Computers, Materials & Continua

Tech Science Press

Doi:10.32604/cmc.2025.066310

ARTICLE

Schweizer-Sklar T-Norm Operators for Picture Fuzzy Hypersoft Sets: Advancing Suistainable Technology in Social Healthy Environments

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Received: 04 April 2025; Accepted: 29 April 2025; Published: 09 June 2025

ABSTRACT: Ensuring a sustainable and eco-friendly environment is essential for promoting a healthy and balanced social life. However, decision-making in such contexts often involves handling vague, imprecise, and uncertain information. To address this challenge, this study presents a novel multi-criteria decision-making (MCDM) approach based on picture fuzzy hypersoft sets (PFHSS), integrating the flexibility of Schweizer-Sklar triangular norm-based aggregation operators. The proposed aggregation mechanisms—weighted average and weighted geometric operators—are formulated using newly defined operational laws under the PFHSS framework and are proven to satisfy essential mathematical properties, such as idempotency, monotonicity, and boundedness. The decision-making model systematically incorporates both benefit and cost-type criteria, enabling more nuanced evaluations in complex social or environmental decision problems. To enhance interpretability and practical relevance, the study conducts a sensitivity analysis on the Schweizer-Sklar parameter (Δ). The results show that varying Δ affects the strictness of aggregation, thereby influencing the ranking stability of alternatives. A comparative analysis with existing fuzzy and hypersoft-based MCDM methods confirms the robustness, expressiveness, and adaptability of the proposed approach. Notably, the use of picture fuzzy sets allows for the inclusion of positive, neutral, and negative memberships, offering a richer representation of expert opinions compared to traditional models. A case study focused on green technology adoption for environmental sustainability illustrates the real-world applicability of the proposed method. The analysis confirms that the approach yields consistent and interpretable results, even under varying degrees of decision uncertainty. Overall, this work contributes an efficient and flexible MCDM tool that can support decision-makers in formulating policies aligned with sustainable and socially responsible outcomes.

KEYWORDS: Hypersoft set; picture fuzzy set; Schweizer-Sklar norms; aggregation operators; decision-making; green technology adoption

1 Introduction

Modeling a decision science problem primarily involves optimizing beneficial outcomes within the constraints of preferences specified by decision-makers, based on the given attribute values. However, effectively processing these preferences is often complex due to the inherent vagueness and uncertainty found in real-world problems. One of the significant areas of application in decision science lies in addressing human-centric environmental and social concerns. There is a growing global need for clean, eco-friendly, and sustainable solutions in production, energy, and transportation to tackle challenges such as climate change, global warming, and resource/waste management.

Three core pillars of green technology adoption aimed at improving social well-being include: promoting digital equity and environmental responsibility through strategic policies, fostering green alliances, and embedding sustainability as a societal value. These efforts contribute to cultivating responsible behaviors, sustainable practices, and lifestyles that enhance human health and well-being.

Fuzzy systems have been widely employed in real-life applications such as solar photovoltaic systems [1] and robotic manipulators [2,3]. Fuzzy decision-making, in particular, plays a vital role in evaluating performance indicators that are typically affected by uncertain data. For instance, Bhatia and Diaz-Elsayed [4] applied fuzzy TOPSIS techniques to support smart manufacturing adoption for small and medium-sized enterprises. Similarly, Aytekin and colleagues [5] evaluated sustainable green strategies in logistics using T-spherical fuzzy methods, and TODIM-based approaches have been implemented for green supplier selection under type-2 neutrosophic environments [6]. Moreover, complex q-rung picture fuzzy frameworks have been applied to power and energy decision-making [7], while interval-valued fermatean neutrosophic super hypersoft sets have been introduced in healthcare assessments [8].

Numerous researchers have developed methodologies rooted in fuzzy set theory and its extensions—such as picture fuzzy sets [9], fuzzy soft sets [10], and fuzzy hypersoft sets [11]. Naeem et al. [12] proposed sigma-algebraic measures for fuzzy neutrosophic soft sets, and later introduced picture fuzzy soft sigma-algebra measures with practical implications [13]. Aggregation operators for picture fuzzy soft sets have also been investigated using weighted average and hybrid models [14].

In certain decision-making scenarios, attributes require further categorization, where conventional soft sets are insufficient. Hypersoft set theory, as introduced by Smarandache [11], becomes essential in such cases. Over time, various hybrid fuzzy-hypersoft models have emerged. Saqlain et al. [15] presented neutrosophic hypersoft extensions of the TOPSIS method. Plithogenic hypersoft sets [16] and intuitionistic fuzzy hypersoft models [17,18] have also been proposed, including Pythagorean fuzzy hypersoft sets with Einstein-based aggregation operators [19] and their application in COVID-19 safety assessment [20].

The generalization of picture fuzzy soft sets by Khan et al. [21], and the development of q-rung orthopair fuzzy hypersoft sets [22] further enriched this domain. Chinnadurai and Robin [23] introduced picture fuzzy hypersoft sets (PFHSS), which were later refined by Dhumras and Bajaj [24] to address the limitations of earlier models.

Recent advancements include the modified MARCOS method in a 2-tuple linguistic q-rung PF environment [25] and possibilistic simulation-based group decision-making for evaluating educational program efficiency [26]. Neutrosophic-fuzzy blended hypersoft models have also contributed to healthcare analytics and green supplier selection using Hellinger divergence and$R$-norm information measures [27–29].

The Schweizer-Sklar t-norm and t-conorm, introduced by Schweizer and Sklar [30], incorporate a parameter Δ to enable flexible handling of imprecise data. This parameter generalizes various t-norms, including Hamacher and Lukasiewicz. Liu and Wang [31] developed q-rung orthopair fuzzy Archimedean t-norms and t-conorms using weighted aggregation. In parallel, Schweizer-Sklar operators have been applied to COPRAS methods [32], Maclaurin symmetric aggregations [33], and power aggregation operators for intuitionistic fuzzy sets [34]. While these approaches are useful, they assume equal priority among decision-makers, a limitation addressed through priority-based aggregation schemes. Yet, there is a noticeable gap in defining aggregation operators for PFHSS using Schweizer-Sklar norms, owing to the structural complexity of integrating multiple fuzzy components.

1.1 Motivation and Research Gap

This study aims to develop a novel decision-making tool tailored for real-life, sub-parameterized complex information scenarios. A core focus is to support green technology adoption within socially healthy frameworks by proposing a new score function that incorporates Schweizer-Sklar t-norm and t-conorm aggregation operators under the picture fuzzy hypersoft environment.

The main motivations include:

- The PFHSS model effectively incorporates an additional refusal/abstain component along with sub-parameterized attributes, offering deeper insight for decision-making problems.
- Schweizer-Sklar-based aggregation operators provide increased flexibility for modeling uncertain data within PFHSS structures.
- The proposed set-theoretic properties for PFHSS aggregation (e.g., weighted average and weighted geometric forms) enable more robust and interpretable aggregation.
- To date, no studies have explored Schweizer-Sklar norm/co-norm based aggregation operators within the PFHSS framework—highlighting a significant research gap addressed in this paper.

1.2 Novelty and Contributions of the Present Study

This paper introduces Schweizer-Sklar-based weighted average and geometric aggregation operators within the PFHSS framework for the first time. These operators provide a flexible and powerful foundation for sustainable decision-making in human-centric applications. The paper presents their set-theoretical properties—such as idempotency, boundedness, homogeneity, and monotonicity—in detail.

The PFHSS-based formulation addresses real-world uncertainty and offers decision-makers the freedom to assign uncertainty components based on expert insights using the adjustable Δ parameter. A comprehensive comparative and graphical analysis is presented to validate the proposed methodology.

The rest of the manuscript is organized as follows:

- Section 2 reviews core definitions related to soft/hypersoft sets, PFHSS, score functions, and Schweizer-Sklar operations.
- Section 3 presents the proposed aggregation operators and their theoretical properties.
- Section 4 outlines the algorithmic framework and procedural flowchart for solving MCDM problems.
- Section 5 presents a detailed case study on green technology adoption.
- Section 6 includes graphical analysis based on the Δ parameter.
- Section 7 provides comparative insights.
- Section 8 concludes the study and summarizes key contributions.

2 Fundamental Concepts & Definitions

In this section, some basic and fundamental definitions which are necessary to understand the propositions of aggregation operators for $PFHSSs$ have been presented as follows.

