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A Review on Fretting Wear Mechanisms, Models and Numerical Analyses

Tongyan Yue[1, 2] and Magd Abdel Wahab[3, 4, *]

Abstract: Fretting wear is a material damage in contact surfaces due to micro relative displacement between them. It causes some general problems in industrial applications, such as loosening of fasteners or sticking in components supposed to move relative to each other. Fretting wear is a complicated problem involving material properties of tribo-system and working conditions of them. Due to these various factors, researchers have studied the process of fretting wear by experiments and numerical modelling methods. This paper reviews recent literature on the numerical modelling method of fretting wear. After a briefly introduction on the mechanism of fretting wear, numerical models, which are critical issues for fretting wear modelling, are reviewed. The paper is concluded by highlighting possible research topics for future work.

Keywords: Fretting wear, wear models, wear mechanisms, numerical modelling.

1 Introduction

In tribology field, fretting is a small oscillatory motion between contact surfaces. Depending on the range of this oscillatory motion at the contact interface, the fretting regime is categorized into three types: stick regime where there is no relative slip at the interface, partial slip regime in which condition sticking regime exists at the centre of the interface with sliding approaching contact edges and gross sliding regime where sliding occurs alone the whole contact surface. Usually, this movement is attributed to the deflection of machine components with clamped joints or press fits. Therefore, unlike rolling or reciprocating, fretting occurs where contact surfaces are not supposed to move relatively to each other. Occasionally, this movement is very small as in the case of gear couplings and spline couplings [Neale and Gee (2001)]. Fretting wear, as one of problems induced by fretting, is wear usually happens in the gross sliding regime [ASTM International (2013)]. Due to its micro scale relative movement, wear debris generated from contact surfaces is difficult to expel from interfaces during wear process. In the following paragraphs, some of most typical occasions of fretting wear existing in industry are introduced.

[1] State Grid Xinyuan Maintenance Branch, Beijing, China.

[2] Department of Electrical Energy, Metals, Mechanical Constructions & Systems, Soete Laboratory, Faculty of Engineering and Architecture, Ghent University, Ghent, Belgium.

[3] Division of Computational Mechanics, Ton Duc Thang University, Ho Chi Minh City, Vietnam.

[4] Faculty of Civil Engineering, Ton Duc Thang University, Ho Chi Minh City, Vietnam.

[*] Corresponding Author: Magd Abdel Wahab. Email: magd.abdelwahab@tdt.edu.vn.

Fretting wear could be found in the blade/disc dovetail connection. This connection is an important element in the fan and compressor rotor assemblies of an aero-engine [Anandavel and Prakash (2011)]. When the engine is rotating, this connection is subjected to fretting induced by the centrifugal blade load and the aero-dynamical high frequency vibrations acting on the blade as shown in Fig. 1. This vibration is in a wide range based on various working conditions. For instance, the stroke is up to 200 µm during engine starts and stops, while it usually reduces to less than 10 µm during the fight in which case the micro-sliding is induced by the aerodynamic perturbation [Miyoshi, Lerch and Draper (2003); Fouvry, Paulin and Deyber (2009); Gallego, Fulleringer, Deyber et al. (2010)]. Therefore, both gross sliding and partial slip may happen at the blade/disk contact when engine is working.

Ropes is widely applied in the industrial field taking the advantage of their high axial strength and bending flexibility. Structurally, as shown in Fig. 2, one rope consists of strands of wires wound together in a variety of arrangements, generating plenty of contact interfaces between wire/wire and strands/strands. Hence the size of wires, the number of wires in one strand and the wind pattern of wires in the strand also affect the mechanical properties of the rope, besides the material properties of wires and the core.

