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ARTICLE

Magnificent Frigatebird Optimization: A New Bio-Inspired Metaheuristic Approach for Solving Optimization Problems

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ABSTRACT

This paper introduces a groundbreaking metaheuristic algorithm named Magnificent Frigatebird Optimization (MFO), inspired by the unique behaviors observed in magnificent frigatebirds in their natural habitats. The foundation of MFO is based on the kleptoparasitic behavior of these birds, where they steal prey from other seabirds. In this process, a magnificent frigatebird targets a food-carrying seabird, aggressively pecking at it until the seabird drops its prey. The frigatebird then swiftly dives to capture the abandoned prey before it falls into the water. The theoretical framework of MFO is thoroughly detailed and mathematically represented, mimicking the frigatebird's kleptoparasitic behavior in two distinct phases: exploration and exploitation. During the exploration phase, the algorithm searches for new potential solutions across a broad area, akin to the frigatebird scouting for vulnerable seabirds. In the exploitation phase, the algorithm fine-tunes the solutions, similar to the frigatebird focusing on a single target to secure its meal. To evaluate MFO's performance, the algorithm is tested on twenty-three standard benchmark functions, including unimodal, high-dimensional multimodal, and fixed-dimensional multimodal types. The results from these evaluations highlight MFO's proficiency in balancing exploration and exploitation throughout the optimization process. Comparative studies with twelve well-known metaheuristic algorithms demonstrate that MFO consistently achieves superior optimization results, outperforming its competitors across various metrics. In addition, the implementation of MFO on four engineering design problems shows the effectiveness of the proposed approach in handling real-world applications, thereby validating its practical utility and robustness.

KEYWORDS

Optimization; metaheuristic; magnificent frigatebirds; exploration; exploitation

1 Introduction

In the realm of scientific inquiry, optimization problems present a crucial challenge, defined by an objective function and constraints, with multiple feasible solutions. The quest for the optimal solution among these options is recognized as optimization. Across various domains like mathematics, engineering, industry, and economics, a myriad of optimization problems necessitate tailored solutions [1]. Approaches to solving these problems are broadly categorized into deterministic and stochastic methods [2]. Deterministic strategies, subdivided into gradient-based and non-gradient-based methods, excel in addressing linear, convex, continuous, and differentiable problems, particularly those with lower dimensions [3,4]. However, deterministic approaches falter in tackling higher-dimensional, non-linear, non-convex, and discontinuous problems often encountered in real-world applications, compelling the exploration of stochastic techniques [5,6].

Among stochastic approaches, metaheuristic algorithms stand out for their efficacy in navigating complex problem spaces through random search and trial-and-error processes [7]. Renowned for their conceptual simplicity, universality, and effectiveness in handling intricate, high-dimensional, non-deterministic polynomial (NP)-hard, and non-linear problems, metaheuristic algorithms have garnered substantial research interest [8]. These algorithms initiate optimization by randomly generating a set of candidate solutions, which are iteratively refined based on algorithmic instructions, ultimately converging to the best solution [9].

An effective metaheuristic algorithm must strike a delicate balance between global exploration and local exploitation [10]. Global exploration entails a comprehensive search of the problem space to avoid local optima and identify the main optimal region, while local exploitation focuses on refining solutions in promising areas to converge towards a global optimum. Despite the stochastic nature of metaheuristic algorithms, they provide near-optimal solutions without guaranteeing a global optimum [11]. Consequently, the quest for more effective optimization solutions has spurred the continual design of new metaheuristic algorithms.

The No Free Lunch (NFL) theorem underscores the necessity for diverse metaheuristic algorithms, as no single algorithm universally excels across all optimization problems [12]. While an algorithm may converge to a global optimum for one problem, it may fail for another. Therefore, the NFL theorem fosters ongoing exploration in metaheuristic algorithm design to devise more effective optimization strategies.

Although numerous metaheuristic algorithms have been designed and introduced, based on the best knowledge obtained from the literature review, it is confirmed that no metaheuristic algorithm has been designed based on simulating the natural behavior of magnificent frigatebirds. This is observed although the kleptoparasitic behavior of the magnificent frigatebird is an intelligent strategy that has a special potential for designing a new metaheuristic algorithm. Therefore, the originality and novelty of the design of the proposed approach are guaranteed.

The primary contribution of this paper is the introduction of Magnificent Frigatebird Optimization (MFO), a novel metaheuristic algorithm inspired by the behavior of magnificent frigatebirds in nature. Key features of MFO include its emulation of frigatebirds' kleptoparasitic behavior and its mathematical modeling, which encompasses exploration and exploitation phases. Evaluation of MFO's performance on standard benchmark functions demonstrates its efficacy in optimization tasks, outperforming twelve established metaheuristic algorithms.

The remainder of this paper unfolds as follows: Section 2 provides a literature review, Section 3 introduces and models the proposed MFO approach, Section 4 presents simulation studies and results, Section 5 evaluates the effectiveness of MFO for handling optimization tasks in real-world applications, and Section 6 concludes with reflections and suggestions for future research directions.

2 Literature Review

Metaheuristic algorithms are derived from a vast array of natural phenomena, encompassing the behaviors of living organisms, fundamental principles in physics, mathematics, and human decision-making strategies. These algorithms are typically organized into five primary categories: swarm-based approaches, which draw from the collective behavior observed in groups of animals, insects, or aquatic life; evolutionary-based approaches, which are inspired by the concepts of natural selection and genetics; physics-based approaches, which model various physical laws and phenomena; human-based approaches, which emulate human behavior and social dynamics in problem-solving scenarios, and mathematics-based approaches, which employ mathematical concepts and operators.

Swarm-based metaheuristic algorithms emulate the collective behaviors of animals, insects, and birds in nature. Notable examples include Particle Swarm Optimization (PSO) [13], Ant Colony Optimization (ACO) [14], Artificial Bee Colony (ABC) [15], and Firefly Algorithm (FA) [16]. These algorithms simulate behaviors such as foraging and communication, offering efficient search strategies for optimization problems. Frilled Lizard Optimization (FLO) is a recently published swarm-based approach whose design is inspired by the sit-and-wait strategy observed in frilled lizards during hunting [17]. Additionally, algorithms like the African Vultures Optimization Algorithm (AVOA) [18], White Shark Optimizer (WSO) [19], Beluga whale optimization [20], and Emperor Penguin Optimizer (EPO) [21] draw inspiration from various animal behaviors to enhance optimization performance.

Evolutionary-based metaheuristic algorithms are inspired by biological concepts such as natural selection and genetic evolution. Genetic Algorithm (GA) [22] and Differential Evolution (DE) [23] are prominent examples, mimicking reproduction and survival processes observed in nature. Other algorithms like Genetic Programming (GP) [24] leverage biological principles to explore optimization spaces effectively.

Physics-based metaheuristic algorithms simulate physical laws and phenomena to navigate optimization landscapes. Simulated Annealing (SA), for instance, replicates the annealing process in metallurgy, while algorithms like Spring Search Algorithm (SSA) [25] and Gravitational Search Algorithm (GSA) [26] simulate forces and transformations from physics. These algorithms capitalize on physical principles to guide search processes efficiently. Equilibrium Optimizer (EO) [27], and Water Flow Optimizer (WFO) [28] are other examples of physics-based metaheuristic algorithms.

Human-based metaheuristic algorithms model human behaviors and decision-making strategies. Teaching-Learning Based Optimization (TLBO) [29] mirrors the educational environment, while Following Optimization Algorithm (FOA) [30] replicates societal influences on individual progress. Language Education Optimization (LEO) [31] and Election Based Optimization Algorithm (EBOA) [32] draw inspiration from language learning and electoral processes, respectively. These algorithms harness human-centric strategies to tackle optimization challenges.

Mathematics-based metaheuristic algorithms have been developed by using the concepts and operators of mathematics. Arithmetic Optimization Algorithm (AOA) [33] and One-to-One Based Optimizer (OOBO) [34] are examples of algorithms of this group that are inspired by mathematics.

Despite the wealth of existing metaheuristic algorithms, none have been specifically designed based on the natural behavior of magnificent frigatebirds. The kleptoparasitic behavior of these birds presents an intelligent strategy ripe for algorithmic exploration. Thus, this paper introduces a new metaheuristic algorithm inspired by the mathematical modeling of magnificent frigatebird behavior, addressing a notable gap in existing research.

3 Magnificent Frigatebird Optimization

3.1 Inspiration

The Magnificent Frigatebird Optimization (MFO) is introduced in this section, drawing inspiration from the behavior of magnificent frigatebirds in their natural habitat. These seabirds, found in tropical and subtropical waters off the coasts of America, exhibit distinctive characteristics such as kleptoparasitic behavior. An image of the magnificent frigatebird is shown in Fig. 1.

