicle routing problem with time windows and partial recharging, addressing both economic and environmental considerations. Additionally, Fan [16] developed a hybrid adaptive large neighborhood search method for the time-dependent open electric vehicle routing problem with hybrid energy replenishment strategies, contributing to the advancement of sustainable transportation solutions.

2.5 Adaptive and Hybrid Metaheuristics

The integration of adaptive mechanisms into hybrid metaheuristics has shown promise in enhancing the flexibility and efficiency of VRPTW solutions. Chen et al. [7] proposed an Improved Genetic Ant Colony Optimization (IGA-ACO) algorithm to efficiently solve VRPTW, integrating Solomon's insertion heuristic for population initialization and accelerating convergence while optimizing route planning to meet vehicle capacity and time window constraints. This approach highlights the potential of combining adaptive strategies with hybrid metaheuristics to tackle complex routing problems effectively.

2.6 Summary

The landscape of VRPTW research has evolved from classical exact methods to sophisticated hybrid metaheuristics that incorporate adaptive mechanisms and uncertainty modeling. Recent advancements have also integrated sustainability considerations, reflecting the multifaceted challenges of modern logistics. Despite these developments, opportunities remain to further enhance the adaptability and robustness of VRPTW solutions, particularly through the integration of real-time data and machine learning techniques.

To provide a concise and comparative overview of recent contributions to the VRPTW and its variants, Table 1 summarizes the key characteristics of relevant studies, including the year of publication, problem variants addressed, uncertainty and sustainability aspects, the metaheuristic techniques employed, and a brief evaluation of their advantages and disadvantages.

Table 1: Summary of recent studies related to the VRPTW, including publication year, problem variants, uncertainty modeling, green logistics integration, metaheuristic strategies, and key advantages and limitations

Reference	Year	Approach	Uncertainty	Green	VRP Variant	Metaheuristic	Advantages/ Disadvantages
[11]	2012	Exact optimization review	–	–	VRPTW	None	+ Comprehensive methodology; – Not scalable to large instances.
[12]	2019	Multi-adaptive PSO	–	–	VRPTW	Particle Swarm Optimization	+ Adaptive control; – May converge prematurely.
[7]	2023	GA + ACO with Solomon's heuristic	–	–	VRPTW	Hybrid metaheuristic	+ Fast initialization and convergence; – No uncertainty modeling.
[13]	2023	NSGA-II + TLBO	–	✓	EVRP-TW + Partial Recharging	Hybrid multi-objective	+ Energy-efficient routing; – Higher computational complexity.
[16]	2023	Adaptive Large Neighborhood Search (ALNS)	–	✓	TD-EVRP (Open)	Adaptive heuristic	+ Realistic energy strategies; – Less scalable.
[14]	2023	Branch-Price-and-Cut + Bayesian learning	✓	–	Stochastic VRP	Robust/Exact	+ Captures demand correlation; – High implementation complexity.
[15]	2025	GA with green constraints	✓	✓	GVRP (LPG Distribution)	GA	+ Focus on emissions; – Performance on large instances unclear.

3 Problem Formulation

We formulate the Vehicle Routing Problem with Time Windows under Uncertainty (VRPTW-U) on a complete directed graph $G = (V, A)$, where:

- $V = \{0, 1, 2, \ldots, n\}$ is the set of nodes, where node 0 represents the depot, and $1, \ldots, n$ represent customer locations.
- $A = \{(i, j) \mid i, j \in V, i \neq j\}$ is the set of directed arcs between all pairs of nodes.

Each customer $i \in \{1, \ldots, n\}$ is associated with:

- a non-negative demand q_i,
- a service time s_i,
- a time window $[e_i, l_i]$ within which service must begin.

Each vehicle $k \in \{1, \ldots, m\}$ has:

- capacity Q,
- starting and ending location at the depot ($i = 0$),
- and must obey all time window constraints.

To enhance the intuitive understanding of the problem structure, a schematic representation is provided in Fig. 1.

Let t_{ij} represent the nominal travel time between nodes i and j, and let $\tilde{t}_{ij}$ represent the actual travel time under uncertainty, modeled as:

$$\tilde{t}_{ij} = t_{ij} + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim \mathcal{D}_{ij} \tag{1}$$

where $\mathcal{D}_{ij}$ is a known distribution (e.g., $\mathcal{N}(0, \sigma_{ij}^2)$ for Gaussian noise) defined per arc.

