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ARTICLE

Sufficient and Necessary Conditions for Leader-Following Consensus of Second-Order Multi-Agent Systems via Intermittent Sampled Control

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ABSTRACT: Continuous control protocols are extensively utilized in traditional MASs, in which information needs to be transmitted among agents consecutively, therefore resulting in excessive consumption of limited resources. To decrease the control cost, based on ISC, several LFC problems are investigated for second-order MASs without and with time delay, respectively. Firstly, an intermittent sampled controller is designed, and a sufficient and necessary condition is derived, under which state errors between the leader and all the followers approach zero asymptotically. Considering that time delay is inevitable, a new protocol is proposed to deal with the time-delay situation. The error system's stability is analyzed using the Schur stability theorem, and sufficient and necessary conditions for LFC are obtained, which are closely associated with the coupling gain, the system parameters, and the network structure. Furthermore, for the case where the current position and velocity information are not available, a distributed protocol is designed that depends only on the sampled position information. The sufficient and necessary conditions for LFC are also given. The results show that second-order MASs can achieve the LFC if and only if the system parameters satisfy the inequalities proposed in the paper. Finally, the correctness of the obtained results is verified by numerical simulations.

KEYWORDS: Intermittent sampled control; leader-following consensus; time delay; second-order multi-agent system

1 Introduction

In recent years, MASs have always been at the forefront of control and network science research. The rapid development of MASs owes mainly to their widespread applications in formation control of UAVs [1], spacecraft attitude coordination control [2], distributed control in microgrids [3], and so on.

Serving as a basis for the control of MASs, studies on consensus are of great practical and theoretical significance. Up to now, many meaningful research results have been obtained. For example, studies [4] and [5] discussed the consensus in first-order MASs under switching and dynamically changing topologies, respectively. Since second-order dynamics is more usual than first-order in applications, consensus on second-order MASs has drawn the interest of an increasing number of researchers. Authors in [6] proposed several second-order algorithms to solve the consensus problems. The consensus of heterogeneous second-order nonlinear MASs was studied in [7–9], whereas studies [10] and [11] discussed the consensus problems for discrete-time MASs under directed information exchange.

Under the continuous control framework, communication between agents must always be maintained. The intermittent control strategy was proposed to overcome the excessive consumption of limited resources in continuous control. Using the intermittent control technique, studies [12] and [13] studied the consensus problems of linear and nonlinear MASs, respectively. Intermittent control was also used in [14–16] to study the second-order consensus of time-delay MASs. The distributed consensus problem for leader-following MASs was investigated in [17] via directed intermittent communication.

Although intermittent control can shorten the controller's running time, the frequency of information transmission cannot be decreased. Sampled-based intermittent controllers were introduced to improve control efficiency and solve second-order consensus problems [18–20]. LFC in second-order MASs was analyzed in [18] by designing an intermittent controller based on a filter with relative state information. In particular, the topology graph in [18] is undirected; however, it is virtually impossible to guarantee bidirectional communication between agents. Therefore, studying MASs with directed topologies of one-way communication is more practical. Reference [19] discussed the consensus of directed second-order MASs by employing current and sampled information. For the containment control problem, the proof of consensus for MASs has been given in [20] under a directed communication topology. Studies [19] and [20] both used the current and sampled state information to design the intermittent communication control protocols.

Sampled data can be used to design control protocols when real-time information is unavailable. For second-order leaderless MASs, periodic intermittent control protocols using sampled information were designed in [21] and [22]. Study [21] considered a consensus protocol with sampled position and velocity information. A protocol with sampled and past sampled position information was proposed in [22] to overcome the dependence on velocity information. By analyzing the relationship of the system parameters, sufficient and necessary conditions have been obtained to ensure the consensus of MASs.

It should be pointed out that none of the above references explored the LFC problems for directed MASs by taking an intermittent sampled control strategy. Based on these analyses, it is valuable to delve deeply into the intermittent sampled LFC control problems for second-order MASs under weighted directed graphs.

The key contributions made by this paper are outlined below. Firstly, a distributed intermittent controller is proposed, which guarantees the second-order LFC and effectively reduces the energy loss of the MAS. Secondly, time delay is considered when designing the controller. Based on the relations of the communication bandwidth, the sampling period, and the time delay, sufficient and necessary conditions for LFC are derived by analyzing the error dynamics of the MAS. Furthermore, following the idea in [23], an intermittent controller based only on sampled position information is designed, under which LFC can still be reached if and only if control parameters, time delay, communication bandwidth, and sampling period satisfy certain conditions.

Before proceeding, we need to introduce some symbols: R, R^n, $R^{n\times n}$, and N indicate the set of real numbers, the n-dimensional real vector space, the $n\times n$ real matrix set, and the natural number set, respectively. Let 0 stand for an appropriate dimension zero vector or zero matrix, and I_n refers to the $n\times n$ identity matrix. For a complex number u, its real part, imaginary part, and modulus are denoted by $\mathfrak{R}(\mu)$, $\mathfrak{I}(\mu)$, and $|\mu|$, respectively. $\|z\|$ means the Euclidean norm of a vector z. $|X|$ denotes the determinant of a square matrix X. $\mathbf{i}$ and diag$(\cdot)$ are the imaginary unit and the diagonal matrix, respectively.

2 Preliminaries

This part will briefly introduce some necessary basics.

2.1 Graph Theory

Basic graph theory is introduced by considering a MAS, which includes one leader and N followers.

$\tilde{G}$ represents the network topology of the considered MAS. $G = \{W, E, A\}$ is a subgraph of $\tilde{G}$, which describes the data exchange among all the followers, where $W = \{\omega_1, \omega_2, \cdots, \omega_N\}$, E, and A are the set of follower nodes, the set of directed edges, and the adjacency matrix, respectively. A directed path from node ω_j to ω_i in G is composed of a series of directed edges $(\omega_i, \omega_{i1}), (\omega_{i1}, \omega_{i2}), \cdots, (\omega_{il}, \omega_j)$, where $\omega_{ik}(k = 1, 2, \cdots, l)$ are distinct from each other.

If there is a node (defined as the root node) such that directed paths exist from this node to every other node in the graph, we say that the directed graph has a directed spanning tree. If $e_{ji} \in E$, then $a_{ij} > 0$; or else $a_{ij} = 0$; $a_{ii} = 0$ for all $i = 1, 2, \cdots, N$. $L = [l_{ij}] \in R^{N\times N}$ denotes the Laplacian matrix associated with graph G, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}(i \neq j)$. $D = \text{diag}(d_1, d_2, \cdots, d_N)$ is utilized to state whether the followers can receive the information from the leader. $d_i > 0$ means that the ith follower can obtain the information from the leader directly; otherwise, $d_i = 0$.

2.2 Useful Lemmas

Lemma 1 ([24]): *$L + D$ is nonsingular if $\tilde{G}$ has a spanning tree where the leader node is the root node.*

Lemma 2 ([25]): *Consider a block matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, $\det(S) = \det(S_{11}S_{22} - S_{21}S_{12})$ *if S_{11} and S_{21}* are commutable, where $S_{11}, S_{12}, S_{21}, S_{22} \in R^{n\times n}$.

Lemma 3 ([26]): *Consider a polynomial of order two with complex coefficients: $l(s) = s^2 + (p_1 + \mathbf{i}q_1)s + p_2 + \mathbf{i}q_2$, where p_1, q_1, p_2, and q_2 are real constants. Then, $l(s)$ is stable if and only if $p_1 > 0$ and $p_1q_1q_2 + p_1^2p_2 - q_2 > 0$.*

Lemma 4 ([27]): *For a third-order complex coefficient polynomial: $m(s) = s^3 + (p_1 + \mathbf{i}q_1)s^2 + (p_2 + \mathbf{i}q_2)s + p_3 + \mathbf{i}q_3$, where p_i and $q_i (i = 1, 2, 3)$ are real constants. Then, $m(s)$ is stable if and only if $p_1 > 0$, $p_1q_1q_2 + p_1^2p_2 - q_2^2 - p_1p_3 > 0$, and* $p_1 \begin{vmatrix} q_1 & -p_2 & -p_3 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix} - \begin{vmatrix} q_2 & -p_3 & 0 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix} > 0.$

2.3 Problem Formulation

Consider a second-order MAS, which includes N followers and one leader. The ith follower's dynamical model can be described as

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \quad i = 1, 2, \cdots, N \end{cases} \tag{1}$$

where $x_i(t) \in R$, $v_i(t) \in R$, and $u_i(t) \in R$ are the ith agent's position, velocity, and control input, respectively. Moreover, the leader's dynamics can be expressed by

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = 0, \quad i = 1, 2, \cdots, N \end{cases} \tag{2}$$

where $x_0(t) \in R$ and $v_0(t) \in R$ stand for the leader's position and velocity, respectively.