Figure 1: Magnificent frigatebird took from: free media wikimedia commons

The magnificent frigatebird exhibits a kleptoparasitic strategy, engaging in behavior where it aggressively attacks and pecks at other seabirds, compelling them to relinquish their prey. Subsequently, the frigatebird swiftly descends towards the discarded food, seizing the prey before it descends to the water's surface. This kleptoparasitic behavior stands out prominently among the natural tendencies of the magnificent frigatebird. Leveraging the mathematical modeling of this kleptoparasitic behavior forms a foundational aspect of the design of the Magnificent Frigatebird Optimization (MFO), as elaborated below.

3.2 Initialization

MFO is a population-based metaheuristic algorithm designed for iterative optimization processes. Each magnificent frigatebird in the MFO population represents a candidate solution, with its position in the problem-solving space modeled as a vector according to Eq. (1). The initial positions of the frigatebirds are determined within specified bounds using Eq. (2).

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m} \tag{1}$$

$$X_i:\ x_{i,j} = lb_j + r \cdot (ub_j - lb_j),\ i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, m, \tag{2}$$

where X is the population matrix of the proposed MFO, N denotes the number of magnificent frigatebirds, m represents the number of decision variables, X_i is the ith magnificent frigatebird (i.e., a candidate solution), $x_{i,j}$ denotes its jth variable, r is a random number from the interval [0, 1], lb_j is a lower bound and ub_j is an upper bound on the jth decision variable.

Subsequently, the objective function corresponding to each frigatebird's position is evaluated that the set of these evaluated values for the objective function can be represented using a vector according to Eq. (3).

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1}, \tag{3}$$

where F is the vector of values of the objective function and F_i is the value of the objective function for the ith magnificent frigatebird (i.e., the candidate solution).

In the design of MFO, in each iteration, each magnificent frigatebird is updated in two phases of exploration and exploitation described below.

3.3 Phase 1: Selecting, Attacking, and Pecking at Seabirds Carrying Prey (Exploration Phase)

The kleptoparasitic behavior of the magnificent frigatebird is a defining characteristic of this species. This strategy begins with the magnificent frigatebird attacking another seabird that is carrying prey. The frigatebird repeatedly pecks at the seabird, forcing it to release the prey. This aggressive interaction results in substantial and abrupt movements of the magnificent frigatebird as it maneuvers to intercept the dropped prey. These significant displacements are mathematically modeled in the Magnificent Frigatebird Optimization (MFO) algorithm to simulate global search and exploration within the problem-solving space.

In the design of MFO, each magnificent frigatebird considers the positions of other population members that have achieved better objective function values as the targets, akin to seabirds carrying prey. The positions of these targeted birds for each frigatebird are determined using Eq. (4). The algorithm assumes that each magnificent frigatebird randomly selects one of these potential targets and initiates an attack. The movement of the magnificent frigatebird towards the selected seabird is then mathematically modeled, resulting in a new position, which is calculated using Eq. (5). If the new position yields an improved objective function value, this new position replaces the previous one as per the rules defined in Eq. (6).

$$CS_i = \{X_k,\quad F_k < F_i \text{ and } k \in \{1, 2, \ldots, N\}\},\ \text{where } i = 1, 2, \ldots, N, \tag{4}$$

$$x_{i,j}^{P1} = x_{i,j} + \left(1 - 2r_{i,j}\right) \cdot \left(SS_{i,j} - I_{i,j} \cdot x_{i,j}\right), \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} \le F_i, \\ X_i, & else, \end{cases} \tag{6}$$

where CS_i is the set of presumed food-carrying seabirds for the ith magnificent frigatebird, X_k denotes the kth row of the matrix X which has a better objective function value than the ith one, where CS_i is the set of presumed food-carrying seabirds for the ith magnificent frigatebird, SS_i represents the selected food-carrying seabird for the ith magnificent frigatebird, $SS_{i,j}$ gives its jth dimension, X_i^{P1} is the new position calculated for the ith magnificent frigatebird based on the first phase of the proposed MFO, $x_{i,j}^{P1}$ gives its jth dimension, F_i^{P1} denotes its objective function value, $r_{i,j}$ are random numbers from the interval [0, 1], and $I_{i,j}$ are numbers which are randomly selected as 1 or 2.

3.4 Phase 2: Diving towards the Dropped Prey (Exploitation Phase)

In the second stage of the kleptoparasitic behavior, the magnificent frigatebird dives toward the prey released by the attacked seabird, catching it before it reaches the water's surface. This behavior is simulated in the Magnificent Frigatebird Optimization (MFO) algorithm through small, precise displacements near the bird's current position. These minor adjustments are mathematically modeled to facilitate local search and exploitation within the problem-solving space.

In the design of MFO, these displacements are assumed to occur within a neighborhood centered around the magnificent frigatebird, with a radius defined by $(\frac{best_j - x_{i,j}}{t})$, where $best_j$ represents the best-known position in the jth dimension, $x_{i,j}$ is the current position of the ith bird in the jth dimension, and t is a control parameter. To model this process, a new position for each magnificent frigatebird is calculated based on its simulated movement toward the abandoned prey, using Eq. (7). If this new position yields an improved value for the objective function, it replaces the bird's previous position according to the update rule in Eq. (8). This method ensures that the MFO algorithm effectively performs local search, refining solutions and enhancing the optimization process by exploiting promising areas in the search space.

$$x_{i,j}^{P2} = x_{i,j} + \left(1 - 2r_{i,j}\right) \cdot \frac{best_j - x_{i,j}}{t} \tag{7}$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} \le F_i \\ X_i, & else \end{cases} \tag{8}$$

where X_i^{P2} is the new position calculated for the ith magnificent frigatebird based on the second phase of the proposed MFO, $x_{i,j}^{P2}$ represents its jth dimension, $best_j$ is the jth dimension of the best member, F_i^{P2} denotes its objective function value, $r_{i,j}$ are random numbers from the interval [0, 1], and t is the iteration counter.

3.5 Repetition Process, Flowchart, and Pseudo-Code of MFO

Upon completing the first iteration, the Magnificent Frigatebird Optimization (MFO) algorithm updates the positions of all magnificent frigatebirds by applying both the exploration and exploitation phases. After these updates, the algorithm proceeds to the next iteration using the newly acquired values. This iterative process continues, adhering to the guidelines set forth by Eqs. (4) to (8) until the predetermined maximum number of iterations is reached.

Throughout each iteration, the best solution found so far is continually updated based on the latest results. At the end of the entire iterative process, MFO provides the best solution obtained during its execution as the final solution to the problem. The steps of the MFO implementation are presented in the form of a flowchart in Fig. 2, and its pseudo-code is presented in Algorithm 1.

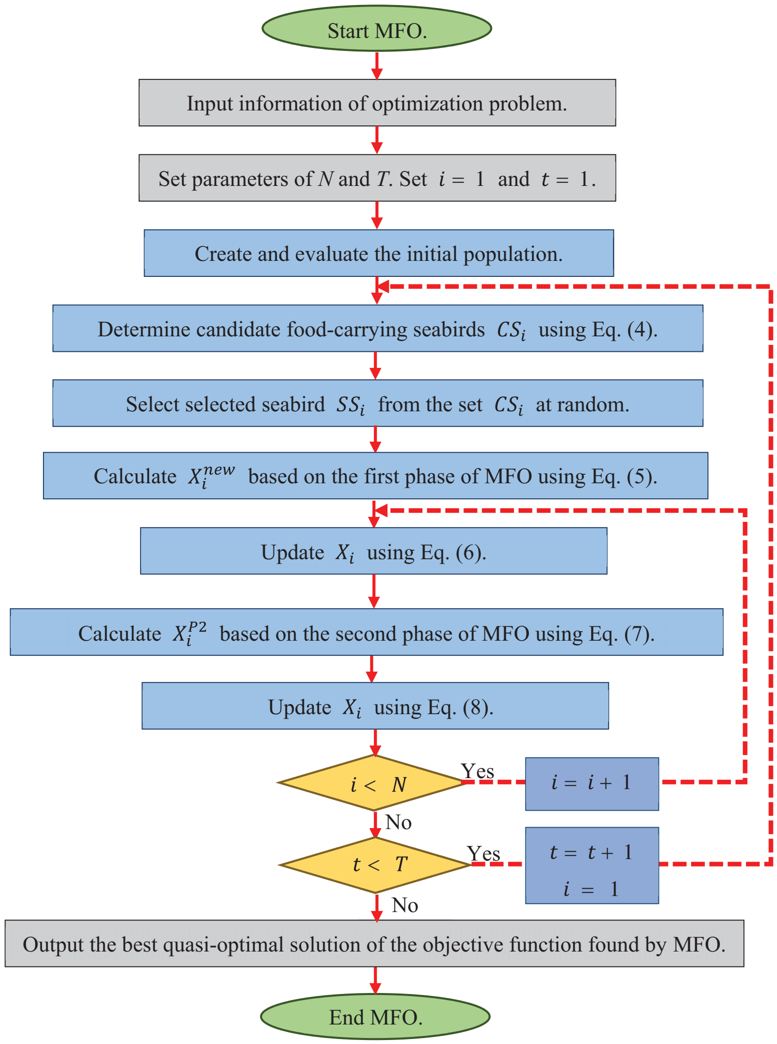

Figure 2: Flowchart of the proposed MFO

Algorithm 1: Pseudo-code of the proposed MFO

Start MFO.
1. Input problem details.
2. Specify iteration count (T) and magnificent frigatebird count (N).
3. Randomly generate initial population by Eq. (2).
4. Assess initial population.
5. For $t = 1: T$