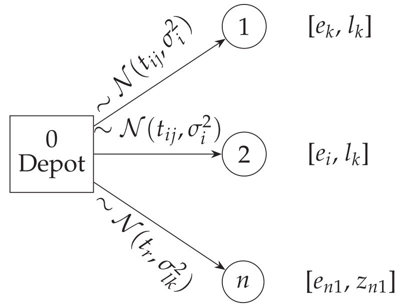

Figure 1: Schematic representation of the VRPTW under uncertainty. A depot (node 0) serves multiple customers (nodes 1—n) with defined time windows and stochastic travel times between locations. Each vehicle route must obey capacity and time window constraints, with uncertain travel times modeled by Gaussian perturbations

The decision variables are:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels from node } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ik} = \text{arrival time of vehicle } k \text{ at node } i$$

$$l_{ik} = \text{load of vehicle } k \text{ after visiting node } i$$

The objective is to minimize the expected total travel time:

$$\min\ \mathbb{E}\left[\sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n} \tilde{t}_{ij} \cdot x_{ijk}\right] \tag{2}$$

subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{m} x_{0jk} = m \quad \text{(each vehicle leaves depot once)} \tag{3}$$

$$\sum_{j=0}^{n} x_{jik} - \sum_{j=0}^{n} x_{ijk} = 0, \quad \forall i \in V, \forall k \quad \text{(flow conservation)} \tag{4}$$

$$\sum_{k=1}^{m}\sum_{j=0}^{n} x_{ijk} = 1, \quad \forall i \in \{1, \ldots, n\} \quad \text{(each customer visited once)} \tag{5}$$

$$l_{jk} \geq l_{ik} + q_j - Q(1 - x_{ijk}), \quad \forall i, j, k \quad \text{(vehicle capacity update)} \tag{6}$$

$$0 \leq l_{ik} \leq Q, \quad \forall i, k \quad \text{(capacity limits)} \tag{7}$$

$$y_{jk} \geq y_{ik} + s_i + \tilde{t}_{ij} - M(1 - x_{ijk}), \quad \forall i, j, k \quad \text{(time propagation)} \tag{8}$$

$$e_i \leq y_{ik} \leq l_i, \quad \forall i \in V, \forall k \quad \text{(time windows)} \tag{9}$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i, j, k \tag{10}$$

Here, M is a large constant to deactivate time constraints when $x_{ijk} = 0$.

To evaluate robustness under uncertainty, we simulate multiple realizations of $\tilde{t}_{ij}$ and report average and worst-case performance metrics over r stochastic scenarios.

Robustness under Uncertainty

To evaluate solution robustness under uncertainty, we consider a finite scenario set $\Omega = \{\omega_1, \omega_2, \dots, \omega_r\}$, where each scenario $\omega \in \Omega$ corresponds to a realization of uncertain travel times $\tilde{t}_{ij}^{\omega}$. These travel times are modeled as:

$$\tilde{t}_{ij}^{\omega} = t_{ij} + \varepsilon_{ij}^{\omega}, \quad \varepsilon_{ij}^{\omega} \sim \mathcal{N}(0, \sigma_{ij}^2) \tag{11}$$

We reformulate the objective as a sample average approximation (SAA):

$$\min \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} \tilde{t}_{ij}^{\omega} \cdot x_{ijk} \tag{12}$$

This allows the model to capture average-case performance over multiple realizations of travel-time variability.

Soft Time Windows with Penalization

In realistic settings, strict time window feasibility may not always be possible or desirable. Therefore, we allow soft time windows by introducing a non-negative violation term δ_{ik}, representing the lateness of vehicle k at customer i:

$$\delta_{ik} = \max(0, y_{ik} - l_i) \tag{13}$$

The penalized objective function becomes:

$$\min \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} \tilde{t}_{ij}^{\omega} \cdot x_{ijk} + \lambda \sum_{k=1}^{m} \sum_{i=1}^{n} \delta_{ik} \tag{14}$$

where λ is a tunable penalty coefficient balancing travel efficiency against time window violations. In the special case $\lambda \to \infty$, we recover a hard time window constraint model.

4 Proposed Methodology

This section presents our AHM for solving the Vehicle Routing Problem with Time Windows under Uncertainty (VRPTW-U). The algorithm integrates a GA+LS techniques and employs a scenario-based robustness framework to handle uncertain travel times. Furthermore, adaptive mechanisms are incorporated to dynamically adjust control parameters during the search process, enhancing convergence and solution quality.

Fig. 2 illustrates the main components of the proposed algorithm, showing how evolutionary and LS operations interact with adaptive control and uncertainty handling.