Definition 1: *For any $x_i(0)$ and $v_i(0)$ $(i = 0, 1, 2, \cdots, N)$, if there exists a control input $u_i(t)$ which makes $\lim_{t\to\infty} |x_i(t) - x_0(t)| = 0$ and $\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0$ hold, then the second-order MAS (1)–(2) is said to attain the LFC.*

The following ISC protocol is introduced to reduce energy consumption:

$$u_i(t) = \begin{cases} \gamma_1(\sum_{j=1, j\neq i}^{N} a_{ij}(x_j(t_k) - x_i(t_k)) + d_i(x_0(t_k) - x_i(t_k))) + \gamma_2(\sum_{j=1, j\neq i}^{N} a_{ij}(v_j(t_k) \\ -v_i(t_k)) + d_i(v_0(t_k) - v_i(t_k))), \ t \in (t_k, t_k + \delta], \\ 0, \ t \in (t_k + \delta, t_{k+1}], \quad i = 1, 2, \cdots, N \end{cases} \tag{3}$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ are the coupling gains, δ, t_k, and T are the communication bandwidth, the sampling instant, and the sampling period, respectively. And the sampling instants satisfy $t_{k+1} - t_k = T$, $\lim_{k\to\infty} t_k = +\infty$, $0 = t_0 < t_1 < \cdots < t_k < \cdots$, and $0 < \delta < T$.

Remark 1: *Continuous control protocols, which are widely applied in the study of consensus-related problems of second-order MASs, will cause excessive consumption of limited resources. Controllers using current and sampled state information were designed in* [18–20] *to decrease the control cost. Reference* [18] *considered the LFC problems of undirected MASs; however, the obtained results do not apply to MASs under directed graphs. References* [19] *and* [20] *address the leaderless consensus of MASs in directed topologies. The controller* (3) *proposed in this paper only uses the sampled data to deal with the LFC problems of directed MASs.*

3 Leader-Following Consensus without Time Delay

In this part, ISC protocol (3) is applied to study the LFC problem of MAS (1)–(2).

Let $\zeta_i(t) = x_i(t) - x_0(t)$ and $\eta_i(t) = v_i(t) - v_0(t)$. With (3), we obtain that

$$\begin{cases} \dot{\zeta}_i(t) = \eta_i(t) \\ \dot{\eta}_i(t) = \begin{cases} -\gamma_1(\sum_{j=1}^{N} l_{ij}\zeta_j(t_k) + d_i\zeta_i(t_k)) - \gamma_2(\sum_{j=1}^{N} l_{ij}\eta_j(t_k) + d_i\eta_i(t_k)), \ t \in (t_k, t_k + \delta] \\ 0, \ t \in (t_k + \delta, t_{k+1}], \ i = 1, 2, \cdots, N. \end{cases} \end{cases} \tag{4}$$

Let $\zeta(t) = [\zeta_1(t), \zeta_2(t), \cdots, \zeta_N(t)]^T, \eta(t) = [\eta_1(t), \eta_2(t) \cdots, \eta_N(t)]^T$ and $H = L + D$, (4) can be converted to

$$\begin{cases} \dot{\zeta}(t) = \eta(t) \\ \dot{\eta}(t) = \begin{cases} -\gamma_1 H\zeta(t_k) - \gamma_2 H\eta(t_k), \ t \in (t_k, t_k + \delta] \\ 0, \ t \in (t_k + \delta, t_{k+1}], \ i = 1, 2, \cdots, N. \end{cases} \end{cases} \tag{5}$$

Eq. (5) can be further reworded to

$$\dot{\xi}(t) = \begin{cases} P_1\xi(t) + P_2\xi(t_k), \ t \in (t_k, t_k + \delta] \\ P_1\xi(t), \ t \in (t_k + \delta, t_{k+1}] \end{cases} \tag{6}$$

where $\xi(t) = [\zeta^T(t), \eta^T(t)]^T$, $P_1 = \begin{bmatrix} 0 & I_N \\ 0 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 & 0 \\ -\gamma_1 H & -\gamma_2 H \end{bmatrix}$.

For matrix H, there is a nonsingular matrix K such that $H = KJK^{-1}$, and J denotes the Jordan form corresponding to H. Let $\hat{\xi}(t) = \begin{bmatrix} K^{-1} & 0 \\ 0 & K^{-1} \end{bmatrix} \xi(t)$, we can derive that

$$\dot{\hat{\xi}}(t) = \begin{cases} P_1\hat{\xi}(t) + \tilde{P}_2\hat{\xi}(t_k), \ t \in (t_k, t_k + \delta] \\ P_1\hat{\xi}(t), \ t \in (t_k + \delta, t_{k+1}] \end{cases} \tag{7}$$

where $\tilde{P}_2 = \begin{bmatrix} 0 & 0 \\ -\gamma_1 J & -\gamma_2 J \end{bmatrix}$. From Lemma 1, if G is directed, some of the eigenvalues of H may be complex and $J = \mathrm{diag}(J_1, J_2, \cdots, J_w)$, where $J_v(v = 1, 2, \cdots, w)$ have the following form: $J_v = \begin{bmatrix} \mu_v & 0 & 0 & 0 \\ 1 & \ddots & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & 1 & \mu_v \end{bmatrix}_{N_v \times N_v}$, in which μ_v are the nonzero eigenvalues of H with algebraic multiplicity $N_v(v = 1, 2, \cdots, w)$ and $N_1 + N_2 + \cdots + N_w = N$. Besides, if G is undirected, J becomes a diagonal matrix with all diagonal elements being positive.

Before proceeding, the following assumption and proposition need to be given.

Assumption 1: *$\tilde{G}$ is a digraph and it has a directed spanning tree with the leader as the root node.*

Proposition 1: *When Assumption* 1 *holds, system* (1)–(2) *with protocol* (3) *can reach LFC if and only if* $\lim_{t\to\infty} \hat{\xi}_i(t) = 0(i = 1, 2, \cdots, N)$, *and* $\hat{\xi}_i(t)$ *is the ith component of* $\hat{\xi}(t)$.

Proof of Proposition 1 (Necessity): If $\lim_{t\to\infty} x_i(t) = x_0(t)$ and $\lim_{t\to\infty} v_i(t) = v_0(t)$ for $i = 1, 2, \cdots, N$, then $\zeta_i(t) \to 0$ and $\eta_i(t) \to 0$ as $t \to \infty$ for $i = 1, 2, \cdots, N$. Thus, the system (7) exhibits asymptotic stability.

(Sufficiency): If $\lim_{t\to\infty} \hat{\xi}_i(t) = 0$ for $i = 1, 2, \cdots, N$, then $\lim_{t\to\infty} \|\xi(t)\| = 0$ since K is nonsingular. It means that system (1)–(2) realizes LFC. □

Corollary 1: *Under Assumption* 1, *MAS* (1)–(2) *can reach LFC if and only if the following N systems are asymptotically stable:*

$$\dot{h}_i(t) = \begin{cases} A_1 h_i(t) + \mu_i A_2 h_i(t_k), \ t \in (t_k, t_k + \delta] \\ A_1 h_i(t), \ t \in (t_k + \delta, t_{k+1}] \end{cases} \tag{8}$$

where $h_i(t) = [\hat{\xi}_i(t), \hat{\xi}_{i+N}(t)]^T$, $\hat{\xi}_i(t)$ *and* $\hat{\xi}_{i+N}(t)$ *represent the ith and the* $(i + N)$*th components of* $\hat{\xi}(t)$, *respectively;* $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 0 & 0 \\ -\gamma_1 & -\gamma_2 \end{bmatrix}$.

Proof of Corollary 1 (Necessity): If MAS (1)–(2) reaches LFC, it follows from Proposition 1 that $\lim_{t\to\infty} \hat{\xi}_i(t) = 0$ for $i = 1, 2, \cdots, N$. For $t \in (t_k, t_{k+1}]$, because the variables in the system (8) are the first terms of the Jordan blocks in (7), it is straightforward to get that $\lim_{t\to\infty} h_i(t) = 0$ for $i = 1, 2, \cdots, N$, which means that system (8) is asymptotically stable.

(Sufficiency): When the N systems in the (8) are asymptotically stable, then $\lim_{t\to\infty} h_i(t) = 0$ for $i = 1, 2, \cdots, N$. Based on the characteristics of the Jordan form, the LFC of (8) is determined by the diagonal entries. Thus, $\lim_{t\to\infty} \hat{\xi}_i(t) = 0$ holds for $i = 1, 2, \cdots, N$. □

Although Proposition 1 and Corollary 1 provide some conditions for ensuring LFC, they do not show how consensus behavior is affected by relevant parameters of the system. Therefore, the following theorem is given to show the relationship among them.

Theorem 1: *With* (3), *MAS* (1)–(2) *can reach the LFC if and only if*

$$\begin{cases} \delta < \dfrac{2\gamma_2}{\gamma_1} \\ T < \Xi_i \end{cases} \tag{9}$$

where $\Xi_i = [\delta^2(\gamma_1\delta - 2\gamma_2)^3|\mu_i|^4 + 4\delta(2\gamma_2 - \gamma_1\delta)^2\Re(\mu_i)|\mu_i|^2]/[\gamma_1\delta^2(2\gamma_2 - \gamma_1\delta)^2|\mu_i|^4 + 16\gamma_1\Im^2(\mu_i)]$, μ_i *is the eigenvalue of* H.