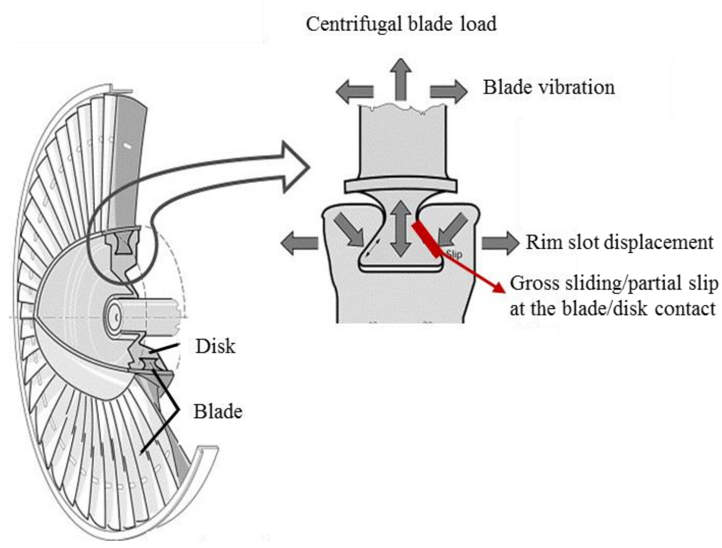

Figure 1: The blade/disk contact of a turbine engine [Miyoshi, Lerch and Draper (2003)]

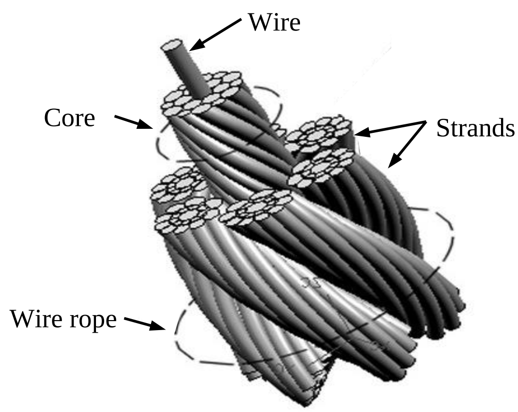

Figure 2: Schematic of a wire rope composed of different strands [Cruzado, Urchegui and Gómez (2012)]

Fretting wear in rope contact is another classic example of fretting problems in practical applications. Wind and the atmospheric corrosion by the pollution induce fretting wear of overhead conductor. By investigating the failure of the ACSR conductor reported in reference [Azevedo and Cescon (2002)], both partial slip and gross sliding region were found. This fretting damage could lead to the strand failure, even to a blackout and a collapse of the power transition line [Chen, Wang, Wang et al. (2012)].

Total Tip Replacement (THR) is a surgical procedure to relief pains for the large majority of patients suffered from most kinds of hip arthritis [UW Medicine (2015)]. Fig. 3 illustrates the components and structure of a THR, and the position of implants in the hip. Many contact surfaces generates during assembling these components and inserting them to the hip, such as acetabular cup/plastic line/femoral head contacts and femoral stem/bone contacts.

Fretting wear occurs when patients bear stresses during walking due to different material properties and geometries among these fretting couples of contact interfaces. In this case, the fixation/loosening related to the implant/bone interaction, and the wear of the articulating surfaces are two critical issues limiting the service life of an artificial hip joint [Mattei, Di Puccio, Piccigallo et al. (2011)]. If the metallic debris from fretting wear and corrosion goes in and around the hip joint, patients may suffer serious problems, for instance, inflammation, Adverse Local Tissue Reactions (ALTRs), hypersensitivity/allergic reactions and the bone loss. As reported in Wang [Wang (2012)], for a metal-UHMWPE artificial hip joint, its maximum biological life is reduced to no more than 10-15 years instead of the normal mechanistic life which can reach 40 years.