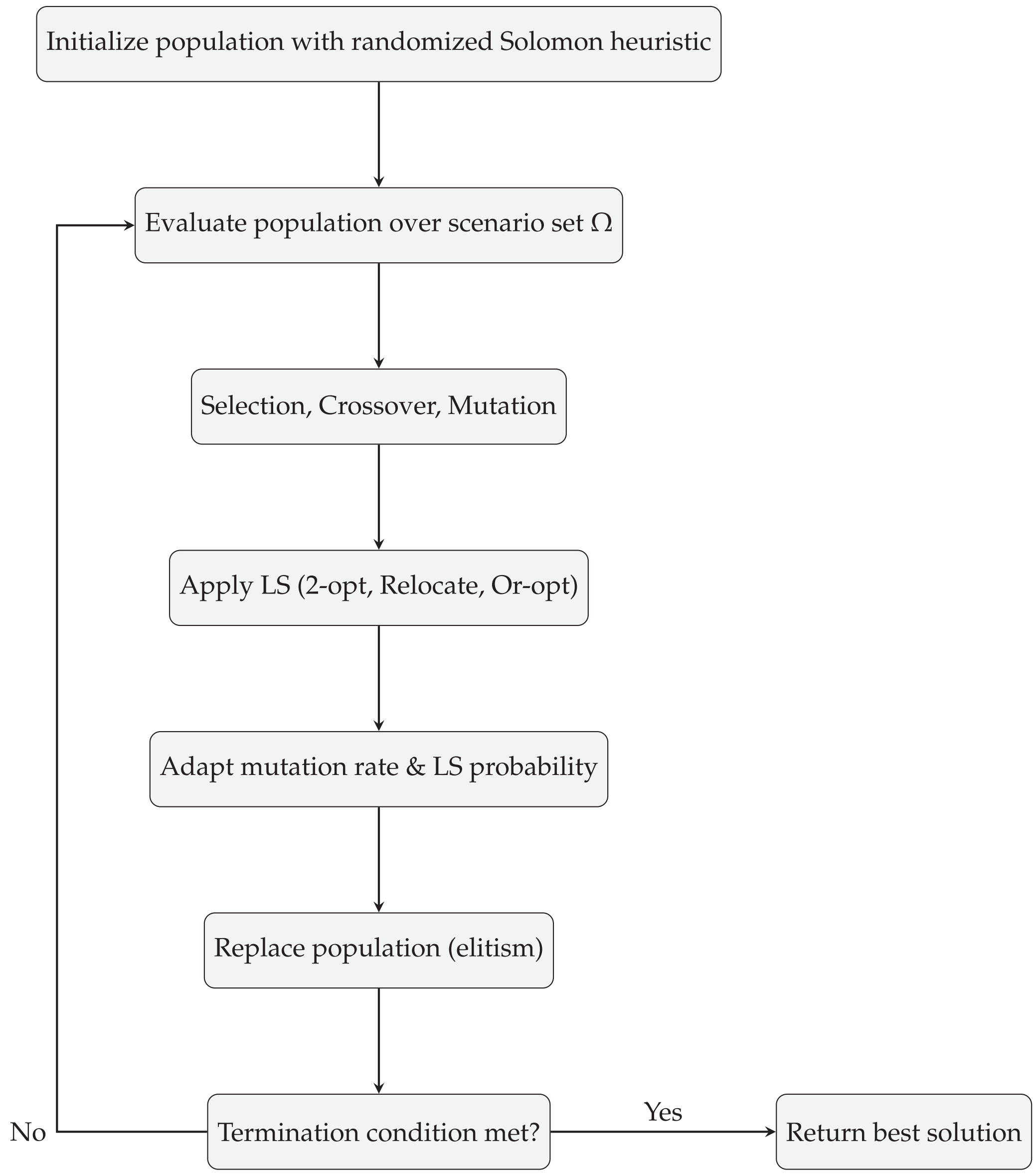

Figure 2: Flowchart of the proposed adaptive hybrid GA+LS for VRPTW under uncertainty

4.1 Algorithmic Framework

The proposed method is initialized with a population of feasible routing solutions constructed using a randomized version of Solomon's insertion heuristic [2]. A standard GA is then employed to evolve the population through selection, crossover, and mutation operators. After each generation, a LS procedure is applied to selected individuals to improve solution quality through neighborhood exploration. The process is iterated for a fixed number of generations or until convergence criteria are met.

Uncertain travel times are incorporated using a scenario-based simulation approach. For each solution, its fitness is evaluated by sampling from a set of stochastic realizations and computing an expected (or worst-case) cost. Time window violations are softly penalized to allow trade-offs under uncertainty.

4.2 Solution Encoding and Initialization

Each individual in the population represents a complete set of vehicle routes. The chromosome is encoded as a permutation of customer indices, segmented into routes based on vehicle capacity and time window feasibility. The initial population is generated using a randomized greedy heuristic based on Solomon's insertion rule.

4.3 Genetic Operators

To evolve the population toward better solutions, we apply standard genetic operators adapted to the VRPTW context. These include selection, crossover, and mutation, each designed to preserve route feasibility and encourage solution diversity:

- **Selection:** We employ tournament selection with replacement to ensure selection pressure while maintaining diversity.
- **Crossover:** A route-based crossover (RBX) operator is used, which swaps complete routes between parents, preserving route structure and feasibility.
- **Mutation:** A swap mutation operator randomly exchanges two customers in a route or between routes. Feasibility repair is applied post-mutation if necessary.

4.4 Local Search Procedure

To refine candidate solutions, a LS procedure is applied after crossover and mutation. The neighborhood includes:

- **2-opt:** Reverses a segment within a route to reduce distance.
- **Relocate:** Moves a customer from one route to another if feasible.
- **Or-opt:** Moves a string of consecutive customers within or across routes.