Definition 1: Picture Fuzzy Set (PFS) [9]. *"For a universe of discourse V a picture fuzzy set R* in *V represented as*

$R = \{\nu, \rho_R(\nu), \tau_R(\nu), \omega_R(\nu) | \nu \ \epsilon \ V\},$

where $\rho_R : V \to [0,1]$, $\tau_R : V \to [0,1]$ *and* $\omega_R : V \to [0,1]$ *indicates the degree of positive, neutral and negative membership of* v in R *respectively, along with the constraints* $\rho_R, \tau_R,\ \omega_R$ *satisfies the constraint*

$$\rho_R(v) + \tau_R(v) + \omega_R(v) \le 1\ (\forall\ v \in V);$$

and, $\beth_R(v) = (1 - (\rho_R(v) + \tau_R(v) + \omega_R(v))$ *indicates refusal membership degree."*

Definition 2: Soft Set (SS) [35]. *"For a universal set V and K be a set of parameters. Then the pair (R,K) is known as a soft set over the universe of discourse V, where R is a function from R: K→* $P(V)$*."*

Definition 3: Hypersoft Set (HSS) [11]. *"For a universal set V be the universal set and P(V) be the set of all subsets of V. Let* $k_1, k_2, \ldots k_n$ for $n \ge 1$, *be* n *set of parameters, whose corresponding parameters values belong to the collection* $K_1, K_2, \ldots, K_n$ with $K_i \cap K_j = \varphi$ for $i \ne j$ and $i, j \in \{1, 2, \ldots, n\}$*. Then the pair* $(R, K_1 \times K_2 \times \ldots K_n)$ *is known as hypersoft collection over the universal set V where* $R : K_1 \times K_2 \times \cdots \times K_n \to P(V)$*."*

Definition 4: (Picture Fuzzy Hypersoft Set) [24]. *"Consider V be a universe of discourse and PFS(V) be the collection of all picture fuzzy subsets from the universal of discourse V. Let* $k_1, k_2, \ldots, k_n$ for $n \ge 1$, *be* n *be the collection of all parameters, whose parameter values belongs to the collection* $K_1, K_2, \ldots, K_n$ with $K_i \cap K_j = \varphi$ for $i \ne j$ and $i, j \in \{1, 2, \ldots, n\}$*. Let* B_i *be the non-void collection of* K_i *for every* $i = 1, 2, \ldots, n$*. A picture fuzzy hypersoft set (PFHSS) is described as follows* $(R, B_1 \times B_2 \times \cdots \times B_n)$*; where* $R : K_1 \times K_2 \times \ldots \times K_n \to \text{PFS}(V)$ *and*

$$R(B_1 \times B_2 \times \ldots . \times B_n) = \left\{ < \vartheta, \left(\frac{v}{\rho_{R(\vartheta)}(v), \tau_{R(\vartheta)}(v), \omega_{R(\vartheta)}(v)} \right) > |\ v\ \epsilon\ V \right\},$$

where $\vartheta \in B_1 \times B_2 \times \cdots \times B_n \subseteq K_1 \times K_2 \times \ldots K_n$ *&* $\rho\ \tau$ and ω *indicates the positive, neutral & negative membership degrees respectively with the additional condition*

$$\rho_{R(\vartheta)}(v) + \tau_{R(\vartheta)}(v) + \omega_{R(\vartheta)}(v) \le 1 \text{ where } \rho_{R(\vartheta)}(v), \tau_{R(\vartheta)}(v), \omega_{R(\vartheta)}(v) \in [0,1].$$

The term $\complement_{R(\vartheta)}(v) = 1 - \rho_{R(\vartheta)}(v) - \tau_{R(\vartheta)}(v) - \omega_{R(\vartheta)}(v)$ *is known as the refusal membership degree of* v in $\text{PFS}(V)$*. To make the mathematical computations simpler, the collection of picture fuzzy hypersoft set may also be described in terms of picture fuzzy hypersoft number (PFHSN):*

$$R_{v_i}(\vartheta_j) = \left\{ \rho_{R(\vartheta_j)}(v_i),\ \tau_{R(\vartheta_j)}(v_i),\ \omega_{R(\vartheta_j)}(v_i)\ |v_i \in V \right\}.$$

Also, the picture fuzzy hypersoft number can be defined as $I_{\vartheta_{ij}} = \left(\rho_{R(\vartheta_{ij})},\ \tau_{R(\vartheta_{ij})}\ \omega_{R(\vartheta_{ij})} \right)$*, where the subscript* ϑ_{ij} *is utilized to build up a relation between the available alternatives with the attributes for the computational processes."*

Definition 5: [36] *"Let* $I_{\vartheta_{ij}} = \left(\rho_{R(\vartheta_{ij})}, \tau_{R(\vartheta_{ij})}\ \omega_{R(\vartheta_{ij})} \right)$ *be a PFHSN. The* **score function** *of* $I_{\vartheta_{ij}}$ *is given by* $\mathbb{S}(I_{\vartheta_{ij}}) = \rho_{R(\vartheta_{ij})} - \omega_{R(\vartheta_{ij})}; \mathbb{S}(I_{\vartheta_{ij}}) \in [-1,1]$*."*

In literature, Schweizer-Sklar [30] has recommended some special types of algebraic operations, i.e., triangular norms for which definitions may be written as follows:

Definition 6: [30] *"Let r and s be any two real numbers. Then, Schweizer-Sklar $t-norms$ and $t-conorms$ are defined as*

$SS_\Delta(r,s) = (r^\Delta + s^\Delta - 1)^{1/\Delta}; \Delta < 0$

$SS^*_\Delta(r,s) = 1 - [(1-r)^\Delta + (1-s)^\Delta - 1]^{1/\Delta}; \Delta < 0$

where, $r, s \in [0,1]$."

3 Average/Geometric Aggregation Operators

The concept of an aggregation operator logically combines the related numerous inputs into a single output value, is a crucial tool in the information fusion process and is frequently applied to a wider range of decision science problems. The issues are not exclusive to mathematics; they are also extensively present in the area of physical sciences, socio-economic fields, engineering applications and other related fields. In this section, we develop two kinds of aggregation operators based on Schweizer-Sklar triangular norms for picture fuzzy hypersoft numbers and describe some outcomes based on them.

For proposing the Schweizer-Sklar based picture fuzzy hypersoft weighted averaging operator/geometric operator, it is required to understand some basic operations of $PFHSNs$ which have been defined below:

3.1 Schweizer-Sklar Operations on $PFHSNs$

In this section, we discuss some Schweizer-Sklar (SS) operations and some of its fundamental notions. Suppose that the t-norms (SS_Δ) and the t-conorms (SS^*_Δ) represents the SS sum and SS product respectively are given as

- $I_{\vartheta_{11}} \oplus_{SS} I_{\vartheta_{12}} = (SS^*_\Delta(\rho_{\vartheta_{11}}, \rho_{\vartheta_{12}}), SS_\Delta(\tau_{\vartheta_{11}}, \tau_{\vartheta_{12}}), SS_\Delta(\omega_{\vartheta_{11}}, \omega_{\vartheta_{12}}))$;
- $I_{\vartheta_{11}} \otimes_{SS} I_{\vartheta_{12}} = (SS_\Delta(\rho_{\vartheta_{11}}, \rho_{\vartheta_{12}}), SS^*_\Delta(\tau_{\vartheta_{11}}, \tau_{\vartheta_{12}}), SS^*_\Delta(\omega_{\vartheta_{11}}, \omega_{\vartheta_{12}}))$.

Definition 7: *Let $I_{\vartheta_d} = (\rho_{\vartheta_d}, \tau_{\vartheta_d}, \omega_{\vartheta_d})$, $I_{\vartheta_{11}} = (\rho_{\vartheta_{11}}, \tau_{\vartheta_{11}}, \omega_{\vartheta_{11}})$ and $I_{\vartheta_{12}} = (\rho_{\vartheta_{11}}, \tau_{\vartheta_{12}}, \omega_{\vartheta_{12}})$ are PFHSNs and $\kappa \in \mathbb{R}^+$. The algebraic operations for PFHSNs may be understood as follows:*

(a) $I_{\vartheta_{11}} \oplus I_{\vartheta_{12}} = \left\langle 1 - ((1-\rho_{\vartheta_{11}})^\Delta + (1-\rho_{\vartheta_{12}})^\Delta - 1)^{1/\Delta}, (\tau^\Delta_{\vartheta_{11}} + \tau^\Delta_{\vartheta_{12}} - 1)^{1/\Delta}, (\omega^\Delta_{\vartheta_{11}} + \omega^\Delta_{\vartheta_{12}} - 1)^{1/\Delta} \right\rangle$.

(b) $I_{\vartheta_{11}} \oplus I_{\vartheta_{12}} = \left\langle (\rho^\Delta_{\vartheta_{11}} + \rho^\Delta_{\vartheta_{12}} - 1)^{1/\Delta}, 1 - ((1-\tau_{\vartheta_{11}})^\Delta + (1-\tau_{\vartheta_{12}})^\Delta - 1)^{1/\Delta}, 1 - ((1-\omega_{\vartheta_{11}})^\Delta + (1-\omega_{\vartheta_{12}})^\Delta - 1)^{1/\Delta} \right\rangle$.

(c) $\kappa I_{\vartheta_d} = \left\langle 1 - (\kappa(1-\rho_{\vartheta_d})^\Delta - (\kappa-1))^{1/\Delta}, (\kappa\tau^\Delta_{\vartheta_d} - (\kappa-1))^{1/\Delta}, (\kappa\omega^\Delta_{\vartheta_d} - (\kappa-1))^{1/\Delta} \right\rangle$.

(d) $I^\kappa_{\vartheta_d} = \left\langle (\kappa\rho^\Delta_{\vartheta_d} - (\kappa-1))^{1/\Delta}, 1 - (\kappa(1-\tau_{\vartheta_d})^\Delta - (\kappa-1))^{1/\Delta}, 1 - (\kappa(1-\omega_{\vartheta_d})^\Delta - (\kappa-1))^{1/\Delta} \right\rangle$.