Proof of Theorem 1: From (8), we can obtain

$$h_i(t) = \begin{cases} e^{A_1(t-t_k)}h_i(t_k) + \mu_i \int_{t_k}^{t} e^{A_1(t-n)}A_2h_i(t_k)dn, \ t \in (t_k, t_k + \delta] \\ e^{A_1(t-t_k-\delta)}h_i(t_k + \delta), \ t \in (t_k + \delta, t_{k+1}]. \end{cases} \tag{10}$$

When $t = t_k + \delta$, it is obvious that $h_i(t_k + \delta) = e^{A_1\delta}h_i(t_k) + \mu_i \int_{t_k}^{t_k+\delta} e^{A_1(t_k+\delta-n)}A_2h_i(t_k)dn$.

Solving the first equation of (10) and substituting its solution into the second equation of (10), it can be obtained that

$$h_i(t) = \begin{cases} E_i(t - t_k)h_i(t_k), \ t \in (t_k, t_k + \delta] \\ F_i(t - t_k - \delta)h_i(t_k), \ t \in (t_k + \delta, t_{k+1}] \end{cases} \tag{11}$$

where $E_i(s) = -\begin{bmatrix} \gamma_1\mu_i s^2/2 - 1 & \gamma_2\mu_i s^2/2 - s \\ \gamma_1\mu_i s & \gamma_2\mu_i s - 1 \end{bmatrix}$, $F_i(s) = \begin{bmatrix} Q_{1i}(s) & Q_{2i}(s) \\ -\gamma_1\mu_i\delta & 1 - \gamma_2\mu_i\delta \end{bmatrix}$ with $Q_{1i}(s) = 1 - \gamma_1\mu_i\delta(\delta/2 + s)$ and $Q_{2i}(s) = s + \delta - \gamma_2\mu_i\delta(s + \delta/2)$.

Hence, we have

$$h_i(t) = \begin{cases} E_i(t - t_k)F_i^k(T - \delta)h_i(t_0), \ t \in (t_k, t_k + \delta] \\ F_i(t - t_k - \delta)F_i^k(T - \delta)h_i(t_0), \ t \in (t_k + \delta, t_{k+1}]. \end{cases} \tag{12}$$

When $t_k < t \le t_{k+1}$, we can easily know that $E_i(t - t_k)$ and $F_i(t - t_k - \delta)$ are bounded. $h_i(t) \to 0$ if and only if all the eigenvalues of $F_i(T - \delta)$ have modulus less than 1. Then, let $|\lambda I_2 - F_i(T - \delta)| = 0$, by Lemma 2, we get

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{13}$$

where $a_1 = \mu_i\delta(\alpha T + \beta - \alpha\delta/2) - 2$ and $a_2 = \alpha\mu_i\delta^2/2 - \beta\mu_i\delta + 1$. In order to analyze the Schur stability of Eq. (13), let $\lambda = (m + 1)/(m - 1)$, then

$$m^2 + b_1m + b_2 = 0 \tag{14}$$

where $b_1 = (2\gamma_2 - \gamma_1\delta)/(\gamma_1 T)$ and $b_2 = \delta/T + 4/(\gamma_1\mu_i\delta T) - 2\gamma_2/(\gamma_1 T) - 1$. It follows from Lemma 3 that (14) is asymptotically stable if and only if (9) holds. Based on the aforementioned analysis, (9) guarantees that $h_i(t) \to 0$ for $i = 1, 2, \cdots, N$. So, MAS (1)–(2) reaches LFC under protocol (3). The proof is complete. □

Corollary 2: *When* $T = \delta$, *system* (1)–(2) *achieves the LFC if and only if*

$$\begin{cases} \delta < 2\gamma_2/\gamma_1 \\ \Xi_i > 1 \end{cases} \tag{15}$$

where $\Xi_i = [T(\gamma_1 T - 2\gamma_2)^3|\mu_i|^4 + 4(2\gamma_2 - \gamma_1 T)^2\Re(\mu_i)|\mu_i|^2]/[\gamma_1 T^2(2\gamma_2 - \gamma_1 T)^2|\mu_i|^4 + 16\gamma_1\Im^2(\mu_i)]$.

Proof of Corollary 2: When $T = \delta$, system (1)–(2) becomes a general sampled system. It is straightforward to get condition (15) from (9). □

Undirected graphs can be considered unique directed graphs. For undirected graphs, the subsequent corollary can be derived.

Corollary 3: *If the communication topology of all the followers is undirected, system (1)–(2) can reach the LFC if and only if*

$$\begin{cases} \delta < 2\gamma_2/\gamma_1 \\ T < \Xi_i \end{cases} \tag{16}$$

where $\Xi_i = [\delta^2(\gamma_1\delta - 2\gamma_2)^3|\mu_i| + 4\delta(2\gamma_2 - \gamma_1\delta)^2]/[\gamma_1\delta^2(2\gamma_2 - \gamma_1\delta)^2|\mu_i|]$.

Proof of Corollary 3: When G is undirected, the eigenvalues $\mu_i(i = 1, 2, \cdots, N)$ of H are real. Then, $\Re(\mu_i) = \mu_i$ and $\Im(\mu_i) = 0$ for $i = 1, 2, \cdots, N$. Thus, (9) is equivalent to (16) for undirected graphs, then LFC is reached if and only if (16) holds. □

4 Leader-Following Consensus with Time Delay

It is widely known that time delay has always been present in actual systems and can not be ignored. For time delay situations, a kind of ISC algorithm is proposed.

$$u_i(t) = \begin{cases} \gamma_1(\sum_{j=1, j\neq i}^{N} a_{ij}(x_j(t_k - \iota) - x_i(t_k - \iota)) + d_i(x_0(t_k - \iota) - x_i(t_k - \iota))) + \gamma_2(\sum_{j=1, j\neq i}^{N} \\ a_{ij}(v_j(t_k - \iota) - v_i(t_k - \iota)) + d_i(v_0(t_k - \iota) - v_i(t_k - \iota))),\ t \in (t_k, t_k + \delta] \\ 0,\ t \in (t_k + \delta, t_{k+1}],\ i = 1, 2, \cdots, N \end{cases} \tag{17}$$

where ι represents the time delay and $0 < \iota < T$.

Remark 2. *Since time delay always exists in engineering applications, based on* [18], *we further consider the time-delay system and propose a time-delay controller that only uses the sampled information. Literature* [13] *and* [14] *considered the leaderless consensus of linear and nonlinear MASs, respectively, where intermittent control protocols were designed using sampled information. As a comparison, this paper discusses the ISC problem of second-order leader-following MASs. An ISC protocol (17) is designed for MAS (1)–(2) with time delay to achieve LFC.*

With (17), by using procedures similar to those in Section 3, we obtain that

$$\dot{\xi}(t) = \begin{cases} P_1\xi(t) + P_2\xi(t_k - \iota),\ t \in (t_k, t_k + \delta] \\ P_1\xi(t),\ t \in (t_k + \delta, t_{k+1}]. \end{cases} \tag{18}$$

Let $\hat{\xi}(t) = \begin{bmatrix} K^{-1} & 0 \\ 0 & K^{-1} \end{bmatrix} \xi(t)$, system (18) can be cast to

$$\dot{\hat{\xi}}(t) = \begin{cases} P_1\hat{\xi}(t) + \tilde{P}_2\hat{\xi}(t_k - \iota),\ t \in (t_k, t_k + \delta] \\ P_1\hat{\xi}(t),\ t \in (t_k + \delta, t_{k+1}]. \end{cases} \tag{19}$$

Let $h_i(t) = [\hat{\xi}_i(t), \hat{\xi}_{i+N}(t)]^T$. (19) can be further reformulated as

$$\dot{h}_i(t) = \begin{cases} A_1h_i(t) + \mu_iA_2h_i(t_k - \iota),\ t \in (t_k, t_k + \delta] \\ A_1h_i(t),\ t \in (t_k + \delta, t_{k+1}]. \end{cases} \tag{20}$$

Theorem 2: *When Assumption* 1 *holds, MAS* (1)–(2) *with the protocol* (17) *achieves the LFC if and only if one of the following conditions hold:*

1) $\iota < \min\{\delta, T-\delta\}$ *or* $\delta < \iota < t-\delta(\delta < t/2)$,

$$\begin{cases} \delta < 2\gamma_2/\gamma_1 - 2\iota \\ T < \Phi_{1i}/\Phi_{2i} \end{cases} \tag{21}$$

where $\Phi_{1i} = \delta^2(\gamma_1\delta - 2\gamma_2 + 2\gamma_1\iota)^3|\mu_i|^4 + 4\delta(2\gamma_2 - \gamma_1\delta - 2\gamma_1\iota)^2\Re(\mu_i)|\mu_i|^2$ *and* $\Phi_{2i} = \gamma_1\delta^2(2\gamma_2 - \gamma_1\delta - 2\gamma_1\iota)^2|\mu_i|^4 + 16\gamma_1\Im^2(\mu_i)$.