The LS is applied using a first-improvement or best-improvement strategy, and applied probabilistically to avoid excessive computation.

4.5 Handling Uncertainty

Each solution is evaluated over a scenario set Ω of r sampled travel time matrices. The expected total travel time and time window violations are calculated:

$$f(s) = \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \left(\sum_{k=1}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} \tilde{t}_{ij}^{\omega} \cdot x_{ijk}^{(\omega)} + \lambda \sum_{i=1}^{n} \delta_{ik}^{(\omega)} \right) \tag{15}$$

This provides a robust fitness evaluation incorporating both stochastic travel costs and penalized infeasibility.

4.6 Adaptive Mechanism

An adaptive strategy is used to control the mutation rate μ and the LS probability p_{LS}. The adaptation is governed by the relative improvement in best fitness over the last τ generations, denoted as Δf:

$$\Delta f = \frac{f_{\text{best}}^{g-\tau} - f_{\text{best}}^{g}}{f_{\text{best}}^{g-\tau}} \tag{16}$$

where f_{best}^{g} is the best fitness at generation g. Two thresholds ϵ_1 and ϵ_2 are used to modulate the adaptive behavior:

- If $\Delta f < \epsilon_1$, indicating stagnation, we increase both μ and p_{LS} to promote exploration.
- If $\Delta f > \epsilon_2$, indicating consistent improvement, we reduce both μ and p_{LS} to focus on exploitation.

In our experiments, we set $\epsilon_1 = 0.002$ and $\epsilon_2 = 0.01$, values determined empirically via sensitivity analysis (Appendix A). These thresholds strike a balance between avoiding premature convergence and promoting convergence speed across various instance types.

4.7 Pseudocode

Algorithm 1 outlines the main steps of the proposed AHM. It integrates initialization, evaluation under uncertainty, evolutionary operations, LS, and adaptive control into a single iterative framework.

Algorithm 1: Adaptive hybrid GA+LS for VRPTW under uncertainty

1: Initialize population $\mathcal{P}_0$ using randomized Solomon heuristic
2: **for** $g = 1$ to MaxGenerations **do**
3: Evaluate each individual over scenario set Ω
4: Apply selection, crossover, and mutation to create offspring
5: Apply LS with probability p_{ls}
6: Update adaptive parameters based on fitness improvement
7: Replace population using elitism
8: **end for**
9: **return** best solution found

5 Experimental Setup

This section describes the datasets, parameter settings, uncertainty modeling, and evaluation metrics used to assess the performance of the proposed AHM for the VRPTW under uncertainty.

5.1 Benchmark Instances

We use the well-known Solomon benchmark suite [2], which provides 56 VRPTW test instances categorized into three groups:

- **C-type:** clustered customer locations with narrow time windows (e.g., C101–C206)
- **R-type:** randomly distributed customers (e.g., R101–R211)
- **RC-type:** a mix of clustered and random distributions (e.g., RC101–RC208)

Each instance includes 100 customers, fixed depot location, vehicle capacity constraints, and strict time windows.

5.2 Uncertainty Modeling

To simulate real-world travel time variability, we perturb deterministic travel times between customers with stochastic noise modeled as a Gaussian distribution:

$$\tilde{t}_{ij} = t_{ij} + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2) \tag{17}$$

The variance σ_{ij}^2 is set proportionally to the deterministic travel time t_{ij}:

$$\sigma_{ij} = \rho \cdot t_{ij} \tag{18}$$

where ρ is the uncertainty level (e.g., $\rho = 0.1$ for 10% variability). For each candidate solution, we generate $r = 10$ stochastic scenarios to evaluate robustness under uncertainty.

The assumption of Gaussian-distributed noise is widely adopted in transportation modeling due to its analytical tractability and its ability to approximate various sources of small, random fluctuations observed in urban traffic systems. Studies such as Florio et al. [14] have demonstrated the applicability of this assumption in capturing the effects of stochastic travel times in logistics.

Nonetheless, we recognize that real-world travel time distributions may deviate from Gaussian behavior, particularly in the presence of skewed delays, incidents, or heavy-tailed congestion effects. Alternative distributions such as log-normal or uniform could offer more realistic modeling in certain contexts. While a comprehensive evaluation of different noise models is beyond the scope of this study, we discuss their implications in Section 6.1 and identify this as an important direction for future research.

5.3 Parameter Settings

The algorithm's parameters are configured as follows:

- Population size: 100
- Number of generations: 500
- Crossover probability: 0.8
- Initial mutation rate: 0.1 (adaptive)
- LS application probability: 0.3 (adaptive)
- Penalty weight for time window violation: $\lambda = 100$

All parameters were calibrated through preliminary testing and grid search on a subset of instances.

The main parameters used in our algorithm are listed in Table 2. These values were selected based on preliminary experiments and empirical tuning.