(e) $I^c_{\vartheta_d} = (\omega_{\vartheta_d}, \tau_{\vartheta_d}, \rho_{\vartheta_d})$.

3.2 PFHS Schweizer-Sklar Weighted Averaging Aggregation Operators ($PFHSSSWA$)

Definition 8: *Suppose $I_{\vartheta_d} = (\rho_{\vartheta_d}, \tau_{\vartheta_d}, \omega_{\vartheta_d})$ is a picture fuzzy hypersoft number. Let λ_i (experts) & δ_j (attributes) be the respective weights. Also, $\lambda_i > 0$, $\sum_{i=1}^n \lambda_i = 1$ and $\delta_j > 0$, $\sum_{i=1}^n \delta_j = 1$. The **PFHSS Schweizer-Sklar Weighted Average AO (PFHSSSWAAO)** is a function $\mathfrak{M}^n \to \mathfrak{M}$ defined as*

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \oplus_{SS}{}^m_{j=1}\delta_j \left(\oplus^n_{i=1}\lambda_i I_{\vartheta_{ij}}\right), \tag{1}$$

where $\mathfrak{M}^n = (I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$ is a set of PFHSNs.

Theorem 1: *Suppose $I_{\vartheta_d} = (\rho_{\vartheta_d}, \tau_{\vartheta_d}, \omega_{\vartheta_d})$ is a picture fuzzy hypersoft number. Then on the basis of above definition, we get*

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$$
$$= \left\langle 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i(1-\rho_{\vartheta_{ij}})^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$
$$\left. \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle. \tag{2}$$

And λ_i (experts) & δ_j (attribute's) are the respective weight vectors. Also, $\lambda_i > 0$, $\sum_{i=1}^{n}\lambda_i = 1$ and $\delta_j > 0$, $\sum_{i=1}^{n}\delta_j = 1$.

Proof: Here, we use the technique of mathematical induction to carry out the proof.

$n = 1 \Rightarrow \lambda_1 = 1$ (as $\sum_{i=1}^{n}\lambda_i = 1$).

By definition (8), we have $PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \oplus_{j=1}^{m}\delta_j I_{\vartheta_{1j}}$.

Now, by using the above-stated operations (a)–(e), we get

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \left\langle 1 - \prod_{j=1}^{m}\left(1-\rho_{\vartheta_{1j}}\right)^{\delta_j}, \prod_{j=1}^{m}\left(\tau_{\vartheta_{1j}}\right)^{\delta_j}, \prod_{j=1}^{m}\left(\omega_{\vartheta_{1j}}\right)^{\delta_j}\right\rangle$$
$$= \left\langle 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i(1-\rho_{\vartheta_{ij}})^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$
$$\left. \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

Also, For $m = 1$, we get $\delta_1 = 1$ (because $\sum_{j=1}^{m}\delta_j = 1$).

Then, from Eq. (1), we have $PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \oplus_{i=1}^{n}\lambda_i I_{\vartheta_{i1}}$. From operations (a)–(e), we get

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$$
$$= \left\langle 1 - \left\{\left(\prod_{i=1}^{n}\lambda_i(1-\rho_{\vartheta_{ij}})^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}, \left\{\left(\prod_{i=1}^{n}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}, \right.$$
$$\left. \left\{\left(\prod_{i=1}^{n}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}\right\rangle.$$
$$= \left\langle 1 - \prod_{j=1}^{1}\left\{\left(\prod_{i=1}^{n}\lambda_i(1-\rho_{\vartheta_{ij}})^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{1}\left\{\left(\prod_{i=1}^{n}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$
$$\left. \prod_{j=1}^{1}\left\{\left(\prod_{i=1}^{n}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

Hence, Eq. (5) is satisfied for the initial values of n and m. Further, by hypothesis, let the Eq. (5) is satisfied for $m = \gamma_1 + 1$, $n = \gamma_2$ and $m = \gamma_1$, $n = \gamma_2 + 1$, i.e.,

$$\oplus_{j=1}^{\gamma_1+1}\delta_j\left(\oplus_{i=1}^{\gamma_2}\lambda_i I_{\vartheta_{ij}}\right)$$

$$=\left\langle 1-\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\right.$$

$$\left.\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

$$\oplus_{j=1}^{\gamma_1}\delta_j\left(\oplus_{i=1}^{\gamma_2+1}\lambda_i I_{\vartheta_{ij}}\right)$$

$$=\left\langle 1-\prod_{j=1}^{\gamma_1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\prod_{j=1}^{\gamma_1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\right.$$

$$\left.\prod_{j=1}^{\gamma_1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

Now for $m=\gamma_1+1, n=\gamma_2+1$, we get

$$\oplus_{j=1}^{\gamma_1+1}\delta_j\left(\oplus_{i=1}^{\gamma_2+1}\lambda_i I_{\vartheta_{ij}}\right)=\oplus_{j=1}^{\gamma_1+1}\delta_j\left(\oplus_{i=1}^{\alpha_2}\lambda_i I_{\vartheta_{ij}}\oplus\lambda_{\gamma_2+1}I_{\vartheta_{(\gamma_2+1)j}}\right)$$

$$\oplus_{j=1}^{\gamma_1+1}\oplus_{i=1}^{\gamma_2}\delta_j\lambda_i I_{\vartheta_{ij}}\oplus_{j=1}^{\gamma_1+1}\delta_j\lambda_{\gamma_2+1}I_{\vartheta_{(\gamma_2+1)j}}$$

$$=\left\langle\left(1-\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right)\oplus\left(1-\prod_{j=1}^{\gamma_1+1}\left\{\left(\lambda_{(\gamma_2+1)}\left(1-\rho_{\vartheta_{(\gamma_2+1)j}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right),\right.$$

$$=\left(\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right)\oplus\left(\prod_{j=1}^{\gamma_1+1}\left\{\left(\lambda_{(\gamma_2+1)}\tau_{\vartheta_{(\gamma_2+1)j}}{}^{\Delta}-(\lambda_{(\gamma_2+1)}-1)\right)^{1/\Delta}\right\}^{\delta_j}\right),$$

$$\left.\left(\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right)\oplus\left(\prod_{j=1}^{\gamma_1+1}\left\{\left(\lambda_{(\gamma_2+1)}\omega_{\vartheta_{(\gamma_2+1)j}}{}^{\Delta}-(\lambda_{(\gamma_2+1)}-1)\right)^{1/\Delta}\right\}^{\delta_j}\right)\right\rangle.$$

$$=\left\langle 1-\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\tau_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j},\right.$$

$$\left.\prod_{j=1}^{\gamma_1+1}\left\{\left(\prod_{i=1}^{\gamma_2+1}\lambda_i\omega_{\vartheta_{ij}}{}^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

Thus, the proposition is valid for $m=\gamma_1+1, n=\gamma_2+1$. Hence the theorem. □

Properties of *PFHSSSWA* Operator

- **Idempotency**

If $I_{\vartheta_{ij}}=I_{\vartheta_\alpha}=\left(\rho_{\vartheta_{ij}},\tau_{\vartheta_{ij}},\omega_{\vartheta_{ij}}\right)\ \forall i,j$, then $PFHSSSWA(I_{\vartheta_{11}},I_{\vartheta_{12}},\ldots,I_{\vartheta_{nm}})=I_{\vartheta_\alpha}$.

Proof. Let $I_{\vartheta_{ij}} = I_{\vartheta_\alpha} = \left(\rho_{\vartheta_{ij}}, \tau_{\vartheta_{ij}}, \omega_{\vartheta_{ij}}\right)$ be a set of PFHSNs. From Eq. (5), we get

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$$
$$= \left\langle 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i \tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j},\right.$$
$$\left.\prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i \omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$
$$= \left\langle 1 - \left\{\left(\sum_{i=1}^{n}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\sum_{j=1}^{m}\delta_j}, \left\{\left(\sum_{i=1}^{n}\lambda_i \tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\sum_{j=1}^{m}\delta_j},\right.$$
$$\left.\left\{\left(\sum_{i=1}^{n}\lambda_i \omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\sum_{j=1}^{m}\delta_j}\right\rangle.$$
$$= \left\langle 1 - \left(\left(1-\rho_{\vartheta_{ij}}\right), \ \left(\tau_{\vartheta_{ij}}\right), \ \omega_{\vartheta_{ij}}\right\rangle = \left(\rho_{\vartheta_{ij}}, \tau_{\vartheta_{ij}}, \omega_{\vartheta_{ij}}\right) = I_{\vartheta_\alpha}.$$

Hence, the idempotency holds.

- **Boundedness**

Suppose $I_{\vartheta_{ij}}$ be a set of $PFHSNs$.

Let $I^{-}_{\vartheta_{ij}} = \left\langle \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}, \max_j \max_i \left\{\tau_{\vartheta_{ij}}\right\}, \max_j \max_i \left\{\omega_{\vartheta_{ij}}\right\}\right\rangle$ and

$I^{+}_{\vartheta_{ij}} = \left\langle \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}, \min_j \min_i \left\{\tau_{\vartheta_{ij}}\right\}, \min_j min_i \left\{\omega_{\vartheta_{ij}}\right\}\right\rangle$, then

$$I^{-}_{\vartheta_{ij}} \leq PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}} \leq I^{+}_{\vartheta_{ij}}.$$

Proof.

