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ARTICLE

Barber Optimization Algorithm: A New Human-Based Approach for Solving Optimization Problems

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ABSTRACT: In this study, a completely different approach to optimization is introduced through the development of a novel metaheuristic algorithm called the Barber Optimization Algorithm (BaOA). Inspired by the human interactions between barbers and customers, BaOA captures two key processes: the customer's selection of a hairstyle and the detailed refinement during the haircut. These processes are translated into a mathematical framework that forms the foundation of BaOA, consisting of two critical phases: exploration, representing the creative selection process, and exploitation, which focuses on refining details for optimization. The performance of BaOA is evaluated using 52 standard benchmark functions, including unimodal, high-dimensional multimodal, fixed-dimensional multimodal, and the Congress on Evolutionary Computation (CEC) 2017 test suite. This comprehensive assessment highlights BaOA's ability to balance exploration and exploitation effectively, resulting in high-quality solutions. A comparative analysis against twelve widely known metaheuristic algorithms further demonstrates BaOA's superior performance, as it consistently delivers better results across most benchmark functions. To validate its real-world applicability, BaOA is tested on four engineering design problems, illustrating its capability to address practical challenges with remarkable efficiency. The results confirm BaOA's versatility and reliability as an optimization tool. This study not only introduces an innovative algorithm but also establishes its effectiveness in solving complex problems, providing a foundation for future research and applications in diverse scientific and engineering domains.

KEYWORDS: Optimization; metaheuristic; barber; hairstyle; human-based algorithm; exploration; exploitation

1 Introduction

Optimization is a fundamental concept in mathematics and various applied sciences, representing problems where more than one feasible solution exists. In these cases, the task is to identify the best solution among all available options. An optimization problem is characterized by having at least two feasible solutions, but it may also have an infinite number of feasible solutions. The systematic process of determining an optimal solution is known as optimization [1]. These problems can be formulated mathematically using three critical components: decision variables, constraints, and objective functions. The primary aim of optimization is to find the values of the decision variables that satisfy the constraints while optimizing the objective function [2]. Approaches to solving optimization problems generally fall into two broad categories: deterministic and stochastic methods [3].

Deterministic approaches, further subdivided into gradient-based and non-gradient-based methods, demonstrate efficiency in solving problems that are linear, convex, continuous, and differentiable. However, as optimization problems become increasingly complex, involving nonlinear, nonconvex, discontinuous, and high-dimensional features, deterministic approaches often fail. These methods may become trapped in unsuitable local optima, rendering them ineffective for practical applications [4]. The inherent limitations of deterministic methods, combined with the complexity of many real-world optimization challenges, have necessitated the development of stochastic approaches [5].

Among the stochastic methods, metaheuristic algorithms have gained significant popularity due to their ability to tackle complex optimization problems. These algorithms employ random search techniques within the solution space and utilize random operators to enhance their performance. Metaheuristic algorithms are inspired by various sources, such as nature, physics, human behavior, etc. For example, the Orangutan Optimization Algorithm (OOA) is a nature-inspired metaheuristic that mimics orangutans' foraging and nesting behaviors, ensuring an efficient exploration and exploitation for engineering optimization problems [6]. The Artificial Satellite Search Algorithm (ASSA) is a physics-based algorithm that mimics satellite motion, utilizing orbit control and quantum computing for an improved exploration and efficiency [7]. Inspired by human behavior, Enterprise Development Optimization (EDO) is a metaheuristic algorithm inspired by enterprise development, integrating tasks, structure, technology, and human interactions with an activity-switching mechanism for solution updates [8]. Other recently published metaheuristic algorithms include Tactical Flight Optimizer (TFO) [9], Paper Publishing Based Optimization (PPBO) [10], and Revolution Optimization Algorithm (ROA) [11].

Metaheuristic algorithms, including improved, hybrid, and integrated variations, have gained significant traction in solving complex real-world problems across a wide range of fields. These algorithms are particularly valuable in optimization tasks, where traditional methods may fail due to the high computational complexity or nonlinearity of the problem [12]. One of the most prominent applications of metaheuristics is in engineering optimization, where they are used to design structures, optimize manufacturing processes, and improve product quality [13]. In the energy sector, metaheuristics have been employed in power generation and distribution systems [14]. Hybrid algorithms that combine features of different metaheuristics, have been used for an optimal placement of distributed generation sources in electrical grids, improving efficiency and reducing operational costs [15]. Transportation and logistics industries also benefit from metaheuristics in vehicle routing, scheduling, and traffic management [16]. Metaheuristic algorithms like ACO, PSO, GA, and SA, when integrated with the adaptive neuro-fuzzy inference system (ANFIS) in the AnFiS-MoH framework, significantly enhance parameter tuning and improve the accuracy and generalization of models for complex, high-dimensional, nonlinear problems, demonstrating their practical utility [17]. In summary, metaheuristic algorithms, especially their improved and hybrid forms, offer a flexible and powerful approach to solving real-world optimization problems across various industries, demonstrating their practical relevance and adaptability [18].

Metaheuristic algorithms are widely appreciated for their simplicity, ease of implementation, and efficiency in addressing nonlinear, discontinuous, and NP-hard problems. They also perform well in unknown and nonlinear search spaces [19]. The optimization process in metaheuristic algorithms begins by generating a random set of candidate solutions that adhere to the given constraints. Through iterative update mechanisms, these candidate solutions are progressively refined. At the end of the algorithm's execution, the best candidate is presented as a near-optimal solution to the problem [20]. While metaheuristic algorithms do not guarantee global optima due to their stochastic nature, the solutions they generate are typically close to the global optimal solution, making them suitable for practical applications.

For metaheuristic algorithms to be effective, they must exhibit robust global and local search capabilities. Global search, or exploration, enables the algorithm to identify promising regions within the search space and avoid being trapped in suboptimal solutions. Conversely, local search, or exploitation, allows the algorithm to thoroughly examine promising regions to converge towards the global optimum [21]. The success of metaheuristic algorithms hinges on their ability to balance exploration and exploitation throughout the optimization process [22].

Different metaheuristic algorithms employ varying strategies for exploration and exploitation, leading to diverse performances across the same optimization problem. The quest for more effective solutions has driven researchers to develop numerous metaheuristic algorithms.

A critical research question in the field of metaheuristic algorithms is whether the introduction of new algorithms is still necessary, given the plethora of existing methods. The No Free Lunch (NFL) theorem [23] provides insight into this question by stating that no single algorithm can outperform all others across every optimization problem. Consequently, the effectiveness of a metaheuristic algorithm for one problem does not guarantee its success for another one. This theorem underscores the importance of continued innovation in designing new algorithms to address the unique challenges of diverse optimization problems.

In this context, this paper introduces an innovative metaheuristic algorithm, the Barber Optimization Algorithm (BaOA), inspired by the dynamic interactions between a barber and their customer. The BaOA draws fundamental inspiration from the processes of selecting and refining a hairstyle during a haircut. This concept is mathematically modeled in two key phases: exploration and exploitation, which simulate the interactions between the barber and the customer.

The BaOA's effectiveness is evaluated using 52 benchmark functions, including unimodal, high-dimensional multimodal, fixed-dimensional multimodal ones, and the CEC 2017 test suite. Furthermore, its performance is compared against 12 well-established metaheuristic algorithms. Additionally, the BaOA's capabilities in solving real-world optimization problems are demonstrated through four engineering design case studies.

Accordingly, the key contributions of this research can be described in completely different and more detailed terms as follows:

- The Barber Optimization Algorithm (BaOA) draws inspiration from the intricate human interactions observed between a barber and a customer, emphasizing their dynamic relationship during the haircut process.
- The fundamental basis of BaOA originates from two key processes: the customer's selection of a desired hairstyle and the refinement or correction of hairstyle details during the haircut, simulating real-world decision-making and problem-solving behaviors.
- The theoretical structure of BaOA is comprehensively articulated and mathematically formulated to represent two distinct phases: exploration, which mimics the creative selection of solutions, and exploitation, which focuses on refining and improving the selected solutions to achieve optimal results.
- BaOA's effectiveness is extensively evaluated using a completely diverse set of fifty-two benchmark functions. These include unimodal functions for testing convergence speed, high-dimensional multimodal

functions for global search capability, fixed-dimensional multimodal functions for specific challenges, and the comprehensive CEC 2017 test suite for advanced performance analysis.

- The algorithm's results are rigorously compared against the performance of twelve widely recognized metaheuristic algorithms, showcasing BaOA's superior ability to solve optimization problems and its competitive edge in achieving better solutions.
- Finally, BaOA's capability to address real-world challenges is validated by applying it to four distinct engineering design problems, demonstrating its practicality and versatility in solving complex optimization applications across various domains.

The remainder of this paper is structured as follows: Section 2 presents a comprehensive literature review. Section 3 introduces and mathematically models the proposed BaOA approach. Section 4 discusses simulation studies and results. Section 5 evaluates the BaOA's performance in real-world applications. Finally, Section 6 provides some conclusions and suggestions for future research directions.

2 Literature Review

Metaheuristic algorithms have emerged as powerful computational tools inspired by a wide array of completely different and intriguing phenomena observed in nature, science, and human behavior. These algorithms draw inspiration from diverse sources, including the complex dynamics of natural phenomena, the organized and collective behavior of animals, the intricate mechanisms of biological processes, the fundamental laws governing physics, strategic principles derived from games, and even the rich spectrum of human interactions and cultural practices. Each source offers unique insights and methodologies for solving challenging optimization problems across various domains.

To better understand their underlying principles, metaheuristic algorithms are categorized into four distinct groups based on the foundational ideas they emulate.

Swarm-based metaheuristic algorithms are completely different from other groups, as they are inspired specifically by swarming phenomena and collective behaviors observed in the natural life of animals, aquatic creatures, insects, reptiles, plants, and various other living organisms. Some of the most prominent and widely used swarm-based metaheuristic algorithms include Particle Swarm Optimization (PSO) [24], Ant Colony Optimization (ACO) [25], the Artificial Bee Colony (ABC) [26], and Firefly Algorithm (FA) [27]. PSO's fundamental concept is derived from the coordinated swarm movement of birds and fish as they search for food sources. Similarly, ACO is inspired by the remarkable ability of ants to find the shortest communication path between their nest and food sources. In the case of ABC, the foraging activities of honey bee colonies have been the core inspiration, while the optical communication observed among fireflies has influenced the design of FA. The hunting strategies, foraging, and migratory behaviors commonly observed in wildlife have inspired the development of several other swarm-based algorithms, such as the Emperor Penguin Optimizer (EPO) [28], the Reptile Search Algorithm (RSA) [29], Grey Wolf Optimization (GWO) [30], the Tunicate Swarm Algorithm (TSA) [31], the White Shark Optimizer (WSO) [32], the African Vultures Optimization Algorithm (AVOA) [33], and the Marine Predators Algorithm (MPA) [34] which further demonstrate the diversity of swarm-based algorithms.

Evolutionary-based metaheuristic algorithms are fundamentally different in their inspiration, as they are based on biological sciences, genetic processes, and the principles of natural selection and survival of the fittest. These algorithms often simulate evolutionary concepts to solve optimization problems. Two of the most notable evolutionary-based algorithms are a Genetic Algorithm (GA) [35] and Differential Evolution (DE) [36]. The design of GA and DE incorporates elements such as reproduction, genetic inheritance, Darwinian evolutionary theory, and stochastic operators like selection, crossover, and mutation. Other examples in this category include Genetic Programming (GP) [37], the Cultural Algorithm (CA) [38], the Artificial Immune System (AIS) [39], the Evolution Strategy (ES) [40], and the Biogeography-based Optimizer (BBO) [41]. These algorithms provide unique frameworks to address optimization challenges by mimicking the complex mechanisms of evolution and natural adaptation.

Physics-based metaheuristic algorithms, as their name suggests, are derived from completely different sources of inspiration—namely, the fundamental laws, phenomena, transformations, and forces in physics. Simulated Annealing (SA), one of the most prominent physics-based algorithms, takes its cue from the process of annealing metals, where a controlled cooling process enables the material to reach a state of minimal energy and maximum structural integrity [42]. Various physical forces have inspired other algorithms, such as the Spring Search Algorithm (SSA) based on tensile force of springs [1], which draws on the tensile force of springs, and the Gravitational Search Algorithm (GSA) [43] models the gravitational pull as a means of guiding optimization. Furthermore, cosmological concepts play a significant role in algorithms like cosmological concepts are employed in the design of algorithms such as the Galaxy-based Search Algorithm (GbSA) [44], Black Hole (BH) [45], and the Multi-Verse Optimizer (MVO) [46]. Other physics-based algorithms include the Artificial Chemical Reaction Optimization Algorithm (ACROA) [47], the Small World Optimization Algorithm (SWOA) [48], the Ray Optimization (RO) [49] algorithm, and the Magnetic Optimization Algorithm (MOA) [50]. These algorithms reflect the application of physical theories to computational problem-solving.

Human-based metaheuristic algorithms are entirely different in their foundation, as they are inspired by human thoughts, interactions, and social dynamics. Teaching-Learning Based Optimization (TLBO) [51] is perhaps the most well-known example, modeled after the knowledge transfer between teachers and students in an educational setting. The Mother Optimization Algorithm (MOA) is a novel human-based metaheuristic approach that draws inspiration from the nurturing relationship between Eshra's moher and her children, simulating the phases of education, advice, and upbringing to guide the optimization process [52]. Other examples include Brain Storm Optimization (BSO) [53] and War Strategy Optimization (WSO) [54].

Despite the extensive diversity of inspirations, a completely different approach has yet to be explored: designing a metaheuristic algorithm based on the dynamic interactions between a barber and a customer in a barbershop. Activities such as selecting a hairstyle, making detailed adjustments, and finalizing the haircut represent intelligent and iterative processes with significant potential for computational modeling. This research paper addresses this gap by presenting a novel human-based metaheuristic algorithm inspired by the mathematical modeling of barber-customer interactions, as elaborated in the subsequent section.

3 Barber Optimization Algorithm

In this section, the newly developed Barber Optimization Algorithm (BaOA) is comprehensively introduced, and its underlying mathematical framework is elaborated in detail.

3.1 Inspiration

Hairstyles and haircuts have been an important part of the tradition and culture of societies since ancient times. Photographs, texts, and descriptions indicate that over the centuries, women's and men's hair has been seen in various ways, such as curled, styled, arranged, and colored, or even enhanced by the use of wigs [55]. This shows that barbering is a long-standing profession that has a special impact on people's culture. People are looking for a skilled barber to provide various hairdressing services to customers based on their needs, tastes and preferences. When the customer visits the barbershop, she/he asks the barber to suggest several suitable hairstyles so that he can choose one among them. It is also possible that the customer has already chosen a hairstyle and asks the barber to use that hairstyle for her/him. After the customer chooses a hairstyle, the barber starts her/his work and cuts the hair according to the hairstyle. In the second step, during the haircut, the customer pays attention to the details of the hairstyle and asks the barber to apply corrections to make the hairstyle more attractive. Therefore, the barber must be able to establish a strong relationship with the customer and follow the customer's instructions to perform the desired hairstyle on the customer's hair.

The inspiration behind BaOA lies in the intelligent decision-making process involved in hairstyling. The algorithm models the two key stages of hairstyling—initial selection and refinement—as exploration and exploitation phases in the optimization. By formalizing these human-driven selection and refinement processes, BaOA introduces a novel approach that aligns intuitive decision-making with systematic search mechanisms.

Among the human interactions between the barber and the customer, (i) choosing a hairstyle by the customer and (ii) correcting the details of the hairstyle during the haircut are the most prominent intelligent processes. A mathematical modeling of these intelligent behaviors is employed in the BaOA design, which is discussed below. These processes are translated into computational operations to enhance BaOA's efficiency in solving optimization problems, as elaborated in the following sections.

3.2 Initialization

The proposed BaOA operates as a population-based optimization algorithm, utilizing iterative processes to identify optimal solutions within a problem space. Each BaOA member represents a potential solution, modeled mathematically as a vector. The dimensionality of this vector corresponds to the number of decision variables in the problem, with each element representing a specific variable.

Collectively, the BaOA members form a population represented by a matrix, initialized randomly at the start of the algorithm. This initialization adheres to Eqs. (1) and (2):

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m}, \tag{1}$$

$$X_i\colon x_{i,j} = lb_j + r\cdot\left(ub_j - lb_j\right), \quad i = 1,2,\ldots,N, \quad j = 1,2,\ldots,m, \tag{2}$$

where X is the population matrix of the proposed BaOA, N is the number of population members, m is the number of decision variables, X_i is the ith candidate solution, $x_{i,j}$ is its j-th variable, r is a random number in the interval $[0,1]$, lb_j is a lower bound, and ub_j is an upper bound on the j-th decision variable.

The objective function values of the problem are evaluated for all BaOA members, forming a vector as shown in Eq. (3):

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1}, \tag{3}$$

where F is the vector of the objective function values and F_i is the objective function value for the i-th candidate solution.

The algorithm identifies the best and worst members based on their respective objective function values. Iteratively, the BaOA population undergoes updates through two distinct phases: exploration and exploitation, which are modeled mathematically based on barber-customer interactions. The initialization process ensures a diverse population, thereby preventing premature convergence and improving the robustness of the search process.