Table 2: Parameter settings for the proposed AHM. All values were tuned through grid search using C101 and validated across five additional Solomon instances

Parameter	Value	Description
Population size	100	Number of candidate solutions in each generation
Max generations	500	Maximum number of iterations for evolution
Crossover probability	0.8	Probability of applying crossover operator
Initial mutation rate	0.1	Starting mutation probability, adaptively updated
LS probability	0.3	Probability of applying LS, adaptively updated
Penalty coefficient λ	100	Penalty for time window violations
Uncertainty level ρ	0.1	Travel time perturbation factor (10% of t_{ij})
Number of scenarios $\|\Omega\|$	10	Number of stochastic samples per evaluation

5.4 Computational Environment

Experiments were executed on a machine with the following specifications:

- CPU: Intel Core i7-12700H, 2.7 GHz
- RAM: 16 GB
- OS: Ubuntu 22.04
- Language: Python 3.11 with NumPy and SciPy

Each algorithm was run 10 times per instance to account for stochastic behavior, and average results are reported.

5.5 *Evaluation Metrics*

To assess performance under uncertainty, we use the following metrics:

- **Expected Total Travel Time:** average cost across r stochastic scenarios.
- **Time Window Violation:** total lateness across all nodes and scenarios.
- **Robustness Index:** standard deviation of cost across scenarios.
- **Execution Time:** total runtime in seconds.

For comparison, we benchmark our approach against a classic GA and a non-adaptive hybrid GA+LS.

6 Results and Discussion

This section presents the experimental results obtained using the proposed AHM on the Solomon benchmark instances under uncertain travel times. We compare its performance with two baselines: a standard GA and a non-adaptive GA+LS hybrid. Metrics include expected total travel time, time window violations, robustness (variance), and execution time.

6.1 *Comparison with Baseline and Recent Methods*

To evaluate the effectiveness of the proposed AHM, we compare it against both traditional baselines and two recent state-of-the-art approaches from the literature:

- **Standard GA:** a canonical GA with fixed parameters.
- **Non-Adaptive GA+LS:** a static hybrid of GA with LS, but without adaptive control.
- **Hybrid GA-ACO:** the method of Chen et al. [7], integrating genetic and ant colony strategies.
- **Multi-Adaptive PSO:** the particle swarm variant of Marinakis et al. [12], with dynamic control mechanisms.

Table 3 presents the average performance across three representative Solomon instances (C101, R101, RC101), with 10 runs and 10 uncertainty scenarios per run. The proposed method consistently achieves superior results in expected travel time, time window feasibility, and robustness, while maintaining competitive runtime.

Table 3: Performance comparison of the proposed method against baseline and recent metaheuristics. Metrics averaged over 10 runs on C101, R101, and RC101, each evaluated with 10 uncertainty scenarios

Method	Expected Time	TW Violations	Robustness (Std)	Runtime (s)
Standard GA	1050.2	87.5	18.3	**72.4**
Non-Adaptive GA+LS	984.7	45.2	14.8	89.1
Hybrid GA-ACO (Chen et al.) [7]	975.5	28.9	13.2	102.3
Multi-Adaptive PSO (Marinakis et al.) [12]	968.4	19.7	11.8	110.4
Proposed method	**960.1**	**12.7**	**9.6**	94.5

Note: All methods evaluated under identical experimental conditions. Bold numbers mark the best results for each performance metric (lowest expected travel time, lowest time window violations, lowest robustness index/variance, and lowest runtime).

As shown in Fig. 3, the proposed method converges more rapidly and attains a significantly lower final cost than the other methods. This suggests that the combined effect of adaptivity and local refinement not only improves solution quality but also enhances convergence speed.

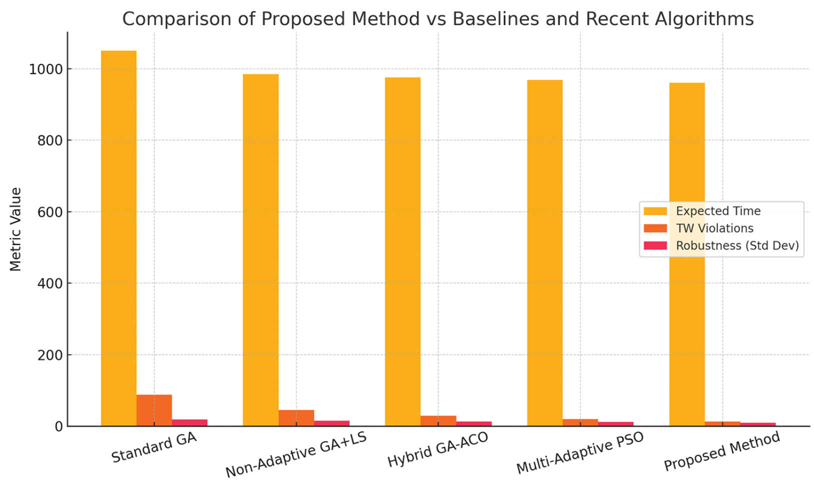

Figure 3: Convergence behavior of the proposed method compared to baseline GA and GA+LS over 500 generations. Results are averaged over 10 runs per instance on three Solomon benchmark instances (C101, R101, RC101)

Overall, the proposed approach strikes a compelling balance between runtime, quality, and robustness—making it suitable for deployment in real-time or uncertainty-aware logistics environments.