Let $I_{\vartheta_{ij}} = \left(\rho_{\vartheta_{ij}}, \tau_{\vartheta_{ij}}, \omega_{\vartheta_{ij}}\right)$ be a *PFHSN*, then $\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\} \leq \left\{\rho_{\vartheta_{ij}}\right\} \leq \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}$

$$\Longrightarrow 1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\} \leq \left\{1-\rho_{\vartheta_{ij}}\right\} \leq 1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}$$

$$\Longrightarrow \left(1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} \leq \left\{1-\rho_{\vartheta_{ij}}\right\}^{\Delta} \leq \left(1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta}$$

$$\Longleftrightarrow \lambda_i\left(1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} \leq \lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} \leq \lambda_i\left(1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}^{\Delta}\right)$$

$$\Longleftrightarrow \lambda_i\left(1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} - (\lambda_i - 1) \leq \lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \leq \lambda_i\left(1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} - (\lambda_i - 1)$$

$$\Longleftrightarrow \sum_{i=1}^{n}\lambda_i\left(1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} - \left(\sum_{i=1}^{n}\lambda_i - 1\right) \leq \prod_{i=1}^{n}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \leq \sum_{i=1}^{n}\lambda_i\left(1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta}$$
$$- \left(\sum_{i=1}^{n}\lambda_i - 1\right)$$

$$\Longleftrightarrow \left(1 - \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta} \leq \prod_{i=1}^{n}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \leq \left(1 - \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\Delta}\left(as \sum_{i=1}^{n}\lambda_i = 1\right)$$

$$\Longleftrightarrow \left(1-\max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right) \le \left(\prod_{i=1}^{n} \lambda_i \left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta} \le \left(1-\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)$$

$$\Longleftrightarrow \left(1-\max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\delta_j} \le \left(\left(\prod_{i=1}^{n} \lambda_i \left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(1-\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\delta_j}$$

$$\Longleftrightarrow \left(1-\max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\sum_{j=1}^{m}\delta_j} \le \prod_{j=1}^{m}\left(\left(\prod_{i=1}^{n} \lambda_i \left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(1-\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)^{\sum_{j=1}^{m}\delta_j}$$

$$\Longleftrightarrow \left(1-\max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right) \le \prod_{j=1}^{m}\left(\left(\prod_{i=1}^{n} \lambda_i \left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(1-\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right)\left(as\sum_{j=1}^{m}\delta_j = 1.\right)$$

$$\Longleftrightarrow \left(\min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}\right) \le 1-\prod_{j=1}^{m}\left(\left(\prod_{i=1}^{n} \lambda_i \left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(\max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}\right)$$

Similarly,

$$\left(\min_j \min_i \left\{\tau_{\vartheta_{ij}}\right\}\right) \le \prod_{j=1}^{m}\left(\left(\prod_{i=1}^{n} \lambda_i \left(\tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(\max_j \max_i \left\{\tau_{\vartheta_{ij}}\right\}\right). \tag{3}$$

$$\left(\min_j \min_i \left\{\omega_{\vartheta_{ij}}\right\}\right) \le \prod_{j=1}^{m}\left(\left(\prod_{i=1}^{n} \lambda_i \left(\omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right)^{\delta_j} \le \left(\max_j \max_i \left\{\omega_{\vartheta_{ij}}\right\}\right). \tag{4}$$

Then, by definition of order relation,

$I^{-}_{\vartheta_{ij}} \le \text{PFHSSSWA}\ (I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) \le I^{+}_{\vartheta_{ij}}$.

- **Homogeneity**

For $\kappa \in \mathbb{R}^{+}$,

$$PFHSSSWA(\kappa I_{\vartheta_{11}}, \kappa I_{\vartheta_{12}}, \ldots, \kappa I_{\vartheta_{nm}}) = \kappa PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}).$$

Proof. Suppose $I_{\vartheta_{ij}}$ is a picture fuzzy hypersoft numnber and $\kappa \in \mathbb{R}^{+}$, using (c) we have

$$\kappa I_{\vartheta_d} = \left\langle 1-(\kappa(1-\rho_{\vartheta_d})^{\Delta} - (\kappa-1))^{1/\Delta}, (\kappa\tau^{\Delta}_{\vartheta_d} - (\kappa-1))^{1/\Delta}, (\kappa\omega^{\Delta}_{\vartheta_d} - (\kappa-1))^{1/\Delta}\right\rangle.$$

Thus,

$$PFHSSSWA(\kappa I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$$

$$= \left\langle 1-\prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\kappa\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\kappa\lambda_i {\tau_{\vartheta_{ij}}}^{\Delta} - (\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$
$$\left. \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\kappa\lambda_i {\omega_{\vartheta_{ij}}}^{\Delta} - (\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$
$$= \kappa PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}).$$

- **Monotonicity** Let $I_{\vartheta_{ij}}$ and $I'_{\vartheta_{ij}}$ be the collection of two $PFHSNs$. If $I_{\vartheta_{ij}} \leq I'_{\vartheta_{ij}}$ then,

$$PFHSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) \leq PFHSWA\left(I_{\vartheta'_{11}},\ I_{\vartheta'_{12}}, \ldots, I_{\vartheta'_{nm}}\right).$$

Proof. By using definitions it can be easily proved on similar lines.

3.3 PFHS Schweizer-Sklar Weighted Geometric Aggregation Operators ($PFHSSSWG$)

Definition 9: *Suppose $I_{\vartheta_d} = (\rho_{\vartheta_d}, \tau_{\vartheta_d}, \omega_{\vartheta_d})$ is a picture fuzzy hypersoft number. Let λ_i (experts) & δ_j (attributes) are the respective weight vectors. Also, $\lambda_i > 0,\ \sum_{i=1}^{n} \lambda_i = 1$ and $\delta_j > 0,\ \sum_{i=1}^{n}\ \delta_j = 1$. The **PFHS Schweizer-Sklar Weighted Geometric AO (PFHSSSWGAO)** is a function $\mathfrak{M}^n \rightarrow \mathfrak{M}$ defined as*

$$PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \otimes_{SS}{}_{j=1}^{m}\delta_j\left(\otimes_{i=1}^{n}\lambda_i I_{\vartheta_{ij}}\right). \tag{5}$$

where $\mathfrak{M}^n = (I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$ is a set of picture fuzzy hypersoft numbers.

Theorem 2: *Suppose $I_{\vartheta_d} = (\rho_{\vartheta_d}, \tau_{\vartheta_d}, \omega_{\vartheta_d})$ is a picture fuzzy hypersoft number. Then based on the above definition, we get*

$$\begin{aligned} &PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) \\ &= \left\langle \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\left(1-\tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right. \\ &\quad \left. 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\left(1-\omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle. \end{aligned} \tag{6}$$

And λ_i (experts) & δ_j (attribute's) are the respective weight vectors. Also, $\lambda_i > 0,\ \sum_{i=1}^{n} \lambda_i = 1$ and $\delta_j > 0,\ \sum_{i=1}^{n}\ \delta_j = 1$.

Proof: Here, we use the technique of mathematical induction to carry out the proof.

$n = 1 \Rightarrow \lambda_1 = 1$ (as $\sum_{i=1}^{n} \lambda_i = 1$).

By definition (9), we have $PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \otimes_{j=1}^{m}\delta_j I_{\vartheta_{1j}}$.

Now, by using the above-stated operations (a)–(e), we get

$$\begin{aligned} &PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots., I_{\vartheta_{nm}}) = \left\langle \prod_{j=1}^{m}\left(\rho_{\vartheta_{1j}}\right)^{\delta_j},\ 1 - \prod_{j=1}^{m}\left(1-\tau_{\vartheta_{1j}}\right)^{\delta_j}, 1 - \prod_{j=1}^{m}\left(1-\omega_{\vartheta_{1j}}\right)^{\delta_j}\right\rangle \\ &= \left\langle \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i\left(1-\tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right. \\ &\quad \left. 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{1}\lambda_i\left(1-\omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right\rangle. \end{aligned}$$

Also, For $m = 1$, we get $\delta_1 = 1$ (as $\sum_{j=1}^{m} \delta_j = 1$).

