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ARTICLE

Learning Vector Quantization-Based Fuzzy Rules Oversampling Method

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ABSTRACT

Imbalanced datasets are common in practical applications, and oversampling methods using fuzzy rules have been shown to enhance the classification performance of imbalanced data by taking into account the relationship between data attributes. However, the creation of fuzzy rules typically depends on expert knowledge, which may not fully leverage the label information in training data and may be subjective. To address this issue, a novel fuzzy rule oversampling approach is developed based on the learning vector quantization (LVQ) algorithm. In this method, the label information of the training data is utilized to determine the antecedent part of If-Then fuzzy rules by dynamically dividing attribute intervals using LVQ. Subsequently, fuzzy rules are generated and adjusted to calculate rule weights. The number of new samples to be synthesized for each rule is then computed, and samples from the minority class are synthesized based on the newly generated fuzzy rules. This results in the establishment of a fuzzy rule oversampling method based on LVQ. To evaluate the effectiveness of this method, comparative experiments are conducted on 12 publicly available imbalance datasets with five other sampling techniques in combination with the support function machine. The experimental results demonstrate that the proposed method can significantly enhance the classification algorithm across seven performance indicators, including a boost of 2.15% to 12.34% in Accuracy, 6.11% to 27.06% in G-mean, and 4.69% to 18.78% in AUC. These show that the proposed method is capable of more efficiently improving the classification performance of imbalanced data.

KEYWORDS

Oversampling; fuzzy rules; learning vector quantization; imbalanced data; support function machine

1 Introduction

As one of the important research directions of machine learning, classification is a key technology for extracting useful information from massive data. However, the imbalance in data distribution affects the accuracy and effectiveness of classification, so it is a major challenge. Imbalanced datasets are datasets in which the number of samples in one class is significantly different from the number of samples in other classes [1]. Such datasets are frequently encountered in real-world scenarios including fault detection [2], medical intelligent diagnosis [3], and text categorization [4]. Traditional classification algorithms, such as decision tree, neural networks and support vector machines (SVM), etc., typically operate under the assumption of a balanced data set where the number of samples for each category is roughly equal. Nevertheless, when encountering imbalanced data, algorithms tend to prioritize accuracy by favoring the majority class, leading to misclassification of the minority class. Therefore, it is crucial to address the issue of imbalanced data distribution effectively in order to successfully accomplish tasks such as classification.

There are two primary approaches for addressing imbalanced data classification: One at the algorithm level and the other at the data processing level. At the algorithm level, modifications are made to algorithms to address the limitations of classifiers in handling imbalanced data, allowing them to better handle classification issues related to imbalanced data. These algorithmic modifications typically involve cost-sensitive learning [5], feature selection and extraction [6], ensemble learning [7], among others. The cost-sensitive analysis seeks to reduce the expense associated with misclassification, specifically the high cost of misclassifying a minority class as a majority class. However, this approach may not always be feasible. Feature selection and extraction involve selecting a subset of features based on predefined rules to enhance the classification accuracy of the classifier, but this process may result in information loss. Ensemble learning can improve the generalizability of classifiers, but often at the expense of increased time complexity. On the other hand, techniques at the data processing level concentrate on adjusting the distribution of training samples to reduce the imbalance within datasets, which is a straightforward approach to achieving balance. A typical data-oriented approach is the resampling method that mainly encompasses the undersampling [8] and the oversampling techniques [9]. Oversampling is frequently preferred over other data-level methods due to its capacity to improve classification accuracy by augmenting the minority class instances. Conversely, undersampling may lead to the elimination of crucial information during the reduction process, consequently diminishing the classifier's ability to generalize effectively.

At present, significant advancements have been achieved in the research methodologies for imbalanced data, with sampling techniques displaying promising potential. The data-level sampling approach functions independently of the classifier, providing greater applicability and improved suitability. In particular, oversampling methods create new minority class samples without sacrificing important information from the original data. Nonetheless, only a limited number of studies have explored leveraging the correlation between attributes in fuzzy rules during data preprocessing. Liu et al. [10] introduced the fuzzy rules based oversampling technique (FRO), which considers the attribute correlation and yields better results for imbalanced data. However, the generation of fuzzy rules in FRO relies on expert knowledge leading to subjectivity.

In order to tackle this issue, the current study endeavors to develop a novel fuzzy rule-based oversampling technique that leverages the correlation among attributes and maximizes the utilization of label information in training data to minimize the influence of expert subjectivity. The key contributions of this research are as follows:

- An improved method for generating fuzzy rules is proposed to learn the distribution of training data and effectively protect the information of minority instances.
- Utilizing the learning vector quantization (LVQ) algorithm, the data attribute interval is dynamically partitioned, leveraging the label information from the training data to mitigate the influence of expert subjectivity.
- By utilizing the correlation between attributes demonstrated by fuzzy rules, the minority class data is synthesized based on the guidance of fuzzy rules to guarantee the generation of new data within a rational range.
- The introduction of the support function machine, comparison experiments carried out on 12 publicly accessible imbalanced datasets against 5 other sampling methods demonstrate that the proposed method significantly enhances the classification performance of imbalanced data.

The remainder of this paper is structured as follows: In Section 2, previous studies relevant to this paper are discussed. Section 3 introduces the details of the proposed method. Section 4 presents the comparison experiments results and analysis. Section 5 summarizes the paper and suggests potential avenues for future research.

