Computers, Materials & Continua

Tech Science Press

DOI: 10.32604/cmc.2023.041508

ARTICLE

Solving Arithmetic Word Problems of Entailing Deep Implicit Relations by Qualia Syntax-Semantic Model

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Received: 25 April 2023 Accepted: 25 July 2023 Published: 31 October 2023

ABSTRACT

Solving arithmetic word problems that entail deep implicit relations is still a challenging problem. However, significant progress has been made in solving Arithmetic Word Problems (AWP) over the past six decades. This paper proposes to discover deep implicit relations by qualia inference to solve Arithmetic Word Problems entailing Deep Implicit Relations (DIR-AWP), such as entailing commonsense or subject-domain knowledge involved in the problem-solving process. This paper proposes to take three steps to solve DIR-AWPs, in which the first three steps are used to conduct the qualia inference process. The first step uses the prepared set of qualia-quantity models to identify qualia scenes from the explicit relations extracted by the Syntax-Semantic (S^2) method from the given problem. The second step adds missing entities and deep implicit relations in order using the identified qualia scenes and the qualia-quantity models, respectively. The third step distills the relations for solving the given problem by pruning the spare branches of the qualia dependency graph of all the acquired relations. The research contributes to the field by presenting a comprehensive approach combining explicit and implicit knowledge to enhance reasoning abilities. The experimental results on Math23K demonstrate hat the proposed algorithm is superior to the baseline algorithms in solving AWPs requiring deep implicit relations.

KEYWORDS

Arithmetic word problem; implicit quantity relations; qualia syntax-semantic model

1 Introduction

Solving arithmetic word problems entailing deep implicit relations is a critical branch problem of solving arithmetic word problems. Arithmetic word problems entailing deep implicit relations are the AWPs that can be solved only after adding the deep implicit relations. Two example types of DIR-AWPs are the AWPs of entailing commonsense or specific domain knowledge. "Chicken and rabbit in the same cage" is an example of entailing commonsense. People use this problem as a touchstone to judge whether the solver is powerful. In other words, people think that DIR-AWPs are the most difficult AWPs. However, there are only so many satisfactory algorithms for solving DIR-AWPs. The paper has studied this problem. The reason is that the type of problem represents the highest degree of difficulty in solving AWPs. This paper proposes discovering deep implicit relations by qualia inference to solve DIR-AWPs, such as entailing commonsense or subject-domain knowledge.

In response to this issue, deep implicit knowledge has been proposed to tackle the challenge of multi-steps of implicit relations reasoning during the problem-solving of DIR-AWPs. The syntax-semantic relation graph (S^2RG) is a middle-state that facilitates the qualia structure from the S^2 model. The S^2RG enhances the knowledge representation and reasoning capability of solving DIR-AWPs in real-world scenarios. This paper proposes the three steps used to conduct the inference process.

Yu et al. proposed a Vectorized Syntactic-Semantic (V-S^2) method [1,2] for solving word problems. This method encapsulates mathematical knowledge into S^2 models and leverages a neural network miner to discover implicit quantity relations. The authors extend this paper by proposing a novel approach called the Qualia Syntax-Semantic Model (QS^2M). The QS^2M method leverages qualia-based relation inference to discover deep implicit relations. Compared to traditional similarity matching and pattern recognition-based inference approaches, the graph-based inference method QS^2M provides a more logically controllable and understandable solution to solving word problems. This modification offers a more sophisticated approach to discovering implicit relations in word problems. This understanding is achieved through QS^2M methods, which extract relations from external qualia-based datasets and uncover relations from an expanded understanding of the problem. The proposed algorithm highlights the AWP text, scenario understanding and inference of deep implicit knowledge.

This paper uses a tutorial based on an algorithm approach that leverages the generalized problem-solving principle. They contend that learners can more effectively learn by focusing on relational operations instead of concentrating only on the system of equations. This approach divides the task of obtaining a system of equations into two simpler sub-tasks: identifying relations and their transformation into equations. The proposed algorithm, known as the "relation-centric solving algorithm", addresses the growing demand for advanced intelligent tutoring systems [3]. The contributions of this paper can be summarized as follows:

1. The QS^2M has been proposed for solving DIR-AWP characterized by complex problem scenarios. The QS^2M approach utilizes graph-based inference, which provides a logically controlled and coherent framework compared to traditional methods.

2. Implicit knowledge addition by the QS^2M model represents the relationships between mathematical entities and their attributes. The qualia role patterns in different problem scenarios are to extract the DIR-AWP quantity relations from fully connected S^2RG.