Then, from Eq. (5), we have $PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \otimes_{i=1}^{n}\lambda_i I_{\vartheta_{i1}}$. From operations (a)–(e), we get

$$PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}})$$

$$= \left\langle \left\{ \left(\prod_{i=1}^{n} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}, 1 - \left\{ \left(\prod_{i=1}^{n} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}, \right.$$
$$\left. 1 - \left\{ \left(\prod_{i=1}^{n} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\} \right\rangle.$$
$$= \left\langle \prod_{j=1}^{1} \left\{ \left(\prod_{i=1}^{n} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, 1 - \prod_{j=1}^{1} \left\{ \left(\prod_{i=1}^{n} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, \right.$$
$$\left. 1 - \prod_{j=1}^{1} \left\{ \left(\prod_{i=1}^{n} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right\rangle.$$

Hence, Eq. (6) is satisfied for the initial values of n and m. Further, by hypothesis, let the (6) is satisfied for $m = \gamma_1 + 1, n = \gamma_2$ and $m = \gamma_1, n = \gamma_2 + 1$, i.e.,

$$\otimes_{j=1}^{\gamma_1+1} \delta_j \left(\otimes_{i=1}^{\gamma_2} \lambda_i I_{\vartheta_{ij}} \right)$$
$$= \left\langle \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, 1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, \right.$$
$$\left. 1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right\rangle.$$
$$\otimes_{j=1}^{\gamma_1} \delta_j \left(\otimes_{i=1}^{\gamma_2+1} \lambda_i I_{\vartheta_{ij}} \right)$$
$$= \left\langle \prod_{j=1}^{\gamma_1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, 1 - \prod_{j=1}^{\gamma_1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, \right.$$
$$\left. 1 - \prod_{j=1}^{\gamma_1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right\rangle.$$

Now for $m = \gamma_1 + 1, n = \gamma_2 + 1$, we get

$$\otimes_{j=1}^{\gamma_1+1} \delta_j \left(\otimes_{i=1}^{\gamma_2+1} \lambda_i I_{\vartheta_{ij}} \right) = \otimes_{j=1}^{\gamma_1+1} \delta_j \left(\otimes_{i=1}^{\alpha_2} \lambda_i I_{\vartheta_{ij}} \otimes \lambda_{\gamma_2+1} I_{\vartheta_{(\gamma_2+1)j}} \right)$$
$$= \otimes_{j=1}^{\gamma_1+1} \otimes_{i=1}^{\gamma_2} \delta_j \lambda_i I_{\vartheta_{ij}} \otimes_{j=1}^{\gamma_1+1} \delta_j \lambda_{\gamma_2+1} I_{\vartheta_{(\gamma_2+1)j}}$$
$$= \left\langle \left(\prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right) \otimes \left(\prod_{j=1}^{\gamma_1+1} \left\{ \left(\lambda_{(\gamma_2+1)} \rho_{\vartheta_{(\gamma_2+1)j}}{}^{\Delta} - (\lambda_{(\gamma_2+1)} - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right), \right.$$
$$\left(1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right) \otimes \left(1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\lambda_{(\gamma_2+1)} \left(1 - \tau_{\vartheta_{(\gamma_2+1)j}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right),$$
$$\left. \left(1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right) \otimes \left(1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\lambda_{(\gamma_2+1)} \left(1 - \omega_{\vartheta_{(\gamma_2+1)j}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right) \right\rangle.$$

$$= \left\langle \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, 1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \left(1 - \tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j}, \right.$$

$$\left. 1 - \prod_{j=1}^{\gamma_1+1} \left\{ \left(\prod_{i=1}^{\gamma_2+1} \lambda_i \left(1 - \omega_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1) \right)^{1/\Delta} \right\}^{\delta_j} \right\rangle.$$

Thus, the proposition is valid for $m = \gamma_1 + 1, n = \gamma_2 + 1$. Hence the theorem. □

Properties of *PFHSSSWG* Operator

- **Idempotency**

If $I_{\vartheta_{ij}} = I_{\vartheta_\alpha} = \left(\rho_{\vartheta_{ij}}, \tau_{\vartheta_{ij}}, \omega_{\vartheta_{ij}}\right) \forall i, j$, then $PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = I_{\vartheta_\alpha}$.

Proof. The proof of idempotency is done in the same way as done in the weighted averaging case.

- **Boundedness**

Suppose $I_{\vartheta_{ij}}$ be a set of PFHSNs.

Let $I^-_{\vartheta_{ij}} = \left\langle \min_j \min_i \left\{\rho_{\vartheta_{ij}}\right\}, \max_j \max_i \left\{\tau_{\vartheta_{ij}}\right\}, \max_j \max_i \left\{\omega_{\vartheta_{ij}}\right\} \right\rangle$ and

$I^+_{\vartheta_{ij}} = \left\langle \max_j \max_i \left\{\rho_{\vartheta_{ij}}\right\}, \min_j \min_i \left\{\tau_{\vartheta_{ij}}\right\}, \min_j min_i \left\{\omega_{\vartheta_{ij}}\right\} \right\rangle$, then

$I^-_{\vartheta_{ij}} \leq PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}} \leq I^+_{\vartheta_{ij}}$.

Proof. Proof can be done in the same way as done in a weighted averaging case.

- **Homogeneity** For any positive real number κ,

$$PFHSSSWA(\kappa I_{\vartheta_{11}}, \kappa I_{\vartheta_{12}}, \ldots, \kappa I_{\vartheta_{nm}}) = \kappa PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}).$$

Proof. The proof of monotonicity is done in the same way as done in weighted averaging case.

- **Monotonicity** Let $I_{\vartheta_{ij}}$and $I'_{\vartheta_{ij}}$ be the collection of two $PFHSNs$. If $I_{\vartheta_{ij}} \leq I'_{\vartheta_{ij}}$ then,

$$PFHSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) \leq PFHSWA\left(I_{\vartheta'_{11}}, I_{\vartheta'_{12}}, \ldots I_{\vartheta'_{nm}}\right).$$

Proof. By using definitions it can be easily proved on similar lines.

4 Decision-Making Methodology Based on Schweizer-Sklar Aggregation Operators inPFHS Environment

This section proposes a novel scheme for solving an MCDM problem based on proposed SSAOs in $PFHSNs$.

Consider $A = \{A_1, A_2, ..., A_s\}$ is a collection having s alternatives with $E = \{E_1, E_2, ..., E_n\}$ being a group of n decision makers. The weights of decision-maker's are given by $\lambda = \lambda_1, \lambda_2, ..., \lambda_n^T$ along with the constraint $\sum_{i=1}^{n} \lambda_i = 1; \lambda_i \in [0, 1]$. Let $C = \{C_1, C_2, ..., C_m\}$ be the set of m criterions whose weights are given by $\delta = \delta_1, \delta_2, ..., \delta_m^T$ such that $\sum_{j=1}^{n} \delta_j = 1; \delta_j \in [0, 1]$. Now, after assessing the alternatives under the required criteria, suppose the decision-makers give the information in terms of*PFHS* decision matrix i.e., *PFHSDM*. Let $\mathbb{E}^l = \left[I_{\vartheta^{(l)}_{ij}}\right]_{n\times m} = \left(\rho^l_{\vartheta_{ij}}, \tau^l_{\vartheta_{ij}}, \omega^l_{\vartheta_{ij}}\right)$ for every alternative which is expressed in terms of $PFHSNs$. The uncertainty components $\left(\rho^l_{\vartheta_{ij}}, \tau^l_{\vartheta_{ij}}, \omega^l_{\vartheta_{ij}}\right)$ represents the standard notions termed as *"degree of positive membership, degree of neutral membership and degree of negative membership"* respectively of i^{th} alternative for j^{th} criterion by the l^{th} expert.

Apply the*PFHSSSWA* and*PFHSSSWG* aggregation operators to aggregate the *PFHSNs* $(I_{\vartheta_{ij}})$ which is based on the decision-makers preferences for each alternative. Finally, utilize the score function for prioritizing the alternatives. Further, the methodology defined above is also listed as follows:

Phase 1. Construct an expert matrix $\left[I_{\vartheta_{ij}^{(l)}}\right]_{n\times m} = \left(\rho^{l}_{\vartheta_{ij}}, \tau^{l}_{\vartheta_{ij}}, \omega^{l}_{\vartheta_{ij}}\right)$ in terms of *PFHSNs* for the alternatives as suggested by the decision-makers.

Phase 2. In this phase, normalization of the cost-type parameters into benefit-type parameters is done and the normalized aggregated matrices are obtained.

$$\varsigma^{l}_{ij} = \begin{cases} I_{\vartheta_{ij}}{}^{c} = \left(\omega_{\vartheta_{ij}}{}^{(l)},\ \tau_{\vartheta_{ij}}{}^{(l)},\ \rho_{\vartheta_{ij}}{}^{(l)}\right);\ \text{cost type parameter,} \\ I_{\vartheta_{ij}} = \left(\rho_{\vartheta_{ij}}{}^{(l)},\ \tau_{\vartheta_{ij}}{}^{(l)},\ \omega_{\vartheta_{ij}}{}^{(l)}\right);\ \text{benefit type parameter.} \end{cases}$$

Phase 3. Now, use the normalized *PFHSNs*ς^{l}_{ij} for each alternative $A = \{A_1, A_2, \ldots, A_s\}$ into an aggregated*PFHSN* by making use of the devised*PFHSSSWA*/*PFHSSSWG* operators as defined in Definition 8/Definition 9.

Phase 4. In the next phase, compute the score values of the alternatives by making use of Definition 2.

Phase 5. Select the alternative having the highest score value and prioritize them accordingly.

Also, a detailed diagram based on these methodological phases is shown in Fig. 1.