2) $T-\delta < \iota < T(\delta > T/2)$ *or* $\iota > \max\{\delta, T-\delta\}$,

$$\begin{cases} T - 2\iota/3 + 2\gamma_2/(3\gamma_1) > 0 \\ p_1^2 p_2 - q_2^2 - p_1 p_3 > 0 \\ \Lambda_i > 0 \end{cases} \tag{22}$$

where $\Lambda_i = p_1 \begin{vmatrix} q_1 & -p_2 & -p_3 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix} - \begin{vmatrix} q_2 & -p_3 & 0 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix}$ *with* $p_1 = 2\gamma_1 T(\gamma_2 - \gamma_1\iota) + 3, q_1 = 0, p_2 = 4\Re(\mu_i)/(\gamma_1\delta T|\mu_i|^2) + 2[(T-\iota)^2 - 2\gamma_2]/(\gamma_1 T) + 4\gamma_2(T-\iota)/(\gamma_1\delta T) + 2(\delta + 2\iota)/T - 5, q_2 = q_3 = 4\Im(\mu_i)/(\gamma_1\delta T|\mu_i|^2)$ *and* $p_3 = 4\Re(\mu_i)/(\gamma_1\delta T|\mu_i|^2) + 2[(T-\iota)^2 - \gamma_2]/(\gamma_1 T) + 4\gamma_2/(\gamma_1\delta) - 4\gamma_2\iota/(\gamma_1\delta T) + (\delta + \iota)/T - 3$.

Proof of Theorem 2: Based on Eq. (20), we deduce that

$$h_i(t) = \begin{cases} e^{A_1(t-t_k)}h_i(t_k) + \int_{t_k}^{t} e^{A_1(t-n)}A_{2i}h_i(t_k-\iota)dn, \ t \in (t_k, t_k+\delta] \\ e^{A_1(t-t_k-\delta)}h_i(t_k+\delta), \ t \in (t_k+\delta, t_{k+1}]. \end{cases} \tag{23}$$

Therefore, $h_i(t_k+\delta) = e^{A_1\delta}h_i(t_k) + \int_{t_k}^{t_k+\delta} e^{A_1(t_k+\delta-n)}A_{2i}h_i(t_k-\iota)dn$. For $t \in (t_k+\delta, t_{k+1}]$, it is obvious that $h_i(t) = \Phi_3(t-t_k-\delta)h_i(t_k) + \mu_i\Phi_4(t-t_k-\delta)h_i(t_k-\iota)$ where $\Phi_3(s) = \begin{bmatrix} \gamma_1\delta(\delta/2+s) & \gamma_2\delta(\delta/2+s) \\ \gamma_1\delta & \gamma_2\delta \end{bmatrix}$ and $\Phi_4(s) = \begin{bmatrix} 1 & s+\delta \\ 0 & 1 \end{bmatrix}$. Then

$$h_i(t) = \begin{cases} \Phi_1(t-t_k)h_i(t_k) + \mu_i\Phi_2(t-t_k)h_i(t_k-\iota), \ t \in (t_k, t_k+\delta] \\ \Phi_3(t-t_k-\delta)h_i(t_k) + \mu_i\Phi_4(t-t_k-\delta)h_i(t_k-\iota), \ t \in (t_k+\delta, t_{k+1}] \end{cases} \tag{24}$$

where $\Phi_1(s) = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$ and $\Phi_2(s) = \begin{bmatrix} \gamma_1 s^2/2 & \gamma_2 s^2/2 \\ \gamma_1 s & \gamma_2 s \end{bmatrix}$.

Let $\hat{h}_i(t) = [h_i(t), h_i(t-\iota)]^T$. According to the relationship of ι, δ, and T, the following four cases are discussed to obtain the detailed expressions of $h_i(t_k-\iota)$.

Case 1: $\iota < \min\{\delta, T-\delta\}$.

For $t \in [t_k, t_k+\iota)$, $h_i(t) = \Phi_1(t-t_k)h_i(t_k) + \mu_i\Phi_2(t-t_k)h_i(t_k-\iota)$, $h_i(t-\iota) = \Phi_3(t-\iota-t_{k-1}-\delta)h_i(t_{k-1}) + \mu_i\Phi_4(t-\iota-t_{k-1}-\delta)h_i(t_{k-1}-\iota)$. For $t \in [t_k+\iota, t_k+\delta)$, $h_i(t) = \Phi_1(t-t_k)h_i(t_k) + \mu_i\Phi_2(t-t_k)h_i(t_k-\iota)$, $h_i(t-\iota) = \Phi_1(t-\iota-t_k)h_i(t_k) + \mu_i\Phi_2(t-\iota-t_k)h_i(t_k-\iota)$. For $t \in [t_k+\delta, t_k+\delta+\iota)$, $h_i(t) = \Phi_3(t-t_k-\delta)h_i(t_k) + \mu_i\Phi_4(t-t_k-\delta)h_i(t_k-\iota)$, $h_i(t-\iota) = \Phi_1(t-\iota-t_k)h_i(t_k) + \mu_i\Phi_2(t-$

$\iota - t_k)h_i(t_k - \iota)$. For $t \in [t_k + \delta + \iota, t_{k+1})$, $h_i(t) = \Phi_3(t - t_k - \delta)h_i(t_k) + \mu_i\Phi_4(t - t_k - \delta)h_i(t_k - \iota)$, $h_i(t - \iota) = \Phi_3(t - \iota - t_k - \delta)h_i(t_k) + \mu_i\Phi_4(t - \iota - t_k - \delta)h_i(t_k - \iota)$.

By letting $V_{1i}(s) = \begin{bmatrix} E_{1i}(s) & F_{1i}(s) \\ \Phi_3(s - \iota + T - \delta) & \mu_i\Phi_4(s - \iota + T - \delta) \end{bmatrix}$, $V_{2i}(s) = \begin{bmatrix} \Phi_1(s) & \mu_i\Phi_2(s) \\ \Phi_1(s - \iota) & \mu_i\Phi_2(s - \iota) \end{bmatrix}$, $V_{3i}(s) = \begin{bmatrix} \Phi_3(s) & \mu_i\Phi_4(s) \\ \Phi_1(s - \iota) & \mu_i\Phi_2(s - \iota) \end{bmatrix}$, $V_{4i}(s) = \begin{bmatrix} \Phi_3(s - \delta) & \mu_i\Phi_4(s - \delta) \\ \Phi_3(s - \delta - \iota) & \mu_i\Phi_4(s - \delta - \iota) \end{bmatrix}$, $E_{1i}(s) = \Phi_1(s)\Phi_3(T - \delta) + \mu_i\Phi_2(s)\Phi_4(T - \delta - \iota)$ and $F_{1i}(s) = \mu_i(\Phi_1(s)\Phi_4(T - \delta) + \Phi_2(s)\Phi_4(T - \iota - \delta))$, we have $h_i(t_{k+1}) = \Phi_3(T - \delta)h_i(t_k) + \mu_i\Phi_2(t - t_k)h_i(t_k - \iota)$ and $h_i(t_{k+1} - \iota) = \Phi_3(T - \delta - \iota)h_i(t_k) + \mu_i\Phi_4(t - t_k)h_i(t_k - \iota)$.

Therefore, for $t \in [t_k, t_k + \iota)$, $\hat{h}_i(t) = V_{1i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{1i}(t - t_k)V_{4i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \iota, t_k + \delta)$, $\hat{h}_i(t) = V_{2i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{2i}(t - t_k)V_{4i}^{k}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta, t_k + \delta + \iota)$, $\hat{h}_i(t) = V_{3i}(t - t_k)\hat{h}_i(t_k) = V_{3i}(t - t_k)V_{4i}^{k}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta + \iota, t_{k+1})$, $\hat{h}_i(t) = V_{4i}(t - t_k)\hat{h}_i(t_k) = V_{4i}(t - t_k)V_{4i}^{k}(T)\hat{h}_i(t_0)$.

Case 2: When $T - \delta < \iota < \delta(\delta > T/2)$, the analysis is similar to that in Case 1.

Let $V_{6i}(s) = \begin{bmatrix} E_{2i}(s) & F_{2i}(s) \\ \Phi_3(s - \iota + T - \delta) & \mu_i\Phi_4(s + T - \iota - \delta) \end{bmatrix}$, $V_{7i}(s) = \begin{bmatrix} \Phi_3(s - \delta) & \mu_i\Phi_4(s - \delta) \\ \Phi_1(s - \iota) & \mu_i\Phi_2(s - \iota) \end{bmatrix}$, $V_{5i}(s) = \begin{bmatrix} E_{2i}(s) & F_{2i}(s) \\ \Phi_1(s + T - \iota) & \mu_i\Phi_2(s - \iota + T) \end{bmatrix}$, $E_{2i}(s) = \Phi_1(s)\Phi_3(T - \delta) + \mu_i\Phi_2(s)\Phi_4(T - \iota)$ and $F_{2i}(s) = \mu_i(\Phi_2(s)\Phi_2(T - \iota) + \Phi_1(s)\Phi_4(T - \delta))$.

For $t \in [t_k, t_{k-1} + \iota + \delta)$, $\hat{h}_i(t) = V_{5i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{5i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_{k-1} + \iota + \delta, t_k + \iota)$, $\hat{h}_i(t) = V_{6i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{6i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \iota, t_k + \delta)$, $\hat{h}_i(t) = V_{2i}(t - t_k)\hat{h}_i(t_k) = V_{2i}(t - t_k)V_{7i}^{k}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta, t_{k+1})$, $\hat{h}_i(t) = V_{7i}(t - t_k)\hat{h}_i(t_k) = V_{7i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$.