These industrial examples described above reveal the importance of predicting fretting wear and reducing wear damage before fretting couples fail. However, in order to study fretting wear, two difficulties of transforming engineering problems to research should be noticed. Firstly, fretting wear could be found in most quasi-static loaded assemblies under vibration, from traditional industrial application, such as engine, to biomedicine cases as the artificial hip joint replacement. Secondly, it is a complicated damage phenomenon relating material properties, working environment, loading conditions, etc. In the laboratory it is not easy to create a general test rig to reproduce and study different practical fretting wear problems. Moreover, parametric study of fretting wear raises higher requirements for the test rig design and selection. Contact variables are also difficult to measure during experiments, while they are essential parameters to investigate wear damage, and evolution of wear scars

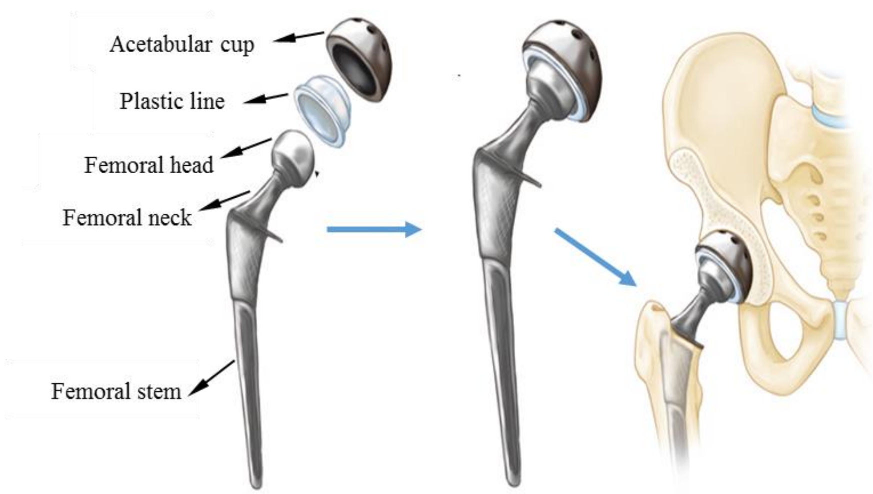

Figure 3: Left: individual component of a total hip artificial prosthesis, centre: the assembly, right: the implant as it fits into the hip [UW Medicine (2015)]

To avoid difficulties encountered in the experimental study of fretting wear, numerical methods are chosen to fretting wear study by more and more researchers, which thanks to the fast development and the more popularity of computer science. As a numerical method, FEM is extensively applied to simulate complicated physical problems. FEM discretize continuous domain to calculate approximation and analyse behaviours of objects. Many different practical applications are studied under fretting condition by FEM, for instance, the aero-engine blade/disc dovetail [Anandavel and Prakash (2011); Golden and Naboulsi (2012)] and the total hip replacement [English, Ashkanfar and Rothwell (2015); English, Ashkanfar and Rothwell (2016)].

This review gives an up-to-date overview of fretting wear research involving wear mechanics and numerical modelling of fretting wear. In Section 2, the literature review on experimental studies for fretting wear mechanism is discussed from two aspects: a) debris effects and b) evolution of Coefficient of Friction (CoF) during wear process. Following these studies, two wear models, namely Archard model and energy model, are presented. Both models are widely implemented to the modelling of fretting wear. Various numerical modelling research for fretting wear in last ten years are reviewed. In the last section, possible topics for improving numerical modelling on fretting wear are proposed to drive future work.

2 Experimental study of fretting wear

2.1 Wear mechanism

Although ASTM Committee G02 [Budinski (2007)] defined fretting wear as non-abrasive wear, the process of fretting wear is very complicated due to different fretting couples. Hence one certain wear mechanism could not explain all various fretting wear processes. Based on experimental studies of fretting wear, debris stays more easily in the contact surfaces of fretting wear than reciprocating wear. Therefore, a description of fretting wear process focusing on the participation of debris was proposed by Hurricks [Hurricks (1970)]. In this description, fretting wear process of metallic material was divided into three stages: (a) initial adhesion and metal transfer, (b) generation of debris and (c) steady-state wear. To date, abrasive wear [Colombié, Vincent, Godet et al. (1984)] and delamination [Suh (1973)] have been found in the process depending on various loading conditions and fretting couples