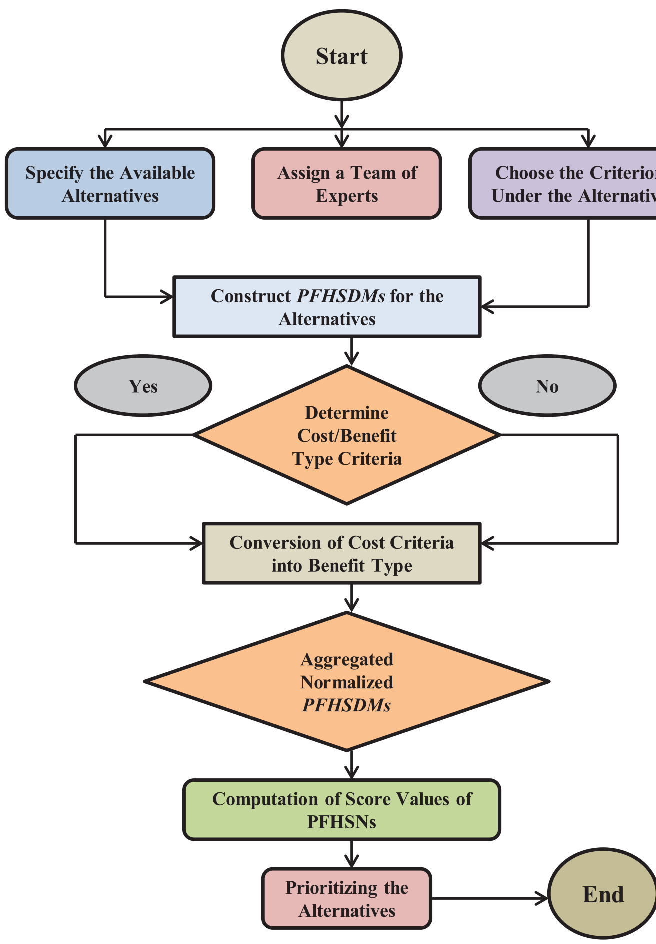

Figure 1: Flow diagram of the proposed methodology

5 Utilization of Proposed Decision-Making Methodology in Green Technology Adoption for Healthy Social Environment

Human living environments are shaped by their surroundings because the behaviour of humanity depends on all environmental elements-such as air, food, goods, locations, and a host of other things—a clean environment (biotic/abiotic) is necessary for a healthy and trouble-free existence. A clean environment is essential to the global advancement of lifestyles. However, the globe is currently dealing with several environmental problems, such as pollution, solid waste, water supplies, global warming, temperature increases, and expanding populations. A sustainable green environment, green production, green energy, and eco-friendly transportation are what the public needs.

All countries on the earth are attempting, via the use of their resources, to address environmental challenges. In recent years, the clean environment has gained international attention. The contrastive features of such important concern are re-iteratively pushing for improvements to the clean, eco-friendly environment. The unfavorable state of the ecosystem has altered traditional wisdom, which could spell doom for a clean environment. While each nation has unique problems with maintaining a clean environment, most of the problems are global.

Suppose that a committee of experts has been formed to examine and decide on the key issues relating to environmental protection. The group of experts looked at many fundamental problems and expressed them in the form of available alternatives. The five possible alternatives that need to be assessed by the experts are; Rise in population (A_1), Environmental shifts (A_2), Global warming (A_3), Ecological harm (A_4) and Exhaustion of resources (A_5). Further, these five alternatives are evaluated under four criteria which are as follows: Use of natural resources (C_1), Exploring environment friendly suppliers (C_2), Work on the strategies to resolve disputes (C_3) and Competent manufacturing policies (C_4). Now, the sub-criterions for these criteria are

- Use of natural resources = $\vartheta_1 = \{\vartheta_{11}$ = optimum use, ϑ_{12} = conservative approach$\}$,
- Exploring environment friendly suppliers = $\vartheta_2 = \{\vartheta_{21}\}$,
- Work on the strategies to resolve disputes = $\vartheta_3 = \{\vartheta_{31}\}$,
- Competent manufacturing policies = $\vartheta_4 = \{\vartheta_{41}$ = internal, ϑ_{42} = external$\}$.

Suppose $\mathfrak{D}' = \vartheta_1 \times \vartheta_2 \times \vartheta_3 \times \vartheta_4$ is a set of sub-criterions defined as

$$= \{((\vartheta_{11}, \vartheta_{21}, \vartheta_{31}, \vartheta_{41}),\ (\vartheta_{11}, \vartheta_{21}, \vartheta_{31}, \vartheta_{42}), (\vartheta_{12}, \vartheta_{21}, \vartheta_{31}, \vartheta_{41}), (\vartheta_{12}, \vartheta_{21}, \vartheta_{31}, \vartheta_{42}))\}.$$

Now, for the simplification processes the set of all sub-criterions can be redefined as

$$\mathfrak{D}' = \{\wp_1,\ \wp_2, \wp_3, \wp_4\},$$

along with their respective weight vectors are $(0.2, 0.2, 0.2, 0.4)^T$. Further, the available alternatives under these sub-criterions are assessed by a team of $E = \{E_1, E_2, E_3, E_4\}$ decision-makers along with their experts weights are $(0.1, 0.3, 0.3, 0.3)^T$.

Evaluation and selection of choices have gotten harder over the past few years because of the fuzziness of the data that is now accessible and the necessity for more accuracy when analyzing qualities. To manage these situations, decision-making mechanisms must be enhanced. This picture fuzzy hypersoft paradigm can take into account a variety of sub-attributes and both perspectives of the three-dimensional information associated with the inclusion of three important uncertainty parameters which are very useful for making decisions. Experts provide their preferences in the form of *PFHSNs* to help choose the optimal alternative after taking all of these factors into account. Now, we present the procedural steps of the proposed methodology in a phase-wise manner to compute the most suitable alternative.

5.1 By Utilizing PFHSSSWA Operators

Phase 1. In the first phase, all the picture fuzzy hypersoft expert matrices for the alternatives are listed from Tables 1–5.

Table 1: *PFHS* expert matrix given for alternative A_1

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
E_1	(0.2,0.5,0.1)	(0.3,0.4,0.2)	(0.4,0.1,0.2)	(0.3,0.5,0.1)
E_2	(0.4,0.3,0.2)	(0.2,0.4,0.1)	(0.1,0.2,0.3)	(0.3,0.2,0.1)
E_3	(0.1,0.2,0.5)	(0.4,0.1,0.2)	(0.3,0.2,0.1)	(0.2,0.4,0.3)
E_4	(0.3,0.5,0.1)	(0.2,0.4,0.1)	(0.3,0.4,0.2)	(0.1,0.3,0.5)

Table 2: *PFHS* expert matrix given for alternative A_2

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
E_1	(0.4,0.2,0.3)	(0.1,0.2,0.6)	(0.2,0.5,0.1)	(0.1,0.2,0.3)
E_2	(0.1,0.2,0.5)	(0.2,0.4,0.1)	(0.3,0.5,0.1)	(0.2,0.4,0.3)
E_3	(0.4,0.1,0.2)	(0.1,0.2,0.4)	(0.3,0.5,0.1)	(0.4,0.3,0.2)
E_4	(0.4,0.3,0.2)	(0.2,0.1,0.5)	(0.1,0.3,0.5)	(0.1,0.2,0.4)

Table 3: *PFHS* expert matrix given for alternative A_3

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
E_1	(0.1,0.3,0.4)	(0.4,0.3,0.2)	(0.2,0.6,0.1)	(0.3,0.2,0.1)
E_2	(0.3,0.5,0.1)	(0.4,0.3,0.2)	(0.1,0.5,0.3)	(0.2,0.4,0.1)
E_3	(0.4,0.1,0.2)	(0.1,0.2,0.3)	(0.2,0.1,0.5)	(0.7,0.1,0.1)
E_4	(0.3,0.5,0.1)	(0.2,0.4,0.1)	(0.1,0.2,0.4)	(0.3,0.4,0.2)

Table 4: *PFHS* expert matrix given for alternative A_4

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
E_1	(0.3,0.5,0.1)	(0.2,0.3,0.4)	(0.1,0.3,0.2)	(0.3,0.2,0.1)
E_2	(0.2,0.3,0.4)	(0.1,0.2,0.3)	(0.3,0.4,0.2)	(0.1,0.2,0.6)
E_3	(0.2,0.1,0.5)	(0.2,0.5,0.2)	(0.1,0.5,0.2)	(0.3,0.2,0.1)
E_4	(0.1,0.5,0.2)	(0.2,0.3,0.1)	(0.2,0.5,0.1)	(0.1,0.2,0.3)

Table 5: *PFHS* expert matrix given for alternative A_5

	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
E_1	(0.1,0.2,0.5)	(0.2,0.4,0.3)	(0.1,0.2,0.4)	(0.1,0.3,0.5)
E_2	(0.2,0.3,0.4)	(0.1,0.2,0.4)	(0.1,0.3,0.2)	(0.3,0.1,0.2)
E_3	(0.5,0.2,0.1)	(0.4,0.2,0.1)	(0.3,0.1,0.2)	(0.2,0.4,0.3)
E_4	(0.3,0.1,0.5)	(0.2,0.1,0.4)	(0.3,0.2,0.4)	(0.1,0.5,0.3)

Phase 2. As every criterion is of benefit type, therefore normalization is not necessary.