2 Related Works

2.1 Sampling Techniques for Imbalanced Data

Random under sampling (RUS) [11] is a nonheuristic method that randomly selects a subset of the majority class and removes the remaining samples to balance the class distribution. A study investigating the application of RUS on tweet data to address class imbalances demonstrated its effectiveness [12]. However, it is important to note that this method may result in the loss of crucial information and potentially lead to underfitting. To address this issue, a clustering technique was integrated into the undersampling approach. Considering the computational complexity, Ofek et al. [13] proposed an undersampling method based on fast clustering to effectively manage class imbalances while achieving a balance between computational efficiency and classification performance.

Oversampling methods alleviate the negative effects of imbalanced distribution by augmenting the number of instances in the minority class while maintaining the size of the majority class constant. Random oversampling (ROS) [11] is a methodology utilized to address imbalanced datasets by replicating instances from minority classes, however, there exists a potential danger of over-fitting. Chawla et al. [14] introduced the synthetic minority oversampling technique (SMOTE) which is based on the concept of K-nearest neighbors. SMOTE generates new instances of the minority class by brief linear interpolating between existing positive samples and their K-nearest neighbors, without considering the majority class sample distribution. Han et al. [15] introduced the borderline SMOTE technique to enhance the minority class samples near the decision boundary, resulting in an improvement in recall performance compared to the traditional SMOTE method. Adaptive synthetic sampling (ADASYN) was introduced by He et al. [16] to address the density distribution of data, automatically assigning different weights to positive samples based on their surrounding context. Borderline SMOTE and ADASYN took into account the distribution of majority class samples in order to minimize the generation of noisy samples. However, these methods had not effectively addressed the issue of increased overlap between classes. Barua et al. [17] proposed the majority weighed minority (MWM) oversampling method, utilizing the Euclidean distance between majority and minority class samples to assign weights to minority class samples, ensuring that the newly created samples fall within the distribution of minority samples in order to mitigate the growth of overlap.

2.2 Fuzzy Methods with Class Imbalance Problem

If-Then fuzzy rules, as a crucial component of fuzzy set theory research, are effective in illustrating data distribution and causal characteristics. They have been employed to tackle the classification challenges associated with imbalanced datasets. Xu et al. [18] expanded upon the fuzzy classification algorithm based on If-Then fuzzy rules, resulting in a significant enhancement in classification performance by minimizing the impact of imbalanced data. Alshomrani et al. [19] introduced a classification system leveraging If-Then fuzzy rules to improve classification accuracy by smoothing the boundary region through feature weighting, particularly in cases of class overlap within imbalanced datasets. Fernández et al. [20] explored various configurations of fuzzy rule-based classification systems and preprocessing methods, demonstrating synergies that highlight the positive impact of applying data sampling techniques to imbalanced datasets in generating If-Then fuzzy rules.

Nevertheless, the effectiveness of the relationship between characteristics demonstrated in the fuzzy rule has not been extensively applied to the data preprocessing phase. Liu et al. [21] introduced the fuzzy information decomposition (FID) method, which effectively utilizes fuzzy information to address class imbalance. However, the method does not fully exploit the attribute correlation issue. Therefore, Liu et al. [10] further proposed an FRO method based on If-Then fuzzy rule to effectively tackle class imbalance. Table 1 presents a summary of the advantage and disadvantage of several traditional methods for addressing class imbalances as discussed earlier.

Table 1: Summary of some representative methods

Methods	Advantage	Disadvantage	Year
Random under sampling (RUS) [11]	High efficiency	Deletion of important information, with the risk of under-fitting	2009
Random oversampling (ROS) [11]	Retain complete data information	Potential risk of over-fitting	2009
Synthetic minority oversampling technique (SMOTE) [14]	Reduce the risk of over-fitting	Generate noise instances	2002
Adaptive synthetic sampling (ADASYN) [16]	Reduce noise	Easy to cause class overlap problems	2008
Fuzzy information decomposition (FID) [21]	Solve the problem of missing and class imbalance at the same time	Inter-attribute dependencies are not taken into account	2017
Fuzzy rules based oversampling technique (FRO) [10]	Correlation between attributes can be leveraged to address class imbalances	The generation of fuzzy rules has the subjectivity of expert experience	2018

3 Learning Vector Quantization-Based Fuzzy Rule Oversampling Method (LVQFRO)

The symbols and the corresponding meanings utilized in this article are presented in Table 2.

Table 2: Symbols and their meanings

Symbols	Meanings
L	Fuzzy division of granularity
η	Learning rate
s_j	The number of samples synthesized
μ, σ	The mean and standard deviation of the Gaussian membership function
D, D^*	Imbalanced training set and balanced data set
x_i, x_{ik}	The instance and its attribute value
C_i, C_j	Instance label and fuzzy rule consequent label
R_j, rw_j	Fuzzy rule and the rule weight

(Continued)

Table 2 (continued)

Symbols	Meanings
A_{jk},f_{jk}	The antecedent linguistic value of rule and its corresponding membership function
c_q, t_q	Prototype vectors and the corresponding label
NR, PR	Rule set of majority and minority class
Num_{maj}, $Num_{\min}$	The number of samples in majority class and minority class
$\left[\alpha_{jk}, \beta_{jk}\right]$	Safety interval

3.1 LVQ-Based Fuzzy Rules

LVQ [22] operates as a prototype clustering algorithm, in contrast to alternative prototype clustering approaches like the K-means algorithm, LVQ makes use of labeled training data. The prototype vectors are adjusted using the label information in order to establish a set of prototype vectors, which act as the centers of each cluster. Each sample point x_i is then assigned to a cluster represented by the prototype vector c_i^* that is closest in distance. For a more detailed explanation of the LVQ algorithm, refer to Algorithm 1.