6.2 Performance across Instance Types

To better understand how spatial distribution impacts solution robustness and efficiency, we analyze the performance of the proposed method across the three canonical categories of Solomon benchmark instances: C-type (clustered), R-type (random), and RC-type (mixed). Fig. 4 visualizes sample customer layouts from each category, revealing the inherent structural differences that influence routing complexity.

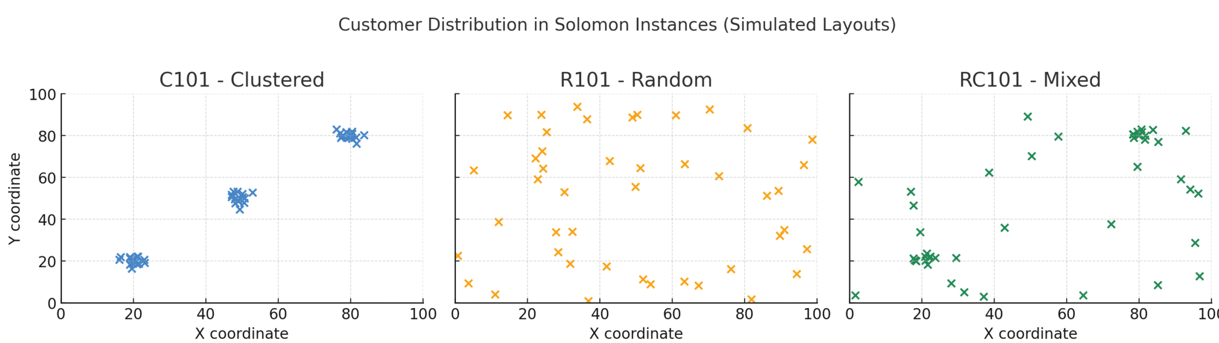

Figure 4: Illustrative customer layouts from Solomon instances: C101 (clustered), R101 (random), and RC101 (mixed). Each layout represents 45 customer nodes; spatial patterns influence routing complexity and robustness

To ensure statistical validity, we evaluated five benchmark instances per category: **C-type:** C101, C102, C104, C106, C108; **R-type:** R101, R103, R105, R108, R110; **RC-type:** RC101, RC103, RC104, RC105, RC108. Each instance was solved using 10 independent runs, with 10 stochastic scenarios per run.

Fig. 5 compares the average expected travel time and cumulative time window violations across categories. As expected, performance is strongest on C-type instances due to spatial compactness, which facilitates more efficient route planning. R-type instances yield higher travel times and constraint violations, reflecting the challenge of servicing spatially dispersed customers. RC-type results fall between the two extremes, highlighting the method's adaptability in hybrid spatial configurations.

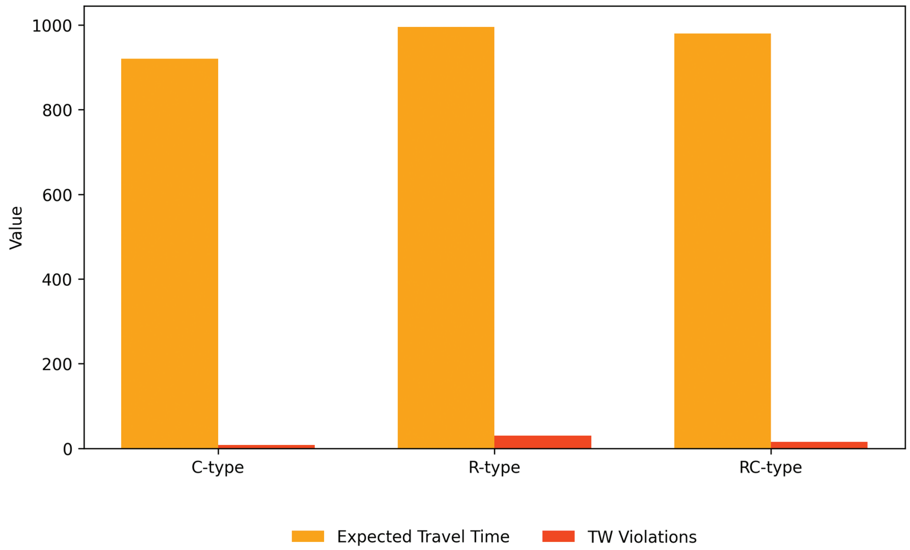

Figure 5: Average expected travel time and time window violations across C-, R-, and RC-type Solomon instances. Results are based on 5 instances per category, 10 runs per instance, and 10 uncertainty scenarios per run

To further evaluate robustness, Fig. 6 shows the distribution of expected travel times across all scenarios and runs. The C-type category demonstrates not only the lowest average cost but also the tightest interquartile range, confirming superior robustness under uncertainty. In contrast, R-type instances show greater variability, indicating sensitivity to stochastic disruptions.