Case 3: $\delta < \iota < T - \delta(\delta < T/2)$.

Let $V_{8i}(s) = \begin{bmatrix} E_{3i}(s) & F_{3i}(s) \\ \Phi_3(s - \iota + T - \delta) & \mu_i\Phi_4(s - \iota + T - \delta) \end{bmatrix}$, $E_{3i}(s) = \Phi_3(s - \delta)\Phi_3(T - \delta) + \mu_i\Phi_4(s - \delta)\Phi_3(T - \iota - \delta)$ and $F_{3i}(s) = \mu_i(\Phi_3(s - \delta)\Phi_4(T - \delta) + \Phi_4(s - \delta)\Phi_4(T - \iota - \delta))$.

For $t \in [t_k, t_k + \delta)$, $\hat{h}_i(t) = V_{1i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{1i}(t - t_k)V_{4i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta, t_k + \iota)$, $\hat{h}_i(t) = V_{8i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{8i}(t - t_k)V_{4i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \iota, t_k + \delta + \iota)$, $\hat{h}_i(t) = V_{7i}(t - t_k)\hat{h}_i(t_k) = V_{7i}(t - t_k)V_{4i}^{k}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta + \iota, t_{k+1})$, $\hat{h}_i(t) = V_{4i}(t - t_k)\hat{h}_i(t_k) = V_{4i}(t - t_k)V_{4i}^{k}(T)\hat{h}_i(t_0)$.

Case 4: $\iota > \max\{\delta, T - \delta\}$.

Let $V_{9i}(s) = \begin{bmatrix} E_{2i}(s) & F_{2i}(s) \\ \Phi_1(s + T - \iota) & \mu_i\Phi_2(s - \iota) \end{bmatrix}$, $V_{10i}(s) = \begin{bmatrix} E_{2i}(s) & F_{2i}(s) \\ \Phi_3(s + T - \iota - \delta) & \mu_i\Phi_4(s + T - \iota - \delta) \end{bmatrix}$, $V_{11i}(s) = \begin{bmatrix} E_{4i}(s) & F_{4i}(s) \\ \Phi_3(s - \iota + T - \delta) & \mu_i\Phi_4(s - \iota + T - \delta) \end{bmatrix}$, $E_{4i}(s) = \Phi_3(s - \delta)\Phi_3(T - \delta) + \mu_i\Phi_1(T - \iota)\Phi_4(s - \delta)$ and $F_{4i}(s) = \mu_i(\Phi_3(s - \delta)\Phi_4(T - \delta) + \Phi_1(T - \iota)\Phi_4(s - \delta))$.

For $t \in [t_k, t_{k-1} + \delta + \iota)$, $\hat{h}_i(t) = V_{9i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{9i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_{k-1} + \delta + \iota, t_k + \delta)$, $\hat{h}_i(t) = V_{10i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{10i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta, t_k + \delta + \iota)$, $\hat{h}_i(t) = V_{11i}(t - t_k)\hat{h}_i(t_{k-1}) = V_{11i}(t - t_k)V_{7i}^{k-1}(T)\hat{h}_i(t_0)$. For $t \in [t_k + \delta + \iota, t_{k+1})$, $\hat{h}_i(t) = V_{7i}(t - t_k)\hat{h}_i(t_k) = V_{7i}(t - t_k)V_{7i}^{k}(T)\hat{h}_i(t_0)$.

If $t \in (t_k, t_{k+1}]$, we have that $V_{ji}(t - t_k)(j = 1, 2, \cdots, 11, j \neq 4, 7)$ are all bounded. $h(t) \to 0$ if and only if $\hat{h}(t) \to 0$, in other words, $h(t) \to 0$ is equivalent to the modulus of $\lambda(V_{4i}(T))$ or $\lambda(V_{7i}(T))$ being less than 1.

For Cases 1 and 3, let $|\lambda I_4 - V_{4i}(T)| = 0$, from Lemma 2, we have

$$\lambda^4 + c_1\lambda^3 + c_2\lambda^2 = 0 \tag{25}$$

where $c_1 = \mu_i\delta(\gamma_1 T - \gamma_1\iota + \gamma_2 - \gamma_1\delta/2) - 2$ and $c_2 = \mu_i\delta(\gamma_1\iota - \gamma_2 + \gamma_1\delta/2) + 1$. Obviously, V_{4i} has two eigenvalues $\lambda_1 = \lambda_2 = 0$. Let $\lambda = (m + 1)/(m - 1)$, we get

$$m^2 + d_1 m + d_2 = 0 \tag{26}$$

where $d_1 = 2(\gamma_2/\gamma_1 - \delta/2 - \iota)/T$ and $d_2 = 4/(\gamma_1\delta T\mu_i^2) + 2(\delta/2 + \iota - \gamma_2)/T - 1$. It is apparent that the modulus of λ is less than 1 if and only if $\Re(m) < 0$. So, a sufficient and necessary condition for $h_i(t) \to 0$ is that all the roots of (26) have negative real parts. According to Lemma 3, the system (26) is stable if and only if (21) holds. The LFC of MAS (1)–(2) with protocol (17) can be reached if and only if (21) holds.

For Cases 2 and 4, let $|\lambda I_4 - V_{7i}(T)| = 0$, based on Lemma 2, we get

$$\lambda^4 + e_1\lambda^3 + e_2\lambda^2 + e_3\lambda = 0 \tag{27}$$

where $e_1 = \gamma_1\mu_i(\iota - T)(2 + \iota - T)/2$, $e_2 = \mu_i(\iota - T)(2\gamma_2 - \gamma_1(1 + \iota - T)) + (\gamma_1\delta/2 + \gamma_1(T - \delta) + \gamma_2)\delta\mu_i$ and $e_3 = \mu_i(T - \iota)(\gamma_2 - \gamma_1\delta + \gamma_1(T - \iota)) + (\gamma_1\delta/2 - \gamma_2)\delta\mu_i$. Clearly, $\lambda_1 = 0$ is an eigenvalue of V_{7i}. Let $\lambda = (m + 1)/(m - 1)$, we have

$$m^3 + f_1 m^2 + f_2 m + f_3 = 0 \tag{28}$$

where $f_1 = \mu_i[\gamma_2(T - \iota) + \gamma_1(T - \iota)^2/2] - 2$, $f_2 = \mu_i(T - \iota)[\gamma_1 - 2\gamma_2 - \gamma_1(T - \iota)] + \mu_i\delta(\gamma_1 T - \gamma_1\delta/2) + 1$ and $f_3 = \mu_i(T - \iota)[\gamma_2 - \gamma_1\delta + \gamma_1(T - \iota)/2] - \mu_i(\gamma_2 - \gamma_1\delta/2)$. $h_i(t) \to 0$ holds if and only if all the roots of (28) have negative real parts. By Lemma 4, the polynomial in (28) is stable if and only if (22) holds. The proof is complete. □

Remark 3: *In Theorem 2, sufficient and necessary conditions are provided under which system (1)–(2) achieves LFC with protocol (17). From the relationship of ι, δ, and $T - \delta$, four cases are considered, i.e., $\iota < min\{\delta, T - \delta\}$, $\delta < \iota < t - \delta(\delta < t/2)$, $T - \delta < \iota < \delta(\delta > T/2)$, and $\iota > max\{\delta, T - \delta\}$. Notably, for $0 = \iota < min\{\delta, T - \delta\}$, the condition (21) reduces to (9). Thus, Theorem 1 is a special case that fits within the framework of Theorem 2.*

Remark 4: *The second-order LFC of MAS (1)–(2) with time delay can be reached if and only if (21) holds. In Case 1, we can first set the values of γ_1, γ_2, and ι, then select the value of δ satisfying $\delta > \iota$. By combining the network topology, we can select the range of the sampling period T so that (21) is satisfied. Finally, further refine the extent of the sampling period such that $\iota < \min\{\delta, T - \delta\}$. In the other three cases, the parameters can be chosen similarly.*

Corollary 4: *For the undirected topology, the second-order MAS (1)–(2) reaches the LFC if and only if one of the following is satisfied:*

1) $\iota < min\{\delta, T - \delta\}$ or $\delta < \iota < t - \delta(\delta < t/2)$,

$$\begin{cases} \delta < 2\gamma_2/\gamma_1 - 2\iota \\ T < \Phi_{1i}/\Phi_{2i} \end{cases} \tag{29}$$

where $\Phi_{1i} = \delta^2(\gamma_1\delta - 2\gamma_2 + 2\gamma_1\iota)^3\mu_i + 4\delta(2\gamma_2 - \gamma_1\delta - 2\gamma_1\iota)^2$ and $\Phi_{2i} = \gamma_1\delta^2(2\gamma_2 - \gamma_1\delta - 2\gamma_1\iota)^2\mu_i$.