(Continued)

Algorithm 1 (continued)

6. For $i = 1 : N$
7. **Phase 1:** ***Selecting, Attacking, and Pecking at Seabirds Carrying Prey***.
8. Identify candidate food-carrying seabirds for the ith frigatebird based on Eq. (4).
9. Compute new position of ith frigatebird via Eq. (5).
10. Update position of ith frigatebird using Eq. (6).
11. **Phase 2:** *Diving Towards Dropped Prey*.
12. Calculate a new proposed position for the ith magnificent frigatebird based on Eq. (7)
13. Update the position of ith magnificent frigatebird using Eq. (8).
14. end
15. Save best solution attained.
16. end
17. Output best solution obtained.

End MFO.

3.6 Computational Complexity of MFO

In this subsection, the computational complexity of MFO is evaluated. The preparation and initialization process of MFO has a complexity equal to $O(Nm)$, where N is the number of magnificent frigatebirds and m is the number of decision variables of the problem. Each magnificent frigatebird is updated in each iteration during two phases of exploration and exploitation. Therefore, the process of updating magnificent frigatebirds has a complexity equal to $O(2NmT)$, where T is the maximum number of iterations of the algorithm. Therefore, the computational complexity of MFO is $O(Nm(1+2T))$.

4 Simulation Studies

In this section, the effectiveness of the Magnificent Frigatebird Optimization (MFO) algorithm in addressing optimization problems is thoroughly evaluated. For this assessment, a set of twenty-three standard benchmark functions, which include unimodal, high-dimensional multimodal, and fixed-dimensional multimodal types [35] is utilized. The mathematical model, the best value (which is denoted by the symbol $\boldsymbol{F_{min}}$), and information about these functions are provided in Appendix A and Tables A1 to A3. The performance of MFO is compared against twelve well-known metaheuristic algorithms: Genetic Algorithm (GA) [22], PSO [13], GSA [26], TLBO [29], Multi-Verse Optimization (MVO) [36], Grey Wolf Optimizer (GWO) [37], Whale optimization Algorithm (WOA) [38], Marine Predators Algorithm (MPA) [39], Tunicate Swarm Algorithm (TSA) [40], Rivest-Shamir-Adleman (RSA) [41], AVOA [18], and WSO [19]. The values of the control parameters of the metaheuristic algorithms are specified in Table 1. The optimization results are reported using six statistical indicators: mean, best, worst, standard deviation, median, and rank. The ranking criterion of the metaheuristic algorithms for each of the benchmark functions is the value of the mean index.

Table 1: Parameter values for the competitive algorithms

Algorithm	Parameter	Value
GA		
	Type	Real coded.
	Selection	Roulette wheel (Proportionate).
	Crossover	Whole arithmetic (Probability = 0.8, $\alpha \in [-0.5, 1.5]$).
	Mutation	Gaussian (Probability = 0.05).
PSO		
	Topology	Fully connected.
	Cognitive and social constant	$(C_1, C_2) = (2, 2)$.
	Inertia weight	Linear reduction from 0.9 to 0.1.
	Velocity limit	10% of the dimension range.
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1.
TLBO		
	T_F: the teaching factor	$T_F = \text{round}\,[(1 + rand)]$.
	Random number *rand*	*rand* is a random number from the interval $[0, 1]$.
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
MVO		
	Wormhole existence probability (WEP)	$\text{Min}(WEP) = 0.2$ and $\text{Max}(WEP) = 1$.
	Exploitation accuracy over the iterations (p)	$p = 6$.
WOA		
	Convergence parameter a	a: Linear reduction from 2 to 0.
	Parameters r and l	r is a random vector in $[0, 1]$, l is a random number in $[-1, 1]$.
TSA		
	P_{min} and P_{max}	1, 4.
	c_1, c_2, c_3	Random numbers lie in the range $[0, 1]$.
MPA		
	Constant number	$P = 0.5$.
	Random vector	R is a vector of uniform random numbers from $[0, 1]$.
	Fish aggregating devices ($FADs$)	$FADs = 0.2$.
	Binary vector	$U = 0$ or 1.
RSA		
	Sensitive parameter	$\beta = 0.01$.
	Sensitive parameter	$\alpha = 0.1$.

(Continued)

Table 1 (continued)

Algorithm	Parameter	Value
	Evolutionary sense (ES)	ES are randomly decreasing values between 2 and −2.
AVOA		
	L_1, L_2	$(L_1, L_2) = (0.8, 0.2)$.
	w	$w = 2.5$.
	P_1, P_2, P_3	$(P_1, P_2, P_3) = (0.6, 0.4, 0.6)$.
WSO		
	F_{min} and F_{max}	$(F_{min}, F_{max}) = (0.07, 0.75)$.
	τ, a_0, a_1, a_2	$(\tau, a_0, a_1, a_2) = (4.125, 6.25, 100, 0.0005)$.

4.1 Results for Unimodal Objective Functions

Table 2 illustrates the performance outcomes for unimodal functions F1 through F7, comparing MFO with various competing algorithms. Since unimodal functions F1 to F7 are devoid of local optima, they provide an ideal testbed for evaluating the local search and exploitation efficiencies of metaheuristic algorithms. Impressively, MFO exhibits exceptional exploitation capabilities, successfully converging to the global optimum for functions F1, F2, F3, F4, F5, and F6. Notably, MFO outperforms all other algorithms as the leading optimizer for function F7. These results highlight MFO's proficiency in managing unimodal functions, surpassing its competitors with its strong exploitation prowess.

Table 2: Performance of the metaheuristic algorithms for unimodal functions

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
F1	Mean	0	0	63.71104	1.85E−49	0.144648	0	4.50E−47	1.29E−16	1.30E−151	0.097592	1.71E−59	2.44E−74	29.48527
	Best	0	0	5.118651	3.68E−52	0.101992	0	1.40E−50	5.18E−17	9.00E−171	0.00047	1.44E−61	5.67E−77	17.3294
	Worst	0	0	230.9467	1.60E−48	0.194588	0	3.19E−46	3.61E−16	2.60E−150	1.351153	7.46E−59	2.51E−73	55.03039
	Std	0	0	52.03747	3.88E−49	0.027372	0	9.87E−47	7.06E−17	5.90E−151	0.30646	2.11E−59	6.07E−74	10.31744
	Median	0	0	43.90597	4.02E−50	0.14551	0	4.13E−48	1.09E−16	2.10E−159	0.009396	1.04E−59	1.64E−75	27.25901
	Rank	1	1	11	5	9	1	6	7	2	8	4	3	10
F2	Mean	0	1.10E−276	2.068512	6.73E−28	0.250534	0	2.04E−28	5.30E−08	2.40E−105	0.865655	1.31E−34	6.54E−39	2.695448
	Best	0	1.30E−306	0.640394	1.78E−29	0.154739	0	1.96E−30	3.37E−08	7.60E−118	0.043772	4.71E−36	8.53E−40	1.687178
	Worst	0	2.10E−275	7.197314	4.55E−27	0.35236	0	1.76E−27	1.19E−07	2.70E−104	2.410205	7.65E−34	2.36E−38	3.679671
	Std	0	0	1.749618	1.08E−27	0.062116	0	5.22E−28	1.85E−08	6.80E−105	0.712678	1.93E−34	5.50E−39	0.537216
	Median	0	6.30E−290	1.479446	3.40E−28	0.259403	0	1.91E−29	4.96E−08	3.30E−108	0.564692	6.29E−35	4.81E−39	2.65017
	Rank	1	2	11	7	9	1	6	8	3	10	5	4	12
F3	Mean	0	0	1726.767	2.43E−12	15.44089	0	1.14E−10	459.6498	19293.91	375.1938	2.10E−14	3.71E−24	2096.683
	Best	0	0	1005.766	5.98E−19	5.775132	0	1.33E−21	237.7651	1996.051	21.04265	2.28E−19	2.13E−29	1376.714
	Worst	0	0	3425.01	1.39E−11	47.30844	0	1.89E−09	1146.773	33532.16	991.2134	3.91E−13	3.49E−23	3343.637
	Std	0	0	619.0675	4.32E−12	10.61524	0	4.30E−10	217.222	8438.217	284.4219	8.89E−14	1.07E−23	630.8007
	Median	0	0	1506.347	1.77E−13	11.48329	0	1.04E−13	386.9903	19646.79	283.2763	4.51E−16	3.91E−26	2030.676
	Rank	1	1	9	4	6	1	5	8	11	7	3	2	10
F4	Mean	0	3.10E−265	16.71946	2.88E−19	0.52888	0	0.004275	1.194685	50.09396	6.070553	1.19E−14	1.78E−30	2.735082
	Best	0	0	11.51766	2.92E−20	0.257061	0	9.33E−05	9.57E-09	0.874419	2.213925	6.34E−16	5.62E−32	2.142586
	Worst	0	4.40E−264	23.04121	9.29E−19	0.930945	0	0.034633	4.763438	88.65274	12.9149	5.55E−14	7.85E−30	3.859647
	Std	0	0	2.84729	2.26E−19	0.189536	0	0.007834	1.367867	29.20309	2.4676	1.44E−14	2.36E−30	0.460447
	Median	0	1.90E−282	17.18027	2.50E−19	0.513343	0	0.001421	0.876717	53.57697	5.686389	6.14E−15	6.31E−31	2.690695
	Rank	1	2	11	4	7	1	6	8	12	10	5	3	9
F5	Mean	0	1.38E−05	10439.42	22.54652	93.01417	12.56534	27.5281	42.58157	26.39938	4458.203	25.69554	25.895	575.5392
	Best	0	1.34E−06	1302.4	22.04833	26.71068	8.41E−29	24.81534	25.02178	25.83133	25.40496	24.71433	24.73574	221.1811