Algorithm 1:

```
Input: Training set $D = \{(x_1, C_1), (x_2, C_2), \cdots, (x_n, C_n)\}$ where $C_i$ is the label of instance $x_i$, $l$ is
       the number of prototype vectors, the corresponding class label of each prototype vector
       marked as $\{t_1, t_2, \cdots, t_l\}$, and learning rate $\eta \in (0, 1)$.
Output: Prototype vector $\{(c_1, t_1), (c_2, t_2), \cdots, (c_l, t_l)\}$.
Randomly generate the initialization prototype vector $\{c_1, c_2, \cdots, c_l\}$.
For randomly selected instances $x_i$, calculate the distance $d_{iq} = \left\|x_i - c_q\right\|_2$ between $x_i$ and each
       prototype vector $c_q$.
    Find the prototype vector $c_q^*$ closest to the instance $x_i$.
        if: $x_i$ and $c_q^*$ are labeled the same, i.e., $y_i = t_q^*$, then perform the "draw" operation on the
            instance $x_i$ and the prototype vector $c_q^*$ and update $c_q^*$ with $c_q^{'} = c_q^* + \eta\left(x_i - c_q^*\right)$.
        else: $x_i$ is not labeled the same as $c_q^*$, i.e., $y_i \neq t_q^*$, then the instance $x_i$ is "away" from the
            prototype vector $c_q^*$ and update $c_q^*$ with $c_q^{'} = c_q^* - \eta\left(x_i - c_q^*\right)$.
End for
```

In previous research [10], the authors utilized the Gaussian membership function as a basis for generating If-Then fuzzy rules. They divided the instance attribute intervals uniformly and assigned the middle value of each interval as the center of the Gaussian membership function μ. The standard deviation σ was set to a fixed value of $0.35/(L-1)$, based on expert knowledge and experience. However, this approach may not fully leverage the underlying characteristics of the dataset. To address this limitation and reduce the reliance on subjective expert experience, this study proposes constructing If-Then fuzzy rules using LVQ. The details are outlined as follows:

- Utilize the K-means algorithm to create the initial prototype vector, which serves as the centroid of the cluster.
- Based on the label information of the data set, the cluster center will be adjusted using LVQ and the instance attribute intervals will be dynamically partitioned.

- The attribute intervals are represented by fuzzy linguistic values, and a membership function is calculated for each interval.

The Gaussian membership function is employed for its wide-ranging support characteristic, which prevents the membership function value from being zero across the entire domain. This feature ensures the fuzzy completeness of the If-Then fuzzy rule, which can be expressed as Eq. (1).

$$f = \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \tag{1}$$

where the parameters σ and μ determine the width and the center of the attribute interval, respectively. Both of them can be determined by the final prototype vector generated by LVQ.

- Divide the instance attribute values into intervals with the highest membership degree, identify the antecedents of If-Then fuzzy rules, incorporate the consequents, and subsequently construct the If-Then fuzzy rule set R.

Fuzzy rule typically appears as Eq. (2), in the context of fuzzy rules, the *jth* rule is denoted as R_j. The antecedent of the rule, referred to as the *If* part, comprises the instance attribute x_{ik} and the linguistic value $A_{jk}\,(k = 1, 2, \cdots, m)$. The determination of the linguistic value A_{jk} is based on its corresponding membership function f_{jk}. The consequent of the rule, known as the *Then* part, includes the sample label C_j and the rule weight rw_j.

$$\begin{aligned}&\mathit{Rule}\ R_j\text{: }\mathit{If}\ x_{i1}\ \mathit{is}\ A_{j1}\ \mathit{and}\ \cdots \mathit{and}\ x_{im}\ \mathit{is}\ A_{jm},\\ &\mathit{Then\ Class}\ C_j\ \mathit{with}\ rw_j,\ j = 1, 2, \cdots, N\end{aligned} \tag{2}$$

- Calculate the rule weights rw_j for each rule R with the Eq. (3).

$$rw_j = \frac{\sum_{x_i \in Class\ C_j} f_{A_j}(x_i)}{\sum_{i=1}^{m} f_{A_j}(x_i)} \tag{3}$$

where $f_{A_j}(x_i) = T\left(f_{j1}(x_{i1}), \cdots, f_{jm}(x_{im})\right)$ is the degree of antecedent matching between the instance x_i and the rule R_j, and T is the product t modulus, the larger the weight the higher the confidence of the rule, and the safer the fuzzy region it represents.

3.2 Synthetic Minority Samples

In the case of imbalanced binary data with n training instances $x_i = (x_{i1}, x_{i2}, \cdots, x_{im}, C_i), i = 1, 2, \cdots, n$, where the majority class has Num_{maj} samples and the minority class has $Num_{\min}$ samples, the minority class samples are synthesized using the rule set R obtained from the learning process detailed in Subsection 3.1. This oversampling technique is guided by fuzzy rules in order to address the data imbalance issue. The details of this process are described below:

- Extract the fuzzy rule PR that represents the minority class from the rule set R.

For $\forall R_j \in PR$, $C_j = 1$, these rules in PR serve as a means to distribute information regarding the minority class, making it justifiable to employ these rules for oversampling in order to augment the quantity of minority class samples within fuzzy regions.

- Calculate the number of samples s_j to be synthesized for each rule R_j with the Eq. (4).

$$s_j = \left(Num_{maj} - Num_{\min}\right) \cdot rw_j^{-1} / \sum_{j=1}^{|PR|} rw_j^{-1} \tag{4}$$

The generation of more minority class samples under the fuzzy rule with reduced weight, thereby enhancing the protection of the minority class samples in the surrounding area.