2 Related Work

The development of methods for acquiring quantity relations from problem texts has involved using manually crafted rules [4,5] or templates in their early stages [6]. Rule-based [7] systems rely on predefined rules, such as predicate logic, for unambiguous deductions. Alternatively, semantic parsing-based methods [8] utilized the semantic structure of problems to retrieve historical knowledge more efficiently. However, this approach came at the expense of ambiguity and inference interpretation [9,10]. Yan et al. [11] proposed a seq2seq model that translated problem sentences into expressions.

Furthermore, Liang et al. [12,13] designed the teacher module to associate the encoding to match the correct solution and analogical pairs in a latent space. Above all, Yu et al. [1] proposed a state-action paradigm that utilized knowledge expressions and action transformations. Advanced methods can be categorized into knowledge-addition and state action-based methods that adopt a relation-centric approach based on this paradigm.

2.1 The Knowledge-Addition Method Solving DIR-AWP

The main objective of the research is to employ S^2RG to bridge the gap of implicit knowledge for DIR-AWP. The paper reviewed related knowledge acquisition and reasoning work to achieve this goal. Walter et al. [14] proposed a two-frame framework for solving AWPs, utilizing knowledge and solution frames to store problem-understanding outcomes. In Natural Language Processing (NLP), knowledge-addition methods have shown the potential to enhance problem comprehension through the utilization of explicit expert knowledge. Several recent studies [15–17] have focused on using additional information to aid in understanding problems. For instance, Graph2Tree [18] was introduced to capture relationships and order information among quantities. In the field of observation, there is a particular emphasis on improving the expression reasoning process [19,20]. Researchers have proposed various methods, such as Goal-Driven Tree-Structured (GTS) Neural Model [21], which utilizes a goal-driven decomposition mechanism to reason an expression tree. Shen et al. [19] also created an ensemble of multiple encoders and decoders, combining semantic understanding and reasoning strengths. The deep learning framework approach to reasoning implicit relations is based on the semantic hint of the shallow implicit knowledge, which directly adding shallow implicit mathematical relationships cannot represent the content of the DIR-AWPs.

Overall, by building on these related studies, the research aims to apply S^2RG to enhance the acquisition of implicit knowledge for solving DIR-AWP. A knowledge-addition solver is an automated system that utilizes a knowledge base to represent subject-domain knowledge. It surpasses the performance of traditional problem-solving methods by leveraging its superior computing capacity. This approach to problem-solving is characterized by its reliance on subject-specific knowledge and its ability to generate innovative solutions.

2.2 The State-Action Framework Reasoning DIR-AWP

This study aims to develop a syntax-semantic relation graph-based approach to enhance the efficiency of quantity relation extraction for resolving DIR-AWPs. The quantity relations and solution goals in such problems are founded on ontology, and hence, recognizing and extracting ontology relations can pave the way for generating the quantity relations. Therefore, the associations between words in the text can be leveraged to obtain ontology relations. Prior research proposes the concept of qualia role [22], a set of relations referred to as qualia that can signify the meaning of a word based on the concept of the words. Furthermore, a set of semantic roles called qualia structure, including formal, constitutive, agentive, and telic roles, is proposed to represent the meaning of nominal and implicit information described in [23].

Knowledge graph completion [24,25] is a relevant task in S^2RG due to the explicit knowledge graph formed. This task involves learning unknown edges in the knowledge graph using existing edges. Various approaches have been proposed, such as Bordes et al. [26] interpreting knowledge semantics through translation operations and Pei [27] capturing structure information and long-range dependencies through a geometric perspective. Other types of knowledge, including background knowledge [28], logical knowledge [29], and implicit knowledge in pre-trained language models, have also been investigated. Special forms of knowledge, such as logic rules [30] and mathematical properties [31], have also been studied in various research works.

The study differs from previous knowledge acquisition research in that it utilizes implicit mathematical knowledge through its reasoning approach S^2RG, a general framework based on a state-action paradigm, and a relation-centric approach. Moreover, the paper introduces a QS^2M within S^2RG, improving solution accuracy and reasoning interpretability. The work contributes to the field by presenting a comprehensive approach that combines explicit and implicit knowledge learning to enhance reasoning abilities.