2) $T-\delta<\iota<\delta(\delta>T/2)$ or $\iota>max\{\delta,T-\delta\}$,

$$\begin{cases} T-2\iota/3+2\gamma_2/(3\gamma_1)>0 \\ p_1^2p_2-p_1p_3>0 \\ \Lambda_i>0 \end{cases} \tag{30}$$

where $\Lambda_i = p_1\begin{vmatrix} q_1 & -p_2 & -p_3 & 0 \\ p_1 & 0 & -p_3 & 0 \\ 1 & q_1 & -p_2 & 0 \\ 0 & p_1 & 0 & -p_3 \end{vmatrix} - \begin{vmatrix} 0 & -p_3 & 0 & 0 \\ p_1 & 0 & -p_3 & 0 \\ 1 & q_1 & -p_2 & 0 \\ 0 & p_1 & 0 & -p_3 \end{vmatrix}$ *with* $p_1=2\gamma_1\gamma_2T-2\gamma_1^2\iota T+3$, $p_2=4/(\gamma_1\delta T\mu_i)+2(T-\iota)^2/(\gamma_1T)+4\gamma_2(T-\iota)/(\gamma_1\delta T)-4\gamma_2/(\gamma_1T)+2\delta/T+4\iota/T-5$ *and* $p_3=4/(\gamma_1\delta T\mu_i)+2(T-\iota)^2/(\gamma_1T)+4\gamma_2/(\gamma_1\delta)-4\gamma_2\iota/(\gamma_1\delta T)-2\gamma_2/(\gamma_1T)+(\delta+\iota)/T-3$.

Proof of Corollary 4: For undirected graph G, we can easily get $\Re(\mu_i)=\mu_i$ and $\Im(\mu_i)=0$ $(i=1,2,\cdots,N)$. So, (21) and (22) are equivalent to (29) and (30), respectively. LFC can be achieved if and only if inequalities (29) and (30) hold. □

5 Leader-Following Consensus Based Only on Position Information

Obtaining velocity information is difficult in engineering applications, so we propose the following ISC protocol based only on position information.

$$u_i(t)=\begin{cases} \gamma_1(\sum_{j=1,j\neq i}^N a_{ij}(x_j(t_k)-x_i(t_k))+d_i(x_0(t_k)-x_i(t_k)))-\gamma_2(\sum_{j=1,j\neq i}^N a_{ij}(x_j(t_k-\iota)- \\ x_i(t_k-\iota))+d_i(x_0(t_k-\iota)-x_i(t_k-\iota))),\ t\in(t_k,t_k+\delta] \\ 0,\ t\in(t_k+\delta,t_{k+1}],\ i=1,2,\cdots,N. \end{cases} \tag{31}$$

Remark 5: *Velocity information was used in designing the controllers in* [18,19] *and* [21]*. However, it is usually hard to get the velocity information. When velocity is unavailable, systems need to rely on position data to reach consensus. This motivates us to explore whether LFC can still be reached when protocols are designed based only on position data. In fact, most studies regard the time delay as a detrimental factor due to its significant impact on system stability (examples include references* [4,14–16]*, and* [21]*). When velocity information is absent, the LFC cannot be achieved by relying solely on position information. In such cases, incorporating time delay information can be beneficial for reaching consensus. Reference* [23] *proved that under some circumstances, time delay is beneficial to achieving the consensus of MASs, in which a continuous time-delay control algorithm was designed to reach second-order consensus. ISC protocol* (31) *employing the time-delay sampled position information is proposed in this paper for second-order MASs to achieve LFC, which can effectively reduce the energy loss of MASs and solve the problem that the velocity information is unavailable.*

After similar calculations as in Theorem 2, we obtain that

$$\dot{h}_i(t)=\begin{cases} A_1h_i(t)+\mu_iB_1h_i(t_k)+\mu_iB_2h_i(t_k-\iota),\ t\in(t_k,t_k+\delta] \\ A_1h_i(t),\ t\in(t_k+\delta,t_{k+1}] \end{cases} \tag{32}$$

where $B_1=\begin{bmatrix} 0 & 0 \\ -\gamma_1 & 0 \end{bmatrix}$ and $B_2=\begin{bmatrix} 0 & 0 \\ 0 & \gamma_2 \end{bmatrix}$.

Theorem 3: *Suppose that Assumption 1 holds. MAS* (1)–(2) *with control protocol* (31) *reaches LFC if and only if one of the following holds:*

1) $\iota < min\{\delta, T-\delta\}$ *or* $\delta < \iota < t-\delta(\delta < t/2)$,

$$\begin{cases} \delta < 2\gamma_2\iota/(\gamma_1-\gamma_2) \\ T < \Delta_{1i} \end{cases} \tag{33}$$

where $\Delta_{1i} = [4\Re(\mu_i)/((\gamma_1-\gamma_2)|\mu_i|^2\delta)] - 16\Im(\mu_i)(\gamma_1-\gamma_2)|\mu_i|^2\delta/[2\gamma_2\iota/(\gamma_1-\gamma_2)-\delta]^2$.

2) $T-\delta < \iota < \delta(\delta > T/2)$ *or* $\iota > max\{\delta, T-\delta\}$,

$$\begin{cases} p_1 > 0 \\ p_1q_1q_2 + p_1^2p_2 - q_2^2 - p_1p_3 > 0 \\ \Delta_{2i} > 0 \end{cases} \tag{34}$$

where $\Delta_{2i} = p_1\begin{vmatrix} q_1 & -p_2 & -p_3 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix} - \begin{vmatrix} q_2 & -p_3 & 0 & 0 \\ p_1 & q_2 & -p_3 & 0 \\ 1 & q_1 & -p_2 & -q_3 \\ 0 & p_1 & q_2 & -p_3 \end{vmatrix}$ *with* $p_1 = [\gamma_2\delta(\delta/2-T) - \gamma_2/2(\iota-T)^2 - \gamma_1/2\delta^2]\Re(\mu_i)/(\kappa|\mu_i|^2/2) - 3$, $q_1 = [\gamma_2\delta(\delta/2-T) - \gamma_2/2(\iota-T)^2 - \delta^2\gamma_1/2]\Im(\mu_i)/(\kappa|\mu_i|^2/2)$, $p_2 = -4[[(\gamma_1-\gamma_2)\delta T + \gamma_2\delta^2]\Re(\mu_i) + [\Re^2(\mu_i) - \Im^2(\mu_i)]|\mu_i|^2]/(\kappa|\mu_i|^4) + 3$, $q_2 = -4[[(\gamma_1-\gamma_2)\delta T + \gamma_2\delta^2]|\mu_i|^2\Im(\mu_i) + 2\Re(\mu_i)\Im(\mu_i)]/(\kappa|\mu_i|^4)$, $p_3 = 2[[(\gamma_1+\gamma_2)\delta(\delta/2-T) + \gamma_2/2(T-\iota) - \gamma_1\delta\gamma_1T]\Re(\mu_i)|\mu_i|^2 + 2[\Re^2(\mu_i) - \Im^2(\mu_i)]]/(\kappa|\mu_i|^4)$, $q_3 = 2[(\gamma_1+\gamma_2)\delta(\delta/2-T) + \gamma_2/2(T-\iota) - \gamma_1\delta\gamma_1T]\Im(\mu_i)|\mu_i|^2 + 4\Re(\mu_i)\Im(\mu_i)]/(\kappa|\mu_i|^4)$ *and* $\kappa = \gamma_1\delta/4(T\delta - 2T^2 - \delta\iota + 2T\iota)(2\gamma_2\delta - \gamma_1T^2 + \gamma_1\iota T)$.

Proof of Theorem 3: The steps of the proof for this part are akin to that of Theorem 2, so it will not be repeated here. □

Remark 6: By following the same line as in Remark 4, we can select proper parameters for (33) and (34) to be satisfied under which LFC in MAS (1)–(2) can still be reached.

Corollary 5: *For undirected graphs, MAS (1)–(2) with the protocol (31) can achieve the LFC if and only if one of the following conditions is valid:*

1) $\iota < min\{\delta, T-\delta\}$ *or* $\delta < \iota < t-\delta(\delta < t/2)$,

$$\begin{cases} \delta < 2\gamma_2\iota/(\gamma_1-\gamma_2) \\ T < \Delta_{1i} \end{cases} \tag{35}$$

where $\Delta_{1i} = 4/((\gamma_1-\gamma_2)\mu_i\delta)$.

2) $T-\delta < \iota < \delta(\delta > T/2)$ *or* $\iota > max\{\delta, T-\delta\}$,

$$\begin{cases} p_1 > 0 \\ p_1^2p_2 - p_1p_3 > 0 \\ \Delta_{2i} > 0 \end{cases} \tag{36}$$

where $\Delta_{2i} = p_1 \begin{vmatrix} 0 & -p_2 & -p_3 & 0 \\ p_1 & 0 & -p_3 & 0 \\ 1 & 0 & -p_2 & 0 \\ 0 & p_1 & 0 & -p_3 \end{vmatrix} - \begin{vmatrix} 0 & -p_3 & 0 & 0 \\ p_1 & 0 & -p_3 & 0 \\ 1 & 0 & -p_2 & 0 \\ 0 & p_1 & 0 & -p_3 \end{vmatrix}$ *with* $p_1 = -[\gamma_2\delta(T-\delta/2) + \gamma_2/2(\iota^2 - 2T\iota + T^2) - \gamma_1/2\delta^2]\mu_i/(\kappa|\mu_i|^2/2) - 3$, $p_2 = -4[(\gamma_1-\gamma_2)\delta T + \gamma_2\delta^2 + \mu_i^2]/(\kappa|\mu_i|^3) + 3$ *and* $p_3 = 2[[(\gamma_1+\gamma_2)\delta(\delta/2-T) + \gamma_2/2(T-\iota) - \gamma_1\delta\gamma_1 T]|\mu_i| + 2]/(\kappa|\mu_i|^2)$.