- Calculate the safety interval $\left[\alpha_{jk}, \beta_{jk}\right]$.

Suppose that the three consecutive Gaussian membership functions are arranged in sequence:

$$f_{ak} = \exp\left(\frac{-(x-\mu_{ak})^2}{2\sigma_a^2}\right), f_{jk} = \exp\left(\frac{-\left(x-\mu_{jk}\right)^2}{2\sigma_j^2}\right), f_{ck} = \exp\left(\frac{-(x-\mu_{ck})^2}{2\sigma_c^2}\right) \tag{5}$$

the safety interval $\left[\alpha_{jk}, \beta_{jk}\right]$ of the function f_{jk} is determined according to the Eq. (6).

$$\exp\left(\frac{-(x-\mu_{ak})^2}{2\sigma_a^2}\right) = \exp\left(\frac{-\left(x-\mu_{jk}\right)^2}{2\sigma_j^2}\right) \tag{6}$$

then we have $\alpha_{jk} = x = \dfrac{\mu_{jk}\sigma_a + \mu_{ak}\sigma_j}{\sigma_a + \sigma_j}$ and $\beta_{jk} = x = \dfrac{\mu_{ck}\sigma_j + \mu_{jk}\sigma_c}{\sigma_c + \sigma_j}$

- Simulate random numbers within the safe interval and synthesize minority data.

The kth antecedent linguistic value of R_j is A_{jk} and the corresponding fuzzy membership function is f_{jk}, and the random number s_j is simulated within the safety interval $\left[\alpha_{jk}, \beta_{jk}\right]$ according to the membership function f_{jk}, which is used as the kth attribute value corresponding to the newly generated s_j minority samples.

When applying the inverse transformation method to generate random numbers, the fuzzy membership function f_{jk} should satisfies that $\int_{\alpha_{jk}}^{\beta_{jk}} \theta \cdot f_{jk}(x)\,dx = 1$, then we have:

$$\theta = \frac{\sqrt{2}}{\sqrt{\pi}\sigma_j\left(Erf\left(\frac{\mu_{jk}-\alpha_{jk}}{\sqrt{2}\sigma_j}\right) - Erf\left(\frac{\mu_{ik}-\beta_{jk}}{\sqrt{2}\sigma_j}\right)\right)} \tag{7}$$

According to the theorem of the inverse transformation method, the corresponding distribution function $F_X = \int_{\alpha_{jk}}^{x} \theta \cdot f_{jk}(y)\,dy$ generates a uniformly distributed random number γ in [0, 1], and calculates the value of x such that $F_X(x) = \gamma$, which is the solution to the integral equation.

$$\int_{\alpha_{jk}}^{x} \theta \cdot f_{jk}(y)\,dy = \gamma \tag{8}$$

then we have:

$$\begin{aligned} x &= \mu_{jk} - \sqrt{2}\sigma_j Erfinv\left[-\frac{\sqrt{\pi}\sigma_j Erf\left(\sqrt{2}\left(\alpha_{jk}-\mu_{jk}\right)/2\sigma_j\right) + \sqrt{2}\frac{\gamma}{\theta}}{\sqrt{\pi}\sigma_j}\right] \\ &= \mu_{jk} - \sqrt{2}\sigma_j Erfinv\left[(1-\gamma)\,Erf\left(\frac{\mu_{jk}-\alpha_{jk}}{\sqrt{2}\sigma_j}\right) + \gamma\,Erf\left(\frac{\mu_{jk}-\beta_{jk}}{\sqrt{2}\sigma_j}\right)\right] \end{aligned} \tag{9}$$

$Erf(x) = \dfrac{2}{\sqrt{\pi}} \int_0^x e^{-z^2}dz$ is the Gaussian error function, which is the inverse function of $Erfinv$, and x is the random number simulated in the safety interval $\left[\alpha_{jk}, \beta_{jk}\right]$ according to the affiliation function f_{jk}.

- Denormalize the simulated random number.

$$x_{ik} = \min\left(x^k\right) + x_{ik}^{new} \cdot \left(\max\left(x^k\right) - \min\left(x^k\right)\right) \tag{10}$$

where x_{ik}^{new} is the kth attribute value for synthesizing the ith new sample of the minority class, $x_{ik}^{new} \in [0, 1]$.

The specific process of the LVQFRO is shown in Algorithm 2.

Algorithm 2:

```
Input: Imbalanced training set $D = \{(x_1, C_1), (x_2, C_2), \cdots, (x_n, C_n)\}$, where $C_i$ is the label of instance,
       fuzzy division of granularity $L$, $LVQ$ algorithm.
Output: Balanced data set $D^*$.
1: Initialize the prototype vectors using K-means clustering.
2: The LVQ algorithm is utilized to dynamically partition the attribute range into $L$ intervals, which
   are respectively represented by $L$ fuzzy linguistic values.
3: Assign a membership function to each interval of the attribute.
4:   for each instance $x_i = (x_{i1}, x_{i2}, \cdots, x_{im}, C_i) \in D$:
5:       Calculate the membership degree of the instance attribute value $x_{ik}$ for each interval based on
         the $L$ membership functions, and assign $x_{ik}$ to the interval with the highest membership degree.
6:       Generate a rule $R_j$ for the instance $x_i$ and calculate the rule weights $rw_j$.
7:   end for
8: Divide the rule set $R$ into two parts: The majority rule set $NR$ and the minority rule integration
   $PR$.
9: Extract the minority rule set $PR$.
10: Calculate the number of samples that need to be synthesized $s_j$ for each rule $R_j$ in the rule set
    $PR$.
11: For $R_j \in PR$:
12:     for $k \leftarrow 1 : n$:
13:         According to the membership function $f_{jk}$, simulate $s_j$ random numbers within the
            specified safety interval, and assign them consecutively to the $kth$ attribute of the
            $s_j$ synthesized sample.
14:     end for
15: End for
16: Denormalize the simulated random number.
17: End
```

4 Experiments

4.1 Experimental Conditions

Considering that support function machine (SFM) [23] serves as an effective generalization of SVM, it not only transforms the distribution of the original datasets but also preserves all the essential information within the initial data. Therefore, this section will assess the efficacy of the LVQFRO algorithm by conducting comparative experiments on publicly accessible datasets, leveraging the capabilities of SFM.