Phase 3. Now, we applied the proposed picture fuzzy hypersoft SSAOs on the obtained expert matrices and acquired the required information from the experts in terms of $PFHSNs(\varsigma_{ij}^{l})$; where $i = 1,2,3,4,5$ & $j,l = 1,2,3,4$ given as

$$PFHSSSWA(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{44}}) = \oplus_{SS}{}_{j=1}^{4}\delta_j\left(\oplus_{i=1}^{4}\lambda_i I_{\vartheta_{ij}}\right).$$

$$= \left\langle 1 - \prod_{j=1}^{4}\left\{\left(\prod_{i=1}^{4}\lambda_i\left(1-\rho_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \prod_{j=1}^{4}\left\{\left(\prod_{i=1}^{4}\lambda_i \tau_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$
$$\left. \prod_{j=1}^{4}\left\{\left(\prod_{i=1}^{4}\lambda_i \omega_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}\right\rangle.$$

For $\Delta = -1$, $A_1 = \langle 0.0814, 0.3832, 0.2559\rangle$, $A_2 = \langle 0.0499, 0.3411, 0.4084\rangle$, $A_3 = \langle 0.0736, 0.3912, 0.2558\rangle$, $A_4 = \langle 0.0381, 0.3441, 0.2772\rangle$, $A_5 = \langle 0.0499, 0.2935, 0.3875\rangle$.

Phase 4. Now utilize the score function formula to compute the scores of all the available alternatives. $\mathbb{S}(A_1) = 0.2311$, $\mathbb{S}(A_2) = 0.0517$, $\mathbb{S}(A_3) = 0.2665$, $\mathbb{S}(A_4) = 0.1302$, $\mathbb{S}(A_5) = 0.0221$.

Phase 5. Based on score values for the alternatives, the prioritization of alternatives can be done as follows: $\mathbb{S}(A_3) > \mathbb{S}(A_1) > \mathbb{S}(A_4) > \mathbb{S}(A_2) > \mathbb{S}(A_5)$. Hence, the alternative A_3 is the most appropriate one.

Further, on similar lines, all the computations can be done for the weighted average aggregation operators.

5.2 By Utilizing PFHSSSWG Operators

Phase 1. This phase is the same as in *PFHSSSWA* operators.

Phase 2. This phase is also the same as in *PFHSSSWA* operators.

Phase 3. Now, we applied the proposed picture fuzzy hypersoft SSAOs on the obtained expert matrices and acquired the required information from the experts in terms of $PFHSNs(\varsigma_{ij}^{l})$; where $i = 1,2,3,4,5$ & $j,l = 1,2,3,4$ given as

$$PFHSSSWG(I_{\vartheta_{11}}, I_{\vartheta_{12}}, \ldots, I_{\vartheta_{nm}}) = \otimes_{SS}{}_{j=1}^{m}\delta_j\left(\otimes_{i=1}^{n}\lambda_i I_{\vartheta_{ij}}\right).$$

$$= \left\langle \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i \rho_{\vartheta_{ij}}{}^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, 1 - \prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\left(1-\tau_{\vartheta_{ij}}\right)^{\Delta} - (\lambda_i - 1)\right)^{1/\Delta}\right\}^{\delta_j}, \right.$$

$$1-\prod_{j=1}^{m}\left\{\left(\prod_{i=1}^{n}\lambda_i\left(1-\omega_{\vartheta_{ij}}\right)^{\Delta}-(\lambda_i-1)\right)^{1/\Delta}\right\}^{\delta_j}\Bigg\rangle.$$

For $\Delta=-1$, $A_1=\langle 0.0814, 0.3832, 0.2559\rangle$, $A_2=\langle 0.0499, 0.3411, 0.4084\rangle$, $A_3=\langle 0.0736, 0.3912, 0.2558\rangle$, $A_4=\langle 0.0381, 0.3441, 0.2772\rangle$, $A_5=\langle 0.0499, 0.2935, 0.3875\rangle$.

Phase 4. Now utilize the score function formula to compute the scores of all the available alternatives. $\mathbb{S}(A_1)=-0.1822$, $\mathbb{S}(A_2)=-0.3654$, $\mathbb{S}(A_3)=-0.1746$, $\mathbb{S}(A_4)=-0.2391$, $\mathbb{S}(A_5)=-0.3377$.

Phase 5. Based on score values for the alternatives, the prioritization of alternatives can be done as follows: $\mathbb{S}(A_3)>\mathbb{S}(A_1)>\mathbb{S}(A_4)>\mathbb{S}(A_5)>\mathbb{S}(A_2)$. Hence, the alternative A_3 is the most appropriate one.

6 Overview of Schweizer-Sklar Parameter (Δ) on Results

To demonstrate the impact of the SS (Δ) parameter, phases 3 and 4 are repeated several times in the previous example, each time with a different value. For both the SSAOs, the SS parameter is set to (Δ) = −1. Tables 6 and 7 provide the results and ranks for the *PFHSSSWA* and *PFHSSSWG* operators, respectively. Tables 6 and 7 show that various SS (Δ) parameter settings have resulted in numerous score values and various rankings of *PFHSNs* for assessing the best possible available alternative.

Table 6: Analysis of SS parameter (Δ) on rankings in*PFHSSSWA* operator

Parameter	$\mathbb{S}(A_1)$	$\mathbb{S}(A_2)$	$\mathbb{S}(A_3)$	$\mathbb{S}(A_4)$	$\mathbb{S}(A_5)$	Rankings
$\Delta=-1$	0.2311	0.0517	0.2665	0.1302	0.0221	$A_3>A_1>A_4>A_2>A_5$
$\Delta=-2$	0.3001	0.1903	0.3549	0.1954	0.1787	$A_3>A_1>A_4>A_2>A_5$
$\Delta=-5$	0.3982	0.3246	0.4652	0.2588	0.3188	$A_3>A_1>A_2>A_5>A_4$
$\Delta=-10$	0.5052	0.4384	0.5621	0.3358	0.4312	$A_3>A_1>A_2>A_5>A_4$
$\Delta=-20$	0.5946	0.5322	0.6406	0.4202	0.5238	$A_3>A_1>A_2>A_5>A_4$
$\Delta=-50$	0.6548	0.5982	0.6934	0.4984	0.5918	$A_3>A_1>A_2>A_5>A_4$

Table 7: Analysis of SS parameter (Δ) on rankings in *PFHSSSWG* operator

Parameter	$\mathbb{S}(A_1)$	$\mathbb{S}(A_2)$	$\mathbb{S}(A_3)$	$\mathbb{S}(A_4)$	$\mathbb{S}(A_5)$	Rankings
$\Delta=-1$	−0.1822	−0.3654	−0.1746	−0.2391	−0.3377	$A_3>A_1>A_4>A_5>A_2$
$\Delta=-2$	−0.2566	−0.4525	−0.2562	−0.3039	−0.4233	$A_3>A_1>A_4>A_5>A_2$
$\Delta=-5$	−0.3680	−0.5928	−0.3583	−0.4110	−0.5772	$A_3>A_1>A_4>A_5>A_2$
$\Delta=-10$	−0.4572	−0.6872	−0.4474	−0.5091	−0.6969	$A_3>A_1>A_4>A_2>A_5$
$\Delta=-20$	−0.5286	−0.7461	−0.5214	−0.5864	−0.7627	$A_3>A_1>A_4>A_2>A_5$
$\Delta=-50$	−0.5898	−0.7846	−0.5837	−0.6449	−0.79918	$A_3>A_1>A_4>A_2>A_5$

It is clear from Fig. 2 that with the decreasing value of the SS parameter, the score values are increasing based on the *PFHSSSWA* operator. Numerous prioritization orderings can be used with the same *PFHSSSWA* operator. For the (Δ) value −1 and −2 the ranking order of alternatives is $A_3>A_1>A_4>A_2>A_5$, and when (Δ) value is −5, −10, −20 and −50 the ranking outcome slightly differs as $A_3>A_1>A_2>A_5>A_4$.

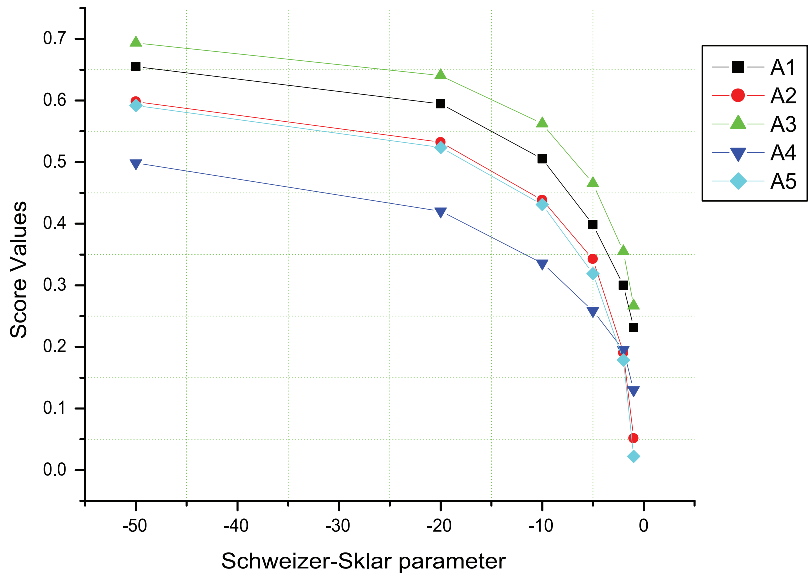

Figure 2: Impact of SS (Δ) parameter on scores of *PFHSSSWA* operator

Similarly, from Fig. 3 with the decreasing value of the SS parameter the score values are decreasing based on the *PFHSSSWG* operator. And different ranking orders can be utilized with the same *PFHSSSWG* operator. Further, For the (Δ) value -1 and -2 the ranking order of alternatives is $A_3 > A_1 > A_4 > A_5 > A_2$, and when (Δ) value is -5, -10, -20 and -50 the ranking outcome slightly differs as $A_3 > A_1 > A_4 > A_2 > A_5$.