Proof of Corollary 5: A similar proof follows from Corollary 4. □

Remark 7: *Both* [19] *and* [20] *used the current and sampled state information to design the control protocols, while this paper only uses the sampled information to improve the control efficiency further. In addition, considering that velocity information is difficult to obtain, this paper proposes an ISC protocol* (31) *that only utilizes the sampled position information, and the sufficient and necessary conditions for LFC are obtained. This approach can avoid the control cost resulting from the velocity measurement.*

6 Simulation

Consider a second-order MAS, which includes 5 followers (nodes 1–5) and 1 leader (node 0). Different from the undirected graph in [18], this paper employs a weighted directed graph, whose communication topology is illustrated in Fig. 1. $\tilde{x}_i$ and $\tilde{v}_i (i = 1, 2, \cdots, 5)$ stand for position errors and velocity errors between the followers and the leader, respectively. To show the effectiveness of the obtained theories, we select several scenarios from the proof of Theorems 1–3 as representatives for numerical simulations.

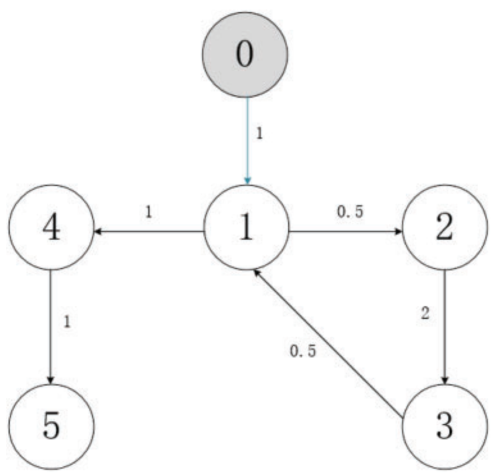

Figure 1: Communication graph of MAS (1)–(2)

Example 1: Consider system (1), (2) with protocol (3). According to Theorem 1, we choose $\gamma_1 = 0.7$, $\gamma_2 = 1.5$, and $\delta = 0.5$ in the simulation. Obviously, the first inequality in (9) holds. We get $T < 1.62$ from the second inequality of (9). Thus, MAS (1)–(2) can reach LFC if and only if $0.5 < T < 1.62$. The position and velocity errors between the followers and the leader are presented in Figs. 2 and 3, respectively. LFC in MAS (1)–(2) can be achieved when $T = 1.1$. However, it cannot be reached when $T = 1.63$.

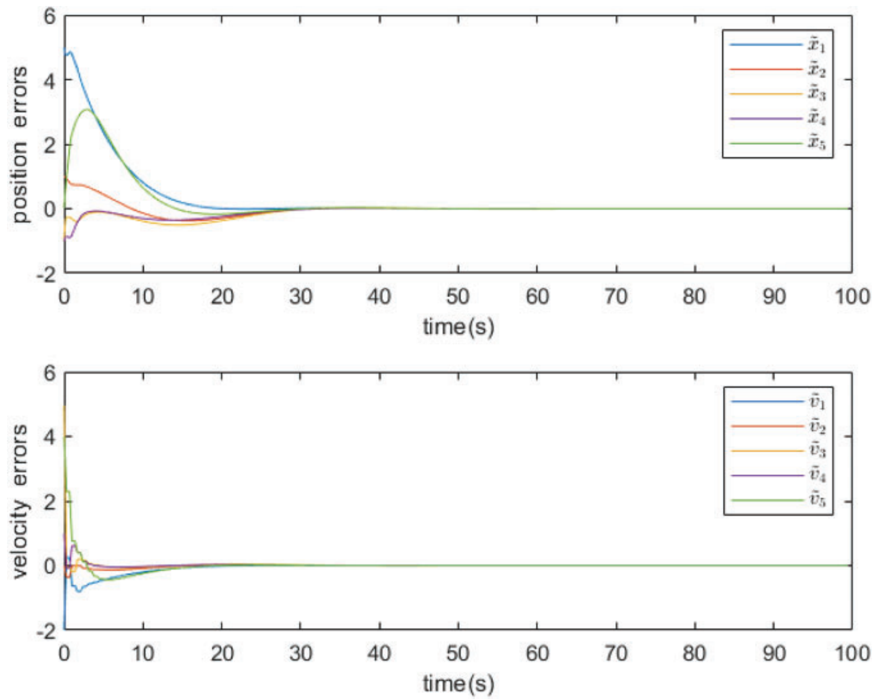

Figure 2: State errors between the followers and the leader when $T = 1.1$

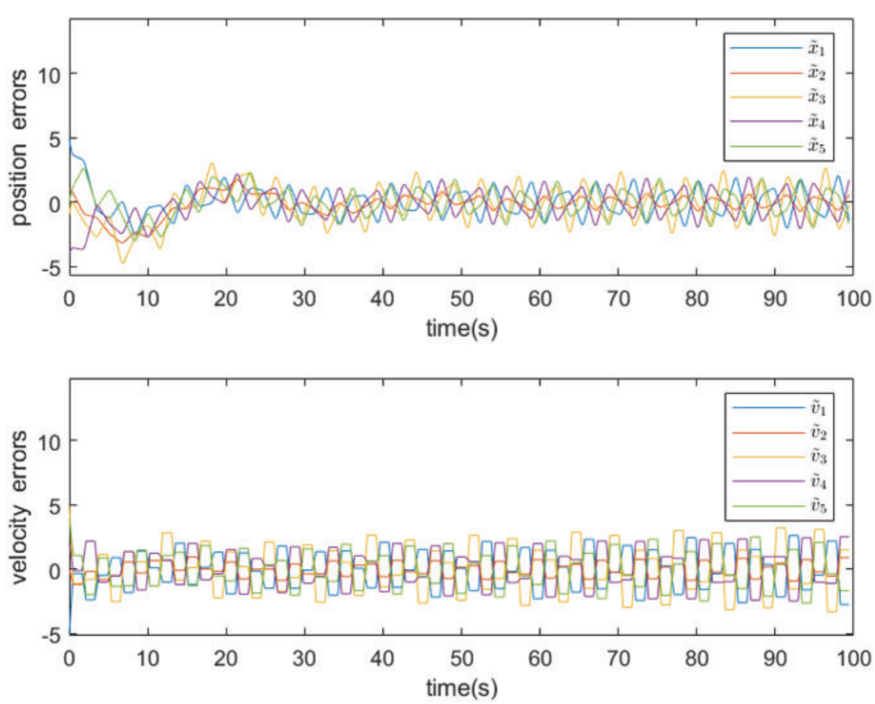

Figure 3: State errors between the followers and the leader when $T = 1.63$

Example 2: In this example, simulations in two different cases (Cases 1 and 2 in Theorem 2) are given to check the validity of the theoretical results.

When $\iota < \min\{\delta, T - \delta\}$, we select $\gamma_1 = 0.5$, $\gamma_2 = 1.9$ and $\iota = 0.1$. Since $\iota < \min\{\delta, T - \delta\}$, $\delta = 0.3$ is selected. According to (21), $0.4 < T < 1.12$ should be satisfied. From Fig. 4, LFC can be achieved when $T = 0.8$.

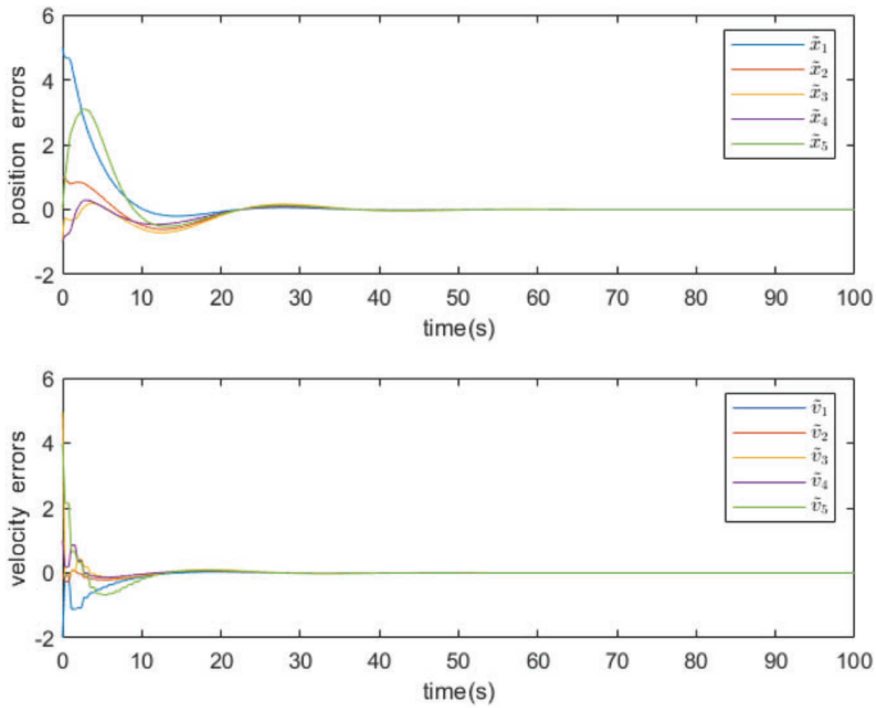

Figure 4: The error trajectories of system (1)–(2) with protocol (17) in Case 1

When $T - \delta < \iota < \delta(\delta > T/2)$, $\gamma_1 = 0.3$, $\gamma_2 = 1.5$ and $\iota = 0.5$ are chosen to verify the correctness of Case 2 in Theorem 2. From (22), we have $0.6 < T < 1.1$ when $\delta = 0.6$. Fig. 5 shows the position and velocity error trajectories when $T = 0.9$.