(Continued)

Table 2 (continued)

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
	Worst	0	5.71E−05	89625.35	23.24763	365.3073	28.02387	27.92861	161.6694	27.77752	87074.7	26.25085	27.79425	2181.823
	Std	0	1.43E−05	19789.47	0.383231	100.0539	14.5397	0.777176	43.70737	0.569688	19837.02	0.519015	0.923329	419.0801
	Median	0	9.07E−06	5422.705	22.51843	29.01745	1.18E−28	27.86183	25.4682	26.18393	83.2281	25.35729	25.45025	459.7206
	Rank	1	2	13	4	10	3	8	9	7	12	5	6	11
F6	Mean	0	4.81E−08	97.54328	1.75E−09	0.145969	6.242621	3.559177	1.01E−16	0.078854	0.061331	0.638821	1.219358	33.00921
	Best	0	6.87E−09	16.3879	7.81E−10	0.076592	3.541149	2.467719	5.34E−17	0.01017	1.84E−06	0.238505	0.22535	15.09203
	Worst	0	1.32E−07	369.7445	4.64E−09	0.241773	7.008337	4.628086	1.75E−16	0.315856	0.523673	1.210535	2.092633	60.67478
	Std	0	3.24E−08	94.15036	9.23E−10	0.046722	1.013656	0.683722	3.66E−17	0.100208	0.146498	0.302348	0.490315	13.36167
	Median	0	4.46E−08	67.25731	1.54E−09	0.154817	6.655456	3.669462	9.16E−17	0.030554	0.001989	0.703072	1.176844	30.62611
	Rank	1	4	13	3	7	11	10	2	6	5	8	9	12
F7	Mean	2.54E−05	6.13E−05	8.79E−05	0.000529	0.011228	2.99E−05	0.004199	0.05105	0.001236	0.178004	0.000804	0.001479	0.010236
	Best	2.35E−06	1.40E−06	1.12E−05	0.000108	0.00384	3.61E−06	0.001444	0.013655	2.08E−05	0.066717	0.000176	8.74E−05	0.002932
	Worst	6.89E−05	0.000255	0.000328	0.00087	0.021817	0.000129	0.009642	0.092389	0.00522	0.397642	0.001892	0.002849	0.021209
	Std	2.02E−05	7.23E−05	8.82E−05	0.000212	0.004964	3.41E−05	0.002309	0.024611	0.001425	0.077917	0.00046	0.000867	0.004752
	Median	1.83E−05	3.94E−05	6.31E−05	0.000516	0.010939	1.51E−05	0.003598	0.050106	0.000792	0.171807	0.000817	0.001457	0.00984
	Rank	1	3	4	5	11	2	9	12	7	13	6	8	10
Sum rank		7	15	72	32	59	20	50	54	48	65	36	35	74
Mean rank		1	2.142857	10.28571	4.571429	8.428571	2.857143	7.142857	7.714286	6.857143	9.285714	5.142857	5	10.57143
Total ranking		1	2	12	4	10	3	8	9	7	11	6	5	13

4.2 Results for High-Dimensional Multimodal Objective Functions

Table 3 showcases the performance results for functions F8 to F13, evaluated using the MFO algorithm and its competing counterparts. These functions, characterized by numerous local optima alongside the primary optima, serve as ideal benchmarks for assessing the global search and discovery capabilities of metaheuristic algorithms. The findings underscore MFO's exceptional discovery abilities, particularly its successful convergence to the global optimum for functions F9 and F11, and its effective identification of the primary optimal region within the problem-solving space. Furthermore, MFO distinguishes itself as the leading optimizer for functions F8, F10, F12, and F13. These results highlight MFO's superior performance in managing high-dimensional multimodal functions, showcasing its robust exploration capabilities and its ability to consistently outperform other algorithms.

Table 3 : Performance of metaheuristic algorithms for high-dimensional multimodal functions

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
F8	Mean	−12498.6	−12471.6	−7232.86	-9.78E+03	−7988.47	−5671.63	−6.35E+03	−3.11E+03	−1.11E+04	−6745.79	−6.29E+03	−5.83E+03	−8557.4
	Rank	1	2	7	4	6	12	9	13	3	8	10	11	5
	Std	194.2272	192.1199	725.2644	3.64E+02	720.1164	222.0492	7.21E+02	4.91E+02	1.71E+03	735.248	4.72E+02	6.02E+02	632.7039
	Best	−12622.8	−12571.3	−9119.64	-1.05E+04	−9302.53	−5888.22	−7.49E+03	−4.26E+03	−1.26E+04	−8371.19	−7.04E+03	−7.21E+03	−9777.72
	Median	−12577.8	−12567.9	−7160.74	−9.81E+03	−7871.29	−5716.74	−6.31E+03	−3.02E+03	−1.21E+04	−6887.85	−6.29E+03	−5.84E+03	−8538.1
	Worst	−11936.3	−11919.4	−6290.08	−9.20E+03	−7069.45	−5156.23	−4.64E+03	−2.47E+03	−7.90E+03	−5242.88	−5.30E+03	−4.82E+03	−7213.97
F9	Mean	0	0.00E+00	23.80915	0.00E+00	94.56873	0	1.67E+02	2.76E+01	0.00E+00	65.45725	1.65E−14	0.00E+00	52.85852
	Rank	1	1	3	1	7	1	8	4	1	6	2	1	5
	Std	0	0	8.498359	0.00E+00	24.84678	0	5.03E+01	9.04E+00	0.00E+00	18.57926	3.20E−14	0.00E+00	13.61568
	Best	0	0.00E+00	14.13232	0.00E+00	51.02728	0	8.68E+01	1.35E+01	0.00E+00	38.47175	0.00E+00	0.00E+00	22.45798
	Median	0	0.00E+00	21.93243	0.00E+00	93.84687	0	1.61E+02	2.55E+01	0.00E+00	62.89961	0.00E+00	0.00E+00	50.86062
	Worst	0	0.00E+00	44.41892	0.00E+00	144.3046	0	2.79E+02	4.71E+01	0.00E+00	110.7433	1.10E−13	0.00E+00	74.3375
F10	Mean	8.88E−16	8.88E−16	5.115004	4.15E−15	0.558636	8.88E−16	1.20E+00	7.94E−09	3.98E−15	2.636326	1.62E−14	4.32E−15	3.45593
	Rank	1	1	11	3	7	1	8	6	2	9	5	4	10
	Std	0	0	1.204492	7.83E−16	0.667773	0	1.55E+00	2.31E−09	2.24E−15	0.845876	3.50E−15	0.00E+00	0.391131
	Best	8.88E−16	8.88E−16	3.270175	8.88E−16	0.097247	8.88E−16	7.76E−15	4.50E−09	8.88E−16	1.637001	7.76E−15	4.32E−15	2.785897
	Median	8.88E−16	8.88E−16	5.00683	4.32E−15	0.187838	8.88E−16	2.15E−14	7.47E−09	4.32E−15	2.64279	1.46E−14	4.32E−15	3.508594
	Worst	8.88E−16	8.88E−16	7.925416	4.32E−15	2.431349	8.88E−16	3.26E+00	1.40E−08	7.76E−15	4.888503	2.15E−14	4.32E−15	4.487235
F11	Mean	0	0.00E+00	1.658952	0.00E+00	0.386353	0	0.008548	6.967747	0	0.17909	1.30E−03	0.00E+00	1.424355