3.3 Phase 1: Choice of Hairstyle by the Customer (Exploration Phase)

Choosing a suitable hairstyle is the most important step for the customer in the barbershop. With the help of the barber, the customer chooses a hairstyle among the hairstyles offered by the barber according to her/his appearance and interests. The hairstyle selection simulation is employed in the design of the first phase of the BaOA update. The choice of a hairstyle by the customer phase, by making major changes in the position of the population members in the search space, leads to an increase in the global search capability and BaOA exploration in escaping from locally optimal solutions and identifying the main optimal area in the search space. This mechanism enhances the diversity of candidate solutions, thereby reducing the likelihood of getting trapped in local optima. The schematic of this phase of BaOA is shown in Fig. 1. This figure shows that the customer first selected the hairstyle he wants. Then the barber, based on this hairstyle, has made widespread changes to the customer hair. These widespread changes to customer's hair correspond to widespread changes to the position of population members that represent a global search with the aim of enhancing the exploration ability of the BaOA.

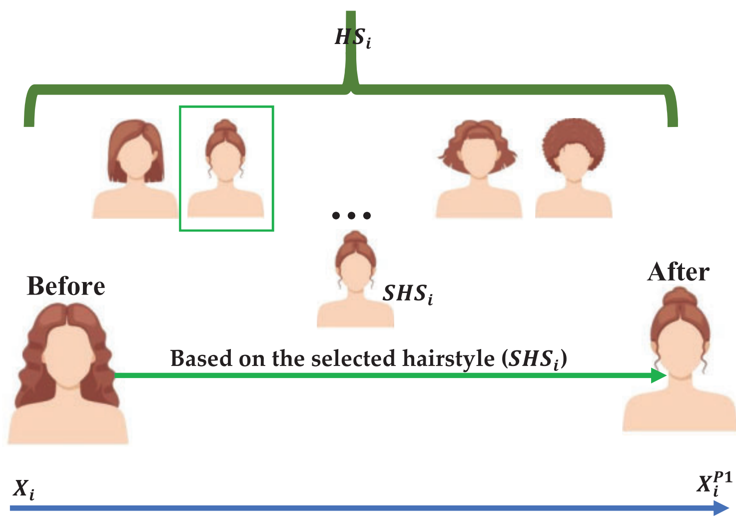

Figure 1: Schematic of the exploration phase of BaOA

In order to model this phase, first, the set of hairstyles offered by the barber to the customer for each BaOA member is determined based on the comparison of the objective function values using Eq. (4). In fact, for each member of BaOA, the position of other population members that have a better objective function value than the corresponding member is considered as a hairstyle.

$$HS_i = \{X_k, F_k < F_i \text{ and } k \in \{1, 2, \ldots, N\}\}, \quad \text{where } i = 1, 2, \ldots, N, \tag{4}$$

here, HS_i is the set of hairstyles for the i-th customer.

In the BaOA design, it is assumed that the customer chooses a hairstyle randomly among the proposed hairstyles. Then, based on modeling the customer's haircut according to the chosen hairstyle, a new position for each BaOA member is calculated using Eq. (5). If the value of the objective function is improved at this new position, this new position replaces the previous position of the corresponding coefficient according to Eq. (6).

$$x_{i,j}^{P1} = x_{i,j} + r_{i,j} \cdot (SHS_{i,j} - I_{i,j} \cdot x_{i,j}), \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} \le F_i, \\ X_i, & else, \end{cases} \tag{6}$$

here, SHS_i is the selected hairstyle for the ith population member, $SHS_{i,j}$ is its j-th dimension, X_i^{P1} is the new position calculated for the ith population member based on first phase of the BaOA, $x_{i,j}^{P1}$ is its jth dimension, F_i^{P1} is its objective function value, $r_{i,j}$ are random numbers from the interval $[0, 1]$, and $I_{i,j}$ are numbers which are randomly selected as 1 or 2. These numbers are used to create a random nature in the performance of metaheuristic algorithms in the search process.

3.4 Phase 2: Correction of Hairstylist Details While Cutting Hair (Exploitation Phase)

An important factor in the success of a barber is that she/he must be detail-oriented and able to establish close relationships with the customers by having strong communication skills. An excellent and professional barber must be able to follow the customer's orders, so that she/he can satisfy the customer by correctly executing the customer's favorite hairstyle. This phase corresponds to the exploitation process in optimization, where local adjustments refine solutions for better accuracy. The schematic of the second phase of BOA is shown in Fig. 2. This figure illustrates that during the haircut, the barber makes small, minor adjustments to the customer's hair in coordination with the customer. These precise and small changes to the customer's hair correspond to small changes to the position of the population members, which indicates a local search aimed at enhancing the exploitation ability of the BOA.

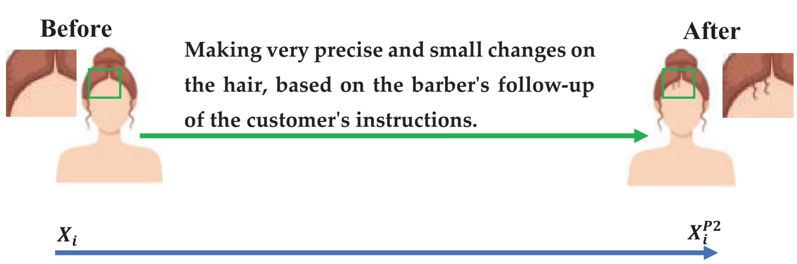

Figure 2: Schematic of the exploitation phase of BaOA

The simulation of the correction of hair style details based on the barber's follow-up of the customer's instructions is employed in the design of the second phase of the BaOA update. The correction of hairstylist details durint the cutting hair phase by making small changes in the position of the population members in the search space, leads to an increase of the local search capability and the exploitation of BaOA in the accurate scanning of the search space in the promising areas and near the discovered solutions with the aim of achieving better solutions.

In order to model this phase of BaOA, for each population member, the small changes in the position of that member in the search space caused by the simulation of the correction of hairstyle details based on the customer's orders, have been calculated using Eq. (7). Then, this new position replaces the previous position of the corresponding member if it improves the value of the objective function according to Eq. (8).

$$x_{i,j}^{P2} = x_{i,j} + \left(1 - 2r_{i,j}\right) \cdot \frac{ub_j - lb_j}{t}, \tag{7}$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} \le F_i \\ X_i, & else \end{cases}, \tag{8}$$

here, X_i^{P2} is the new position calculated for the i-th population member based on second phase of the proposed BaOA, $x_{i,j}^{P2}$ is its j-th dimension, F_i^{P2} is its objective function value, $r_{i,j}$ are random numbers from the interval $[0,1]$, and t is the iteration counter.

By integrating these two phases, BaOA balances exploration and exploitation more effectively than many conventional metaheuristic algorithms. This dual-phase approach enhances convergence speed while maintaining solution diversity, providing a theoretical and practical advantage over existing frameworks.

3.5 Computational Complexity of BaOA

In this subsection, the computational complexity of BaOA is analyzed from a completely different perspective, employing more words and sentences to provide clarity and details. During the initialization phase, BaOA performs several operations such as generating the initial population and setting up necessary parameters, which together contribute to a computational complexity expressed as $O(Nm)$. Here, N denotes the total count of population members involved in the optimization process, while m represents the number of decision variables associated with the problem under consideration.

Furthermore, in the exploration and exploitation phases, the population undergoes iterative updates designed to enhance solution quality and convergence. These updates involve computational tasks proportional to $O(2NmT)$, where T symbolizes the algorithm's maximum iteration count. Combining these contributions yields an overall computational complexity for BaOA, succinctly represented as $O(Nm(1 + 2T))$. This revised analysis underscores the interplay of key algorithmic components and their impact on computational demands.

3.6 Repetitions Process, Flowchart, and Pseudocode of BaOA

The execution of the proposed Barber Optimization Algorithm (BaOA) involves a completely different sequence of steps that are more detailed and elaborate. Initially, the algorithm completes its first iteration by systematically updating all members of the population. This update process, divided into two primary phases, ensures that the solution search is efficient and effective. Once this initial stage is completed, the algorithm transitions into the subsequent iterations. During these iterations, the population members are dynamically updated based on their newly calculated positions. These updates are performed iteratively following the mathematical expressions provided in Eqs. (4)–(8). This iterative cycle continues systematically until the algorithm reaches the final iteration, thereby ensuring a thorough exploration and exploitation of the search space.

With each iteration, the algorithm meticulously evaluates and identifies the best candidate solution. This solution is continuously updated and preserved as the best result discovered up to that point in the execution. By the conclusion of the algorithm's operation, the most refined and near-optimal candidate solution is presented as the final output, representing the resolution of the problem being addressed.

To provide a clearer understanding of the entire process, the implementation steps of the BaOA are illustrated comprehensively in Fig. 3 using a flowchart. Additionally, the pseudo-code representation in Algorithm 1 complements the flowchart by offering a more detailed, step-by-step procedural view of the algorithm's execution. These representations ensure that the algorithm's workings are fully transparent and accessible to readers, providing more words and more sentences to thoroughly describe the methodology.

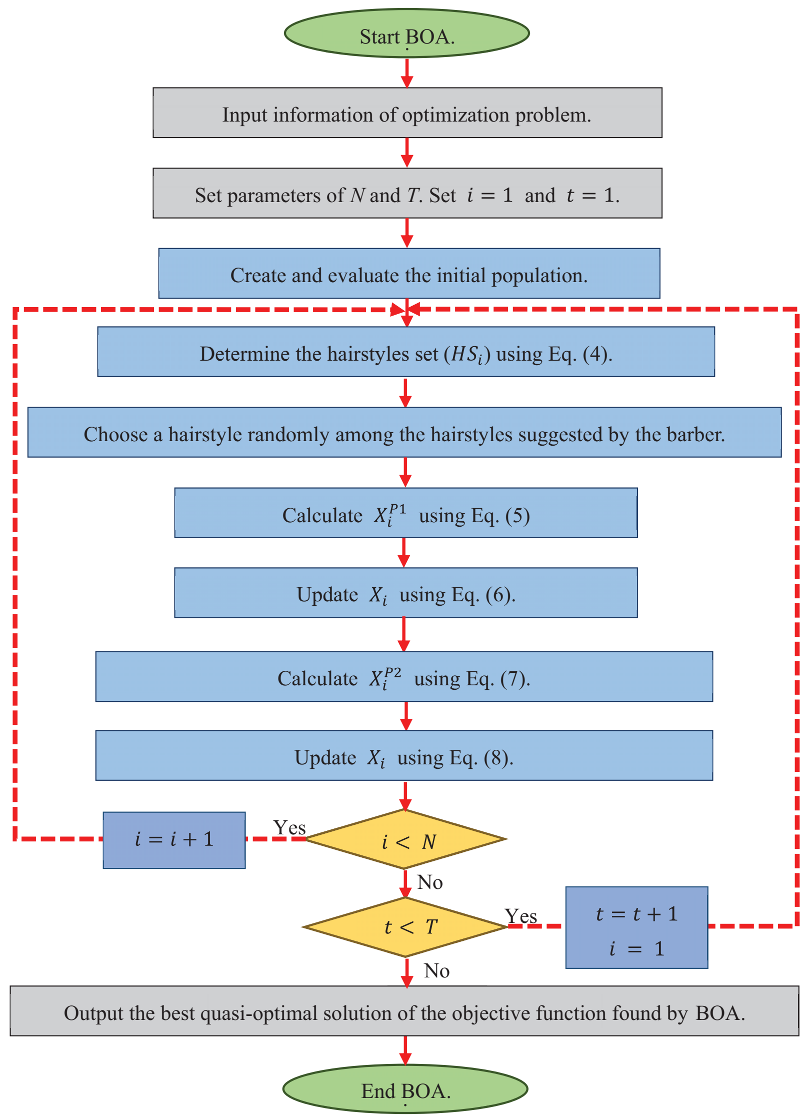

Figure 3: Flowchart of the proposed BaOA

Algorithm 1: Pseudo-code of the proposed BaOA

Start BaOA.

1. Input the optimization problem information.
2. Set the number of iterations T and the number of population members N.
3. Generate the initial population at random based on Eq. (2).
4. Evaluate the initial population.
5. For t = 1: T
7. For i = 1: N
8. **Phase 1:** Choice of Hairstyle by the Customer (Exploration Phase).
9. Determine hairstyles set for the i-th member based on Eq. (4).
10. Select a hairstyle for the i-th member among the hairstyles proposed by the barber at random.
11. Calculate new position of the i-th population member based on Eq. (5).
12. Update the i-th population member using Eq. (6).
13. **Phase 2:** Correction of Hairstylist Details while Cutting Hair (Exploitation Phase).
14. Calculate a new position of the i-th population member based on Eq. (7)
15. Update the i-th population member using Eq. (8).
16. end
17. Save the best proposed solution so far.
18. end
19. Output the best obtained proposed solution.

End BaOA.

4 Simulation Studies

In this section, the evaluation of the Barber Optimization Algorithm (BaOA) for tackling various optimization challenges is presented. To comprehensively assess its performance, the proposed BaOA has been tested on an extensive suite of optimization problems. This evaluation includes fifty-two standard benchmark functions categorized into three distinct types: unimodal functions, which assess convergence accuracy, high-dimensional multimodal functions, which evaluate the algorithm's exploration and exploitation balance, and fixed-dimensional multimodal functions, which measure its ability to escape local optima [56]. Furthermore, the assessment incorporates the CEC 2017 test suite [57], recognized as a challenging benchmark for modern optimization algorithms.

The reasons for choosing the CEC 2017 test suite are as follows:

1. **Benchmark Consistency:** CEC-2017 provides a well-established and standardized set of benchmark functions that are widely recognized in the optimization community. Using CEC-2017 ensures that comparisons between different algorithms are consistent with past studies, which helps validate the results and maintain the integrity of research over time.
2. **Diversity of Problem Types:** The CEC-2017 test suite includes a diverse set of problem types, including unimodal, multimodal, fixed-dimensional, and high-dimensional problems. This variety is important for thoroughly evaluating an algorithm's performance across different problem landscapes, and it has become a reference for testing new algorithms in a comprehensive manner.

3. **Comparison with the Existing Literature:** Since many studies and algorithms have already been evaluated using the CEC-2017 suite, it allows for a direct comparison with existing results. This is important for demonstrating the relative performance of the new algorithm in relation to established methods.
4. **Widely Accepted Validation:** The CEC-2017 suite is considered a reliable and robust validation tool for assessing optimization algorithms. It has been used in numerous publications and competitions, making it a trusted resource for benchmarking.

The effectiveness of BaOA is compared against twelve well-established metaheuristic algorithms, namely GA (1988), PSO (1995), GSA (2009), TLBO (2011), GWO (2014), MVO (2016), WOA (2017), MPA (2020), TSA (2020), RSA (2022), AVOA (2021), and WSO (2022). It should be mentioned that in order to provide a fair comparison, in the simulation studies, the original versions of competing algorithms published by their main researchers have been used. Also, regarding GA and PSO, the standard versions published by Professor Seyed Ali Mirjalili have been used. Moreover, a complete information and details about the experimental test suites and their optimal values are available in their respective references introduced in each subsection.

The experimental results are reported using six critical statistical metrics to provide a more detailed and nuanced understanding of the algorithm's performance. These include the mean, best, and worst values, which demonstrate the overall solution quality, the standard deviation, which indicates solution stability, the median value, which highlights the central tendency, and the rank, which facilitates a comparative analysis. To determine the relative effectiveness of the algorithms on individual benchmark problems, the mean values are employed as the primary ranking index.

4.1 Evaluation of Unimodal Objective Functions

The performance evaluation of the Barber Optimization Algorithm (BaOA) on unimodal objective functions, specifically F1 through F7, is detailed in Table 1. These functions are designed to test the algorithm's exploitation capabilities by focusing on the convergence toward the global optimum. According to the results, BaOA has demonstrated a remarkable exploitation strength, achieving the global optimum for the functions F1, F2, F3, F4, F5, and F6. Furthermore, BaOA has emerged as the top-performing optimizer for the F7 function. A deeper analysis reveals that the BaOA, with its exceptional local search and exploitation capabilities, outperforms the competing algorithms when applied to these unimodal functions. Compared to alternative metaheuristics, BaOA's performance is not only superior but also consistently competitive, highlighting its effectiveness in handling unimodal problems.