To ensure the generalizability of the method, all experiments in this study are conducted on a PC servers. The hardware configuration included an Intel i5-1240P processor running at 1.70 GHz and

16 GB of memory. The software environment consisted of a 64-bit Windows 11 operating system, the Pycharm experimental platform, and Python 3.10.

4.2 Datasets Description

In this study, 12 public datasets for binary classification from the KEEL database are chosen for experimental validation These datasets exhibit variations in attributes, sample sizes, and imbalance ratios (IR). Detailed information regarding the datasets can be found in Table 3. To quantify the level of imbalance in the selected datasets, IR is defined by the following Eq. (11):

$$IR = \frac{Num_{maj}}{Num_{\min}} \tag{11}$$

where Num_{maj} and $Num_{\min}$ denote the number of samples in majority class and minority class, respectively.

Table 3: Datasets information

Number	Datasets	Attributes	Size	Majority	Minority	IR
D1	Iris0	4	150	100	50	2:1
D2	Haberman	3	306	225	81	2.78:1
D3	Vehicle2	18	846	628	218	2.88:1
D4	Glass0123456	9	214	163	51	3.2:1
D5	Vehicle0	18	846	647	199	3.25:1
D6	New-thyroid1	5	215	180	35	5.14:1
D7	Ecoli2	7	336	284	52	5.46:1
D8	Segment0	19	2308	1979	329	6.02:1
D9	Glass016vs2	9	192	175	17	10.29:1
D10	Glass2	9	214	197	17	11.59:1
D11	Glass4	9	214	201	13	15.47:1
D12	Glass5	9	214	205	9	22.78:1

4.3 Evaluation Metrics

In the case of imbalanced datasets, seven performance metrics are employed in this study as the evaluation criteria for various methods: Accuracy, Precision, Specificity, Recall, F1-score, G-mean (Geometric mean), and AUC (Area under curve).

$$Accuracy = \frac{TP + TN}{TP + FN + FP + TN}, Precision = \frac{TP}{TP + FP} \tag{12}$$

$$Specificity = \frac{TN}{TN + FP}, Recall = \frac{TP}{TP + FN}, \tag{13}$$

where true positive (TP) and true negative (TN) represent the count of samples correctly predicted as belonging to the positive (minority) or negative (majority) classes, while false negative (FN) and false positive (FP) represent the count of samples incorrectly predicted as belonging to the positive or negative classes, respectively.

$$F1 - measure = \frac{2 \cdot Recall \cdot Precision}{Recall + Precision} \tag{14}$$

$$G - mean = \sqrt{Recall \cdot Specificity} \tag{15}$$

$$AUC = \frac{1}{2}\left(1 + \frac{TP}{TP + FN} - \frac{FP}{FP + TN}\right) \tag{16}$$

In addition to this study also employs three comprehensive evaluation metrics, F1-measure, G-mean and AUC to evaluate the effectiveness of approaches for addressing imbalanced data classification. The formulas for calculating these metrics are as Eqs. (14)–(16). F1-measure considers both precision and recall, calculating a harmonic average of the two to provide a comprehensive evaluation of the classifier's ability to identify minority classes. G-mean takes into account both Recall and Specificity, and representing the overall performance of the classifier. A higher AUC value indicates a ROC curve closer to the upper left corner, allowing for a simultaneous evaluation of the classifier's ability to classify both classes and providing a reasonable assessment of its classification performance.

4.4 Granularity Analysis of Fuzzy Partitions

This section concentrates on the granularity of fuzzy partitioning in order to facilitate performance comparison with other algorithms. For LVQ, the granularity of fuzzy partitions affects the value of evaluation indexes to a certain extent. A smaller granularity, indicating fewer divisions, results in wider fuzzy regions. This increases the likelihood of synthetic minority class samples overlapping with the majority class sample region. Conversely, a larger granularity of fuzzy partitions may lead to overfitting issues.

By leveraging the granularity of fuzzy partitions 5, 6, 7, 8, and 9 [10], which are widely accepted division values for LVQ, Table 4 presents the performance metrics of F1-measure, G-mean, and AUC when employing SFM across five distinct fuzzy granularities. These results are derived from experiments conducted on 12 imbalanced public datasets.