3 The Qualia DIR-AWPs Solver

3.1 Overview

This section details the proposed qualia-based DIR-AWPs solver to discover implicit quantity relations for solving AWPs with complex problem scenarios. The proposed QS^2M framework represents two main steps in Fig. 1. First, solving AWPs is based on relations from the S^2 model, and the entity relation representation S^2RG is an intermediate state for solving the problem in Fig. 2. Second, based on the S^2 model, the implicit relations are extracted from expanded S^2RG of implicit knowledge.

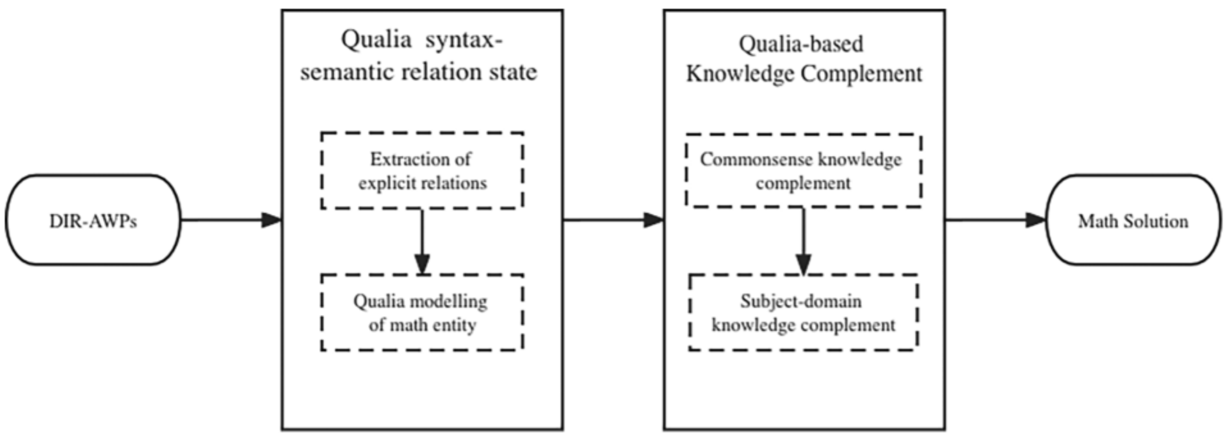

Figure 1: The framework for solving DIR-AWPs using the proposed QS^2M method

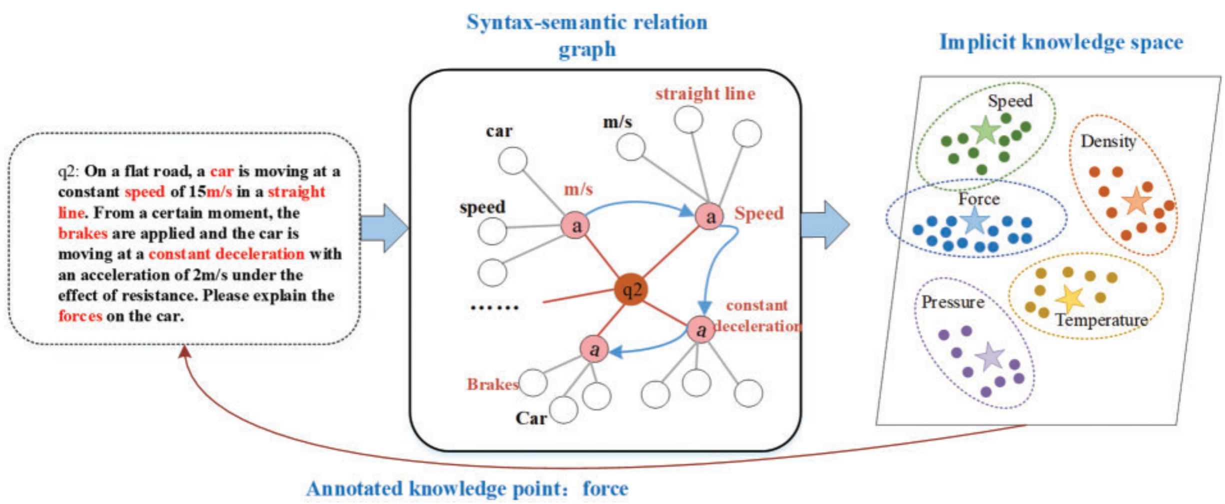

Figure 2: The example of solving DIR-AWPs by using the proposed QS^2M method

Compared to traditional knowledge models, the advantage of QS^2M is the cross-scenario multi-step inference for discovering implicit knowledge entities N and quantity relations R. A given arithmetic word problem *P* could be translated into a triple of <N, R, g> that contains a set of knowledge entities $N = N_e \cup N_i$ and quantity expressions $R = R_e \cup R_i$ as well as the solution goal g to be solved, where N_e and R_e are knowledge entities and quantity expressions that are directly stated in P. N_i and R_i are implicit knowledge entities and quantity expressions indicated by implicit knowledge of DIR-AWPs.