Table 1: Evaluation results for unimodal objective functions

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F1	Mean	0	23.72251	1.968918	1.968918	1.968918	1.968918	1.968918	2.067695	1.968918	1.968918	1.968918	2.035562	22.10391
	Best	0	3.811189	1.158124	1.158124	1.158124	1.158124	1.158124	1.264204	1.158124	1.158124	1.158124	1.158861	12.99207
	Worst	0	80.82577	3.655266	3.655266	3.655266	3.655266	3.655266	3.761769	3.655266	3.655266	3.655266	3.656851	41.23459
	Std	0	32.1263	1.240774	1.240774	1.240774	1.240774	1.240774	1.242581	1.240774	1.240774	1.240774	1.275703	14.06057
	Median	0	16.92623	1.858844	1.858844	1.858844	1.858844	1.858844	1.944731	1.858844	1.858844	1.858844	1.943484	20.47356
	Rank	1	6	2	2	2	2	2	4	2	2	2	3	5
F2	Mean	0	0.958729	0.252453	0.252453	0.252453	0.252453	0.252453	0.423539	0.252453	0.252453	0.252453	0.843594	2.093129
	Best	0	0.459711	0.15478	0.15478	0.15478	0.15478	0.15478	0.304286	0.15478	0.15478	0.15478	0.184671	1.337812
	Worst	0	2.789401	0.369359	0.369359	0.369359	0.369359	0.369359	0.565664	0.369359	0.369359	0.369359	1.977827	2.882144
	Std	0	1.113584	0.101432	0.101432	0.101432	0.101432	0.101432	0.128979	0.101432	0.101432	0.101432	0.965494	0.728006
	Median	0	0.794767	0.247124	0.247124	0.247124	0.247124	0.247124	0.420038	0.247124	0.247124	0.247124	0.631526	2.060467
	Rank	1	6	2	2	2	2	2	4	2	2	3	5	7
F3	Mean	0	754.3303	164.7405	164.7405	164.7405	164.7405	13340.23	175.2848	164.7405	164.7405	478.6276	420.954	1596.53
	Best	0	462.6659	101.846	101.846	101.846	101.846	1503.925	114.5523	101.846	101.846	289.4873	176.7842	1041.981
	Worst	0	1324.452	255.5471	255.5471	255.5471	255.5471	23053.55	260.225	255.5471	255.5471	902.3675	885.0378	2517.54
	Std	0	382.3056	83.54307	83.54307	83.54307	83.54307	10447.68	83.39888	83.54307	83.54307	254.1019	395.71	860.684
	Median	0	719.4145	160.5489	160.5489	160.5489	160.5489	13593.23	173.7475	160.5489	160.5489	439.4014	353.099	1529.12
	Rank	1	8	2	2	3	4	10	5	2	2	7	6	9
F4	Mean	0	6.327246	0.618529	0.618529	0.618529	0.621448	34.82684	0.979692	0.618529	0.618529	1.434359	4.764006	2.48627
	Best	0	4.656376	0.336565	0.336565	0.336565	0.337402	1.188909	0.512108	0.336565	0.336565	0.396635	1.848418	2.044628
	Worst	0	8.387474	1.056	1.056	1.056	1.056804	61.20835	1.375121	1.056	1.056	3.77312	9.875363	3.30458
	Std	0	1.728778	0.301252	0.301252	0.301252	0.30188	36.34347	0.389749	0.301252	0.301252	1.778567	3.362419	0.685587
	Median	0	6.485225	0.59969	0.59969	0.59969	0.601184	37.22256	0.96985	0.59969	0.59969	1.181491	4.506956	2.452487
	Rank	1	9	2	2	2	4	10	5	3	2	6	8	7
F5	Mean	0	3907.533	343.0814	351.662	358.478	361.8798	361.1091	406.5992	360.6284	360.7646	372.1596	3387.513	736.1073
	Best	0	506.0502	26.44798	26.44797	42.28355	45.47428	44.22351	46.02172	43.32497	44.37409	43.61233	57.18925	179.3006
	Worst	0	30643.27	5830.562	5849.699	5845.962	5849.634	5848.443	5907.304	5848.469	5849.024	5940.963	65292.4	6370.673
	Std	0	12269.82	2398.26	2402.82	2398.257	2398.384	2398.193	2409.622	2398.415	2398.599	2434.6	27046.79	2528.141
	Median	0	2014.757	48.18105	58.48506	63.5361	67.23715	66.86702	74.75299	65.43516	65.58872	65.27027	108.2013	366.3377
	Rank	1	13	2	3	4	8	7	10	5	6	9	12	11

(Continued)

Table 1 (continued)

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F6	Mean	0	35.78337	2.478044	6.741024	2.478044	4.908546	2.531893	2.577724	2.914284	3.310723	2.478044	2.519926	25.01947
	Best	0	8.560781	1.320863	4.242565	1.320863	3.598302	1.418552	1.467112	1.646376	1.875509	1.320863	1.322206	11.62695
	Worst	0	128.2901	4.287836	9.073708	4.287836	6.44756	4.366557	4.343217	4.948841	4.952108	4.287836	4.317381	45.72161
	Std	0	58.28614	1.600576	2.392531	1.600576	1.740465	1.588315	1.590812	1.672635	1.606565	1.600576	1.59523	18.20147
	Median	0	25.06396	2.313508	6.585088	2.313508	4.641907	2.33479	2.411352	2.81081	3.370392	2.313508	2.313705	23.22756
	Rank	1	13	4	11	3	10	6	7	8	9	2	5	12
F7	Mean	3.48E−06	0.013581	0.013584	0.013563	0.013904	0.016409	0.014386	0.021209	0.014091	0.014552	0.048403	0.135098	0.020532
	Best	1.02E−06	0.005703	0.005717	0.005705	0.005941	0.009516	0.00714	0.008312	0.006333	0.006826	0.024008	0.05125	0.009763
	Worst	8.78E−06	0.028099	0.028228	0.028069	0.028357	0.029288	0.028721	0.035215	0.028471	0.029843	0.076219	0.299597	0.036952
	Std	9.68E−06	0.00943	0.009449	0.00941	0.009406	0.008978	0.009234	0.011729	0.009375	0.009663	0.032195	0.106191	0.011804
	Median	3.62E−06	0.013477	0.013442	0.013437	0.013857	0.016577	0.013855	0.019088	0.013982	0.01394	0.046303	0.130737	0.020464
	Rank	1	3	4	2	5	9	7	11	6	8	12	13	10
Sum rank		7	58	18	24	21	39	44	46	28	31	41	52	61
Mean rank		1	8.285714	2.571429	3.428571	3	5.571429	6.285714	6.571429	4	4.428571	5.857143	7.428571	8.714286
Total ranking		1	12	2	4	3	7	9	10	5	6	8	11	13

4.2 Evaluation of High-Dimensional Multimodal Objective Functions

The optimization outcomes for high-dimensional multimodal functions, ranging from F8 to F13, are presented in Table 2. These functions are particularly challenging as they evaluate the algorithm's ability to balance exploration and exploitation while avoiding local optima. The simulation results indicate that BaOA successfully converges to the global optimum for F9 and F11, showcasing its strong exploratory capabilities. Additionally, BaOA outperforms all competitor algorithms, claiming the top rank for F8, F10, F12, and F13. The results suggest that BaOA's global search mechanism excels in traversing complex solution landscapes, making it highly effective for these high-dimensional multimodal problems. Compared to other algorithms, BaOA provides a superior performance, achieving high-quality solutions across all tested functions in this category. Although the performance of some competing algorithms is close to that of BaOA based on the simulation results, an important issue is that functions F8 to F13, by their nature, have a large number of local optima, which challenge the exploration ability of metaheuristic algorithms. Therefore, the greater an algorithm's ability to converge to better solutions, the higher its capacity to escape from local optima and explore the search space more effectively. To further confirm the superiority of BaOA over the competing algorithms, this issue is also addressed through a statistical analysis in Section 4.4 "Comprehensive Evaluation of the CEC 2017 Benchmark Suite". In that subsection, it is shown that BaOA has a significant statistical advantage over the competing algorithms.

4.3 Evaluation of Fixed-Dimensional Multimodal Objective Functions

The results of employing BaOA on fixed-dimensional multimodal functions, specifically F14 through F23, are reported in Table 3. These functions challenge the algorithm's robustness and precision in dealing with problems of fixed dimensionality. BaOA emerges as the best-performing optimizer for the functions F14, F15, F21, F22, and F23. For the functions F16 to F20, the proposed BaOA approach achieves comparable mean index values to some competitor algorithms. However, BaOA demonstrates a superior consistency, as evidenced by its lower standard deviation (std) values, which indicate a stable performance across multiple runs. This consistency reinforces BaOA's ability to effectively solve fixed-dimensional multimodal functions. Overall, the BaOA achieves competitive results across this function set, yet BaOA delivers more reliable and efficient solutions, affirming its superiority in optimizing fixed-dimensional problems.

To provide further insight into the comparative performance of BaOA and the other algorithms, boxplot diagrams summarizing the optimization outcomes for the functions F1 through F23 are depicted in Fig. 4. These visualizations highlight the robustness and reliability of BaOA in achieving consistent results across diverse benchmark functions, solidifying its status as a leading metaheuristic algorithm.

4.4 Comprehensive Evaluation of the CEC 2017 Benchmark Suite

This subsection provides a detailed analysis of the performance of the proposed Barber Optimization Algorithm (BaOA) in addressing the challenging functions of the CEC 2017 test suite. The CEC 2017 benchmark suite is widely recognized in the optimization community for its rigor and diversity, consisting of thirty benchmark functions categorized into four distinct types: three unimodal functions (C17-F1 to C17-F3), seven multimodal functions (C17-F4 to C17-F10), ten hybrid functions (C17-F11 to C17-F20), and ten composite functions (C17-F21 to C17-F30). These functions are specifically designed to test various aspects of optimization algorithms, including exploitation, exploration, and the ability to navigate complex, high-dimensional landscapes.

Table 2: Evaluation results for high-dimensional multimodal objective functions

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F8	Mean	−12498.6	−8941.75	−10721.3	−6077.68	−8884.01	−6541.72	−9793.42	−7659.81	−6502.41	−6184.73	−4325.08	−6811.2	−8048.32
	Best	−12622.8	−9726.41	−10901.8	−6411.2	−9423.15	−7383.14	−10899.5	−8664.55	−7022.16	−7039.87	−5010.56	−8011.06	−8995.18
	Worst	−11936.3	−8537.19	−10168.9	−5652.2	−8365.41	−5249.15	−7529.03	−6857	−5931.47	−5459.4	−3919.87	−5693.79	−6971.66
	Std	353.419	530.2574	331.2121	327.9836	520.2865	970.4313	2160.628	1007.717	599.176	736.8139	599.7498	1045.297	938.5856
	Median	−12577.8	−8918.08	−10752.9	−6095.38	−8924.84	−6477.62	−10543.1	−7643.92	−6392.42	−6176.29	−4213.64	−6966.77	−7989.2
	Rank	1	4	2	12	5	9	3	7	10	11	13	8	6
F9	Mean	0	33.31341	25.18398	25.18398	25.18398	139.4668	25.18398	89.76336	25.18398	25.18398	44.00108	69.88363	61.28017
	Best	0	24.27019	19.37701	19.37701	19.37701	79.10611	19.37701	55.21515	19.37701	19.37701	29.04558	48.14058	44.60965
	Worst	0	39.77902	31.64351	31.64351	31.64351	221.1742	31.64351	127.3038	31.64351	31.64351	59.14128	102.5832	79.37385
	Std	0	7.071286	7.197207	7.197207	7.197207	68.90408	7.197207	34.10318	7.197207	7.197207	14.30404	26.98201	17.75416
	Median	0	34.2987	24.92633	24.92633	24.92633	134.7718	24.92633	90.4754	24.92633	24.92633	43.59101	69.07007	62.03562
	Rank	1	4	2	2	2	9	2	8	3	2	5	7	6
F10	Mean	8.88E−16	2.266528	0.520053	0.520053	0.520053	1.340248	0.520053	0.901536	0.520053	0.520053	0.520053	2.320355	2.880049
	Best	8.88E−16	1.731663	0.318221	0.318221	0.318221	0.318221	0.318221	0.388018	0.318221	0.318221	0.318221	1.436101	2.273687
	Worst	8.88E−16	3.077311	0.696184	0.696184	0.696184	2.831494	0.696184	2.136405	0.696184	0.696184	0.696184	3.922512	3.754599
	Std	0	0.718112	0.217563	0.217563	0.217563	2.099988	0.217563	0.872114	0.217563	0.217563	0.217563	1.165111	0.604421
	Median	8.88E−16	2.287669	0.529567	0.529567	0.529567	0.534201	0.529567	0.723232	0.529567	0.529567	0.529567	2.310085	2.9433
	Rank	1	9	2	2	4	8	3	7	5	4	6	10	11
F11	Mean	0	0.698876	0.132441	0.132441	0.132441	0.138278	0.132441	0.396274	0.133325	0.132441	4.890597	0.254738	1.105109
	Best	0	0.484901	0.109693	0.109693	0.109693	0.117251	0.109693	0.277461	0.109693	0.109693	2.097591	0.114231	0.962509
	Worst	0	1.221271	0.181014	0.181014	0.181014	0.181972	0.181014	0.534829	0.181014	0.181014	8.46829	0.759179	1.273353
	Std	0	0.335531	0.034237	0.034237	0.034237	0.032422	0.034237	0.116907	0.033011	0.034237	3.329499	0.312988	0.166826
	Median	0	0.646187	0.128326	0.128326	0.128326	0.137274	0.128326	0.392356	0.128326	0.128326	4.95442	0.209482	1.089957
	Rank	1	7	2	2	2	4	2	6	3	2	9	5	8
F12	Mean	1.57E−32	1.624896	0.545697	1.415482	0.545697	4.369636	0.558963	1.149471	0.572022	0.592783	0.684347	1.536576	0.727161
	Best	1.57E−32	0.594851	0.114329	0.941948	0.114329	0.79878	0.119855	0.150994	0.141716	0.173538	0.132364	0.28033	0.193564
	Worst	1.57E−32	3.099041	0.980575	2.062292	0.980575	10.30725	0.981773	3.051508	1.000782	1.031426	1.246839	3.97665	1.233445
	Std	5.21E−48	1.276806	0.445112	0.603621	0.445112	5.152083	0.443197	1.528167	0.435017	0.442467	0.62543	1.741524	0.493766
	Median	1.57E−32	1.528804	0.521336	1.390071	0.521336	3.391368	0.532781	0.847596	0.548574	0.569188	0.638288	1.504844	0.730844
	Rank	1	12	3	10	2	13	4	9	5	6	7	11	8

(Continued)

Table 2 (continued)

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F13	Mean	1.35E−32	1188.725	0.613286	0.613286	0.614936	2.406761	0.754951	0.634922	0.95247	1.340737	0.650689	2.99475	2.400783
	Best	1.35E−32	5.174082	0.299636	0.299636	0.306889	1.920205	0.340658	0.32433	0.57359	0.911846	0.299636	0.305955	1.257115
	Worst	1.35E−32	20517.73	1.153462	1.153462	1.153462	3.213681	1.253848	1.161495	1.455694	2.020915	1.325414	9.461491	3.293458
	Std	5.21E−48	8487.818	0.359485	0.359485	0.358593	0.69575	0.438813	0.359698	0.45937	0.482319	0.469462	4.058102	1.11076
	Median	1.35E−32	15.07404	0.599334	0.599334	0.599334	2.257213	0.740427	0.624448	0.90653	1.300102	0.60296	2.739434	2.469787
	Rank	1	13	3	2	4	11	7	5	8	9	6	12	10
Sum Rank		6	49	14	30	19	54	21	42	34	34	46	53	49
Mean rank		1	8.166667	2.333333	5	3.166667	9	3.5	7	5.666667	5.666667	7.666667	8.833333	8.166667
Total ranking		1	9	2	5	3	11	4	7	6	6	8	10	9

Table 3: Evaluation results for fixed-dimensional multimodal objective functions

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
F14	Mean	0.998004	1.866902	1.895689	3.222571	1.837983	6.879377	2.867745	1.830202	3.61066	1.830203	3.522296	3.545056	1.863646
	Best	0.998004	1.061646	1.061646	1.265303	1.061646	1.717823	1.061648	1.061646	1.061646	1.061647	1.061673	1.061646	1.061646
	Worst	0.998004	2.634316	3.182106	9.704646	2.712039	12.13217	8.318551	2.556593	8.943761	2.556593	9.674317	9.704646	2.592184
	Std	0	0.971906	1.065105	3.605307	0.929514	6.853633	3.71493	0.90493	5.176118	0.904928	3.659546	5.127513	0.95838
	Median	0.998004	1.87236	1.904181	2.661536	1.87236	9.072971	2.402787	1.87236	2.62556	1.872361	3.269189	2.469238	1.87236
	Rank	1	6	7	9	4	13	8	2	12	3	10	11	5
F15	Mean	0.000307	0.003452	0.00284	0.003347	0.003402	0.013449	0.00314	0.004353	0.004827	0.002998	0.004158	0.004255	0.012764
	Best	0.000307	0.000769	0.000403	0.000876	0.000549	0.000384	0.000456	0.000637	0.000384	0.000419	0.001315	0.000384	0.000716
	Worst	0.000307	0.010134	0.007571	0.008207	0.008472	0.080166	0.007667	0.017579	0.016451	0.007576	0.008933	0.016303	0.049878
	Std	4.71E−19	0.004888	0.003886	0.004059	0.004082	0.040135	0.0039	0.008503	0.009648	0.003843	0.004085	0.00815	0.021895
	Median	0.000307	0.002544	0.002093	0.002502	0.002995	0.003024	0.002447	0.002672	0.002653	0.002608	0.004061	0.002833	0.012124
	Rank	1	7	2	5	6	13	4	10	11	3	8	9	12
F16	Mean	−1.03163	−1.03071	−1.03148	−1.03002	−1.02993	−1.03044	−1.03148	−1.03148	−1.03148	−1.03148	−1.03148	−1.03148	−1.03148
	Best	−1.03163	−1.03163	−1.03163	−1.03162	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163
	Worst	−1.03163	−1.02089	−1.02959	−1.01075	−1.01076	−1.00871	−1.02959	−1.02959	−1.02959	−1.02959	−1.02959	−1.02959	−1.02959
	Std	3.41E−16	0.00452	0.000865	0.008548	0.008713	0.009497	0.000865	0.000865	0.000865	0.000865	0.000865	0.000865	0.000865
	Median	−1.03163	−1.03162	−1.03163	−1.03137	−1.03161	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163
	Rank	1	8	2	10	11	9	3	5	4	7	2	2	6
F17	Mean	0.397887	0.424632	0.424463	0.43285	0.424802	0.424487	0.424463	0.424463	0.424463	0.424511	0.424463	0.653359	0.46944
	Best	0.397887	0.397893	0.397891	0.398531	0.397893	0.3979	0.397891	0.397891	0.397891	0.397893	0.397891	0.397891	0.397893
	Worst	0.397887	0.551148	0.55113	0.608808	0.551166	0.551144	0.551131	0.55113	0.551131	0.551151	0.55113	2.130994	1.378592
	Std	0	0.087589	0.087744	0.104279	0.087437	0.087727	0.087744	0.087744	0.087744	0.087731	0.087744	0.94993	0.405926