(Continued)

Table 3 (continued)

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
	Rank	1	1	7	1	5	1	3	8	1	4	2	1	6
	Std	0	0	0.535069	0.00E+00	0.080719	0	0.006206	2.68309	0	0.225311	4.42E−03	0.00E+00	0.122147
	Best	0	0	1.067082	0.00E+00	0.245676	0	0.00E+00	2.90E+00	0	0.002288	0.00E+00	0.00E+00	1.245159
	Median	0	0.00E+00	1.547617	0.00E+00	0.402634	0	0.008694	7.067426	0	0.118278	0.00E+00	0.00E+00	1.399452
	Worst	0	0.00E+00	3.175238	0.00E+00	0.51812	0	0.019862	12.21652	0	0.846654	1.82E−02	0.00E+00	1.66833
F12	Mean	1.57E−32	2.49E−09	3.160713	1.97E−10	0.884154	1.273695	5.599698	0.203036	0.019427	1.451023	0.038549	0.068951	0.265731
	Rank	1	3	12	2	9	10	13	7	4	11	5	6	8
	Std	2.86E−48	1.63E−09	1.803987	9.47E−11	1.180104	0.299644	3.826503	0.303146	0.039446	1.267759	0.021036	0.020659	0.136721
	Median	1.57E−32	2.31E−09	2.795586	1.99E−10	0.40627	1.34E+00	4.161408	0.077525	0.005591	1.242425	0.036647	0.0664	0.255609
	Best	1.57E−32	3.90E−10	0.921364	5.02E−11	0.000966	7.44E−01	1.002296	4.59E−19	0.001186	0.000103	0.012144	0.023306	0.058812
	Worst	1.57E−32	7.57E−09	7.142397	3.69E−10	3.719777	1.591042	13.6648	0.900712	0.132338	5.045246	0.083891	0.130631	0.629147
F13	Mean	1.35E−32	9.69E−09	3479.692	2.42E−03	0.031682	3.03E−31	2.626328	5.48E−02	0.207451	3.487367	0.496693	1.065263	2.617574
	Rank	1	3	13	4	5	2	11	6	7	12	8	9	10
	Std	2.86E−48	8.66E−09	13661.44	6.26E−03	0.024442	2.22E−31	0.549782	2.11E−01	0.18097	2.988888	0.254246	0.228154	0.74399
	Best	1.35E−32	1.11E−09	13.33768	9.62E−10	0.006227	6.35E−32	1.945369	4.50E−18	0.035963	9.25E−03	4.53E−05	0.568876	1.248893
	Median	1.35E−32	6.30E-09	42.7561	2.73E−09	0.022846	3.88E−31	2.450665	1.72E−17	0.160271	3.195604	0.499913	1.077463	2.771648
	Worst	1.35E−32	3.68E−08	60089.28	2.45E−02	0.088573	5.26E−31	3.590139	9.26E−01	0.677	12.16611	0.918449	1.489831	3.80889
Sum rank		6	11	53	15	39	27	52	44	18	50	32	32	44
Mean rank		1	1.833333	8.833333	2.5	6.5	4.5	8.666667	7.333333	3	8.333333	5.333333	5.333333	7.333333
Total ranking		1	2	11	3	7	5	10	8	4	9	6	6	8

4.3 Results for Fixed-Dimensional Multimodal Objective Functions

Table 4 presents the outcomes of evaluating the MFO algorithm alongside its competitors for functions F14 to F23. These functions are designed to test the balance between exploration and exploitation in metaheuristic algorithms. The results highlight MFO's effectiveness, establishing it as the top optimizer across all functions F14 to F23. Even in cases where MFO matches other algorithms in mean index value, it consistently outperforms them in the standard deviation (std) index, indicating more reliable and consistent performance. These findings emphasize MFO's exceptional capability to balance exploration and exploitation, yielding superior results in comparison to other algorithms when dealing with multi-modal functions of fixed dimensions.

Table 4: Performance of metaheuristic algorithms for fixed-dimensional multimodal functions

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
F14	Mean	0.998004	1.094295	1.094486	1.009791	0.998397	3.037353	8.392305	3.476264	2.517753	3.509593	3.605663	0.998398	1.047371
	Rank	1	6	7	4	2	9	13	10	8	11	12	3	5
	Std	0	0.437402	0.301571	0.053711	1.79E−03	3.01391	4.981633	2.715566	2.905095	3.736073	3.678825	1.79E-03	0.218891
	Best	0.998004	0.998004	0.998004	0.998004	0.998004	0.998034	1.958897	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
	Median	0.998004	0.998004	0.998004	0.998004	0.998004	2.18421	11.35955	2.828583	0.998008	1.958898	2.915968	0.998004	0.998006
	Worst	0.998004	2.915968	1.958897	1.233486	1.005853	12.28142	15.02814	11.50749	10.43767	12.28142	10.43768	1.005853	1.958903
F15	Mean	0.000307	0.000384	0.001352	0.001207	0.002599	0.001126	0.015919	0.002314	0.000822	0.002456	0.003294	0.000615	0.014916
	Rank	1	2	7	6	10	5	13	8	4	9	11	3	12
	Std	2.59E−19	9.47E−05	0.00442	0.000558	0.005974	0.000461	0.029608	0.001352	0.000488	0.006043	0.007221	0.000395	0.015999
	Best	0.000307	0.000311	0.000308	0.000309	0.000311	0.000743	0.000311	0.000871	0.000325	0.000308	0.000311	0.000317	0.000812
	Median	0.000307	0.000354	0.000353	0.0016	0.000693	0.001024	0.000867	0.002148	0.000688	0.000353	0.000353	0.000363	0.01383
	Worst	0.000307	0.000718	0.01974	0.001674	0.019711	0.00284	0.106662	0.006783	0.002233	0.01974	0.01974	0.001264	0.064742
F16	Mean	−1.03163	−1.03155	−1.03155	−1.02929	−1.03155	−1.02941	−1.03002	−1.03155	−1.03155	−1.03155	−1.03155	−1.03155	−1.03155
	Rank	1	2	6	11	5	10	9	2	3	2	4	8	7
	Std	1.87E−16	2.35E−04	2.35E−04	0.007045	2.35E-04	0.006888	0.006966	2.35E−04	2.35E−04	2.35E−04	2.35E−04	2.35E−04	2.34E−04
	Best	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03161	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163
	Median	−1.03163	−1.03163	−1.03163	−1.0316	−1.03163	−1.03129	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163	−1.03163
	Worst	−1.03163	−1.03061	−1.03061	−1.00093	−1.03061	−1.00105	−1.00102	−1.03061	−1.03061	−1.03061	−1.03061	−1.0306	−1.03061
F17	Mean	0.397887	0.397904	0.397905	0.398401	0.397905	0.410187	0.39794	0.397904	0.397905	0.733096	0.397905	0.397975	0.463769
	Rank	1	2	4	9	3	10	7	2	5	12	6	8	11
	Std	0	3.25E−05	3.25E−05	0.000975	3.25E−05	0.019174	7.05E-05	3.25E−05	3.25E−05	0.699436	3.25E−05	6.90E-05	0.29852

(Continued)

Table 4 (continued)