Definition 1 (Knowledge entity): A knowledge entity $e^a = \{e_1, e_2, ..., e_i\}$ mentioned is a word in solving DIR-AWPs, e.g., "speed" and "uniform linear motion" are often used to explain relevant knowledge points in knowledge scenarios. For these terms with a clear knowledge orientation, e_m is a knowledge attribute, where m is the number of knowledge attribute words. These knowledge-attributing words are selected from many teaching resources, including textbooks and test questions.

Definition 2 (Syntax-semantic relation graph): Each DIR-AWP is constructed as a syntax-semantic relation graph, denoted as $S^2RG = <EE, IE, ER, IR>$. which captures the relations between the DIR-AWP knowledge words and their neighbors to highlight the knowledge point. As shown in Fig. 2. The knowledge entity is directly connected to nodes, and neighboring entities are connected to their corresponding knowledge entities. The qualia relations of knowledge entities also form a scenario-aware knowledge representation.

Definition 3 (Implicit knowledge space): The implicit knowledge space S is based on the knowledge point to enrich the connotation and extension of knowledge points. Its knowledge entity combinations in different knowledge scenarios are $S = \{i_1, i_2, ..., i_k\}$, and k is the number of knowledge points. In the hidden knowledge space, the closer the knowledge is to each other, the more similar the knowledge features are.

Algorithm 1: Qualia-based Solver for Solving DIR-AWPs

Input: The DIR-AWP P_{text} and $QS^2M = (n_e, n_i, q_r, q_p, r_i)$, i = 1, 2, . . . , m.
Output: The solution of the DIR-AWP, denoted as $R_{\Delta.}$

State1 (Understanding DIR-AWP text knowledge)
1: Procedure 1: Extracting explicit relations from the text;
2: Procedure 2: Adding qualia relation of math entities to math relations;
State2(Implicit knowledge complement)
3: Procedure 3: Commonsense knowledge complement;
4: Procedure 4: Subject-domain knowledge complement;
State3 (Symbolic solver)
5: Procedure 5: Generating math relations R_Δ from S^2RG.
6: **return** R_Δ

3.2 Qualia Syntax-Semantic Model

The solvable state of DIR-AWPs requires constructing a connected S^2RG of relations, which provides a comprehensive understanding of the process involving using the S^2RG to represent and reason knowledge in knowledge entities. On the S^2RG, quantity relations are represented by a set of connected attribute nodes belonging to one or more entity nodes, which can be viewed as sub-graphs by applying a graph traversal. A quantity relation mining algorithm translates such sub-graphs into quantity relations.

The quantity relations indicated by the relations among knowledge entities and their attributes can be modeled as a set of knowledge models named the QS^2M. The QS^2M is structured as a quintuple,

$$QS^2M = (N_e, N_i, Q_r, Q_p, R_i) \tag{1}$$

where: N_e is the explicit knowledge entity stated in the AWPs to be calculated in the solving process.

N_i is the knowledge entity to link AWP entities in the solving process.

Q_r is the semantic pattern AWPs that constructs the qualia relation to link N_i scenario entities.

Q_p is the syntax-semantic structure pattern for converting math relations from the S^2RG.

R_i is the quantity relations associated with the qualia role pair $<N_e, N_i>$.

The QS^2M linked the AWP explicit relation and the implicit relation. The set of QS^2M, $M_i = (N_e, N_i, Q_r, Q_p, R_i)$, i = 1, 2,..., m denotes the pool of qualia-based knowledge models.

As a syntax relationship between entities, the qualia structure can be incorporated into the existing S^2 model to construct S^2RG. The knowledge description ability of S^2RG lies in the concept network centered on nouns as entities. The QS^2M allows the model to perform multi-step reasoning. The quantity expressions indicated by knowledge entities and attributes can be modelled as a set of S^2RG. Inspired by the qualia structure system [19], the knowledge base uses the QES^2 to represent the structure of AWP, and entities form its object e^o, entity attribution e^a, and values. The entity is independent and used to distinguish different knowledge entities, and attribution is attached to the entity and used to present numeric values. A hierarchical structure of Entity−Attribution−Value can represent the quantity relations. Object entities and their attributions form the basic form for representing quantitative relations. Identifying an application's object entity is the key to extracting quantitative relations and understanding and solving problems.