Under protocol (17), the position and velocity errors between all the followers and the leader tend to 0 when time delay exists, which means that MAS (1)–(2) achieves LFC asymptotically.

Example 3: The other two different cases (Cases 3 and 4 in Theorem 3) are considered to verify the effectiveness of protocol (31) in this example.

When $\delta < \iota < T - \delta(\delta < T/2)$, we choose $\gamma_1 = 2.1$, $\gamma_2 = 1.4$, $\iota = 0.5$, and $\delta = 0.4$. From (33) in Theorem 3, we get $0.9 < T < 2.11$. Fig. 6 illustrates that LFC is reached when $T = 1$.

When $\iota > \max\{\delta, T - \delta\}$, $\gamma_1 = 2.2$, $\gamma_2 = 1.8$, $\iota = 0.65$, and $\delta = 0.54$ are chosen. By (34), MAS (1)–(2) can reach consensus if and only if $0.54 < T < 1.19$. It can be observed from Fig. 7 that LFC is achieved when $T = 0.94$.

In these two cases, the state errors between all the followers and the leader converge to 0, even if the time delay exists. That is to say, MAS (1)–(2) with protocol (31) achieves LFC.

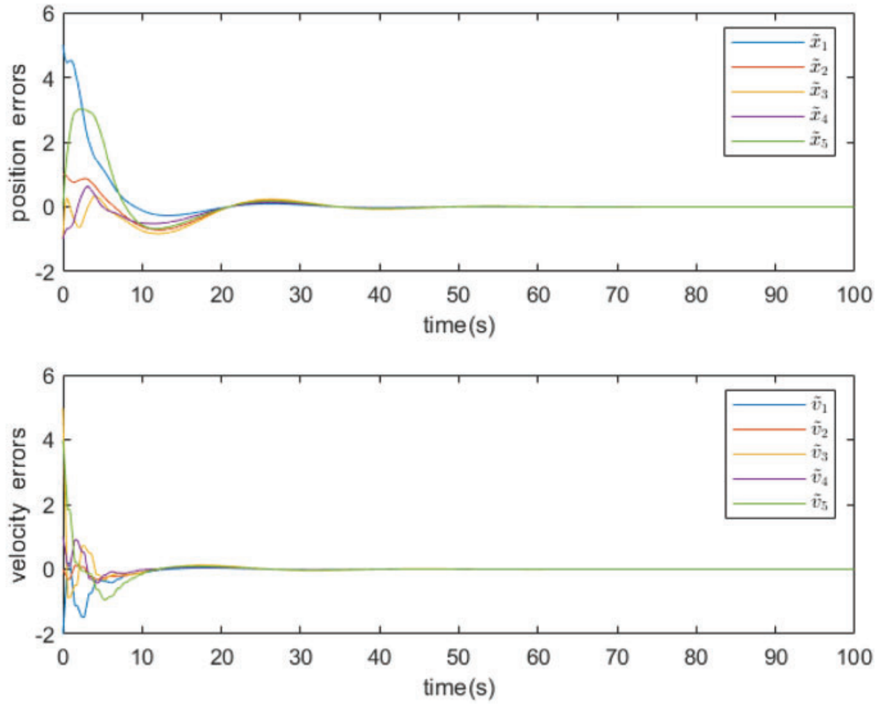

Figure 5: The error trajectories of system (1)–(2) with protocol (17) in Case 2

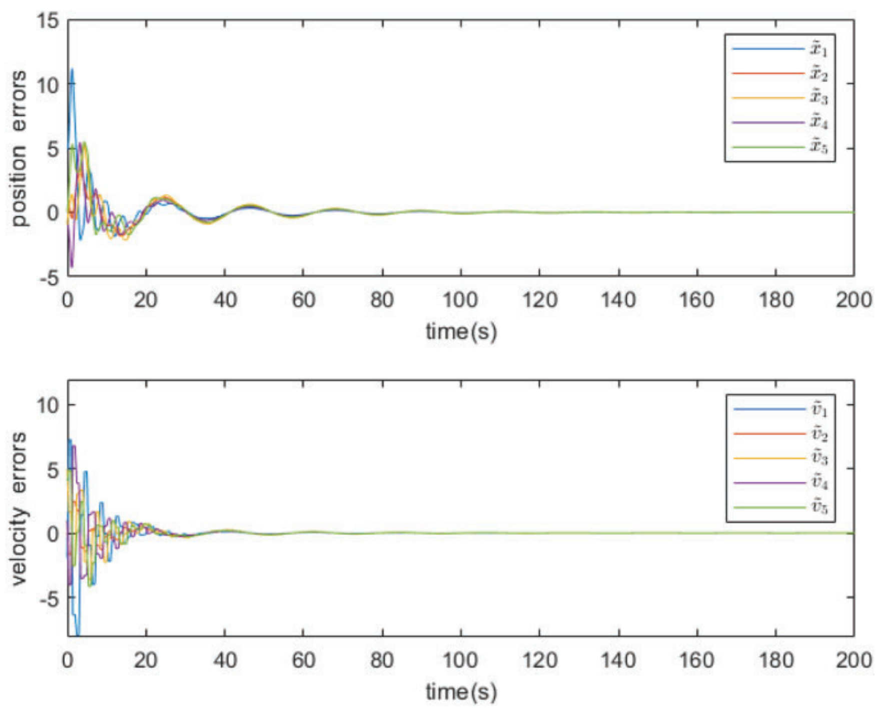

Figure 6: The error trajectories of system (1)–(2) with protocol (31) in Case 3

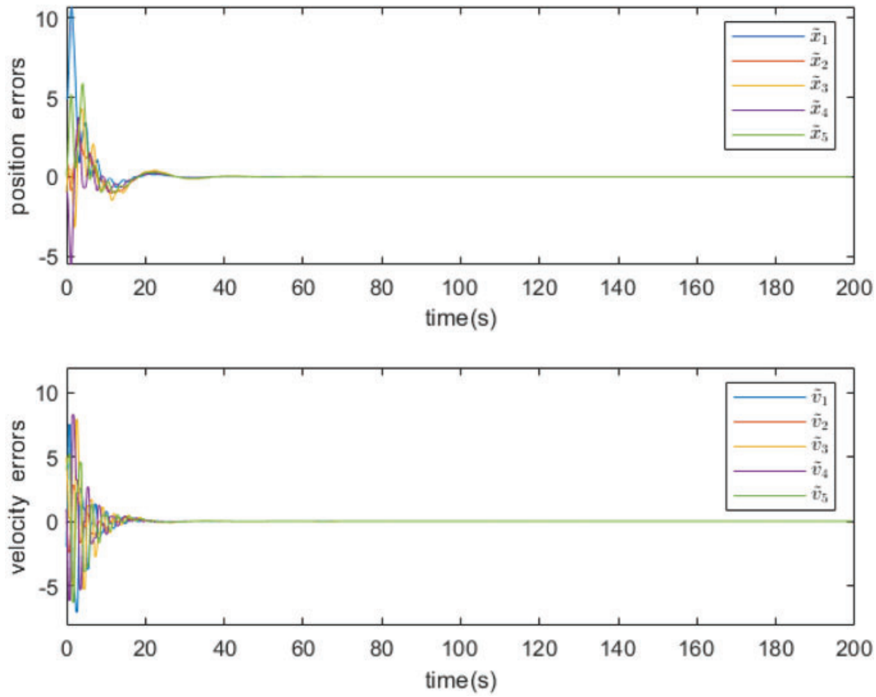

Figure 7: The error trajectories of system (1)–(2) with protocol (31) in Case 4

7 Conclusion

Second-order LFC problems in MASs are investigated in this work. Firstly, a new ISC protocol is proposed, and consensus conditions are analyzed to ensure the LFC. A sufficient and necessary condition dependent on coupling gain, sampling period, communication bandwidth, and network structure is obtained. Besides, input delay is taken into account, and a time-delay protocol is proposed. According to the relationship of system parameters, four cases are discussed separately, and it is concluded that MAS can reach the LFC if and only if system parameters satisfy certain conditions. Furthermore, considering it is hard to get the velocity information, an intermittent sampled protocol only with position information is proposed. A sufficient and necessary condition is also acquired to guarantee the LFC. In the future, we will further investigate the second-order LFC of MASs with quantized communication or stochastic switching topologies, etc.

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Abbreviations

ISC	Intermittent sampled control
LFC	Leader-following consensus
MASs	Multi-agent systems

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