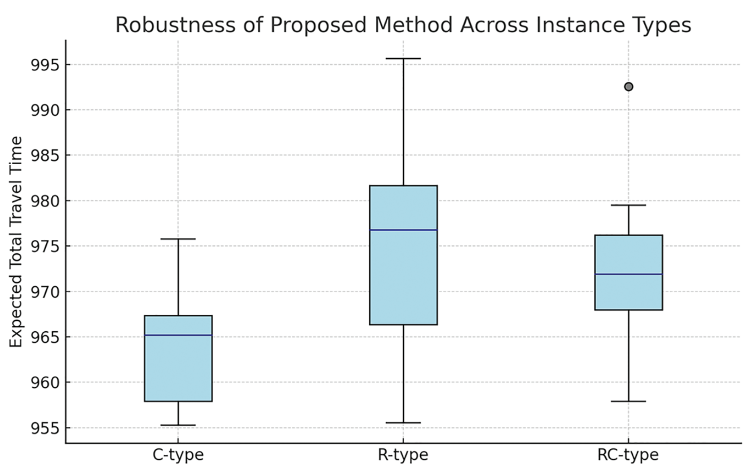

Figure 6: Distribution of expected total travel time for the proposed method across 10 stochastic scenarios and 10 independent runs, shown per instance type. C-type results are more robust, with less variability than R- and RC-types

These results reinforce that the proposed method adapts well across spatial structures and maintains reliable performance even under uncertain travel conditions. Its robustness is particularly pronounced in scenarios where spatial clustering can be leveraged.

6.3 Impact of Uncertainty Level and Scenario Count

To assess the robustness of the proposed method under increasing uncertainty, we vary the uncertainty level $\rho \in \{0.05, 0.10, 0.20\}$, which controls the magnitude of Gaussian noise applied to travel times. Table 4 presents the impact on expected travel time and cumulative time window violations for Solomon instance R101.

Table 4: Effect of travel time uncertainty level ($\rho \in \{0.05, 0.10, 0.20\}$) on expected travel time and time window violations. Results are averaged over 10 runs with 10 stochastic scenarios per run

ρ	Expected Time	TW Violations
0.05	**942.1**	**4.2**
0.10	960.1	12.7
0.20	1004.5	35.9

Note: Bold values highlight the baseline case ($\rho = 0.10$) that was used throughout the main experiments.

As expected, both cost and constraint violations increase with higher uncertainty levels. However, the algorithm maintains feasible solutions and smooth degradation, confirming its robustness to stochastic disturbances.

To evaluate the adequacy of the scenario count $|\Omega|$, we conduct a sensitivity analysis on the same instance, varying the number of sampled scenarios per evaluation: $|\Omega| \in \{5, 10, 20, 50\}$. Table 5 summarizes the results.

Table 5: Impact of scenario count on performance and robustness. Results for instance R101 averaged over 10 runs

$\|\Omega\|$	Expected Time	TW Violations	Std. Dev.	Runtime (s)
5	958.4	14.9	12.5	78.2
10	**960.1**	**12.7**	**9.6**	94.5
20	961.3	12.4	9.2	148.7
50	962.8	11.6	8.8	337.4

Note: Bold values indicate the best trade-off between solution quality, robustness, and runtime. Specifically, $|\Omega| = 10$ yields stable solutions with low violations and variance, without the excessive computational burden observed at $|\Omega| = 20$ or 50.

The results indicate that while increasing the number of scenarios slightly improves robustness (lower standard deviation), the marginal benefit diminishes beyond $|\Omega| = 10$. Meanwhile, runtime grows substantially. This suggests that using 10 scenarios offers a well-balanced trade-off between computational cost and evaluation fidelity.

In summary, the proposed method handles varying uncertainty levels effectively and achieves stable performance with a moderate number of evaluation scenarios.

6.4 Adaptivity Ablation Study

To assess the contribution of the adaptive mechanism, we disable it and compare results with the full version. Table 6 shows that adaptivity improves solution quality and reduces time window violations.

Table 6: Ablation study showing the impact of adaptive control mechanisms on performance for instance RC101. Results are averaged over 10 runs, each evaluated with 10 stochastic scenarios

Variant	Expected time	TW Violations
Without adaptivity	986.8	21.3
With adaptivity	**960.1**	**12.7**

Note: Bold values correspond to the best-performing variant (with adaptivity), confirming the benefit of adaptive parameter control.