The AWPs scenario has three main categories of factual facts: reflexive fact, connective fact, and con-vergence fact. The expressions are presented as facts that facts could further translate into mathematical operations to calculate the final answers. The knowledge entity relation can be described as a qualia structure denoted as R_c. Each element r_c is $<e_{src}, e_{dst}, Q_p, R_i>$, where e_{src} and e_{dst} are two knowledge entities, Q_r denotes the semantic role of e_{dst} associated with e_{src}, Q_p denotes the syntax-semantic pattern related to Q_r. The six kinds of qualia roles Q_r for solving AWPs: **Formal role** (FOR), **Constitutive role** (CON), **Unit role** (UNI), **Material role** (MAT), **Telic role** (TEL), **Evaluation role** (EVA), **Handle role** (HAN), **Action role** (ACT) and **Orientation role** (ORI).

As a result, quantity facts in AWPs could be divided into the following three categories accord-ingly:

Reflexive Fact: reflexive fact presents the expressions amount different attributes e^a of a knowledge entity object e^o. The relation between the target entity and its quantity, length, weight, speed, the relation between the speed-time distance of the target entity, etc., which associates with the qualia roles of FOR and UNI.

Connective Fact: connective fact presents the expressions amount different entity objects $e^o{}_i$, e.g., comparative relations: "there are five more apples than pears", multiplicative/proportional relations: "the number of pears is twice the number of peaches". Which associates with qualia roles of EVA, MAT, and ORI.

Convergence Fact: convergence fact describes the convergence relation between an object entity and two or more object entities, e.g., summation relation: "38 trees were planted in Year 3 and 22 trees were planted in Year 4. How many trees were planted in both years?". which usually associates with qualia roles of CON, TEL, ACT, HAN, MAT, and ORI.

Based on the aforementioned definition, reflexive facts can be represented as constitutive roles linking a mathematical entity with its associated attributes. For instance, the constitutive role C (rabbit, leg) establishes a connection between the knowledge entity "rabbit" and its attribute "legs". Similarly, the constitutive role C (circle, area, radius) links the attributes "area" and "radius" of a "circle" mathematical entity. Unlike reflexive facts, connective facts are context-dependent and may take on various forms, such as "is-a" relationships between knowledge entities, "used-for" relations, "created-by" relations, and so forth. For instance, the source formula "rabbit. Legs = 4" can be deduced from the constitutive role C (rabbit, leg), and the formula "area = PI ∗ radius ∗ radius" can be derived from the C (circle, area, radius).

This study uses the Language Technology Platform (LTP) [32] natural language processing tool for word segmentation and part-of-speech tagging of word problems. For a DIR-AWP text P entered in natural language, a lexical tagging(POS) algorithm uses transformers' tokenizer to separate the AWP into lexical subdivisions of the text andtheir lexical roles. In Algorithm 2, the explicit entities are extracted from the S^2 model, and then the entities are constructed as S^2RG through the entity dependency relation and S^2 relation.

Algorithm 2: Qualia syntax-semantic model for S^2RG construction

Input: The arithmetic word problem text P_{text} and $QS^2M = (N_e, N_i, Q_r, Q_p, R_i)$, $i = 1, 2, \ldots, m$.
Output: Syntax-semantic relation graph $S^2RG = (E, R)$, solution goal g.

Initialize N, E as empty
//The S^2 model extraction of explicit relations and nodes.
Transform P_{text} to part-of-speech annotation $W = w_i|i = 1, 2, \ldots, n$
Extracted explicit R_e from the P_{text} by the S^2 model
for each w_i annotated as noun in W do
  if IsAttribute(w_i) is **true then**
    assign w_i as n^a
  else
    assign w_i as n^e
  end if
    add n^a, n^e to N_e
end for
//The dependency relation generation of S^2RG
for each n_e in N_e **do**
  if exist a qualia relation Q_r from n_i to n_j, **then**
    Add directed edge $<n_i, n_j>$ to S^2RG
  end if
 end for
 return S^2RG

3.3 Implicit Knowledge Addition by Qualia Syntax-Semantic Model

Implicit relation R_i recovery refers to an entity e^o corresponding to attribute e^a in qualia disciplines. However, its value does not explicit in the problem text; the paper defines this knowledge as implicit knowledge fact. AWPs implicit knowledge contains two types of implicit relation sources: missing entity and subject-domain relations. The generated prompt questions Q_r are embedded in the S^2RG and as clues to traverse the nodes in the S^2RG. Implicit quantity relation mining is designed to discover implicit quantity relations from the updated S^2RG.