F		MFO	AVOA	WSO	MPA	MVO	RSA	TSA	GSA	WOA	PSO	GWO	TLBO	GA
	Best	0.397887	0.397887	0.397887	0.397887	0.397887	0.398612	0.397891	0.397887	0.397887	0.397887	0.397888	0.397895	0.397887
	Median	0.397887	0.39789	0.39789	0.397974	0.39789	0.403587	0.397911	0.39789	0.397891	0.3979	0.39789	0.39797	0.397956
	Worst	0.397887	0.397996	0.397996	0.401154	0.397996	0.482351	0.398195	0.397996	0.397997	2.711409	0.397997	0.398163	1.707037
F18	Mean	3	3.10539	3.105389	6.161661	3.105389	5.787686	11.32378	3.105389	3.105414	3.105389	3.105401	3.10539	7.264861
	Rank	1	6	2	11	5	10	13	4	9	3	8	7	12
	Std	1.19E−15	2.16E−01	2.16E−01	6.48712	2.16E−01	8.388869	25.80118	2.16E−01	2.16E−01	2.16E−01	2.16E−01	2.16E−01	10.35747
	Best	3	3.000465	3.000464	3.013933	3.000465	3.00123	3.000472	3.000464	3.000465	3.000464	3.000468	3.000466	3.001383
	Median	3	3.018789	3.018788	3.563655	3.018789	3.05925	3.059243	3.018788	3.018796	3.018788	3.01881	3.018789	3.076207
	Worst	3	3.900043	3.900043	30.00128	3.900043	30.37419	89.07078	3.900043	3.900044	3.900043	3.900054	3.900045	33.90749
F19	Mean	−3.86278	−3.85818	−3.85818	−3.72483	−3.85818	−3.83319	−3.8578	−3.85818	−3.8559	−3.85818	−3.85671	−3.85712	−3.85802
	Rank	1	3	2	11	4	10	6	2	9	2	8	7	5
	Std	2.32E−15	4.67E−03	4.67E−03	0.140189	4.67E-03	0.022469	0.004646	4.67E-03	0.004935	4.67E-03	0.005119	0.004533	0.004817
	Best	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.85462	−3.86267	−3.86278	−3.86276	−3.86278	−3.86278	−3.86262	−3.86276
	Median	−3.86278	−3.85821	−3.85821	−3.72574	−3.85821	−3.8384	−3.8579	−3.85821	−3.85577	−3.85821	−3.85718	−3.8571	−3.85814
	Worst	−3.86278	−3.84379	−3.84379	−3.2931	−3.84379	−3.77725	−3.8437	−3.84379	−3.84337	−3.84379	−3.84367	−3.84354	−3.84346
F20	Mean	−3.322	−3.24396	−3.27844	−2.53258	−3.24961	−2.75749	−3.23097	−3.29568	−3.22598	−3.24022	−3.23481	−3.21907	−3.20511
	Rank	1	5	3	13	4	12	8	2	9	6	7	10	11
	Std	4.53E−16	0.061068	0.043697	0.343753	0.063354	0.310937	0.068242	1.15E-02	0.08269	0.078508	0.079053	0.083616	0.073815
	Best	−3.322	−3.30645	−3.31876	−3.22483	−3.31876	−3.05377	−3.31656	−3.31876	−3.31327	−3.31876	−3.31875	−3.3014	−3.29077
	Median	−3.322	−3.27557	−3.29621	−2.58954	−3.29198	−2.83137	−3.23001	−3.29758	−3.27301	−3.28752	−3.27557	−3.25326	−3.21757
	Worst	−3.322	−3.16228	−3.17906	−1.78365	−3.15575	−1.68042	−3.07807	−3.27072	−3.06908	−3.099	−3.05099	−2.97928	−2.98892
F21	Mean	−10.1532	−10.0667	−8.37825	−7.55876	−8.84127	−5.13865	−5.97966	−7.20628	−9.32454	−5.68831	−9.32929	−6.87626	−6.30352
	Rank	1	2	6	7	5	13	11	8	4	12	3	9	10
	Std	2.12E−15	6.98E−02	3.115756	2.093732	2.21318	6.98E-02	3.201509	3.426227	1.842108	2.834435	1.832443	2.045191	2.715594
	Best	−10.1532	−10.1531	−10.1496	−10.1515	−10.1531	−5.22507	−10.1267	−10.1531	−10.1524	−10.1469	−10.153	−9.40037	−9.67362
	Median	−10.1532	−10.0781	−10.0515	−7.90122	−10.011	−5.15006	−5.05576	−9.98327	−10.0486	−5.12189	−10.0508	−7.23947	−7.06123
	Worst	−10.1532	−9.98327	−2.76194	−5.0552	−5.05519	−5.0552	−2.69157	−2.76194	−5.06542	−2.72152	−5.09897	−3.39667	−2.47476
F22	Mean	−10.4029	−10.3258	−9.95603	−8.0897	−8.4232	−5.18774	−6.92458	−10.0613	−8.10789	−6.43982	−10.3253	−7.95447	−7.3958
	Rank	1	2	5	8	6	13	11	4	7	12	3	9	10
	Std	3.58E−15	7.12E−02	1.672377	2.134656	2.734411	7.12E-02	3.503065	1.204233	3.000267	3.441722	0.071209	1.656718	1.915935
	Best	−10.4029	−10.4029	−10.4029	−10.4005	−10.3929	−5.26476	−10.3285	−10.4029	−10.3921	−10.3945	−10.4026	−9.95397	−9.98558
	Median	−10.4029	−10.3577	−10.3191	−9.04577	−10.2463	−5.21961	−7.57779	−10.3312	−10.2225	−5.16087	−10.3574	−8.30637	−7.84449
	Worst	−10.4029	−10.2258	−2.99691	−5.08767	−2.96311	−5.08767	−1.94131	−5.05462	−1.96279	−2.84329	−10.2244	−4.14987	−2.75718
F23	Mean	−10.5364	−10.4903	−10.4903	−9.15341	−9.45157	−5.26264	−7.47297	−10.2496	−8.60245	−6.51191	−10.4899	−8.12171	−6.45327
	Rank	1	3	2	7	6	13	10	5	8	11	4	9	12
	Std	2.82E−15	5.01E−02	5.01E-02	1.503609	2.185049	5.01E-02	3.406413	1.0987	3.232484	3.793662	0.050104	1.633702	2.557243
	Best	−10.5364	−10.5335	−10.5335	−10.4492	−10.5335	−5.30583	−10.441	−10.5335	−10.5325	−10.5301	−10.5332	−9.7088	−10.016
	Median	−10.5364	−10.5034	−10.5034	−9.54713	−10.4927	−5.27576	−10.2177	−10.5034	−10.484	−4.01965	−10.5032	−8.6941	−6.97761
	Worst	−10.5364	−10.3561	−10.3561	−5.12848	−5.17406	−5.12847	−2.67772	−5.67849	−1.92102	−2.62894	−10.3558	−4.4165	−2.62735
Sum rank		10	33	44	87	50	105	101	47	66	80	66	73	95
Mean rank		1	3.3	4.4	8.7	5	10.5	10.1	4.7	6.6	8	6.6	7.3	9.5
Total ranking		1	2	3	9	5	12	11	4	6	8	6	7	10

The convergence curves of MFO and the competing algorithms in handling functions F1 to F23 are drawn in Fig. 3. The convergence analysis shows that when dealing with unimodal functions F1 to F7, where these functions have no local optimum, MFO has identified the main optimum region in the initial iterations and is converging towards the global optimum with high exploitation ability. When dealing with multimodal functions F8 to F23, where these functions have local optima, the convergence curves show that MFO with the exploration ability, during successive repetitions of the algorithm, has tried to identify the main optimal area by escaping from local optima and then, relying on the exploitation ability, until the last iterations of the algorithm, it goes through the process of convergence towards better solutions.