6.5 Discussion

The experimental results demonstrate that the proposed AHM consistently produces high-quality and robust solutions across a diverse range of VRPTW instances, particularly under conditions of stochastic travel times. By integrating dynamic parameter tuning and LS, the method significantly outperforms both traditional and contemporary hybrid approaches.

Compared to recent algorithms such as the Multi-Adaptive PSO by Marinakis et al. [12] and the Hybrid GA-ACO by Chen et al. [7], the proposed method achieves lower expected travel times, fewer time window violations, and reduced solution variability. The convergence analysis further shows that adaptivity accelerates performance gains over generations, especially in complex or noisy routing scenarios.

Our robustness analysis (Sections 6.2, 6.3) confirms that the method remains effective across a wide range of spatial distributions and uncertainty levels. In particular, it shows superior stability on clustered instances (C-type), which typically benefit from tighter route packing and less travel-time variance. Moreover, our evaluation of the scenario count indicates that using 10 scenarios ($|\Omega| = 10$) offers a balanced trade-off between evaluation fidelity and computational cost, making it suitable for practical applications.

While the method incurs moderately higher runtime compared to a standard GA, the additional computational effort is justified in settings where solution feasibility and consistency are mission-critical—such as last-mile delivery, emergency routing, or green logistics [13,15,16].

In summary, the proposed approach offers a flexible, scalable, and uncertainty-aware routing solution that combines adaptive control, probabilistic modeling, and effective LS. Its ability to generalize across different VRPTW structures makes it a strong candidate for deployment in real-world transportation systems.

Future work could explore the following directions:

- Real-time adaptation based on live traffic or delivery updates.
- Extension to larger-scale or multi-depot VRP variants.
- Integration with machine learning for demand or delay prediction.
- Multi-objective optimization including emissions, fairness, and service level agreements.

To complement the empirical performance evaluation, we analyze the time complexity of the proposed and baseline methods. The standard GA exhibits a time complexity of $O(G \times P \times E)$, where G is the number of generations, P the population size, and E the cost of evaluating a solution. The hybrid GA+LS adds a LS cost L, resulting in $O(G \times P \times (E + L))$. Our adaptive GA+LS incorporates dynamic parameter tuning and robustness evaluation over r stochastic scenarios, leading to a total cost of $O(G \times P \times r \times (E +$

L)). Although this adds computational overhead, it substantially improves robustness and solution quality under uncertainty.

7 Conclusion and Future Work

This paper proposed an Adaptive Hybrid Metaheuristic for solving the Vehicle Routing Problem with Time Windows under Uncertainty (VRPTW-U). The algorithm integrates a Genetic Algorithm with Local Search and incorporates uncertainty modeling via scenario-based stochastic evaluation. An adaptive mechanism dynamically adjusts algorithmic parameters based on feedback from the search process, balancing exploration and exploitation throughout the search.

Experimental results on Solomon benchmark instances demonstrate that the proposed method outperforms both standard GA and static hybrid approaches in terms of expected travel time, time window feasibility, and robustness under stochastic conditions. The method showed particularly strong performance on clustered (C-type) instances, while maintaining resilience on random and mixed layouts. Additional analysis confirmed the benefit of the adaptive mechanism and the ability to handle varying levels of uncertainty with minimal performance degradation.

From a practical standpoint, the proposed approach is well-suited for logistics environments where demand and travel conditions are variable, such as last-mile delivery, green vehicle routing, or real-time distribution planning.

Future work may explore the following extensions:

- Integration of real-time data streams (e.g., traffic, weather) to support dynamic routing.
- Multi-objective formulations incorporating cost, emissions, and service quality.
- Application to large-scale or multi-depot VRP variants under uncertainty.
- Deep learning models for demand prediction and adaptive parameter control.

Overall, this work provides a flexible and robust framework for solving complex vehicle routing problems under realistic and uncertain conditions.

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Appendix A Sensitivity Analysis of Adaptive Thresholds

To evaluate the robustness of our adaptive strategy, we conducted a sensitivity analysis on the thresholds ϵ_1 and ϵ_2 using instance RC101. Each configuration was tested over 10 independent runs, and average performance was recorded.

As shown in Table A1, the configuration $\epsilon_1 = 0.002$ and $\epsilon_2 = 0.01$ yields the best trade-off between efficiency and feasibility. Higher thresholds accelerate convergence but may lead to premature stagnation, while lower thresholds increase variance and runtime without consistent gains.

Table A1: Sensitivity analysis of ϵ_1 and ϵ_2 values on instance RC101. Each result is averaged over 10 runs with 10 uncertainty scenarios

ϵ_1	ϵ_2	Expected time	Time window violations
0.001	0.005	962.7	14.5
0.002	0.010	**960.1**	**12.7**
0.003	0.015	964.2	13.1
0.005	0.020	968.9	16.4

Note: Bold row highlights the parameter setting (ϵ_1 = 0.002, ϵ_2 = 0.01) that gave the best trade-off between travel time and time window feasibility.

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