The S^2RG nodes as an indicator to match the pattern of POS: entities E_e and attributes A_e from the expressions R_e, and solution goals g. The S^2RG generated math relations from explicit entities connect the solution goal g, inference domain knowledge Rd in the knowledge model. The purpose of the S^2RG generation algorithm is to implement the addition and inference of implicit expressions. On the S^2RG, the entities and attribute values are represented as nodes and relations. In the qualia role description system, entity relations are modelled by the qualia roles of entities. Specifically, the qualia roles of entities are determined by the syntactic structures, and the paper uses six qualia roles [19] to describe the knowledge entity relations in Chinese AWPs.

The reasoning of the solution chains is achieved from the discrete S^2RG to be holistic knowledge as a fully connected S^2RG. By modeling the entity relations of the DIR-AWP, the object roles between entities are obtained and added to R. R holds the entity qualia roles obtained after modeling for the input DIR-AWP. The related information in R must be completed by classifying the AWP scenario and obtaining the entity relation combination of the current AWP scenario in Table 1.

Table 1: The six DIR-AWP examples of the implicit knowledge and their corresponding QS^2M models

No.	QS2M model (Pattern R)	DIR-AWP text	DIR-AWP examples model match	Implicit knowledge
1	(CON n v n v: {[n,v,n,v], $f = n * n$})	The relation between the length L and the area A	model 1	$\{A = L * L\}$
2	(n ACT v: {[n,v], $f = k * v$})	*I* as an interest of deposit D	model 2	$\{I = D * rate\}$
3	(m n v ORI: {[m,n,v], $f = m * n$})	A relation between the price P and the weight W	model 3	$\{P = k * W\}$
4	(m n v q: {[m,n,v,q], $f = k * n + m$})	The proportional relation of the distance D with the speed S	model 4	$\{D = S * time\}$
5	(n v increase of n: {[n,v,n], $f = k * n + m$ })	Y is increases with the X increases	model 5	$\{Y = k * X + b\}$
6	(n n m CON: {[n,n,m], $f = k * n$})	V is a proportional decrease function of T	model 6	$\{V=k * T\}$

3.3.1 Commonsense Foreground Knowledge: S^2RG Node Generation

The S^2RG of DIR-AWPs is the solving process state of reasoning missing nodes. Based on the S^2RG scenario feature, the model defines the input position sequence nodes N_e. The knowledge model combined the manual prompt pattern for inquiring about implicit AWP knowledge entity candidates for AWPs from the pre-train language model.

After setting up the template, the explicit entities N_e follow prompt Q_r to complement a node N_i, the attributes are e^a, and the links Q_r. The model needs to fill the candidate entities into the structure of the incomplete triple $u = <N_e, Q_r, N_i>$: the label words match the Chinese pre-trained language model L = chinese-roberta-wwm-ext for MLM (Masked Language Model) to get the Chinese grammatical words [MASK]. Ranking the candidate entities according to scores.

Where N_i represents the output of the prompts f (N_e):

$$N_i = f_{Qr}(N_e) \tag{2}$$

The N_e is the indicator to match pattern Q_r to traverse the nodes in the S^2RG. The implicit nodes N_i and domain R_d through Q_r. The prompt pattern $N_i = f_{Qr}(N_e)$ defines the input position and explicit nodes N_e.

3.3.2 *Subject-Domain Background Knowledge: S^2RG Implicit Relation Generation*

Subject-domain relations exist in the DIR-AWP scenario-solving process, represented as S^2RG complements the DIR-AWP's problem-solution chain. The construction of entities by obtaining the qualia roles of entities based on a syntactic format and then describing the entity relations through the qualia roles.

$$R_i = f_{DomainFormula}(E_1, E_2, \ldots, E_n) \tag{3}$$

Compared with the S^2 model defined in Yu et al. [1], this definition extends the QS^2M method that provides a mechanism for acquiring the knowledge items from function problem text. The paper manually designs 150 logical cues for arithmetic reasoning based on the problem context of the topic by splitting the solution expression into multiple sub-expressions based on different topic contexts and giving logical explanations described in natural language based on each sub-operational unit according to the pattern of thought chain reasoning. which covered the five contextual categories summarised in Table 1, including 20 for the plane problem; 32 for the task problem; 38 for price problems; 36 for task problems; 24 for MovePath problems.