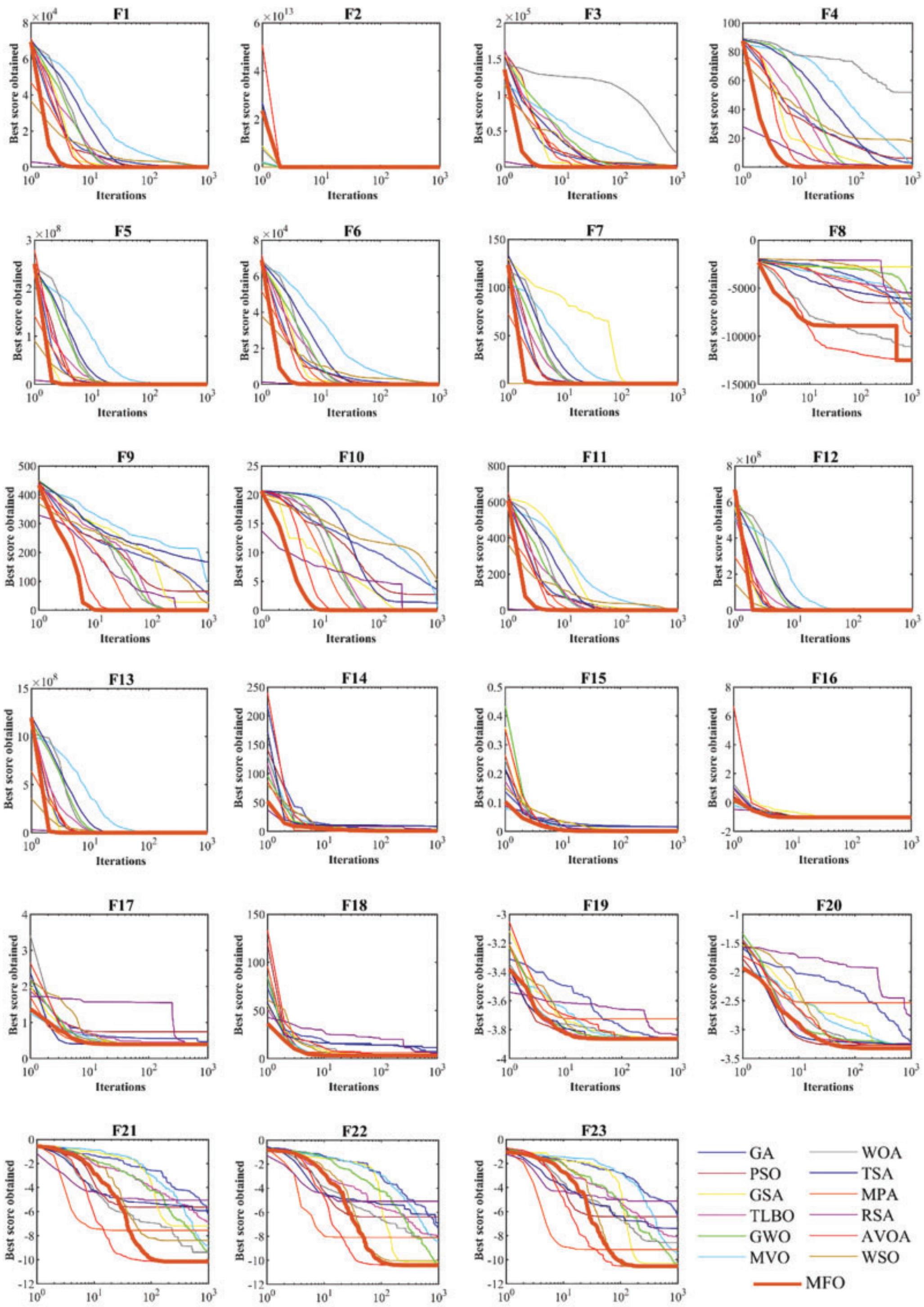

Figure 3: Convergence curves of MFO and the competing algorithms for F1 to F23

4.4 Statistical Analysis

In this subsection, using a statistical analysis, it is checked whether the superiority of MFO compared to the competing algorithms is significant from a statistical point of view. For this purpose, the Wilcoxon sign-rank test is used, which is a non-parametric statistical test and has an application to determine a significant difference between the averages of two data samples. In this test, using an index called *p*-value, it is determined whether there is a significant difference between the performance of the two algorithms or not.

The results of implementing the Wilcoxon sign-rank statistical analysis on the performance of MFO and the metaheuristic algorithms are presented in Table 5. Based on the obtained results, in the cases where the *p*-value is less than 0.05, MFO has a significant statistical superiority compared to the corresponding competing algorithm. Therefore, it can be seen that MFO has a significant statistical superiority against twelve competing metaheuristic algorithms in handling the evaluated benchmark functions.

Table 5: Obtained results from the Wilcoxon sum-rank test

Compared algorithms	Unimodal	High-multimodal	Fixed-multimodal
MFO *vs*. WSO	1.81E−24	1.93E−21	3.61E−06
MFO *vs*. AVOA	2.96E−11	4.89E−05	2.51E−21
MFO *vs*. RSA	4.17E−07	1.60E−11	1.41E−34
MFO *vs*. MPA	9.90E−25	1.02E−14	2.05E−34
MFO *vs*. TSA	9.90E−25	1.28E−20	1.41E−34
MFO *vs*. WOA	2.39E−24	6.01E−11	1.41E−34
MFO *vs*. MVO	9.90E−25	1.93E−21	1.41E−34
MFO *vs*. GWO	9.90E−25	5.23E−16	1.41E−34
MFO *vs*. TLBO	9.90E−25	6.84E−15	1.41E−34
MFO *vs*. GSA	9.90E−25	1.93E−21	4.55E−13
MFO *vs*. PSO	9.90E−25	1.93E−21	3.84E−17
MFO *vs*. GA	9.90E−25	1.93E−21	1.41E−34

5 Application of MFO to Real-World Optimization Problems

In this section, the effectiveness of the MFO proposed approach in dealing with real-world applications is investigated. For this purpose, MFO has been implemented on four engineering design issues: tension/compression spring (TCS) design, welded beam (WB) design, speed reducer (SR) design, and pressure vessel (PV) design. The full description and mathematical model of these problems are provided for TCS in [38], WB in [38], SR in [42,43], and PV in [44].

The results of employing MFO and the competing algorithms in solving the aforementioned engineering problems are reported in Tables 6 and 7. Based on the simulation results, MFO has provided the best design for the TCS problem with the values of the design variables equal to (0.051689061 0.356717739 11.28896583) and the value of the corresponding objective function equal to 0.012665233. In dealing with the WB problem, MFO has presented the best design with the values of the design variables equal to (0.778027075 0.384579186 40.3122837200) and the value of the corresponding objective function equal to 5882.901334. In solving the SR problem, the proposed approach of MFO has provided the best design with the values of the design variables equal to

(3.5 0.7 17 7.3 7.8 3.350214666 5.28668323) and the value of the corresponding objective function equal to 2996.348165. MFO has presented the best design of the PV problem with the values of the design variables equal to (0.20572964 3.470488666 9.03662391 0.20572964) and the value of the corresponding objective function equal to 1.724852309.

Table 6: Evaluation results for real-world applications

DP		MFO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
TCS	Mean	0.012602	0.012675	0.013367	0.013267	0.012663	0.012972	0.013294	0.016619	0.012723	1.83E−02	0.019682	2.17E+13	1.70E+12
	Best	0.012602	0.012663	0.012669	0.013182	0.012663	0.012681	0.012669	0.012752	0.012669	0.017728	0.013093	0.017621	0.018142
	Worst	0.012602	0.01283	0.014208	0.013416	0.012663	0.013557	0.014568	0.018112	0.012955	1.89E−02	0.03281	3.85E+14	1.76E+13
	Std	7.58E−18	0.000041	0.000638	0.0000794	3.26E−09	0.000276	0.000691	0.001885	0.0000633	4.10E−04	0.004875	9.51E+13	5.58E+12
	Median	0.012602	0.012664	0.013296	0.013245	0.012663	0.012896	0.013088	0.01757	0.012721	1.82E−02	0.019239	1.76E−02	2.60E−02
	Rank	1	3	8	6	2	5	7	9	4	10	11	13	12
WB	Mean	5882.895	5892.851	6285.246	13683.51	5882.901	6346.909	8411.601	6642.085	6037.637	32643.66	23524.68	34333.94	29242.67
	Best	5882.895	5882.901	5882.909	8126.174	5882.901	5914.542	6346.587	6027.457	5891.637	11768.56	13175.35	10801.98	11903.52
	Worst	5882.895	5981.068	7273.405	22745.64	5882.901	7156.346	14156.57	7278.287	6824.82	70935.42	37223.74	59462.42	53269.7
	Std	2.06E−12	29.26527	464.1028	4118.759	4.85E−06	438.8137	2215.282	421.9025	315.3024	18170.7	8840.155	17011.18	14262.98
	Median	5882.895	5882.902	6079.861	12480.85	5882.901	6194.503	7910.813	6706.73	5901.603	28702.12	22552.76	37945.51	25804.7
	Rank	1	3	5	9	2	6	8	7	4	12	10	13	11
SR	Mean	2996.348	2996.647	3001.095	3291.635	2996.348	3034.028	3158.194	3031.596	3005.06	7.32E+13	3478.921	1.08E+14	5.20E+13
	Best	2996.348	2996.348	2996.348	3192.706	2996.348	3014.804	3040.469	3008.864	3001.787	5390.679	3178.905	3318.752	3365.418
	Worst	2996.348	2998.929	3011.855	3353.032	2996.348	3048.486	3468.881	3074.088	3011.34	5.30E+14	4132.84	5.48E+14	3.36E+14
	Std	1.03E−12	6.79E−01	4.604449	66.74101	3.70E-06	11.76609	123.3479	15.38387	2.909441	1.34E+14	304.2616	1.44E+14	9.03E+13
	Median	2996.348	2996.365	3000.99	3307.301	2996.348	3035.91	3123.09	3032.059	3004.514	2.87E+13	3342.405	7.73E+13	2.09E+13
	Rank	1	3	4	9	2	7	8	6	5	12	10	13	11
PV	Mean	1.72468	1.724848	1.763113	2.205535	1.724847	1.744102	2.341388	1.742075	1.727373	3.5E+13	2.481338	4.83E+13	1.18E+13
	Best	1.72468	1.724847	1.725971	1.988618	1.724847	1.734312	1.826383	1.728544	1.725554	3.091744	2.103332	4.143315	2.815268
	Worst	1.72468	1.724853	1.848847	2.57137	1.724847	1.753776	4.167689	1.777672	1.731629	3.38E+14	2.80607	2.92E+14	1.28E+14
	Std	2.51E−16	1.45E−06	0.042294	1.67E−01	3.89E−09	0.006501	0.744221	0.015955	1.58E−03	9.41E+13	0.222108	1.02E+14	4.01E+13
	Median	1.72468	1.724847	1.748494	2.179187	1.724847	1.744203	2.104666	1.737791	1.727115	5.899168	2.512428	6.992042	5.864015
	Rank	1	3	7	8	2	6	9	5	4	12	10	13	11
Sum rank		4	12	24	32	8	24	32	27	17	46	41	52	45
Mean rank		1	3	6	8	2	6	8	6.75	4.25	11.5	10.25	13	11.25
Total ranking		1	3	5	7	2	5	7	6	4	10	8	11	9
p-value			7.85E−15	7.85E−15	7.84E−15	7.85E−15	7.85E−15	7.85E−15	7.85E−15	7.85E−15	7.85E−15	7.85E−15	7.47E−15	7.85E−15