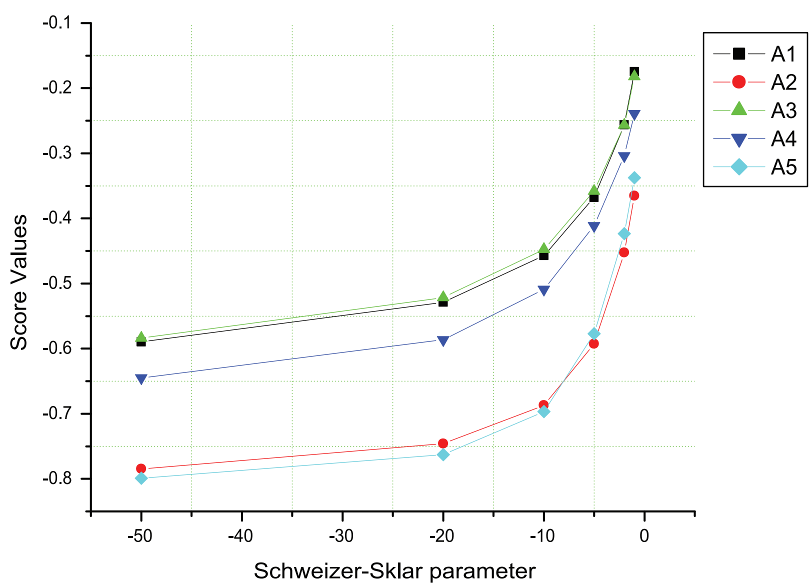

Figure 3: Impact of SS (Δ) parameter on scores of *PFHSSSWG* operator

Now, depending on the perceptions of the experts, a decision-maker may have a positive or negative view. Therefore, choosing a larger value for the SS parameter is advised for decision-makers who have a negative outlook on a viable alternative based on criteria. Also, this suggests that the approach under consideration is supposed to be computationally robust and the process with the obtained results is valid.

7 Comparative Analysis & Advantages

The presented decision-making technique that can be utilized in *PFHSSSWA* or *PFHSSSWG* aggregation operators under a picture fuzzy hypersoft environment. The devised technique which is executed is decisive and useful in practical scenarios. Our anticipated methodology outperforms some of the existing methodologies and is capable of handling even more complex decision-making situations. The presented model performs numerous tasks and is more flexible to incorporate the variations while dealing with the process of uncertain problems. In the literature, there are various Schweizer-Sklar aggregation operators under different fuzzy environments and the evaluation system for every methodology is unique. These research deliberations and analyses have led us to the conclusion that the hybrid decision-making technique produces more reliable results than the conventional one.

Also, the criterion and decision maker's weights for the evaluation of alternatives under these criteria are very important factors for decision-making technique. Further, we compare our methodologies based on these terms with some existing methodologies tabulated in Table 8 and one can say that the proposed decision-making methodologies are equally consistent with some of the existing techniques.

Table 8: Consistency with the MCDM methods

	IVIF-DEMATEL & MOORA [37]	**Spherical fuzzy TOPSIS** [38]	**Fuzzy DEMATEL** [39]	**Fuzzy COPRAS** [40]	**Proposed methods**
A_1	2	2	2	2	2
A_2	5	5	5	5	5
A_3	1	1	1	1	1
A_4	3	3	3	3	3
A_5	4	4	4	4	4

The key distinctions and advantages of our method are as follows:

- **Improved Handling of Uncertainty:**
  - Unlike traditional fuzzy or intuitionistic fuzzy methods, our approach leverages picture fuzzy sets, which incorporate positive, neutral, and negative membership degrees. This allows for a more expressive and realistic modeling of human judgments, especially in complex or ambiguous decision environments.
  - Competing methods often fail to explicitly account for neutrality or hesitation, limiting their effectiveness in real-world uncertain scenarios.
- **Flexible Aggregation with the Schweizer-Sklar Operator:**
  - The use of the parameterized Schweizer-Sklar operator provides a tunable mechanism to control the level of compensation among criteria.
  - This flexibility allows decision-makers to adapt the model based on their preferences (risk-averse, neutral, or compensatory), whereas most traditional methods apply fixed aggregation rules that cannot be adjusted.
- **Computational Efficiency:**
  - Although our method introduces additional components (e.g., picture fuzzy logic and parameterized aggregation), the computational complexity remains linear with respect to the number of criteria and alternatives, which is comparable to or even better than some iterative or optimization-based MCDM techniques.

- Additionally, the closed-form formulations used in the aggregation and scoring steps make the method scalable for large decision problems.

- **Ranking Stability and Robustness:**
  - Through our sensitivity analysis on the Schweizer-Sklar parameter (Δ), we demonstrate that the ranking results of our method are highly stable across a wide range of parameter values.
  - Some benchmark methods show ranking reversals or inconsistencies under slight model perturbations, suggesting lower robustness.
- **Limitations and Scope for Future Work:**
  - We acknowledge that our method requires parameter tuning (Δ), which introduces subjectivity unless guided by expert input or empirical calibration.
  - Moreover, while picture fuzzy sets enhance uncertainty modeling, they also increase the cognitive load on decision-makers during input elicitation.

Furthermore, due to various unique circumstances, the tool of parametrization and sub-parametrization is very useful which is not yet covered by the existing Schweizer-Sklar aggregation operators. As a result, the approach we have devised will be substantially stronger, more reliable, and better than the various existing techniques. Table 9 presented the characteristic comparison analysis of the proposed aggregation operators with existing aggregation operators.

Table 9: Characteristic comparison of existing SSAOs

SSAOs	Parametrization feature	Sub-parametrization feature	Degree of abstain/refusal
SSAOs for *IFS* [41]	×	×	×
SSAOs for $PyFS$ [42]	×	×	×
SSAOs for $PyFS$ [43]	×	×	×
SSAOs for $SVNS$ [44]	×	×	×
SSAOs for $q-ROFS$ [45]	×	×	×
SSAOs for *FFS* [46]	×	×	×
SSAOs for *IV IFS* [47]	×	×	×
SSAOs for *DHFS* [48]	×	×	✓
SSAOs for *PFS* [49]	×	×	×
SSAOs for $q-ROPHFS$ [50]	×	×	×
Proposed SSAOs	✓	✓	✓

Where, $IFS, PyFS, SVNS, q-ROFS, FFs, IVIFS, DHFS, PFS, q-ROPHFS$ are the intuitionistic fuzzy, Pythagorean fuzzy, single-valued neutrosophic, q-rung orthopair fuzzy, Fermatean fuzzy, interval-valued intuitionistic fuzzy, dual-hesitant fuzzy, picture, q-rung probabilistic hesitant fuzzy sets respectively.

8 Conclusion & Future Work

The significant findings and contributions of this study are summarized as follows:

- We proposed novel Schweizer-Sklar-based aggregation operators (both weighted average and geometric average) for picture fuzzy hypersoft information systems. These operators generalize existing ones by incorporating a tunable parameter (Δ), offering greater flexibility in modeling uncertainty. Their key properties—idempotency, boundedness, homogeneity, and monotonicity—were formally established.

- A structured multi-criteria decision-making (MCDM) algorithm was developed using the proposed operators. Its effectiveness was demonstrated through an illustrative example related to green technology adoption in social environments. Comparative analysis highlighted the advantages of the proposed method over existing approaches.
- The sensitivity of the decision-making results to the Schweizer-Sklar parameter was examined, revealing that the PFHSSSWA operator yields increasing scores with decreasing Δ, while the PFHSSSWG operator shows the opposite trend. This provides decision-makers with flexible control over alternative selection.

Future Directions

This work opens several avenues for further research:

- Extending the proposed framework to more generalized fuzzy environments such as T-spherical fuzzy sets, indetermSoft sets, and indetermHyperSoft sets [51] to handle larger and more complex decision problems.
- Applying the aggregation operators in hybrid or ensemble decision models, particularly in AI-driven applications such as intelligent recommendation systems, stochastic simulations, and ML-integrated decision support systems.
- Developing data-driven consensus models and real-time decision tools that incorporate the tunable behavior of the Schweizer-Sklar parameter for adaptive analysis.

Acknowledgement: The authors acknowledge the National Natural Science Foundation of China for creating a conducive and enabling environment that supports this research.

Funding Statement: This work was supported by the National Natural Science Foundation of China (No. 62172095).

Author Contributions: The authors confirm contribution to the paper as follows: study conception and design: Xingsi Xue, Himanshu Dhumras, Garima Thakur, Rakesh Kumar Bajaj, Varun Shukla; analysis and interpretation of results: Xingsi Xue, Himanshu Dhumras, Garima Thakur, Rakesh Kumar Bajaj; draft manuscript preparation: Xingsi Xue, Himanshu Dhumras, Garima Thakur, Rakesh Kumar Bajaj, Varun Shukla. All authors reviewed the results and approved the final version of the manuscript.

Availability of Data and Materials: All data used to justify the proposed model are given in the manuscript.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

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