Abrasive wear: Abrasive wear usually occurs when two contact surfaces have different hardnesses. It is attributed to the indentation of harder asperities or particles to the softer surface in the relative sliding process. Usually, debris trapped in the fretted interfaces is metal oxide which is harder than the matrix material. In this case, abrasion wear, precisely three-body abrasion happens in the contact surfaces even between surfaces of similar materials. Abrasive wear under fretting condition has been studied on different materials of fretting couples and loading conditions. In the early studies, fretting wear tests of different materials, i.e., steel/steel and chalk/glass, were carried out by Colombié et al. [Colombié, Vincent, Godet et al. (1984)]. Results revealed that the wear of the matrix material was governed by the competition of generation and maintenance of the debris layer with abrasion of debris layer. Wear mechanisms can be different according to different types of fretting couples. Varenberg et al. [Varenberg, Halperin and Etsion (2002)] investigated the role of oxide debris in fretting couples of steel/bronze and steel/steel. Experiment results show the abrasive mechanism was prevailing in the pair of steel/steel with accelerating the damage by debris. While for the combination of steel/bronze, the adhesive wear mechanism was predominant and the debris acted as a kind of lubricant reducing the damage of fretting wear. Hardness of fretting couples may affect wear mechanisms. The research of Lemm et al. [Lemm, Warmuth, Pearson et al. (2015)] presented findings that a critical hardness differential threshold existed for fretting couples of steel/steel above which the oxide-based fretting debris was trapped on the surface of softer body of fretting couple and protected the softer body, as shown in Fig. 4. In this case, the wear was predominantly related to the harder specimen, due to this retention of oxide debris resulting in the abrasion wear of the harder counter-face. The experimental studies above demonstrate abrasion wear is a wear mechanism for fretting wear.

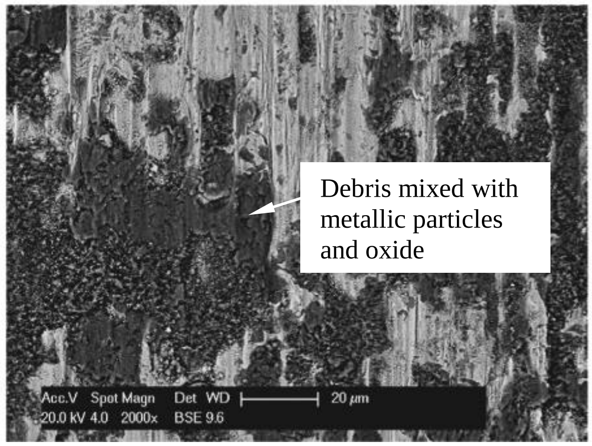

Figure 4: SEM of debris trapped in the matrix of the homo-hardness fretting couples [Lemm, Warmuth, Pearson et al. (2015)]

Elleuch et al. [Elleuch and Fouvry (2002)] studied effects of different displacement amplitudes on fretting behaviour of aluminium alloy (A357)/52100 steel. They found that a displacement amplitude threshold existed relating to the form and composition of the debris which was independent of the sliding velocity and temperature. However, according to the research on the high strength alloy steel SCWV in fretting wear [Pearson, Shipway, Abere et al. (2013)], a significant reduction in wear was found from 25°C to 85°C due to the formation of a protective glaze layer by retention of wear debris. A recent research on the high strength alloy steel S132 reported that the transition temperature, under which range the wear coefficient and CoF decrease significantly with increasing the temperature, related to the displacement amplitude [Hayes and Shipway (2017)]. Based on the study of 304 stainless steel on fretting wear, the fretting frequency affects the transition temperature [Jin, Shipway and Sun (2017)]. Besides steel, the loading conditions (displacement amplitude, temperature, the effect of the normal load, the frequency and the contact size) were also studied on Ti-6Al-4V contact [van Peteghem, Fouvry and Petit (2011); Fouvry, Arnaud, Mignot et al. (2017)]. The normal force sequence governed the interface structure of the contact and the oxidation process, and the frequency controlled the wear rate. Furthermore, the lower loading conditions (low contact pressure, low frequency, small contact size and varying normal force) promoted the contact oxidation favoured a U shape fretting wear scar and a higher abrasive rate.