Table 7: Values of the design variables in real-world applications

DP		MFO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
TCS	x1	0.051689	0.051687	0.051166	0.05005	0.051691	0.050952	0.051139	0.05005	0.05197	0.06857	0.055289	0.068484	0.069062
	x2	0.356718	0.356668	0.34426	0.311938	0.35676	0.339229	0.343629	0.318037	0.363498	0.919695	0.445535	0.91642	0.9279
	x3	11.28897	11.29191	12.05975	14.89054	11.28646	12.40322	12.10105	14.02097	10.90649	2.273998	7.639465	2.273998	2.273998
WB	x1	0.778027	0.778027	0.778032	1.276407	0.778027	0.779787	0.937572	0.845435	0.778544	1.715331	1.198716	1.700751	1.529071
	x2	0.384579	0.384579	0.384581	0.690309	0.384579	0.386061	0.464051	0.422617	0.386053	0.500182	1.308525	0.669686	0.861097
	x3	40.31228	40.31228	40.31251	64.49933	40.31228	40.40101	47.386	43.78489	40.32226	49.13719	44.85134	67.59505	61.75523
	x4	200	200	199.9968	18.3698	200	200	120.9257	156.8619	199.9575	109.9405	188.9895	20.46493	49.36398
SR	x1	3.5	3.5	3.5	3.59705	3.5	3.513581	3.592103	3.502371	3.500675	3.559067	3.524123	3.508617	3.582145
	x2	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.704209	0.702899	0.700076	0.70586
	x3	17	17	17	17	17	17	17	17	17	26.81737	17.38869	18.15368	17.85692
	x4	7.3	7.30001	7.300001	8.270503	7.3	7.3	7.3	7.3	7.305416	8.143807	7.848094	7.404283	7.766019
	x5	7.8	7.8	7.8	8.285251	7.8	8.285251	8.020414	8.083288	7.8	8.16348	7.894355	7.871633	7.858801
	x6	3.350215	3.350215	3.350215	3.355952	3.350215	3.350558	3.362215	3.370621	3.364675	3.680018	3.411876	3.608438	3.720167
	x7	5.286683	5.286683	5.286683	5.493708	5.286683	5.290403	5.28676	5.286892	5.288923	5.342147	5.391191	5.34706	5.349493
PV	x1	0.20573	0.20573	0.204924	0.196219	0.20573	0.204115	0.214149	0.206007	0.205585	0.321003	0.298461	0.381288	0.225283
	x2	3.470489	3.470489	3.487949	3.538053	3.470489	3.496686	3.322339	3.464512	3.473811	4.471499	2.68242	3.422277	7.095157
	x3	9.036624	9.036624	9.036512	9.971583	9.036624	9.065638	8.970518	9.045109	9.03622	6.680154	7.336434	7.255857	7.696643
	x4	0.20573	0.20573	0.205735	0.218432	0.20573	0.206179	0.2218	0.206073	0.205802	0.436605	0.313307	0.593261	0.30954

It can be concluded from the simulation results that MFO has provided a superior performance compared to the competing algorithms by providing better designs and better values for statistical indicators in solving four engineering design problems. Also, the values obtained for the *p*-value index from the Wilcoxon statistical analysis show that MFO has a significant statistical advantage compared to the competing algorithms. Based on the simulation results, MFO has an acceptable efficiency in handling optimization problems in real-world applications.

6 Concluding Remarks and Future Works

This paper introduces a novel metaheuristic algorithm called Magnificent Frigatebird Optimization (MFO), inspired by the natural behaviors of magnificent frigatebirds. MFO draws its fundamental principles from the kleptoparasitic behavior exhibited by these birds in the wild. The theory behind MFO is elucidated and mathematically formulated into two distinct phases: exploration, which simulates the frigatebird's attack on food-carrying seabirds, and exploitation, which mimics its diving towards abandoned prey. The efficacy of MFO in solving optimization problems is evaluated across twenty-three standard benchmark functions, encompassing both unimodal and multimodal types. Results indicate MFO's proficiency in exploration, exploitation, and maintaining a balance between the two, leading to favorable solutions for optimization tasks. Comparative analysis against twelve established metaheuristic algorithms highlights MFO's superior performance across a range of benchmark functions. Also, the application of MFO to four engineering design problems showed the effective capability of the proposed approach in handling optimization problems in real-world optimization applications.

Moreover, future research avenues include extending MFO to binary and multi-objective optimization problems, as well as exploring its applicability in various scientific domains and real-world applications.

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Appendix A. Information about the Test Objective Functions

The information about the objective functions used in the simulation section is presented in Tables A1 to A3.

Table A1: Unimodal functions

Objective function	Range	Dimensions	F_{min}
$F_1(x) = \sum_{i=1}^{m} x_i^2$	$[-100, 100]$	*30*	*0*
$F_2(x) = \sum_{i=1}^{m} \lvert x_i \rvert + \prod_{i=1}^{m} \lvert x_i \rvert$	$[-10, 10]$	30	0
$F_3(x) = \sum_{i=1}^{m} \left(\sum_{j=1}^{i} x_i\right)^2$	$[-100, 100]$	30	0
$F_4(x) = max\{\lvert x_i \rvert, 1 \le i \le m\}$	$[-100, 100]$	30	0
$F_5(x) = \sum_{i=1}^{m-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2)\right]$	$[-30, 30]$	30	0
$F_6(x) = \sum_{i=1}^{m} ([x_i + 0.5])^2$	$[-100, 100]$	30	0
$F_7(x) = \sum_{i=1}^{m} ix_i^4 + random(0, 1)$	$[-1.28, 1.28]$	30	0

Table A2: High-dimensional multimodal functions

Objective function	Range	Dimensions	F_{min}
$F_8(x) = \sum_{i=1}^{m} -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	$[-500, 500]$	30	-12569
$F_9(x) = \sum_{i=1}^{m} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	$[-5.12, 5.12]$	30	0
$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{m}\sum_{i=1}^{m} x_i^2}\right) - \exp\left(\frac{1}{m}\sum_{i=1}^{m}\cos(2\pi x_i)\right) + 20 + e$	$[-32, 32]$	30	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{m} x_i^2 - \prod_{i=1}^{m} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]$	30	0
$F_{12}(x) = \frac{\pi}{m}\left\{10\sin(\pi y_1) + \sum_{i=1}^{m}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{m} u(x_i, 10, 100, 4)$ $u(x_i, a, i, n) = \begin{cases} k(x_i - a)^n & x_i > -a \\ 0 & -a < x_i < a \\ k(-x_i - a)^n & x_i < -a \end{cases}$	$[-50, 50]$	30	0
$F_{13}(x) = 0.1\left\{\sin^2(3\pi x_1) + \sum_{i=1}^{m}(x_i - 1)^2\left[1 + \sin^2(3\pi x_i + 1)\right] + (x_n - 1)^2\left[1 + \sin^2(2\pi x_m)\right]\right\} + \sum_{i=1}^{m} u(x_i, 5, 100, 4)$	$[-50, 50]$	30	0

Table A3: Fixed-dimensional multimodal functions

Objective function	Range	Dimensions	F_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} \left(x_i - a_{ij} \right)^6} \right)^{-1}$	$[-65.53, 65.53]$	2	0.998
$F_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1 \left(b_i^2 + b_i x_2 \right)}{b_i^2 + b_i x_3 + x_4} \right]^2$	$[-5, 5]$	4	0.00030
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	$[-5, 5]$	2	−1.0316
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) cosx_1 + 10$	$[-5, 10] \times [0, 15]$	2	0.398
$F_{18}(x) = \left[1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2 \right) \right] \times \left[30 + (2x_1 - 3x_2)^2 \times \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2 \right) \right]$	$[-5, 5]$	2	3
$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp \left(-\sum_{j=1}^{3} a_{ij} \left(x_j - P_{ij} \right)^2 \right)$	$[0, 1]$	3	−3.86
$F_{20}(x) = -\sum_{i=1}^{4} c_i \exp \left(-\sum_{j=1}^{6} a_{ij} \left(x_j - P_{ij} \right)^2 \right)$	$[0, 1]$	6	−3.22
$F_{21}(x) = -\sum_{i=1}^{5} \left[(X - a_i)(X - a_i)^T + 6c_i \right]^{-1}$	$[0, 10]$	4	−10.1532
$F_{22}(x) = -\sum_{i=1}^{7} \left[(X - a_i)(X - a_i)^T + 6c_i \right]^{-1}$	$[0, 10]$	4	−10.4029
$F_{23}(x) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + 6c_i \right]^{-1}$	$[0, 10]$	4	−10.5364