Table 4: The average performance for different fuzzy partitions granularity (%)

Granularity	F1-measure	G-mean	AUC
5	89.51	90.85	90.85
6	**93.07**	**93.09**	**93.09**
7	90.08	91.54	91.54
8	89.74	91.69	91.70
9	89.24	91.14	91.16

In the LVQFRO method, the parameter for the granularity of fuzzy partitions increases from 5 to 6, resulting in an increase in the average values of F1-measure, G-mean, and AUC. These values reach their peak at a granularity of 6, with percentages of 93.07%, 93.09%, and 93.09%, respectively. These values represented increases of 3.56%, 2.24%, and 2.24% compared to a granularity of 5. As the fuzzy partition granularity increases, there is a decreasing trend in each evaluation metrics, leading to a decline in classifier performance. Specifically, the performance indicators saw increases of 3.33%, 1.38%, and 1.37% with respect to a granularity of 8%, and 3.83%, 1.93%, and 1.91% with respect to a granularity of 9. In conclusion, the best classification outcomes are observed at a granularity of 6. Therefore, the LVQFRO algorithm with a fuzzy partition granularity of 6 is chosen to preprocess the imbalanced datasets.

4.5 Results Analysis

To assess the efficacy of the LVQFRO method constructed in this study, it is compared with four sampling techniques, RUS [11], SMOTE [14], FID [21], FRO [10], and the original dataset (ORI) [21] using the SFM method in a series of comparative experiments. The parameter configurations of each sampling technique remain consistent with those outlined in the original text. Specifically, the SMOTE algorithm utilizes a nearest neighbor count of 5, the FRO algorithm operates with a granularity value of 7, and the LVQFRO algorithm employs a granularity value of 6. Furthermore, the oversampling rate for all oversampling algorithms in the study is established at 1.

To objectively assess the generalization capability of various algorithms and mitigate the impact of randomness on experimental outcomes, this study employs an 80%–20% training-test split and five folds cross-validation methodology. The experiments are iterated five times to ensure unbiased results, with the final outcome being the average of the experimental results.

Fig. 1 presents the Accuracy values for each algorithm. LVQFRO achieves the best results on more than half of the 12 imbalanced public datasets (including a tie for first place), with an average accuracy improvement of 9.97% compared to the benchmark ORI, 12.34%, 8.24%, 8.24%, 2.98% and 2.15% compared to ORI, RUS, SMOTE, FID and FRO, respectively. These findings suggest that the LVQFRO algorithm developed in this study effectively enhances the overall accuracy of imbalanced datasets and demonstrates strong classification performance.

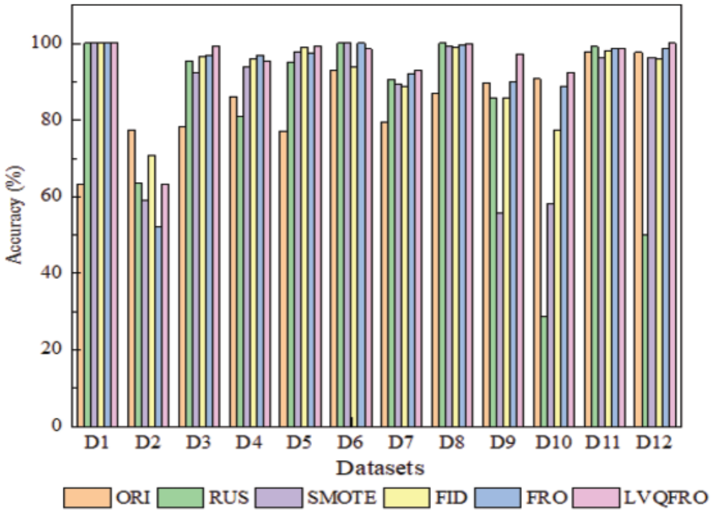


Figure 1: Comparative results for accuracy with different methods

Figs. 2 and 3a display the Precision, Specificity and Recall outcomes for each algorithm. Precision evaluates the predictive model's capability to detect positive class samples, with the LVQFRO algorithm demonstrating the highest Precision across 9 datasets (including a tie for first place), showcasing an average enhancement of 11.04% compared to the ORI benchmark, 13.16% compared to RUS, 7.42% compared to SMOTE, 5.37% compared to FID, and 6.55% compared to the FRO algorithm. Specificity measures the accuracy in classifying negative samples, and the LVQFRO oversampling approach exhibits superior Specificity results across 10 of the 12 datasets, with at least a 2.34% improvement over other sampling methods. Recall represents the accuracy in classifying positive class samples, with LVQFRO achieving significant values on 7 datasets (including a tie for first place), surpassing the ORI, RUS, SMOTE, FID, and FRO algorithms by 10.70%, 9.59%, 6.54%, 5.90%, and 4.99%, respectively. These findings indicate that LVQFRO outperforms other sampling techniques in classification outcomes across most datasets, particularly in scenarios with high data imbalance. This method enhances the detection of minority samples while ensuring high classification accuracy across majority classes, emphasizing high recall and specificity characteristics.