(Continued)

Table 3 (continued)

F		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
	Median	0.397887	0.39812	0.397938	0.406413	0.398282	0.397978	0.397938	0.397938	0.397939	0.397992	0.397938	0.397938	0.398039
	Rank	1	8	2	10	9	6	4	3	5	7	2	12	11
F18	Mean	3	4.925819	3.882281	5.713976	5.969357	9.494482	3.882298	3.882281	3.882289	3.882281	3.882281	3.882281	6.722714
	Best	3	3.005013	3.000415	3.000836	3.009612	3.000419	3.000415	3.000414	3.000417	3.000415	3.000414	3.000414	3.000826
	Worst	3	12.44553	8.702384	23.42621	21.35757	67.47643	8.702417	8.702384	8.702388	8.702384	8.702384	8.702384	26.14969
	Std	2.16E−15	4.577293	3.142879	10.92863	7.898354	35.02497	3.142876	3.142878	3.142875	3.142878	3.142878	3.142878	14.02804
	Median	3	4.127031	3.063663	3.063817	4.754885	3.063663	3.063676	3.06366	3.06367	3.06366	3.06366	3.06366	3.063706
	Rank	1	9	5	10	11	13	8	4	7	6	3	2	12
F19	Mean	−3.86278	−3.81439	−3.85992	−3.84286	−3.76886	−3.85966	−3.85836	−3.85992	−3.85892	−3.8592	−3.85992	−3.85992	−3.85981
	Best	−3.86278	−3.86277	−3.86277	−3.85752	−3.86277	−3.8627	−3.86276	−3.86277	−3.86277	−3.86266	−3.86277	−3.86277	−3.86276
	Worst	−3.86278	−3.66346	−3.85149	−3.8044	−3.47543	−3.85143	−3.8512	−3.85149	−3.85141	−3.85132	−3.85149	−3.85149	−3.85127
	Std	4.23E−15	0.089191	0.005006	0.028033	0.173386	0.005066	0.005422	0.005006	0.005837	0.004893	0.005006	0.005006	0.005186
	Median	−3.86278	−3.8146	−3.85999	−3.84649	−3.76937	−3.85967	−3.85827	−3.85999	−3.85922	−3.85912	−3.85999	−3.85999	−3.85992
	Rank	1	11	3	10	12	6	9	4	8	7	2	2	5
F20	Mean	−3.322	−3.01892	−3.25004	−2.91784	−2.76425	−3.24117	−3.23776	−3.2539	−3.24379	−3.23304	−3.28536	−3.24748	−3.22351
	Best	−3.322	−3.27858	−3.29587	−3.13007	−3.24651	−3.30915	−3.30691	−3.31065	−3.31065	−3.2988	−3.31065	−3.31065	−3.28807
	Worst	−3.322	−2.75742	−3.18647	−2.18029	−2.24967	−3.12147	−3.13354	−3.18644	−3.10786	−3.06727	−3.26496	−3.14902	−3.06059
	Std	8.24E−16	0.220862	0.077265	0.390363	0.427661	0.093682	0.105803	0.082376	0.101558	0.105108	0.019772	0.099587	0.099122
	Median	−3.322	−3.03242	−3.27007	−2.96581	−2.80731	−3.23991	−3.26729	−3.27943	−3.27006	−3.25648	−3.2876	−3.27827	−3.2277
	Rank	1	11	4	12	13	7	8	3	6	9	2	5	10
F21	Mean	−10.1532	−7.72917	−9.162	−5.79671	−7.44936	−6.37102	−8.65518	−8.32517	−8.65843	−6.9833	−7.20866	−6.17206	−6.59218
	Best	−10.1532	−9.44717	−9.76006	−6.39476	−9.39366	−9.42767	−9.59535	−9.76002	−9.75968	−8.85208	−9.76006	−9.76006	−9.36485
	Worst	−10.1532	−4.71593	−8.58873	−5.22343	−5.22343	−3.64187	−5.39767	−5.22343	−5.46508	−4.40399	−3.83164	−3.79706	−3.73671
	Std	3.86E−15	2.828091	0.596947	0.596947	2.792511	4.216189	2.211756	2.904148	2.446993	2.827812	4.397538	3.868156	3.699789
	Median	−10.1532	−8.07143	−9.15985	−5.79456	−7.73431	−5.54451	−9.03162	−8.96918	−9.12189	−7.35654	−8.66596	−5.58861	−7.04811
	Rank	1	6	2	13	7	11	4	5	3	9	8	12	10
F22	Mean	−10.4029	−8.6647	−9.55447	−6.04575	−8.02746	−7.23181	−8.03988	−8.2552	−9.55412	−7.93511	−9.37382	−6.90078	−7.5536
	Best	−10.4029	−10.007	−10.2734	−6.76473	−9.90061	−10.1515	−10.2701	−10.2734	−10.2729	−9.00444	−10.2734	−10.2734	−9.74681
	Worst	−10.4029	−7.21423	−8.96859	−5.45987	−5.45987	−3.60358	−3.38139	−4.26054	−8.96816	−5.36535	−5.6888	−3.92722	−3.86842
	Std	6.51E−15	1.833426	0.692071	0.69207	3.019829	4.53422	4.038436	3.709395	0.691967	1.967971	1.748968	4.757464	2.833088
	Median	−10.4029	−8.61853	−9.54881	−6.04009	−8.33965	−7.67568	−9.16958	−9.2449	−9.54859	−8.28808	−9.54881	−5.90406	−7.93906
	Rank	1	5	2	13	8	11	7	6	3	9	4	12	10
F23	Mean	−10.5364	−9.25306	−9.70953	−6.13964	−8.79659	−7.64904	−8.42034	−9.00019	−9.70922	−8.09206	−9.54514	−6.99275	−6.95271
	Best	−10.5364	−9.90879	−10.3075	−6.73761	−9.88002	−10.2386	−10.088	−10.3075	−10.3067	−9.529	−10.3075	−10.3075	−9.82642
	Worst	−10.5364	−8.14422	−9.09961	−5.52972	−6.46089	−3.98786	−4.35923	−5.5827	−9.09933	−5.58924	−6.72352	−3.81115	−4.03426
	Std	5.13E−15	1.025905	0.713942	0.713942	1.796167	4.758126	3.709472	2.961266	0.713841	2.124015	1.418476	5.15996	3.540944

(Continued)

Table 3 (continued)

F	BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
Median	−10.5364	−9.38471	−9.79072	−6.22083	−9.09828	−9.49487	−9.19256	−9.74149	−9.79055	−8.32489	−9.75315	−5.49596	−7.22335
Rank	1	5	2	13	7	10	8	6	3	9	4	11	12
Sum rank	10	76	31	105	88	99	63	48	62	69	45	78	93
Mean rank	1	7.6	3.1	10.5	8.8	9.9	6.3	4.8	6.2	6.9	4.5	7.8	9.3
Total ranking	1	8	2	13	10	12	6	4	5	7	3	9	11

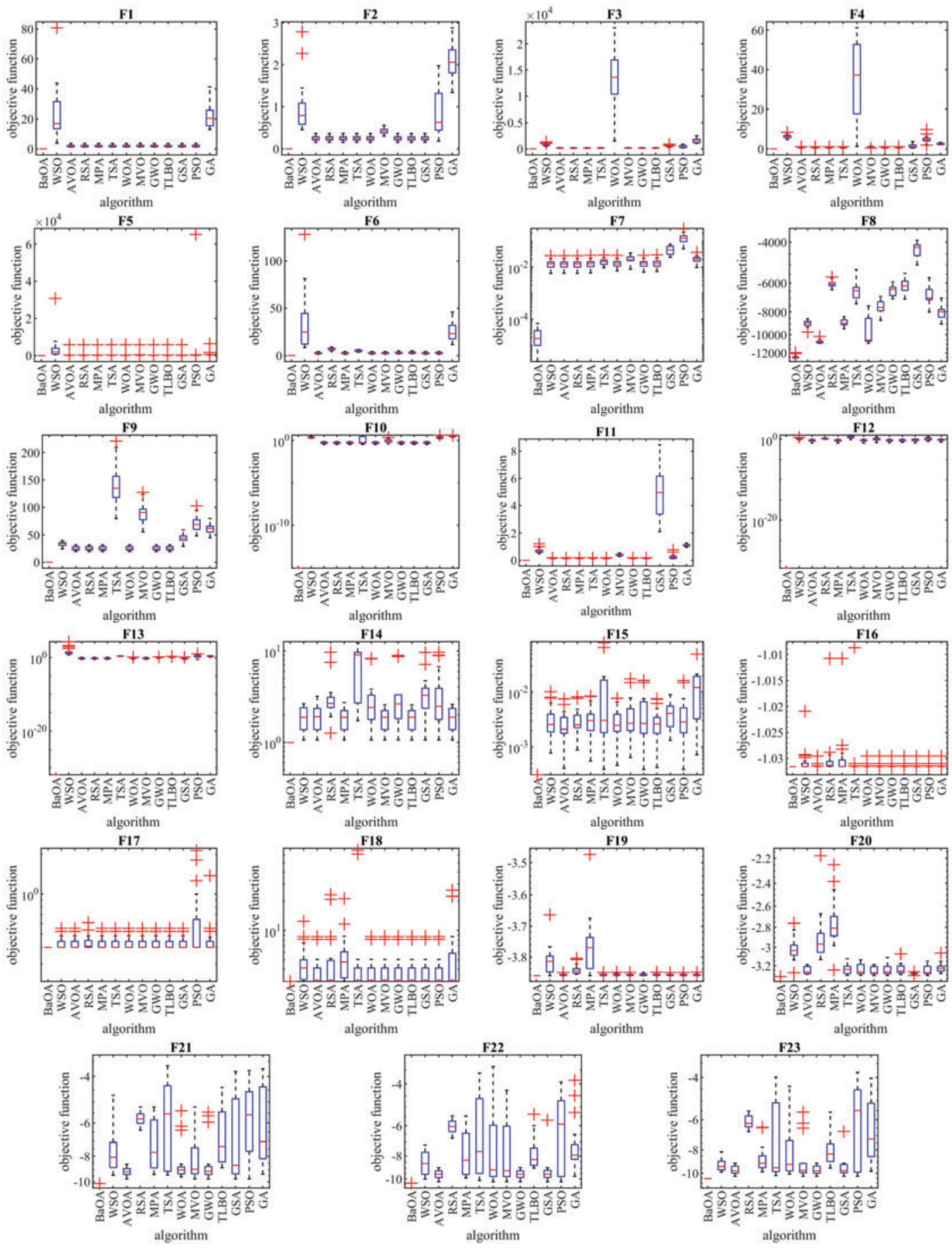

Figure 4: Boxplots of BaOA and the competitor algorithms performances for F1 to F23

It is important to note that the C17-F2 function was excluded from this study due to its unstable behavior during preliminary simulations, which could lead to unreliable results. The performance results of BaOA, along with those of the competing algorithms, are presented in Table 4 for comparison. Additionally, boxplot diagrams summarizing the statistical performance of BaOA and the other algorithms across all test functions are shown in Fig. 5. These diagrams provide a visual representation of BaOA's stability and consistency.

Table 4: Evaluation results for the CEC 2017 test suite

		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	Mean	100	5.44E+08	5.44E+08	6.77E+09	9.44E+09	1.57E+09	5.47E+08	5.44E+08	5.45E+08	5.96E+08	5.44E+08	5.44E+08	5.6E+08
	Best	100	3.82E+08	3.82E+08	4.91E+09	6.67E+09	4.53E+08	3.84E+08	3.82E+08	3.82E+08	4.4E+08	3.82E+08	3.82E+08	4.04E+08
	Worst	100	7.46E+08	7.46E+08	8.64E+09	1.13E+10	3.89E+09	7.47E+08	7.46E+08	7.46E+08	7.91E+08	7.46E+08	7.46E+08	7.57E+08
	Std	2.2E−05	2.08E+08	2.08E+08	2.33E+09	2.84E+09	2E+09	2.07E+08	2.08E+08	2.09E+08	1.96E+08	2.08E+08	2.08E+08	2.03E+08
	Median	100	5.24E+08	5.24E+08	6.77E+09	9.9E+09	9.79E+08	5.28E+08	5.24E+08	5.27E+08	5.77E+08	5.24E+08	5.24E+08	5.38E+08
	Rank	1	5	4	12	13	11	8	6	7	10	3	2	9
C17-F3	Mean	300	1295.217	996.7498	12720.33	8524.98	5506.514	3321.269	990.161	1515.475	1388.057	10052.6	1288.237	24846.08
	Best	300	993.0005	709.6167	11659.86	7845.351	940.4932	1178.304	709.6667	803.0648	1054.667	7745.119	1032.358	13937.15
	Worst	300	2096.572	1164.644	14029.76	8968.18	7992.158	7112.501	1164.663	2787.594	1587.062	14054.3	1902.007	35938.34
	Std	1.15E−10	684.7094	253.6154	1324.607	637.4245	4004.822	3357.841	250.6098	1116.687	300.3388	3567.713	529.0273	11653.15
	Median	300	1045.648	1056.369	12595.84	8643.194	6546.702	2497.135	1043.157	1235.62	1455.249	9205.497	1109.291	24754.41
	Rank	1	5	3	12	10	9	8	2	7	6	11	4	13
C17-F4	Mean	400	449.3409	454.0669	710.347	1124.293	590.9102	503.1852	450.2056	459.2918	460.5677	450.7166	450.7019	458.993
	Best	400	430.562	433.7609	534.9993	850.6618	509.8756	459.176	432.7115	434.3721	436.9376	432.0936	434.7717	445.382
	Worst	400	462.6959	481.7301	962.6778	1320.555	786.2829	591.7821	466.2292	496.467	478.6145	466.5767	463.4011	472.679
	Std	8.57E−08	18.09517	26.90943	260.2927	262.6964	167.3556	77.81765	18.63871	34.18151	26.95201	19.17989	17.85531	15.86301
	Median	400	452.0528	450.3884	671.8554	1162.977	533.7412	480.8914	450.9409	453.1641	463.3594	452.098	452.3174	458.9555
	Rank	1	2	6	12	13	11	10	3	8	9	5	4	7
C17-F5	Mean	510.9445	520.3326	549.8937	559.4236	574.628	554.1115	536.8499	520.0521	517.0164	536.5848	548.0741	525.6915	533.2131
	Best	506.9647	515.3301	537.8946	551.1301	557.3921	542.3573	531.6408	515.3308	516.0048	535.0979	541.023	522.7627	524.9217
	Worst	514.9244	529.7144	581.1529	566.8556	592.7666	565.4743	544.4395	527.1315	518.7686	538.4021	559.7312	530.8326	545.9562
	Std	4.650775	8.195913	26.76058	9.671534	21.55318	13.54318	6.961511	6.417103	1.630945	2.076818	10.40666	4.707707	12.26632
	Median	510.9445	518.1428	540.2636	559.8544	574.1767	554.3072	535.6597	518.8731	516.6461	536.4195	545.771	524.5854	530.9873
	Rank	1	4	10	12	13	11	8	3	2	7	9	5	6
C17-F6	Mean	600.0006	605.3113	624.6944	640.1595	638.3715	625.7453	628.2873	604.8396	606.5547	610.1637	622.8433	615.6928	611.6463
	Best	600.0004	604.499	618.7461	638.6768	634.2453	609.4433	619.0587	604.3337	604.9985	608.8358	617.7678	604.0691	609.1299
	Worst	600.0007	606.5354	633.5167	642.3846	642.9547	637.4717	635.8289	605.1158	610.3661	613.5726	627.7975	628.8507	614.0155
	Std	0.000137	1.171973	8.807828	2.048574	4.746561	17.56068	8.860117	0.471006	3.263105	2.914932	5.250214	15.04596	2.860151
	Median	600.0006	605.1055	623.2575	639.7883	638.143	628.0332	629.1308	604.9544	605.4271	609.1232	622.904	614.9256	611.7199
	Rank	1	3	9	13	12	10	11	2	4	5	8	7	6
C17-F7	Mean	722.5537	727.2092	767.8239	790.9185	792.7836	799.6484	778.9271	738.622	744.8397	758.3244	727.8015	743.3209	741.3442
	Best	719.8043	723.3942	748.4874	788.2504	777.2741	767.3517	757.4845	733.4587	732.3467	750.4986	724.4147	730.3079	731.1375
	Worst	725.4258	729.2857	791.724	793.7643	809.7784	855.5105	794.6481	744.1703	761.969	763.4747	734.0779	769.9332	750.9643
	Std	3.568861	3.359686	23.60066	3.094454	18.76272	49.25479	20.12429	6.441559	17.0522	7.36954	5.509872	23.35164	10.74459
	Median	722.4922	728.0784	765.5421	790.8296	792.0408	787.8658	781.7879	738.4295	742.5216	759.6621	726.3567	736.5213	741.6375
	Rank	1	2	9	11	12	13	10	4	7	8	3	6	5