Solving DIR-AWPs involves combining explicit and implicit knowledge into a fully connected graph S^2RG, which involves searching for a chain of nodes that connect the known information to the solution goal *g*. The S^2RG is enhanced using a structural qualia syntax-semantic pattern, transforming it into a relation-centric representation. This pattern includes various elements such as lexical markers, keywords, dependency relations, and sequential relations within the sentence to construct a semantic scenario. Overall, the approach provides a more comprehensive and rigorous framework for problem-solving in Algorithm 3.

$$R_i = f_{s2}(E_1, E_2, \ldots, E_n) \tag{4}$$

Algorithm 3: Implicit knowledge acquisition and transformation into quantity relations

Input: The syntax-semantic relation graph $S^2RG = (N_e, R_e)$,
The qualia structure patterns $Q_r = (f_a, f_b, f_c)$.
Output: Quantity relation set $R_{\Delta.}$
//Traverse solution goal g and known entities ne to add n_i.

1: **for** each n_e in S^2RG do
2: **if** g not contain relation with n_e **then**
3: Add n_i and Q_r from ne to S^2RG;

(Continued)

Algorithm 3 (continued)

else if g and ne has a loop **then**
continue;
end if
end for
//Traverse connected S^2RG to expand knowledge point
for matching fact sub-graph f_i in S^2RG **do**
if f_i only contain two nodes **then**
add f_i to G_i remove the r_e;
else if f_i has a loop **then**
continue;
end if
end for
// Transform connected S^2RG to math relations
for each sub-graph f_i in F_a U F_b U F_c do
Translate f_i to R_i according to formulas;
end for
return $\mathbf{R}_\Delta$

4 Experiment

The paper presents the "scenario category" for the DIR-AWPs and constructs a comprehensive entity relation graph S^2RG. Specifically, the study investigates five scenario categories of primary school DIR-AWPs and provides a detailed account of each category's entity and qualia role combinations. These findings shed light on the underlying structures of the DIR-AWP types and offer insights into how to model entity relations effectively. This section presents the empirical findings compared to the Math23K, a publicly available dataset.

4.1 Dataset

In this study, the paper employed the Math23K [11] dataset, which is widely used to evaluate math problem solvers, and contains both story and non-story problems (e.g., equations, formulas, numbers). The approach, QS^2M, was explicitly applied to story problems. Previous researches by Mayer [33], Cheng et al. [34], Hong et al. [35], and He et al. [36] have shown that AWPs can be classified into various scenarios based on their storylines, which impact the problem-solving process and the quantity relations involved. To assess the effectiveness of the algorithm on different problem categories, the paper classified 6030 problems from Math23K into five distinct groups to create a new dataset for evaluation. The goal was to evaluate the performance of QS^2M across these different categories. The newly created dataset comprises only one-quarter of the original Math23K dataset and comprises five DIR-AWPs types. The dataset includes 6030 problems, sufficient to demonstrate the universality of AWP solvers, as it represents typical cases encountered in AWPs. Table 2 provides detailed information regarding the new dataset and is available for download.

Baselines. Three methods compared the model as below:

• **S^2 model** [2]: a theoretical framework has been developed for addressing arithmetic word problems that involve explicit statements and require the use of a set of S^2 models. This framework offers a systematic approach to solving such problems by incorporating various linguistic and mathematical concepts to represent and manipulate the problem's elements effectively.

- **GTS** [21]: a math word problem solver, structured as a goal-oriented tree, is utilized for the purpose of producing solution expressions.
- **Graph2Tree** [18]: a deep learning architecture that integrates the strengths of graph-based encoders and tree-based decoders to generate expressive solutions. This model achieves enhanced performance in generating solution expressions by leveraging the inherent structural properties of both graph and tree representations.
- **QS²M:** the model proposed in this paper.

Table 2: The distribution of Math23K over five types of problems

Scenario of DIR-AWP	Number of DIR-AWP	Distribution
Percentage	1379	6.1%
Plane	1106	4.5%
Price	246	1.3%
Task	246	6.5%
MovePath	1830	7.8%
Total	6030	26.2%

4.2 Performance on Quantity Relation Extraction

The evaluation of quantity relation extraction is not commonly performed by all neural solvers, and the lack of large-scale ground truth for quantity relation evaluation posed a significant challenge. To assess the performance of quantity relation extraction, accuracy (Acc), recall (R), and F1-score metrics were compared to the V−S² model. The results of the test are summarized in Table 3 and show that the proposed QS²M model significantly outperforms the V–S² model by 7.8% in overall accuracy. Specifically, the QS²M model outperforms neural models by 11.8% and 11.1% on the Plane and MovePath problems, respectively.