Delamination: In 1973, Suh [Suh (1973)] proposed the delamination theory of wear. This theory argues that the delamination caused adhesive wear and fatigue, and the wear process could be summarized as four stages by this theory:

1) The dislocations at the interface of the soft material are taken into sub-surface;
2) Cracks and voids of subsurface appear;
3) Cracks are gathered by the shear deformation of the surface;
4) The wear sheet is created.

This mechanism is closer to the practical situation, considering actual micro-mechanism based on the failure and damage processes. On year later, Waterhouse and Taylor [Waterhouse and Taylor (1974)] studied fretted surfaces of 0.7 carbon steel, commercially pure titanium and Al-Zn-Mg alloy. The experimental results showed that the wear mechanism was adhesion and abrasion when the applied displacement amplitude was higher than 70 µm. Below 70 µm, results indicated that loose wear debris was generated by the propagation of sub-surface cracks. These results were similar to that postulated in the delamination theory of wear. Li et al. [Li and Lu (2013)] studied the influence of the applied displacement amplitude on fretting wear of Inconel 600 alloy. It is found that wear mechanisms changed to the oxidation and delamination with increasing the applied displacement amplitude to the gross sliding regime.. As shown in Fig. 5, the crack appears at the subsurface of the contact surface. Hence, based on the experiment observation, delamination is demenstrated as one of the wear mechanisms for the fretting wear.

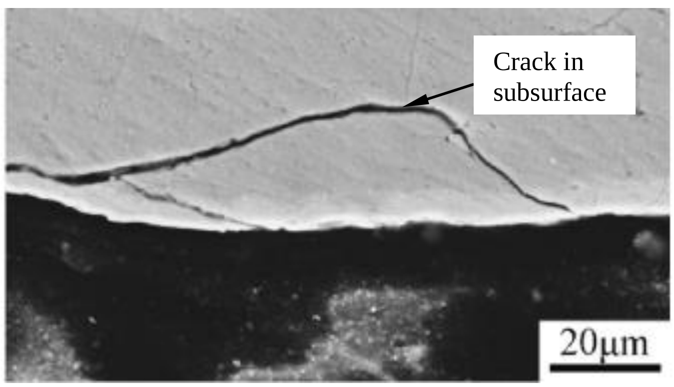

Figure 5: SEM of the crack at the subsurface of the contact under the gross sliding condition: *D*=120 µm, *P*=100 N [Li and Lu (2013)]

These experimental studies presented above brings significant insights into the mechanisms of fretting wear and discovers the important role of the debris playing during wear process. In addition, for a given fretting couples, the fretting regime is directly controlled by mechanical parameters of fretting experiments thus which should be carefully selected in relevant studies.

2.2 Evolution of CoF

Friction is the force of resisting two contact bodies moving on each other, which induces energy loss and leads to some important consequences in the contact surfaces, such as wear. Usually, force of dry friction is supposed to follow Coulomb friction law, of which model the friction force is described by CoF and the applied normal load. CoF is categorized as a systems-dependent parameter rather than an intrinsic property of a material or combination of materials. System variables, such as the sliding distance and the environment properties like the contact pressure and the surface quality, also have considerable influences on the CoF between contact surfaces [Suh and Sin (1981)]. Factors impacting the friction behaviour are grouped by Blau as: the contact geometry, fluid properties and flow, lubricant chemistry, the relative motion, applied forces, third-bodies, temperature, stiffness and vibrations [Blau (2001)].