(Continued)

Table 4 (continued)

		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F8	Mean	807.9597	812.2793	828.8386	851.78	842.6877	845.9023	840.8479	822.2909	816.8824	826.4104	827.2009	825.1992	822.572
	Best	805.9697	810.0706	819.6692	847.605	836.7707	839.3448	835.1031	815.1698	814.3685	820.737	826.2202	815.3763	815.8442
	Worst	809.9496	813.8461	834.572	860.1661	850.0938	853.1293	846.3956	828.4073	821.651	834.379	829.2061	833.363	828.4876
	Std	2.325388	2.034279	8.312649	7.310499	8.520167	8.620398	5.905642	7.863216	4.172452	7.329721	1.73929	9.528206	6.635191
	Median	807.9597	812.6002	830.5567	849.6744	841.9432	845.5675	840.9464	822.7931	815.7549	825.2628	826.6886	826.0288	822.9781
	Rank	1	2	9	13	11	12	10	4	3	7	8	6	5
C17-F9	Mean	900	980.0665	1052.402	1414.973	1515.54	1367.828	1446.93	955.945	968.0407	981.0735	955.7576	956.783	960.7073
	Best	900	942.3022	1003.533	1339.804	1387.219	1003.188	1258.937	931.1347	930.7983	940.3501	930.794	931.5063	933.7131
	Worst	900	1024	1111.066	1555.96	1689.042	1691.682	1618.6	970.3354	1001.043	1004.338	970.3263	972.2993	976.8816
	Std	4.38E−08	43.3408	58.46311	129.8582	161.7285	379.8126	210.7775	23.69327	36.85074	37.53873	23.75895	24.44425	25.33845
	Median	900	976.9818	1047.505	1382.065	1492.95	1388.221	1455.092	961.1549	970.1606	989.8028	960.955	961.6631	966.1173
	Rank	1	7	9	11	13	10	12	3	6	8	2	4	5
C17-F10	Mean	1379.646	1521.826	2082.333	2506.789	2373.843	1930.034	1797.868	1719.113	1778.029	1862.98	2498.138	2147.718	1650.618
	Best	1130.393	1332.48	1810.35	2387.664	2211.813	1539.93	1341.576	1496.036	1421.888	1836.602	2151.082	1724.378	1359.884
	Worst	1591.498	1720.704	2261.764	2600.409	2564.377	2205.947	2301.968	1865.47	2186.823	1921.647	2848.638	2677.929	1853.119
	Std	274.1373	205.0708	252.4361	115.7323	199.0975	371.766	505.9843	215.3596	405.8352	50.50318	364.7472	504.7945	279.2921
	Median	1398.345	1517.06	2128.609	2519.541	2359.591	1987.13	1773.965	1757.473	1751.702	1846.835	2496.416	2094.283	1694.734
	Rank	1	2	9	13	11	8	6	4	5	7	12	10	3
C17-F11	Mean	1101.505	1191.097	1200.834	4265.999	1441.148	2173.456	1241.493	1203.438	1199.876	1202.062	1189.682	1196.501	2865.8
	Best	1100	1131.91	1150.574	1729.235	1315.978	1242.835	1171.904	1136.281	1138.24	1145.039	1139.171	1141.17	1162
	Worst	1102.998	1337.297	1331.787	7835.371	1653.836	4798.618	1401.599	1345.292	1345.536	1334.626	1329.432	1324.975	7621.473
	Std	1.644386	125.255	111.9175	3470.59	206.3596	2241.414	137.7108	122.4521	124.9658	113.734	119.3309	110.588	4063.687
	Median	1101.512	1147.59	1160.487	3749.695	1397.39	1326.186	1196.235	1166.09	1157.863	1164.292	1145.063	1159.929	1339.864
	Rank	1	3	6	13	10	11	9	8	5	7	2	4	12
C17-F12	Mean	1264.785	14669750	16034310	3.21E+08	2.79E+08	16932491	17364113	15064588	15944769	16898460	15059399	16178074	15239556
	Best	1201.415	2281044	2831482	1.17E+08	43333833	2395467	3286203	2954816	2604220	4186601	2521226	2312124	4152621
	Worst	1331.208	26013272	26319686	5.07E+08	4.96E+08	26168878	26538121	26175050	30487205	29407919	26488952	26022449	26093242
	Std	91.70289	12673263	12464801	2.19E+08	2.42E+08	13036511	13530263	12360118	14832386	13788390	12763620	12758555	11731388
	Median	1263.259	15192342	17493036	3.3E+08	2.88E+08	19582809	19816065	15564242	15343826	16999661	15613708	18188862	15356181
	Rank	1	2	7	13	12	10	11	4	6	9	3	8	5
C17-F13	Mean	1305.286	8282.105	14068.37	35642999	122628.3	15175.85	15555.16	13788.65	12650.35	12973.12	16203.84	10194.49	20128.02
	Best	1302.215	3051.749	7903.466	384371.9	24034.34	5858.581	11823.46	5240.153	4196.716	8936.601	8922.808	3599.503	6233.487
	Worst	1309.369	16880.27	18881.09	57855221	286716.2	26058.89	19309.2	21508.32	21464.79	19875.65	23656.5	17159.02	29815.56
	Std	4.214822	8486.004	5929.442	32820769	159383	11888.8	4292.737	9338.547	10645.83	6241.508	7945.453	7397.79	13076.28
	Median	1304.78	6598.202	14744.47	42166201	89881.28	14392.97	15544	14203.05	12469.94	11540.12	16118.03	10009.72	22231.53
	Rank	1	2	7	13	12	8	9	6	4	5	10	3	11

(Continued)

Table 4 (continued)

		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F14	Mean	1404.229	1643.064	3598.432	3657.219	1723.737	3127.34	3490.648	1667.503	4157.559	1742.375	4620.746	4134.724	4359.895
	Best	1400.996	1539.594	1783.688	1685.451	1619.093	1618.679	1866.109	1550.833	3449.986	1633.806	3153.569	1807.854	3002.311
	Worst	1407.96	1732.401	7471.457	5558.669	1833.911	4694.615	4561.212	1772.804	4456.588	1824.832	6896.611	5349.637	7006.41
	Std	4.208266	120.1644	3350.95	2154.175	137.4153	2211.05	1614.615	134.3554	607.2281	104.5919	2167.455	2086.343	2312.803
	Median	1403.98	1650.13	2569.291	3692.378	1720.973	3098.033	3767.635	1673.188	4361.831	1755.431	4216.401	4690.703	3715.43
	Rank	1	2	8	9	4	6	7	3	11	5	13	10	12
C17-F15	Mean	1500.466	2262.598	5157.738	14977.71	8441.192	7406.888	5725.767	2677.83	4286.206	2452.241	13416.17	6798.567	3459.668
	Best	1500.163	1960.306	2845.8	7484.996	4341.457	3701.003	2550.116	1961.815	2462.291	2140.771	8534.56	3054.115	2232.507
	Worst	1500.75	2771.377	9633.351	19305.7	9915.202	17095.11	8371.499	3345.582	5832.524	2926.75	16874.12	14439	6404.011
	Std	0.397471	461.565	3950.321	6619.784	3500.634	8283.781	3494.863	725.3689	1908.485	430.5577	4500.95	6704.528	2536.489
	Median	1500.475	2159.354	4075.901	16560.08	9754.054	4415.72	5990.726	2701.962	4425.004	2370.721	14127.99	4850.578	2601.077
	Rank	1	2	7	13	11	10	8	4	6	3	12	9	5
C17-F16	Mean	1601.334	1703.001	1818.528	2060.236	1988.097	1921.09	1824.194	1763.952	1775.387	1720.148	2111.281	1928.322	1785.296
	Best	1600.604	1627.84	1663.956	1964.279	1893.837	1671.531	1753.403	1714.952	1650.357	1649.304	1942.77	1823.674	1629.994
	Worst	1602.476	1772.932	1913.082	2179.282	2097.694	2088.961	1903.729	1809.261	1967.275	1783.325	2256.974	2059.742	1857.789
	Std	1.117622	85.13982	137.5797	115.0311	111.2665	230.1521	104.8199	55.27779	197.3285	74.33535	186.334	131.5842	134.2605
	Median	1601.128	1705.616	1848.537	2048.693	1980.428	1961.934	1819.821	1765.798	1741.959	1723.981	2122.689	1914.935	1826.7
	Rank	1	2	7	12	11	9	8	4	5	3	13	10	6
C17-F17	Mean	1720.654	1762.064	1766.462	1858.595	1830.768	1896.814	1807.664	1783.282	1805.681	1770.724	1796.929	1774.398	1765.447
	Best	1718.157	1746.783	1753.982	1843.706	1768.837	1806.397	1777.799	1755.525	1735.634	1755.756	1754.35	1738.989	1755.741
	Worst	1722.134	1788.18	1786.364	1870.754	1895.515	2115.038	1871.631	1854.924	1899.812	1787.659	1886.687	1818.404	1778.857
	Std	2.238793	23.5392	19.00873	14.42762	69.59754	186.954	55.53714	61.36905	89.00986	19.14876	78.05942	42.29004	12.51283
	Median	1721.163	1756.646	1762.751	1859.961	1829.36	1832.91	1790.614	1761.339	1793.639	1769.74	1773.34	1770.1	1763.594
	Rank	1	2	4	12	11	13	10	7	9	5	8	6	3
C17-F18	Mean	1800.479	2629638	2639505	19870103	50521691	2650651	2634160	2644356	2645480	2654999	2640279	2639754	2636100
	Best	1800.412	6248.116	10130.17	2100035	41589.14	45620.4	8546.252	20800.99	12043.86	14587.95	13155.6	9382.549	11207.22
	Worst	1800.548	6790806	6814067	55923647	1.31E+08	6797008	6792242	6807366	6818392	6805691	6808344	6798014	6802153
	Std	0.075965	3729512	3741120	31246777	71756833	3714563	3728134	3730717	3740098	3727452	3735486	3729874	3733417
	Median	1800.478	1860749	1866911	10728365	35731682	1879988	1867927	1874628	1875741	1899859	1869807	1875811	1865520
	Rank	1	2	5	12	13	10	3	8	9	11	7	6	4
C17-F19	Mean	1900.702	2385.101	10006.23	323079	5259.044	6016.34	145024.8	2590.301	4506.776	2551.474	26568.05	7048.494	6711.241
	Best	1900.219	1989.436	3055.701	12951.31	3304.114	2073.052	5493.902	2100.239	2004.259	2240.417	14568.3	4211.301	2192.271
	Worst	1901.123	2820.638	22322.1	1063809	10283.86	11803.3	551049.9	2823.603	10002.99	2936.651	38465.32	14186.32	12748.57
	Std	0.554342	550.2165	11629.39	635124.2	4316.929	6104.68	346529.1	423.1207	4732.342	451.0331	12513.78	6118.613	5634.962
	Median	1900.733	2365.164	7323.564	107778.1	3724.102	5094.505	11777.72	2718.681	3009.927	2514.413	26619.29	4898.177	5952.061
	Rank	1	2	10	13	6	7	12	4	5	3	11	9	8

(Continued)

Table 4 (continued)

		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F20	Mean	2019.37	2049.494	2119.774	2200.903	2225.763	2158.454	2194.806	2055.195	2085.634	2103.428	2301.59	2140.196	2070.955
	Best	2017.46	2029.251	2082.619	2180.292	2164.911	2075.612	2161.639	2048.426	2041.728	2069.191	2177.156	2112.487	2041.565
	Worst	2021.304	2073.699	2207.055	2232.681	2284.874	2246.691	2251.64	2072.799	2157.657	2167.117	2364.117	2179.525	2088.41
	Std	2.641738	23.41319	75.00288	31.07705	70.17575	111.8351	50.50729	15.05559	64.78194	55.61776	108.1924	37.99966	28.25022
	Median	2019.358	2047.513	2094.712	2195.32	2226.634	2155.757	2182.973	2049.777	2071.575	2088.703	2332.542	2134.386	2076.923
	Rank	1	2	7	11	12	9	10	3	5	6	13	8	4
C17-F21	Mean	2200	2286.257	2275.476	2288.033	2357.161	2334.768	2308.226	2290.934	2308.438	2298.479	2341.469	2296.845	2278.227
	Best	2200	2224.532	2223.525	2250.238	2344.992	2327.187	2255.915	2223.653	2304.389	2225.976	2331.813	2227.375	2234.676
	Worst	2200	2310.249	2332.837	2356.902	2364.816	2352.275	2336.306	2317.124	2312.062	2326.18	2349.331	2326.153	2321.871
	Std	1.98E−05	52.83315	75.67627	61.65311	10.89968	15.22909	45.67622	57.78424	4.360942	61.96396	11.4093	59.6087	64.11703
	Median	2200	2305.124	2272.77	2272.497	2359.419	2329.806	2320.342	2311.479	2308.651	2320.881	2342.365	2316.927	2278.179
	Rank	1	4	2	5	13	11	9	6	10	8	12	7	3
C17-F22	Mean	2300.224	2344.77	2336.852	3018.296	2774.695	2488.651	2330.178	2340.219	2344.37	2350.856	2337.374	2620.179	2350.715
	Best	2300	2333.874	2325.948	2795.835	2452.748	2368.034	2317.164	2326.728	2325.603	2336.736	2324.653	2332.993	2336.84
	Worst	2300.553	2352.904	2351.859	3256.394	3072.317	2612.332	2352.321	2349	2354.411	2356.208	2345.742	3035.786	2359.381
	Std	0.348972	10.71965	15.58442	272.3896	324.3305	145.3997	21.2527	13.36057	16.45088	12.09722	12.27767	437.319	12.54435
	Median	2300.172	2346.152	2334.8	3010.478	2786.857	2487.12	2325.614	2342.574	2348.732	2355.239	2339.55	2555.969	2353.321
	Rank	1	7	3	13	12	10	2	5	6	9	4	11	8
C17-F23	Mean	2609.635	2646.572	2638.354	2702.906	2703.323	2699.476	2650.402	2637.116	2637.014	2641.55	2719.067	2646.386	2660.276
	Best	2608.441	2621.425	2626.551	2686.198	2680.828	2681.014	2636.266	2628.246	2628.105	2637.469	2706.366	2641.291	2652.074
	Worst	2611.752	2670.821	2651.305	2731.504	2723.574	2751.453	2660.282	2647.405	2642.793	2644.829	2730.502	2657.147	2674.616
	Std	1.864017	32.21371	12.96442	26.46736	26.83667	44.39586	12.94971	11.7251	8.196204	3.987915	14.75882	9.432282	12.62437
	Median	2609.174	2647.021	2637.78	2696.961	2704.445	2682.719	2652.531	2636.408	2638.579	2641.951	2719.7	2643.552	2657.207
	Rank	1	7	4	11	12	10	8	3	2	5	13	6	9
C17-F24	Mean	2525.171	2724.086	2746.569	2818.066	2804.033	2709.925	2735.898	2728.316	2716.16	2737.959	2599.917	2707.211	2657.918
	Best	2500	2710.651	2728.442	2786.465	2755.583	2545.704	2721.681	2725.724	2702.586	2730.469	2529.958	2554.303	2553.39
	Worst	2600.683	2734.093	2774.383	2855.801	2871.402	2785.34	2741.768	2731.604	2726.568	2751.482	2775.329	2794.088	2751.135
	Std	64.44325	13.23059	25.11784	37.89142	62.16549	142.0737	12.1987	3.86516	12.8241	12.64789	150.2447	134.7229	133.4518
	Median	2500	2725.8	2741.725	2814.999	2794.573	2754.329	2740.071	2727.969	2717.744	2734.942	2547.189	2740.226	2663.574
	Rank	1	7	11	13	12	5	9	8	6	10	2	4	3
C17-F25	Mean	2823.318	2948.605	2948.903	3243.373	3438.457	3052.183	2962.708	2946.503	2965.037	2982.481	2962.442	2949.121	2969.702
	Best	2600.042	2923.226	2919.016	3196.835	3220.487	2943.912	2888.993	2882.844	2912.107	2979.383	2915.81	2917.804	2923.948
	Worst	2897.743	2982.438	2989.089	3274.907	3575.25	3208.924	3009.794	2977.913	2987.467	2986.978	2981.491	2978.817	2988.15
	Std	190.5466	31.56957	37.40458	43.76366	195.2994	153.2158	68.20301	57.07266	45.40338	4.123558	39.92394	32.8139	39.16628
	Median	2897.743	2944.378	2943.754	3250.875	3479.046	3027.948	2976.023	2962.628	2980.288	2981.783	2976.233	2949.932	2983.355
	Rank	1	3	4	12	13	11	7	2	8	10	6	5	9

(Continued)

Table 4 (continued)