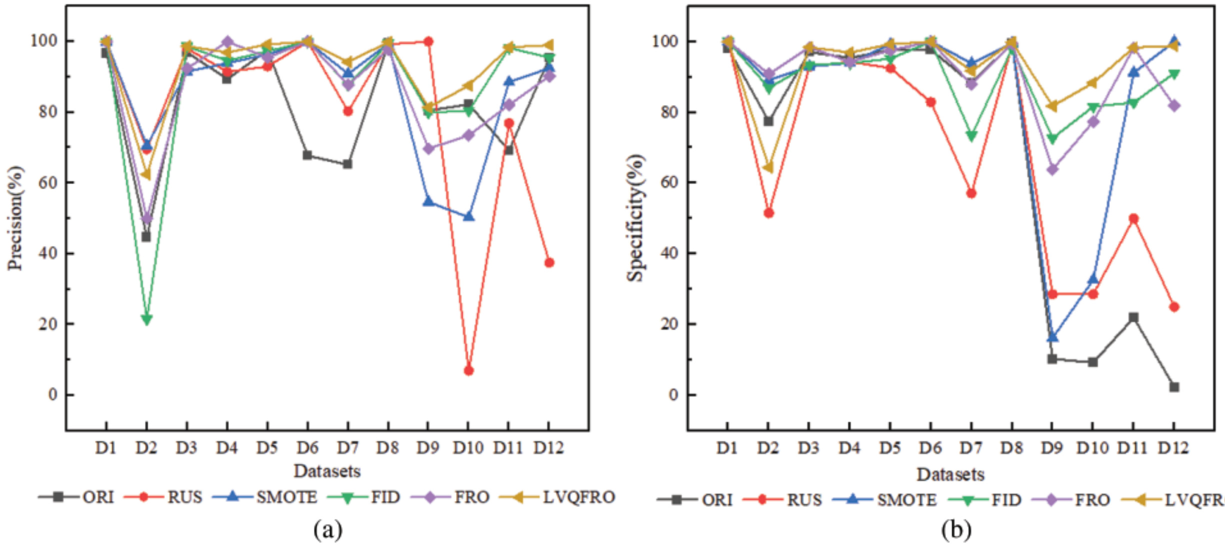


Figure 2: (a) Precision and (b) specificity comparison of different algorithms

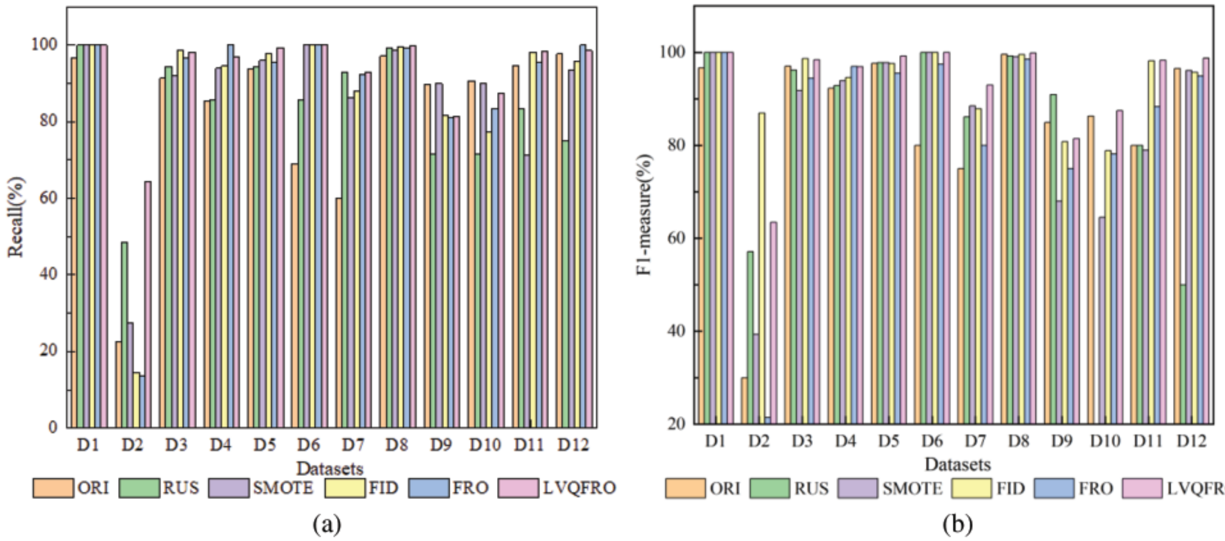
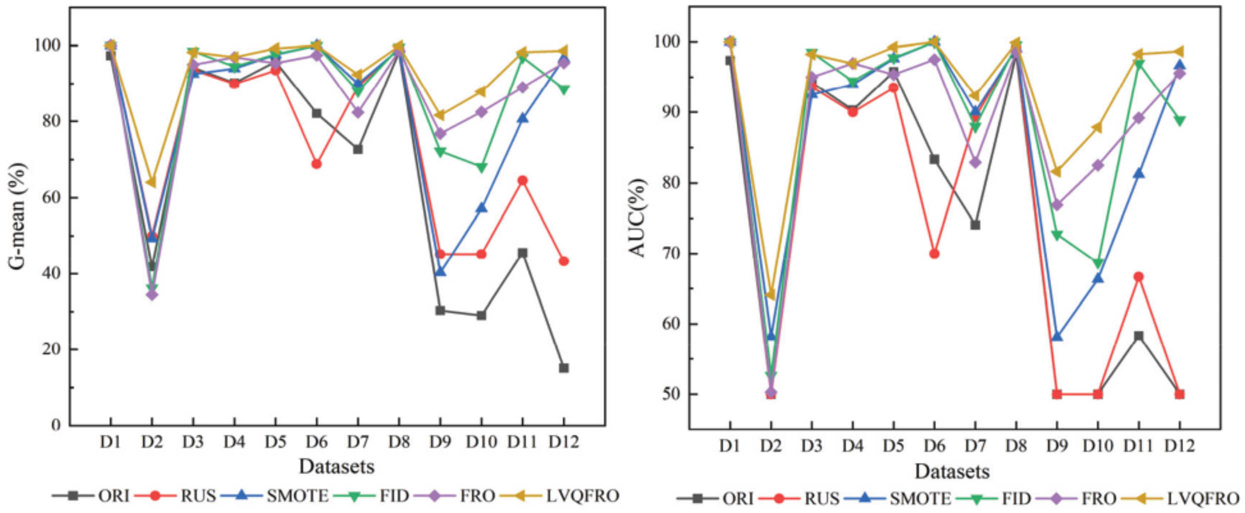