Table 3: The extraction result (%) of math relations compared with the V−S² method

Scenario of DIR-AWP	Number of question	V-S² model			QS²M		
		Acc	*R*	F_1	*Acc*	*R*	F_1
Percentage	152	0.811	0.721	0.733	0.821	0.850	0.835
Plane	223	0.825	0.870	0.847	0.943	0.913	0.927
Price	232	0.812	0.722	0.764	0.975	0.847	0.861
Task	643	0.905	0.779	0.837	0.916	0.943	0.955
Movepath	248	0.755	0.632	0.687	0.866	0.854	0.859
Total	1498	0.845	0.753	0.795	0.923	0.911	0.917

The authors of [37] employed a methodology where they encoded the problem statement and the output of Algorithm 2 using the AWP solvers, such as GTS and Graph2Tree. This approach was designed to evaluate problem-solving accuracy, and the results were reported in Table 4. According to the results presented in Table 4, it can be observed that the incorporation of the QS^{2}M tasks has led to a significant improvement in the average accuracy of both the Graph2Tree and GTS models.

Table 4: The accuracy result (%) of problem-solving

Scenario of DIR-AWP	GTS	Graph2Tree	GTS + QS^{2}M	Graph2Tree + QS^{2}M
Percentage	82.7	80.9	95.0	92.9
Plane	89.6	88.9	91.7	91.2
Price	86.2	82.8	97.9	94.9
Task	27.7	34.7	42.7	62.0
MovePath	43.9	44.3	57.2	59.7
Average	65.8	66.5	78.5	82.0

Specifically, the injection of the extracted quantity expressions has resulted in an average accuracy of 82.0% and 78.5% for the Graph2Tree and GTS models, respectively. These findings suggest that the proposed approach is highly effective in solving AWPs that require more implicit relations.

5 Conclusions and Future Work

This paper acknowledges the notable advancements achieved in solving AWPs. However, it also recognizes the absence of an effective method for uncovering deep implicit relations for addressing DIR-AWPs, including those related to common sense or subject-specific knowledge. The present paper suggests utilizing the three-step qualia-quantity approach for discovering deep implicit relations. In the initial stage, the S^2 method extracts all the explicit relations and identifies scenarios using a pre-existing set of qualia-quantity models. Subsequently, the missing entities are incorporated under the identified scenarios, and qualia-quantity models are employed to establish deep implicit relations. Finally, an S^{2}RG is proposed to represent all the obtained relations, which is then condensed by pruning superfluous branches to solve the given problem. The answers are obtained by solving the distilled relations.

The study proposed a novel method called QS^{2}M that represents quantity expressions linked with uncorrelated entities to address the lack of hidden relations in complex scenarios. In future endeavors, the paper aims to enhance this solver for a broader range of issues and construct a more comprehensive knowledge repository, based on qualia role, to construct problem solvers. Furthermore, the paper has plans to design an intelligent tutoring system and to explore more efficient educational strategies utilizing the system to guide and teach students.

Acknowledgement: I would like to express my deep gratitude to Professor Xinguo Yu, my research supervisors, for his patient guidance, enthusiastic encouragement and useful critiques of this research work. I would also like to thank Dr. Bin He, for her advice and assistance in keeping my progress on schedule. My grateful thanks are also extended to Mr. Litian Huang for his help in doing the meteorological data analysis, to Mr. Liang Xue, who helped me calculate the wind pressure coefficient and to Mr. Zongyou Qiu for their support in the site measurement. I would also like to extend my thanks to the technicians of the laboratory of National Engineering Research Center for E-Learning (NERCEL) for their help in offering me the resources in running the program. Finally, I wish to thank my parents for their support and encouragement throughout my study.

Funding Statement: The National Natural Science Foundation of China (No. 61977029) supported the work. This work was supported partly by Nurturing Program for Doctoral Dissertations at Central China Normal University (No. 2022YBZZ028).

Author Contributions: H. Meng: review and editing, writing original draft, funding acquisition; X. Yu: conceptualization, methodology, funding acquisition; B. He: investigation, project administration, supervision; L. Huang: software, validation, visualization; L. Xue, Z. Qiu: data collection.

Availability of Data and Materials: Data will be made available on request.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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