		BaOA	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F26	Mean	2850.001	3073.198	3162.945	3978.537	4075.856	3985.049	3560.378	3202.629	3197.314	3063.2	3454.487	3038.302	3132.724
	Best	2800.001	3043.63	3022.142	3747.295	3910.785	3439.537	3197.653	3008.043	3008.039	3041.495	2948.113	2948.984	2988.885
	Worst	2900	3094.591	3305.415	4216.485	4269.9	4304.458	3931.2	3758.887	3734.199	3101.059	4085.3	3100.974	3217.004
	Std	73.90655	29.93234	151.6917	268.4011	200.0176	487.3659	505.6202	474.7911	458.2821	33.76647	759.161	87.77049	127.8935
	Median	2850.001	3077.286	3162.11	3975.184	4061.37	4098.101	3556.329	3021.794	3023.51	3055.123	3392.268	3051.624	3162.505
	Rank	1	4	6	11	13	12	10	8	7	3	9	2	5
C17-F27	Mean	3089.072	3113.573	3114.037	3155.498	3150.01	3182.909	3134.3	3105.062	3123.472	3107.036	3211.117	3137.467	3133.09
	Best	3088.978	3110.302	3109.684	3144.841	3119.995	3109.083	3108.95	3098.096	3105.532	3100.305	3184.241	3117.059	3119.356
	Worst	3089.297	3119.164	3119.602	3176.036	3165.439	3255.991	3192.512	3111.814	3170.877	3115.047	3236.304	3176.682	3138.58
	Std	0.193461	5.166452	5.314489	18.21596	26.16647	77.5859	50.3156	7.260809	40.53482	7.802695	27.68099	34.15096	11.75058
	Median	3089.006	3112.413	3113.43	3150.557	3157.303	3183.281	3117.869	3105.169	3108.74	3106.395	3211.961	3128.063	3137.211
	Rank	1	4	5	11	10	12	8	2	6	3	13	9	7
C17-F28	Mean	3100	3240.397	3325.904	3624.216	3655.033	3366.196	3286.51	3336.799	3329.85	3339.737	3434.43	3263.782	3373.348
	Best	3100	3160.378	3143.43	3520.146	3559.492	3235.105	3223.096	3245.047	3247.035	3206.251	3415.822	3143.429	3222.957
	Worst	3100	3371.73	3392.099	3762.404	3733.025	3441.146	3392.24	3371.554	3385.61	3392.288	3454.902	3379.758	3480.319
	Std	7.57E−05	117.6148	155.8721	157.0156	99.28981	120.3861	93.96928	78.66529	76.90014	114.4936	23.71714	165.6677	163.9753
	Median	3100	3214.741	3384.044	3607.157	3663.807	3394.266	3265.352	3365.297	3343.378	3380.203	3433.497	3265.971	3395.058
	Rank	1	2	5	12	13	9	4	7	6	8	11	3	10
C17-F29	Mean	3146.525	3181.421	3248.364	3387.322	3375.58	3286.81	3386.956	3227.784	3213.331	3224.016	3428.135	3246.912	3228.999
	Best	3135.297	3177.011	3199.2	3256.518	3304.149	3210.77	3317.33	3180.42	3174.655	3208.615	3214.362	3229.665	3208.79
	Worst	3157.795	3191.234	3295.675	3554.108	3415.28	3347.566	3556.442	3352.601	3279.572	3242.025	3631.84	3281.221	3261.234
	Std	12.39774	8.611463	67.28112	158.5317	65.07625	75.05204	145.1358	106.7006	61.93079	17.75337	251.1635	30.97391	30.81054
	Median	3146.504	3178.72	3249.29	3369.332	3391.445	3294.453	3337.025	3189.056	3199.549	3222.712	3433.169	3238.381	3222.987
	Rank	1	2	8	12	10	9	11	5	3	4	13	7	6
C17-F30	Mean	3400.543	689473.6	1539649	9025811	7838276	5472442	1014937	1225386	1245165	975902.3	1954141	1096593	2795503
	Best	3395.811	597444	1062804	1997785	1242594	710376.9	720794.5	603815	612546.5	617810.7	1008697	750151.3	917096.7
	Worst	3413.809	810405.7	1966243	14165394	13985279	10762432	1663185	1967275	1899684	1664601	4162332	1751413	5756133
	Std	11.32675	137518.7	577608.7	6510367	7089471	6910352	562727.8	800377	752088.3	600352.7	1903801	586920.7	2660539
	Median	3396.275	675022.3	1564775	9970032	8062616	5208480	837884.2	1165227	1234215	810598.6	1322768	942405	2254391
	Rank	1	2	8	13	12	11	4	6	7	3	9	5	10
Sum rank		29	95	192	343	330	288	242	134	175	187	245	180	199
Mean rank		1	3.275862	6.62069	11.82759	11.37931	9.931034	8.344828	4.62069	6.034483	6.448276	8.448276	6.206897	6.862069
Total rank		1	2	7	13	12	11	9	3	4	6	10	5	8

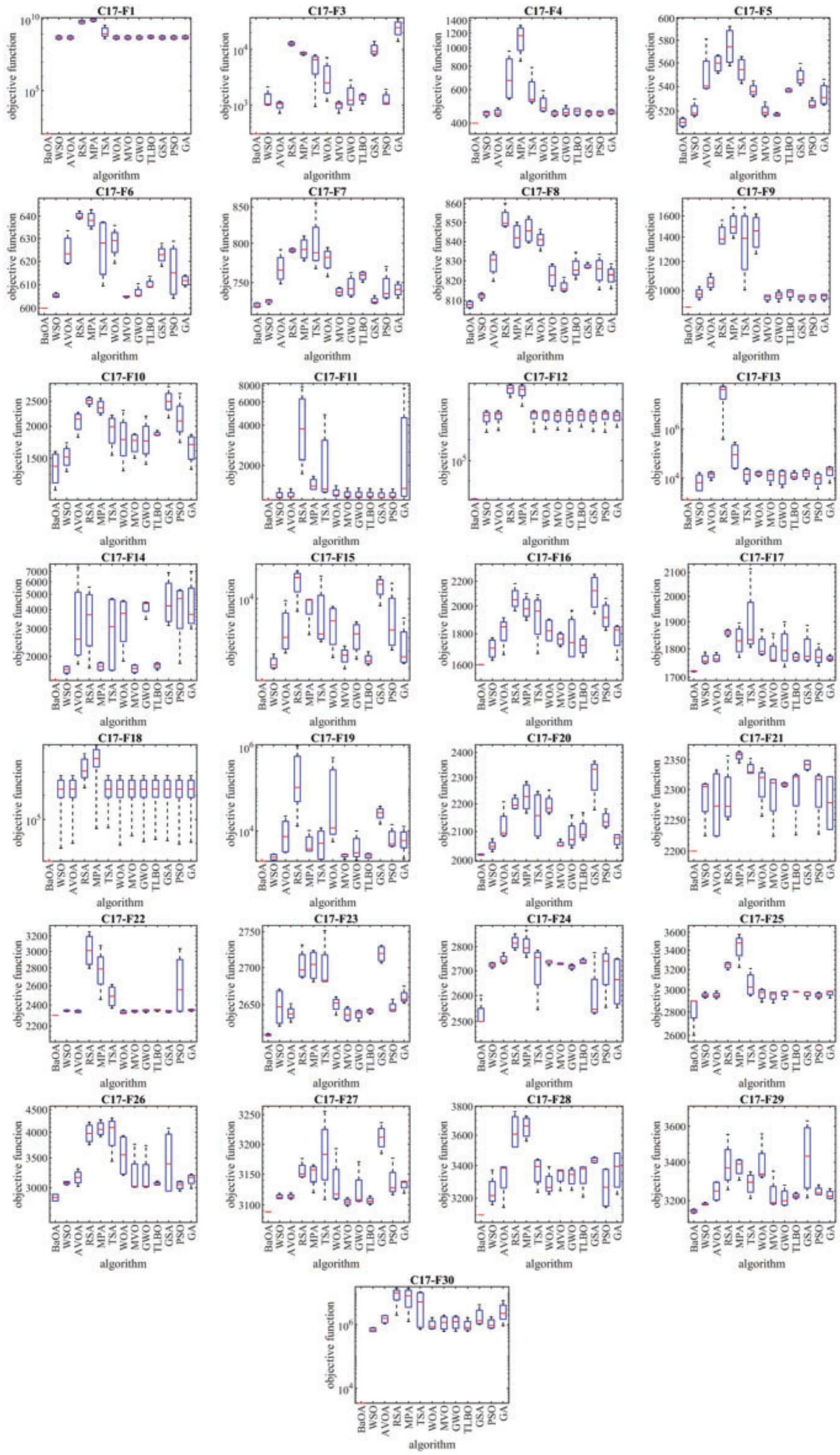

Figure 5: Boxplots of BaOA and the competitor algorithms performances for the CEC 2017 test suite

The optimization results demonstrate that BaOA achieves first-place rankings in the majority of the benchmark functions, specifically for the functions C17-F1, C17-F3 through C17-F6, C17-F8 through C17-F21, and C17-F23 through C17-F30. This highlights BaOA's versatility and capability to handle diverse problem types. For unimodal functions, BaOA excels in a precise convergence toward the global optimum, showcasing its strong exploitation ability. For the multimodal functions, BaOA effectively avoids local optima, thanks to its robust exploration mechanisms. When tackling hybrid and composite functions, which combine the characteristics of multiple landscapes, BaOA demonstrates a remarkable adaptability and efficiency in navigating complex solution spaces.

A comprehensive analysis of these results indicates that BaOA outperforms many well-established metaheuristic algorithms in terms of accuracy, convergence speed, and solution quality. This superior performance is evident in its ability to produce better optimization outcomes for most of the benchmark functions compared to its competitors. The findings reinforce BaOA's potential as a powerful and reliable tool for solving a wide range of real-world and theoretical optimization problems.

4.5 Comprehensive Statistical Evaluation

In this subsection, a completely different approach is taken to statistically analyze the performance of BaOA in comparison with the other metaheuristic algorithms. The primary goal of this statistical evaluation is to determine whether the observed superiority of BaOA over its competitors is statistically significant. To achieve this, the Wilcoxon signed-rank test [58], a widely recognized non-parametric statistical method, is employed. This test is particularly suitable for comparing paired data samples and assessing whether there is a significant difference between their central tendencies.

The Wilcoxon signed-rank test utilizes a key metric known as the p-value to determine statistical significance. A p-value less than 0.05 indicates that there is a statistically significant difference between the two data samples being compared. This threshold provides a rigorous basis for confirming whether BaOA consistently outperforms the other algorithms or if the observed differences could be attributed to random variations.

It is important to note that the simulation studies were conducted using the MATLAB software. In order to report visually and reader-friendly, the results are reported in a simple manner. The important issue in the results obtained from the statistical analysis is that p-values are less than 0.05. The smaller the p-value, the more significant is the superiority of BaOA over the corresponding competing algorithm.

The detailed results of the Wilcoxon signed-rank test, which compare BaOA against each competing algorithm, are comprehensively presented in Table 5. The table highlights cases where BaOA achieves a statistically significant advantage, underscoring its reliability and robustness in solving complex optimization problems. Specifically, for benchmark functions where the p-value is less than 0.05, it can be concluded that BaOA demonstrates a superior performance compared to the corresponding metaheuristic algorithm.

Table 5: Obtained results from the Wilcoxon sum-rank test

Compared algorithms	Unimodal	High-multimodal	Fixed-multimodal	CEC 2017 test suite
BaOA vs. WSO	1.85E−24	1.97E−21	2.09E−34	2.04E−18
BaOA vs. AVOA	3.02E−11	4.99E−05	1.44E−34	3.69E−21
BaOA vs. RSA	4.25E−07	1.63E−11	1.44E−34	1.97E−21
BaOA vs. MPA	1.01E−24	1.04E−14	2.09E−34	1.97E−21
BaOA vs. TSA	1.01E−24	1.31E−20	1.44E−34	1.97E−21
BaOA vs. WOA	2.44E−24	6.13E−11	1.44E−34	3.98E−21

(Continued)

Table 5 (continued)

Compared algorithms	Unimodal	High-multimodal	Fixed-multimodal	CEC 2017 test suite
BaOA vs. MVO	1.01E−24	1.97E−21	1.44E−34	2.18E−21
BaOA vs. GWO	1.01E−24	5.34E−16	1.44E−34	2.54E−21
BaOA vs. TLBO	1.01E−24	6.98E−15	1.44E−34	1.97E−21
BaOA vs. GSA	1.01E−24	1.97E−21	1.44E−34	5.41E−20
BaOA vs. PSO	1.01E−24	1.97E−21	1.44E−34	3.76E−20
BaOA vs. GA	1.01E−24	1.97E−21	1.44E−34	1.97E−21

By incorporating this rigorous statistical analysis, it becomes evident that BaOA's enhanced optimization capabilities are not only apparent in the numerical results but also substantiated through a formal statistical validation. This comprehensive evaluation adds more credibility to the effectiveness of BaOA and further solidifies its position as a leading optimization technique in both theoretical and practical applications.

5 Application of BaOA to Real-World Problems

This section provides a completely different and detailed evaluation of the proposed BaOA approach by applying it to real-world engineering design problems. These problems, which represent practical optimization challenges, include the tension/compression spring design, the welded beam design, the speed reducer design, and the pressure vessel design. The results demonstrate how effectively BaOA addresses these challenges and compares its performance with other metaheuristic algorithms.

5.1 Tension/Compression Spring Design

The tension/compression spring design problem is a classic and widely studied engineering optimization task aimed at minimizing the weight of a spring while satisfying specific constraints. This problem is particularly significant due to its practical implications in the design of lightweight and efficient mechanical components. A completely different perspective can be taken by analyzing both the design variables and constraints to achieve optimal results.

The schematic representation of the tension/compression spring design problem is shown in Fig. 6. The problem can be mathematically formulated as follows [59]:

$Consider\ X = [x_1, x_2, x_3] = [d, D, P].$
$Minimize\ f(x) = (x_3 + 2)\, x_2 x_1^2,$

Figure 6: Schematic of the tension/compression spring design

subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566\left(x_2 x_1^3\right)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0,$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

with

$0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3$ and $2 \leq x_3 \leq 15$.

The BaOA approach, along with the competitor algorithms, was implemented to solve this optimization problem. The numerical results are presented in Tables 6 and 7, showcasing the best found design values and statistical performance. Based on the simulation results, BaOA successfully identified a near-optimal optimal design with the following values for the design variables: (0.0516885, 0.3567142, 11.288853) and the objective function's minimized value was found to be (0.012665233). These results illustrate that BaOA provides a completely different level of precision and efficiency in optimizing this problem compared to other algorithms.

Table 6: Performance of the optimization algorithms for the tension/compression spring design problem

Algorithm	Best values of the variables			Minimal cost
	d	***D***	***P***	
BaOA	0.0516885	0.3567142	11.288853	0.012665233
WSO	0.040585	0.286916	8.723815	0.012672436
AVOA	0.054337	0.437455	10.24699	0.01269265
RSA	0.052952	0.401866	12.90495	0.012947052
MPA	0.054012	0.430304	10.57361	0.0126715
TSA	0.055895	0.477768	8.533558	0.012703804
WOA	0.056193	0.486452	8.178985	0.012714604
MVO	0.053075	0.406034	12.4979	0.012850589
GWO	0.053013	0.407024	12.18248	0.012705471
TLBO	0.065016	0.796741	4.713392	0.015952979
GSA	0.04279	0.32984	7.329621	0.013735892
PSO	0.064921	0.793823	4.713392	0.015891043
GA	0.067532	0.877543	3.11575	0.016337922

The convergence behavior of BaOA during the optimization process is depicted in Fig. 7, highlighting its rapid and stable convergence to an optimal solution. A detailed analysis of the simulation results demonstrates that BaOA outperforms the competitor algorithms in achieving superior design variable values and satisfying statistical performance indicators. This finding underscores the algorithm's ability to handle complex engineering design challenges effectively and with more accuracy.

Table 7: Statistical results of the optimization algorithms for the tension/compression spring design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
BaOA	0.012665233	0.012665233	0.012665233	9.84676E−19	0.012665233	1
WSO	0.012686126	0.012672436	0.012711628	9.67159E−06	0.012686917	3
AVOA	0.012908352	0.01269265	0.013619348	0.000265892	0.012800885	6
RSA	0.015939903	0.012947052	0.063738584	0.011464448	0.013064206	9
MPA	0.012683188	0.0126715	0.012705563	8.51129E−06	0.012684043	2
TSA	0.012852942	0.012703804	0.013132773	0.000107561	0.012859307	5
WOA	0.013203373	0.012714604	0.014463274	0.000598603	0.012985309	7
MVO	0.015489544	0.012850589	0.01609086	0.000989051	0.015931113	8
GWO	0.012721689	0.012705471	0.012746167	1.12674E−05	0.012720547	4
TLBO	0.016323991	0.015952979	0.016702555	0.000232427	0.016283489	10
GSA	0.01740768	0.013735892	0.020759729	0.002284236	0.017177311	11
PSO	2.50294E+13	0.015891043	2.50294E+14	7.78168E+13	0.015907491	13
GA	0.02027763	0.016337922	0.026190791	0.002568846	0.019743449	12

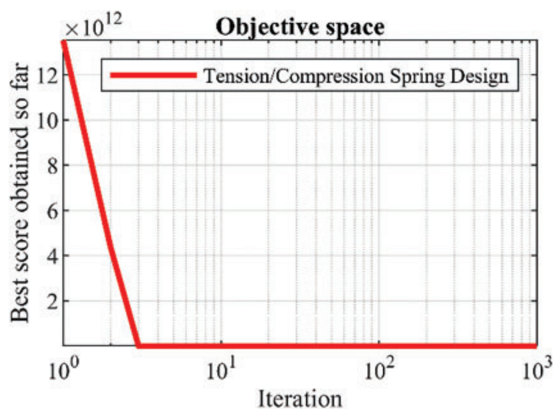

Figure 7: BaOA's performance convergence curve for tension/compression spring

5.2 Welded Beam Design

The welded beam design problem is a completely different type of optimization challenge compared to many other engineering design problems. Its primary objective is to minimize the total cost associated with the fabrication of a welded beam while satisfying multiple constraints related to mechanical and structural performance. This problem is crucial in practical engineering applications where cost efficiency and reliability are critical considerations. To address this, a mathematical model of the welded beam design problem has been formulated, which incorporates more words and details about its components and constraints.