Figure 3: (a) Recall and (b) F1-measure comparison of different algorithms

Figs. 3b and 4 present a summary of the F1-measure, G-mean, and AUC index values for each algorithm across 12 public datasets. Among these datasets, LVQFRO demonstrates the best F1-measure performance on 8 datasets (including a tie for first place). Additionally, its performance on the remaining 3 datasets is also strong, with only a small gap separating it from the optimal results. The LVQFRO sampling method demonstrate superior performance compared to other methods in terms of G-mean. Specifically, 11 datasets (including a tie for first place) achieve high values , that is only slightly below the FID of 0.27% observed in the Vehicle2 data set, with insignificant difference between them. In comparison to the other five sampling methods, LVQFRO achieve the highest AUC values on 10 datasets (including a tie for first place), showing increases of 27.06%, 19.56%, 10.01%, 6.40%, and 6.11% on G-mean, and 18.78%, 17.90%, 6.95%, 4.92%, and 4.96% on AUC, respectively. In terms of the F1-measure, there are varying degrees of improvement ranging from 8.01% to 12.82%.



(a) (b)

Figure 4: (a) G-mean and (b) AUC comparison of different algorithms

In conclusion, LVQFRO enhances the recognition rate of minority samples, and improves classification accuracy. On datasets with high class imbalance ratios, such as Glass016vs2, Glass2, Glass4, and Glass5, the original performance is subpar or even invalid. LVQFRO mitigates classification bias stemming from imbalanced data, exhibits strong generalization capabilities, and yields superior classification outcomes.

Table 5 displays the comparison of LVQFRO with other five algorithms. In this comparison, symbol '+' denotes an improvement in the evaluation index of LVQFRO compared to the current algorithm, while symbol '–' indicates a decrease in the index value. In comparison to the ORI benchmark, LVQFRO shows an average increase of 27.06% in G-mean, 26.92% in Specificity, and 8.41% in F1-measure. When compared to RUS, LVQFRO exhibits improvements of 12.34%, 13.16%, 9.59%, 26.28%, 12.82%, 19.56% and 17.90% in different metrics. Additionally, LVQFRO shows improvements ranging from 6.54% to 10.01% in the seven indicators compared to the SMOTE algorithm. In comparison to the FID and FRO algorithms, LVQFRO's improvements are relatively small, with the lowest average increase at 2.15% and the highest at 8.01%. On the F1-measure, LVQFRO is only 0.17% lower than the FID algorithm, indicating insignificant differences in performance.

Table 5: Percentage improvements attained by LVQFRO (%)

	Accuracy	Precision	Recall	Specificity	F1-measure	G-mean	AUC
ORI	+9.97	+11.04	+10.70	+26.92	+8.41	+27.06	+18.78
RUS	+12.34	+13.16	+9.59	+26.28	+12.82	+19.56	+17.90
SMOTE	+8.24	+7.42	+6.54	+9.09	+8.23	+10.01	+6.95
FID	+2.98	+5.37	+5.90	+4.04	−0.17	+6.40	+4.92
FRO	+2.15	+6.55	+4.99	+2.34	+8.01	+6.11	+4.69

In conclusion, LVQFRO demonstrates superior classification performance across various datasets with different IR. The evaluation metrics show significant improvements compared to other oversampling algorithms, highlighting the effectiveness of dynamically partitioning attribute intervals and generating minority samples within the safe interval, and rational in addressing imbalanced data, ensuring the algorithm's comprehensiveness and stability.

To further illustrate the statistical properties of the experimental results, LVQFRO is selected as the primary control method and a Friedman test [24] is conducted in comparison to five other methods. Table 6 presents the average rankings obtained from the Friedman test on seven evaluation criteria. The findings reveal that LVQFRO consistently achieves the highest rankings, suggesting that LVQFRO performs exceptionally well and consistently in comparison to the other methods, and demonstrates superior statistical significance.

Table 6: Average ranking results of Friedman test

	Accuracy	Precision	Recall	Specificity	F1-measure	G-mean	AUC
ORI	4.58	4.25	4.67	4.33	4.04	5.34	5.33
RUS	3.67	4.17	4.33	5.00	3.83	4.42	4.92
SMOTE	4.25	3.88	3.83	3.13	4.04	3.50	3.33
FID	3.75	3.04	3.00	3.79	2.88	2.83	2.67
FRO	2.75	4.13	3.25	2.96	4.50	3.58	3.33
LVQFRO	**2.00**	**1.54**	**1.92**	**1.79**	**1.71**	**1.33**	**1.42**

5 Conclusions

In order to address the issue of imbalanced data classification more effectively, this study introduces a novel fuzzy rule oversampling technique called LVQFRO based on LVQ. LVQFRO determines the antecedents of If-Then fuzzy rules by taking into account the label information of training samples and dynamically dividing data attribute intervals using LVQ. This method helps to eliminate the subjective nature of expert experience in rule construction. Additionally, LVQFRO utilizes the correlation between data attributes to generate minority samples based on fuzzy rules, thus mitigating class imbalance issues. Experimental results on 12 datasets with varying imbalance ratios, in conjunction with the SFM classifier, demonstrate that the LVQFRO method yields superior classification performance. Furthermore, statistical analysis indicates that this method exhibits greater stability. This paper presents a new approach for binary-class classification of imbalanced data, showing improved performance. However, the extension of this method to multi-class imbalanced data classification warrants further investigation.

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Availability of Data and Materials: The data that support the findings of this study are openly at: https://sci2s.ugr.es/keel/datasets.php.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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