The schematic representation of the welded beam design is shown in Fig. 8. The problem involves optimizing four design variables, which are defined as follows [59]:

$$Consider\ X = [x_1, x_2, x_3, x_4] = [h, l, t, b].$$
$$Minimize\ f(x) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4 (14.0 + x_2),$$

subject to:

$$g_1(x) = \tau(x) - 13600 \le 0,$$
$$g_2(x) = \sigma(x) - 30000 \le 0,$$
$$g_3(x) = x_1 - x_4 \le 0,$$
$$g_4(x) = 0.10471x_1^2 + 0.04811x_3 x_4 (14 + x_2) - 5.0 \le 0,$$
$$g_5(x) = 0.125 - x_1 \le 0,$$
$$g_6(x) = \delta(x) - 0.25 \le 0,$$
$$g_7(x) = 6000 - p_c(x) \le 0$$

where

$$\tau(x) = \sqrt{\tau' + (2\tau\tau')\frac{x_2}{2R} + (\tau'')^2},$$

$$\tau' = \frac{6000}{\sqrt{2}x_1 x_2},$$

$$\tau'' = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{x_1 x_2 \sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\},$$

$$\sigma(x) = \frac{504000}{x_4 x_3^2},$$

$$\delta(x) = \frac{65856000}{(30 \cdot 10^6)\, x_4 x_3^3},$$

$$p_c(x) = \frac{4.013\left(30 \cdot 10^6\right)\sqrt{\frac{x_3^2 x_4^6}{36}}}{196}\left(1 - \frac{x_3}{28}\sqrt{\frac{30 \cdot 10^6}{4\,(12 \cdot 10^6)}}\right),$$

with

$0.1 \le x_1, x_4 \le 2$ and $0.1 \le x_2, x_3 \le 10$.

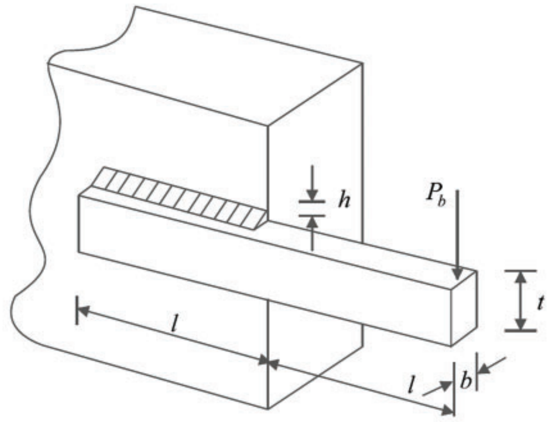

Figure 8: Schematic of welded beam design

The application of BaOA and the competing algorithms to solve this problem has produced significant results, which are summarized in Tables 8 and 9. BaOA demonstrated a superior performance by identifying the best design variables as: (0.2057276, 3.470454, 9.0365335, 0.2057276) and the corresponding minimized cost function value is: (1.724852309). The convergence curve of BaOA during the optimization process is depicted in Fig. 9. A detailed analysis of the simulation results shows that BaOA achieved completely different and superior outcomes compared to other algorithms. It not only provided the best design variables but also excelled in statistical performance indicators. The results underscore BaOA's capability to effectively solve welded beam design problems by delivering highly efficient and cost-effective solutions.

Table 8: Performance of the optimization algorithms for the welded beam design problem

Algorithm	Best values of the variables				Minimal cost
	h	***l***	***t***	***b***	
BaOA	0.2057276	3.470454	9.0365335	0.2057276	1.724852309
WSO	0.1608	2.732405	7.011254	0.16318	1.72546204
AVOA	0.207408	3.714317	8.734867	0.228572	1.72549169
RSA	0.19703	5.234012	8.746253	0.238817	1.860636851
MPA	0.208703	3.686303	8.734314	0.228574	1.725382642
TSA	0.208215	3.702826	8.756985	0.229093	1.730777108
WOA	0.207495	4.003879	8.72099	0.238203	1.772931452
MVO	0.198966	3.917521	8.73814	0.228563	1.729494006
GWO	0.207775	3.70621	8.737046	0.228577	1.726139784
TLBO	0.199016	4.388882	8.326612	0.262123	1.90048239
GSA	0.169682	2.874159	6.876084	0.183816	1.757714435
PSO	0.32574	3.377452	7.277437	0.385917	2.376962812
GA	0.233423	4.542673	7.207004	0.343711	2.307123823

Table 9: Statistical results of the optimization algorithms for the welded beam design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
BaOA	1.724852309	1.724852309	1.724852309	6.90342E−16	1.724852309	1
WSO	1.726230228	1.72546204	1.726671734	0.000335887	1.726273592	3
AVOA	1.740247557	1.72549169	1.777608117	0.015523944	1.734363927	7
RSA	2.100721306	1.860636851	3.165403926	0.279130052	2.04242714	8
MPA	1.726050774	1.725382642	1.726434777	0.000292138	1.726088493	2
TSA	1.738233183	1.730777108	1.742998913	0.003854653	1.738845582	5
WOA	2.191733268	1.772931452	3.508865579	0.512186144	1.94002356	10
MVO	1.740182285	1.729494006	1.761570813	0.009304602	1.737575882	6
GWO	1.727770361	1.726139784	1.730012522	0.001146958	1.727564297	4
TLBO	1.76063E+13	1.90048239	2.96965E+14	6.6917E+13	3.855374861	12
GSA	2.129517756	1.757714435	2.320486324	0.13970294	2.137341547	9
PSO	4.69928E+13	2.376962812	5.69381E+14	1.36193E+14	4.051611977	13
GA	3.91915E+12	2.307123823	7.61258E+13	1.71734E+13	3.976688895	11

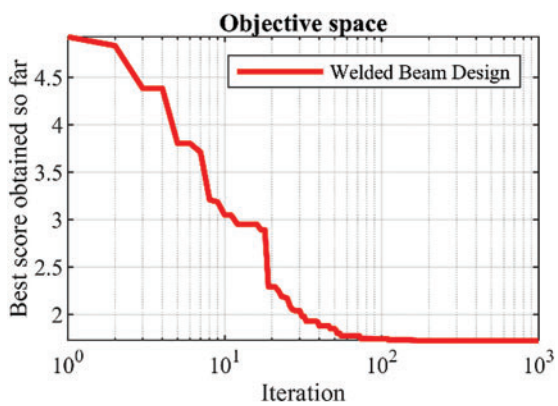

Figure 9: BaOA's performance convergence curve for the welded beam design

5.3 Speed Reducer Design

The speed reducer design presents a significant engineering challenge, with the primary objective being the reduction of weight while maintaining or improving performance. This complex task involves a detailed schematic representation, which can be found in Fig. 10, accompanied by the mathematical model that underpins the design process [60,61]:

$$Consider\ X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] = [b, m, p, l_1, l_2, d_1, d_2]\,.$$

$$Minimize\ f(x) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4x_6^2 + x_5x_7^2\right),$$

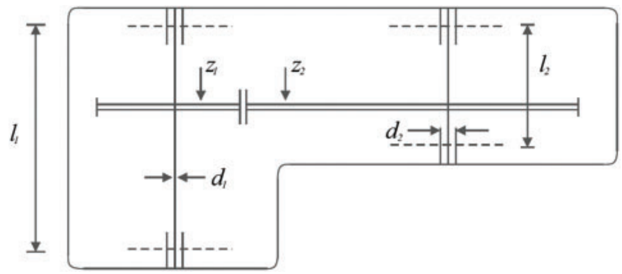

Figure 10: Schematic of the speed reducer design

subject to:

$$g_1(x) = \frac{27}{x_1 x_2^2 x_3} - 1 \leq 0,$$

$$g_2(x) = \frac{397.5}{x_1 x_2^2 x_3} - 1 \leq 0,$$

$$g_3(x) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \leq 0,$$

$$g_4(x) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \leq 0,$$

$$g_5(x) = \frac{1}{110 x_6^3} \sqrt{\left(\frac{745 x_4}{x_2 x_3}\right)^2 + 16.9 \times 10^6} - 1 \leq 0,$$

$$g_6(x) = \frac{1}{85 x_7^3} \sqrt{\left(\frac{745 x_5}{x_2 x_3}\right)^2 + 157.5 \times 10^6} - 1 \leq 0,$$

$$g_7(x) = \frac{x_2 x_3}{40} - 1 \leq 0,$$

$$g_8(x) = \frac{5 x_2}{x_1} - 1 \leq 0,$$

$$g_9(x) = \frac{x_1}{12 x_2} - 1 \leq 0,$$

$$g_{10}(x) = \frac{1.5 x_6 + 1.9}{x_4} - 1 \leq 0,$$

$$g_{11}(x) = \frac{1.1 x_7 + 1.9}{x_5} - 1 \leq 0,$$

with

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3, 2.9 \leq x_6 \leq 3.9, \text{ and } 5 \leq x_7 \leq 5.5.$$

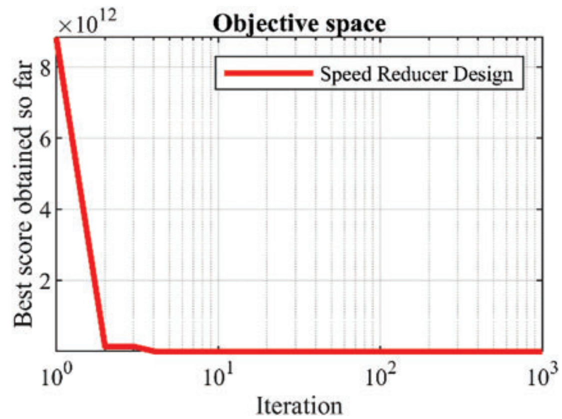

Figure 11: BaOA's performance convergence curve for the speed reducer design

In addressing this challenge, various optimization algorithms have been applied, and their performance compared in terms of how effectively they solve the speed reducer design problem. The results are presented in Tables 10 and 11. One such algorithm, namely BaOA, has shown promising results in the simulation studies, offering the best design solutions. This is evident from the values of the design variables—(3.499965, 0.699993, 16.99983, 7.299927, 7.799922, 3.350181, 5.28663)—and the corresponding objective function value of 2996.348165. Moreover, the convergence curve for BaOA, shown in Fig. 11, illustrates how the algorithm efficiently converges to the optimal solution for the speed reducer design.

Table 10: Performance of the optimization algorithms for the speed reducer design problem

Algorithm	Best values of the variables							Minimal cost
	b	m	p	l_1	l_2	d_1	d_2	
BaOA	3.499965	0.699993	16.99983	7.299927	7.799922	3.350181	5.28663	2996.348165
WSO	2.730607	0.546031	13.34896	5.697578	6.089215	2.614628	4.1255	2998.333593
AVOA	3.508632	0.700974	17.76011	7.373101	7.891722	3.365877	5.307849	2998.606975
RSA	3.551121	0.700974	17.76011	7.967442	8.166541	3.712241	5.442251	3068.311155
MPA	3.508632	0.700974	17.76011	7.337321	7.851481	3.36581	5.307836	2998.06369
TSA	3.52034	0.700974	17.76011	7.404414	7.851481	3.367648	5.343073	3009.309854
WOA	3.516005	0.700974	20.35411	7.671177	7.903289	3.379332	5.307552	3007.144822
MVO	3.512182	0.700974	17.76011	7.504644	8.025625	3.383674	5.308183	3006.509968
GWO	3.510935	0.700974	17.76011	7.429515	7.915947	3.36911	5.308672	3000.597927
TLBO	3.53988	0.702588	22.12687	7.900463	8.050078	3.644236	5.362489	4055.526521
GSA	2.773487	0.548049	14.90346	6.184644	6.339201	2.849397	4.138361	3187.46692
PSO	3.574255	0.721681	21.78677	7.458198	7.907371	3.698288	5.39443	4228.672836
GA	3.548586	0.705884	21.59069	7.358079	7.936775	3.426541	5.414086	3923.446243

A deeper analysis of these simulation results reveals that BaOA consistently outperforms the competing algorithms in terms of achieving better values for the design variables, as well as superior statistical performance indicators. These findings suggest that BaOA not only provides a more efficient solution but also offers a more reliable approach when compared to other optimization methods applied to the speed reducer design problem. Therefore, this research underscores the effectiveness of BaOA in tackling the complexities associated with speed reducer design optimization.

Table 11: Statistical results of the optimization algorithms for the speed reducer design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
BaOA	2996.348165	2996.348165	2996.348165	9.42546E−13	2996.348165	1
WSO	2999.224749	2998.333593	3000.503624	0.569766413	2999.096791	3
AVOA	3002.716642	2998.606975	3006.774134	2.493981374	3002.688553	4
RSA	3161.598674	3068.311155	3231.34145	40.90421349	3154.750717	8
MPA	2998.848764	2998.06369	2999.962202	0.496867977	2998.738542	2
TSA	3021.841298	3009.309854	3034.709636	6.230815509	3021.427834	6
WOA	3175.862584	3007.144822	4052.700839	287.0640984	3092.739521	9
MVO	3023.856405	3006.509968	3046.892061	11.31516643	3025.098902	7
GWO	3004.863422	3000.597927	3010.309431	2.48785059	3004.817052	5
TLBO	3.43005E+13	4055.526521	1.56969E+14	4.16756E+13	1.78072E+13	11
GSA	3355.623244	3187.46692	3790.265505	150.0739866	3330.730137	10
PSO	9.13563E+13	4228.672836	4.42536E+14	1.26774E+14	2.59137E+13	13
GA	6.17884E+13	3923.446243	4.37862E+14	9.9946E+13	3.44654E+13	12

5.4 Pressure Vessel Design

The pressure vessel design is a crucial engineering challenge, especially in practical applications, where the primary objective is to minimize the overall cost of the design while ensuring safety, durability, and efficiency. The complexity of this task is demonstrated by the pressure vessel design schematic, which is shown in Fig. 12, alongside its associated mathematical model, as detailed in [62]:

$$Consider\ X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L].$$
$$Minimize\ f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3,$$

subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \le 0,$$
$$g_2(x) = -x_2 + 0.00954x_3 \le 0,$$
$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0,$$
$$g_4(x) = x_4 - 240 \le 0,$$

with

$$0 \le x_1, x_2 \le 100, \text{and } 10 \le x_3, x_4 \le 200.$$

To solve this complex problem, various optimization algorithms are employed. One of the most effective algorithms in this context is the BaOA, which has shown promising results in comparison to other algorithms. The optimization results using BaOA and the other competing algorithms are presented in Tables 12 and 13, providing a comprehensive comparison of the performance of each method.

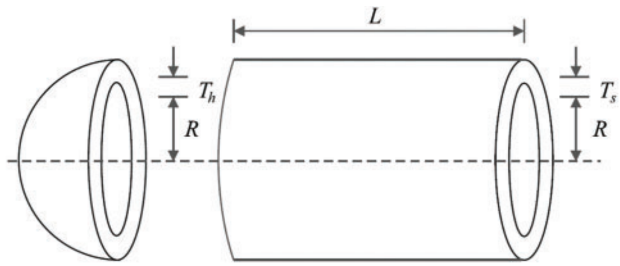

Figure 12: Schematic of the pressure vessel design

Table 12: Performance of the optimization algorithms for the pressure vessel design problem

Algorithm	Best values of the variables				Minimal cost
	T_s	T_h	R	L	
BaOA	0.778019	0.384575	40.31188	199.998	5882.9013
WSO	0.623596	0.310956	31.82911	152.934	5882.9013
AVOA	0.918552	0.47668	43.57248	174.603	5882.9088
RSA	1.297069	0.526887	53.95198	89.6774	6624.5646
MPA	0.918549	0.476679	43.57233	174.6051	5882.9013
TSA	1.156747	0.594301	55.67871	72.42199	5910.2826
WOA	1.104629	0.567759	52.52029	90.22139	6366.9495
MVO	1.178188	0.603885	56.8916	65.79012	5928.9243
GWO	0.919488	0.477444	43.60859	174.1111	5889.473
TLBO	1.073509	2.095782	46.51753	153.2486	13859.799
GSA	0.74818	0.371513	38.20128	99.6093	6797.5343
PSO	1.615235	1.114346	48.83039	131.1491	15409.18
GA	1.367064	0.813607	46.68199	155.4433	13257.991

Table 13: Statistical results of the optimization algorithms for the pressure vessel design problem

Algorithm	Mean	Best	Worst	Std	Median	Rank
BaOA	5882.9013	5882.9013	5882.9013	1.89E–12	5882.9013	1
WSO	5882.9137	5882.9013	5883.1494	0.056037	5882.9013	3
AVOA	6258.4626	5882.9088	7244.3707	396.54414	6184.1124	6
RSA	10714.654	6624.5646	19681.96	2923.6643	10323.778	9
MPA	5882.9014	5882.9013	5882.9014	3.08E–05	5882.9014	2
TSA	6229.4701	5910.2826	7302.4807	412.78514	5985.5448	5
WOA	7802.6641	6366.9495	10354.64	1255.4896	7332.924	8
MVO	6488.7335	5928.9243	7200.1258	354.30925	6461.6725	7
GWO	6068.9666	5889.473	7112.5407	359.23581	5906.2473	4
TLBO	29392.96	13859.799	44713.751	9072.4462	28572.828	11
GSA	21789.052	6797.5343	46711.398	10646.253	20819.584	10
PSO	43681.798	15409.18	91778.529	21609.573	36634.22	13

(Continued)

Table 13 (continued)

Algorithm	Mean	Best	Worst	Std	Median	Rank
GA	33056.188	13257.991	59604.825	11024.575	31793.615	12

The simulation results indicate that BaOA delivers the best design with the values of the design variables being (0.778019, 0.384575, 40.31188, 199.998) and the objective function value equal to 5882.9013. The convergence curve for BaOA, shown in Fig. 13, illustrates how the algorithm efficiently approaches the optimal solution for the design variables. This demonstrates BaOA's superior performance in solving the pressure vessel design problem.

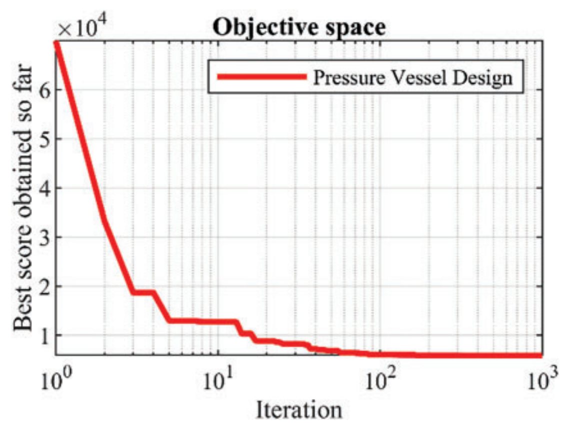

Figure 13: BaOA's performance convergence curve for the pressure vessel design

Upon analyzing the results, it is clear that BaOA outperforms the competing algorithms, not only providing better design values but also offering superior statistical performance indicators. This makes BaOA a more effective and reliable tool for addressing pressure vessel design challenges in engineering applications.

6 Conclusion and Future Works

In this paper, we presented a novel metaheuristic algorithm inspired by human behavior, called the Barber Optimization Algorithm (BaOA). This new approach is designed to solve complex optimization problems across various scientific disciplines. The core inspiration behind BaOA stems from the human interactions between a barber and a customer, which include two key processes: (1) the customer's selection of a hairstyle and (2) the refinement or correction of the hairstyle during the haircut. These human elements form the basis for the two-phase process of BaOA: the exploration phase, modeled on the selection of a hairstyle, and the exploitation phase, which simulates the correction of hairstyle details. Both phases are mathematically modeled to guide the optimization process effectively.

The performance of BaOA in solving optimization problems was rigorously tested on a diverse set of fifty-two benchmark functions. These functions include unimodal, high-dimensional multimodal, fixed-dimensional multimodal ones, and those from the CEC 2017 test suite. The results from these tests demonstrate BaOA's strong capability to balance exploration and exploitation, allowing it to find optimal or near-optimal solutions with high efficiency. In comparison with twelve well-established metaheuristic algorithms, BaOA consistently delivered a superior performance, providing better solutions for the majority of the benchmark functions. This highlights BaOA's competitive edge in solving complex optimization problems.

Furthermore, BaOA was applied to four engineering design problems, where it showcased its potential for real-world applications, solving practical design challenges with remarkable accuracy. The study concludes by suggesting several avenues for future research, including the development of binary and multi-objective versions of BaOA. Additionally, the application of BaOA to further optimization problems in a wide range of scientific and engineering fields presents exciting opportunities for future exploration.

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References

1. Dehghani M, Montazeri Z, Dhiman G, Malik O, Morales-Menendez R, Ramirez-Mendoza RA, et al. A spring search algorithm applied to engineering optimization problems. Appl Sci. 2020;10(18):6173. doi:10.3390/app10186173.
2. Dehghani M, Montazeri Z, Dehghani A, Samet H, Sotelo C, Sotelo D, et al. DM: dehghani method for modifying optimization algorithms. Appl Sci. 2020;10(21):7683. doi:10.3390/app10217683.
3. Coufal P, Hubálovský Š, Hubálovská M, Balogh Z. Snow leopard optimization algorithm: a new nature-based optimization algorithm for solving optimization problems. Mathematics. 2021;9(21):2832. doi:10.3390/math9212832.
4. Kvasov DE, Mukhametzhanov MS. Metaheuristic vs. deterministic global optimization algorithms: the univariate case. Appl Math Comput. 2018;318(9):245–59. doi:10.1016/j.amc.2017.05.014.
5. Mirjalili S. The ant lion optimizer. Adv Eng Softw. 2015;83:80–98. doi:10.1016/j.advengsoft.2015.01.010.
6. Hamadneh T, Batiha B, Gharib GM, Montazeri Z, Werner F, Dhiman G, et al. Orangutan optimization algorithm: an innovative bio-inspired metaheuristic approach for solving engineering optimization problems. Int J Intell Eng Syst. 2025;18(1):47–57. doi:10.22266/ijies2025.0229.05.
7. Cheng MY, Sholeh MN. Artificial satellite search: a new metaheuristic algorithm for optimizing truss structure design and project scheduling. Appl Math Model. 2025;143(3):116008. doi:10.1016/j.apm.2025.116008.
8. Truong DN, Chou JS. Metaheuristic algorithm inspired by enterprise development for global optimization and structural engineering problems with frequency constraints. Eng Struct. 2024;318:118679. doi:10.1016/j.engstruct.2024.118679.
9. Mortazavi A, Moloodpoor M. Tactical flight optimizer: a novel optimization technique tested on mathematical, mechanical, and structural optimization problems. Mater Test. 2025;67(2):330–52. doi:10.1515/mt-2024-0327.

10. Hamadneh T, Batiha B, Gharib GM, Montazeri Z, Dehghani M, Aribowo W, et al. Paper publishing based optimization: a new human-based metaheuristic approach for solving optimization tasks. Int J Intell Eng Syst. 2025;18(2):504–19. doi:10.22266/ijies2025.0331.37.
11. Hamadneh T, Batiha B, Gharib GM, Montazeri Z, Dehghani M, Aribowo W, et al. Revolution optimization algorithm: a new human-based metaheuristic algorithm for solving optimization problems. Int J Intell Eng Syst. 2025;18(2):520–31. doi:10.22266/ijies2025.0331.38.
12. Wang Y, Xiong G. Metaheuristic optimization algorithms for multi-area economic dispatch of power systems: part II—a comparative study. Artif Intell Rev. 2025;58(5):132. doi:10.1007/s10462-025-11125-w.
13. Akçay Ö, İlkılıç C. Light-weight design of aerospace components using genetic algorithm and dandelion optimization algorithm. Int J Aeronaut Space Sci. 2025;26(2):1–12. doi:10.1007/s42405-025-00900-2.
14. Nassar SM, Saleh A, Eisa AA, Abdallah E, Nassar IA. Optimal allocation of renewable energy resources in distribution systems using meta-heuristic algorithms. Results Eng. 2025;25(2):104276. doi:10.1016/j.rineng.2025.104276.
15. Daravath R, Bali SK. Optimal distributed energy resources placement to reduce power losses and voltage deviation in a distribution system. Iran J Sci Technol Trans Electr Eng. 2025;49(1):1–12. doi:10.1007/s40998-024-00780-4.
16. Chau ML, Gkiotsalitis K. A systematic literature review on the use of metaheuristics for the optimisation of multimodal transportation. Evol Intell. 2025;18(2):1–37. doi:10.1007/s12065-025-01020-2.
17. Wang H, Chen B, Sun H, Li A, Zhou C. AnFiS-MoH: systematic exploration of hybrid ANFIS frameworks via metaheuristic optimization hybridization with evolutionary and swarm-based algorithms. Appl Soft Comput. 2024;167(2):112334. doi:10.1016/j.asoc.2024.112334.
18. Jaber I, Hassouneh Y, Khemaja M. A hybrid meta-heuristic algorithm for optimization of capuchin search algorithm for high-dimensional biological data classification. Neural Comput Appl. 2025;37(7):5719–50. doi:10.1007/s00521-024-10815-w.
19. Dokeroglu T, Sevinc E, Kucukyilmaz T, Cosar A. A survey on new generation metaheuristic algorithms. Comput Ind Eng. 2019;137(5):106040. doi:10.1016/j.cie.2019.106040.
20. Dehghani M, Montazeri Z, Dehghani A, Malik OP, Morales-Menendez R, Dhiman G, et al. Binary spring search algorithm for solving various optimization problems. Appl Sci. 2021;11(3):1286. doi:10.3390/app11031286.
21. Hussain K, Mohd Salleh MN, Cheng S, Shi Y. Metaheuristic research: a comprehensive survey. Artif Intell Rev. 2019;52(4):2191–233. doi:10.1007/s10462-017-9605-z.
22. Iba K. Reactive power optimization by genetic algorithm. IEEE Trans Power Syst. 1994;9(2):685–92. doi:10.1109/59.317674.
23. Wolpert DH, Macready WG. No free lunch theorems for optimization. IEEE Trans Evol Comput. 1997;1(1):67–82. doi:10.1109/4235.585893.
24. Kennedy J, Eberhart R. Particle swarm optimization. In: Proceedings of ICNN'95—International Conference on Neural Networks; 1995 Nov 27–Dec 1; Perth, WA, Australia. doi:10.1109/ICNN.1995.488968.
25. Dorigo M, Maniezzo V, Colorni A. Ant system: optimization by a colony of cooperating agents. IEEE Trans Syst Man Cybern Part B. 1996;26(1):29–41. doi:10.1109/3477.484436.
26. Karaboga D, Basturk B. Artificial bee colony (ABC) optimization algorithm for solving constrained optimization problems. In: Foundations of fuzzy logic and soft computing; Berlin/Heidelberg: Springer; 2007. p. 789–98.
27. Yang XS, editor. Firefly algorithms for multimodal optimization. In: International Symposium on Stochastic Algorithms; 2009 Oct 26–28; Sapporo, Japan.
28. Dhiman G, Kumar V. Emperor penguin optimizer: a bio-inspired algorithm for engineering problems. Knowl-Based Syst. 2018;159(2):20–50. doi:10.1016/j.knosys.2018.06.001.
29. Abualigah L, Abd Elaziz M, Sumari P, Geem ZW, Gandomi AH. Reptile Search Algorithm (RSA): a nature-inspired meta-heuristic optimizer. Expert Syst Appl. 2022;191(11):116158. doi:10.1016/j.eswa.2021.116158.
30. Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. Adv Eng Softw. 2014;69:46–61. doi:10.1016/j.advengsoft.2013.12.007.

31. Kaur S, Awasthi LK, Sangal AL, Dhiman G. Tunicate Swarm Algorithm: a new bio-inspired based metaheuristic paradigm for global optimization. Eng Appl Artif Intell. 2020;90(2):103541. doi:10.1016/j.engappai.2020.103541.
32. Braik M, Hammouri A, Atwan J, Al-Betar MA, Awadallah MA. White Shark Optimizer: a novel bio-inspired meta-heuristic algorithm for global optimization problems. Knowl-Based Syst. 2022;243(7):108457. doi:10.1016/j.knosys.2022.108457.
33. Abdollahzadeh B, Gharehchopogh FS, Mirjalili S. African vultures optimization algorithm: a new nature-inspired metaheuristic algorithm for global optimization problems. Comput Ind Eng. 2021;158(4):107408. doi:10.1016/j.cie.2021.107408.
34. Faramarzi A, Heidarinejad M, Mirjalili S, Gandomi AH. Marine predators algorithm: a nature-inspired meta-heuristic. Expert Syst Appl. 2020;152:113377. doi:10.1016/j.eswa.2020.113377.
35. Goldberg DE, Holland JH. Genetic algorithms and machine learning. Mach Learn. 1988;3(2):95–9. doi:10.1023/A:1022602019183.
36. Storn R, Price K. Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. J Glob Optim. 1997;11(4):341–59. doi:10.1023/A:1008202821328.
37. Banzhaf W, Nordin P, Keller RE, Francone FD. Genetic programming: an introduction: on the automatic evolution of computer programs and its applications. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc.; 1998.
38. Reynolds RG. An introduction to cultural algorithms. In: Proceedings of the Third Annual Conference on Evolutionary Programming; 1994 Feb 24–26; San Diego, CA, USA.
39. De Castro LN, Timmis JI. Artificial immune systems as a novel soft computing paradigm. Soft Comput. 2003;7(8):526–44. doi:10.1007/s00500-002-0237-z.
40. Beyer HG, Schwefel HP. Evolution strategies—a comprehensive introduction. Nat Comput. 2002;1(1):3–52. doi:10.1023/A:1015059928466.
41. Simon D. Biogeography-based optimization. IEEE Trans Evol Comput. 2008;12(6):702–13. doi:10.1109/TEVC.2008.919004.
42. Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing. Science. 1983;220(4598):671–80. doi:10.1126/science.220.4598.671.
43. Rashedi E, Nezamabadi-Pour H, Saryazdi S. GSA: a gravitational search algorithm. Inf Sci. 2009;179(13):2232–48. doi:10.1016/j.ins.2009.03.004.
44. Shah-Hosseini H. Principal components analysis by the galaxy-based search algorithm: a novel metaheuristic for continuous optimisation. Int J Comput Sci Eng. 2011;6(1–2):132–40. doi:10.1504/IJCSE.2011.041221.
45. Hatamlou A. Black hole: a new heuristic optimization approach for data clustering. Inf Sci. 2013;222:175–84. doi:10.1016/j.ins.2012.08.023.
46. Mirjalili S, Mirjalili SM, Hatamlou A. Multi-verse optimizer: a nature-inspired algorithm for global optimization. Neural Comput Appl. 2016;27(2):495–513. doi:10.1007/s00521-015-1870-7.
47. Alatas B. ACROA: artificial chemical reaction optimization algorithm for global optimization. Expert Syst Appl. 2011;38(10):13170–80. doi:10.1016/j.eswa.2011.04.126.
48. Du H, Wu X, Zhuang J. Small-world optimization algorithm for function optimization. In: Advances in Natural Computation: Second International Conference, ICNC 2006; 2006 Sep 24–28; Xi'an, China.
49. Kaveh A, Khayatazad M. A new meta-heuristic method: ray Optimization. Comput Struct. 2012;112-113:283–94. doi:10.1016/j.compstruc.2012.09.003.
50. Tayarani NMH, Akbarzadeh TMR. Magnetic optimization algorithms a new synthesis. In: 2008 IEEE Congress on Evolutionary Computation (IEEE World Congress on Computational Intelligence); 2008 Jun 1–6; Hong Kong, China. doi:10.1109/CEC.2008.4631155.
51. Rao RV, Savsani VJ, Vakharia D. Teaching-learning-based optimization: a novel method for constrained mechanical design optimization problems. Comput-Aided Des. 2011;43(3):303–15. doi:10.1016/j.cad.2010.12.015.
52. Matoušová I, Trojovský P, Dehghani M, Trojovská E, Kostra J. Mother optimization algorithm: a new human-based metaheuristic approach for solving engineering optimization. Sci Rep. 2023;13(1):10312. doi:10.1038/s41598-023-37537-8.

53. Shi Y. Brain storm optimization algorithm. In: Advances in Swarm Intelligence: Second International Conference, ICSI 2011; 2011 Jun 12–15; Chongqing, China.
54. Ayyarao TL, RamaKrishna N, Elavarasam RM, Polumahanthi N, Rambabu M, Saini G, et al. War strategy optimization algorithm: a new effective metaheuristic algorithm for global optimization. IEEE Access. 2022;10(4):25073–105. doi:10.1109/ACCESS.2022.3153493.
55. Lawson HM. Working on hair. Qual Sociol. 1999;22(3):235–57. doi:10.1023/A:1022957805531.
56. Yao X, Liu Y, Lin G. Evolutionary programming made faster. IEEE Trans Evol Comput. 1999;3(2):82–102. doi:10.1109/4235.771163.
57. Awad N, Ali M, Liang J, Qu B, Suganthan P, Definitions P. Problem definitions and evaluation criteria for the CEC, 2017 special session and competition on single objective real-parameter numerical optimization. Technol Rep. 2016;1:8.
58. Wilcoxon F. Individual comparisons by ranking methods. In: Kotz S, Johnson N, editors. Breakthroughs in statistics. Berlin/Heidelberg, Germany: Springer; 1992. p. 196–202.
59. Mirjalili S, Lewis A. The whale optimization algorithm. Adv Eng Softw. 2016;95(12):51–67. doi:10.1016/j.advengsoft.2016.01.008.
60. Gandomi AH, Yang XS. Benchmark problems in structural optimization. In: Computational optimization, methods and algorithms. Berlin/Heidelberg, Germany: Springer; 2011. p. 259–81.
61. Mezura-Montes E, Coello CAC. Useful infeasible solutions in engineering optimization with evolutionary algorithms. In: Mexican International Conference on Artificial Intelligence; 2005 Nov 14–18; Monterret, Mexico.
62. Kannan B, Kramer SN. An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design. J Mech Des. 1994;116(2):405–11. doi:10.1115/1.2919393.