During the fretting wear process, loading conditions like the applied normal load and the displacement amplitude have significant influences on CoF. CoF of the steady stage decreases with increasing the normal load under a given displacement amplitude, as shown in Fig. 6. Similar tendency of CoF during fretting wear is also reported in the study of the high strength alloy steel [McColl, Ding and Leen (2004)] and steel wires [Shen, Zhang, Duan et al. (2011)]. This phenomenon may be explained by that the elastic deformation induced by the small normal load causes asperities of rough contact surfaces to interlock with each other, leading to high CoF. Increasing the normal load to activate the plastic deformation of asperities, CoF gets lower because of less effect of interlock [Zhang, Ge and Qiang (2003)]. Under a given normal load, CoF of both dry and lubricated contact surfaces are various depending on the applied displacement amplitude, which is presented in Fig. 7. Besides the continuous evolution of the contact variables caused by the changing of contact geometries, debris also plays a significant role on CoF. A transition to a higher CoF occurs in a critical contact pressure, and this threshold of pressure is related to the composition of the debris [Diomidis and Mischler (2011)].

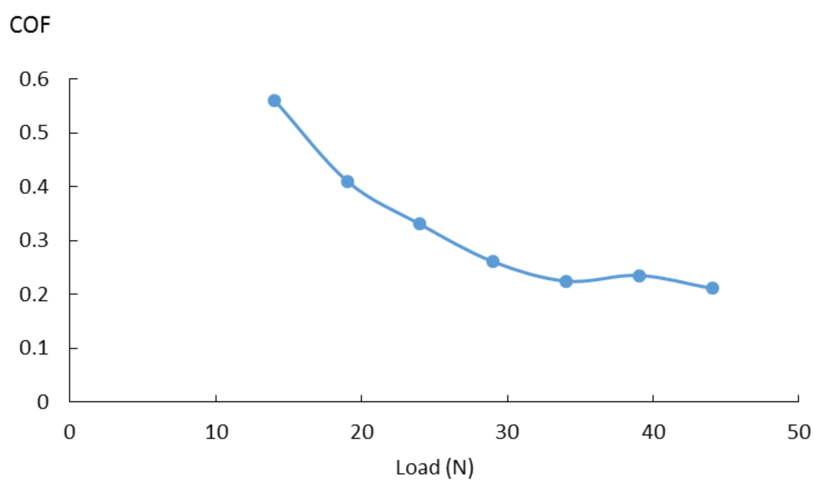

Figure 6: The evolution of CoF of the steady stage for different normal loads, *D*=75 μm [Zhang, Ge and Qiang (2003)]

Usually, the evolution of CoF during fretting wear process could be divided into 3 stages, as shown in Fig. 8. The first stage is running-in stage of which CoF is low due to contact surfaces covered by the oxide and the 'nature pollution' film reducing the adhesion of the interface. With removing of this film and appearing more adhesion and abrasion in substrate interfaces, CoF increases gradually. At the third stage, a balance between generation and ejection of debris reaches and CoF keeps stable [Zhang, Ge and Qiang (2003)].

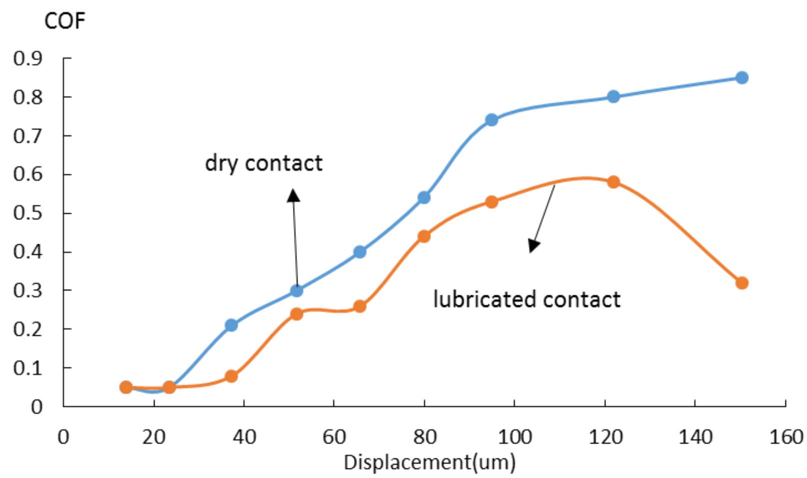

Figure 7: The evolutions of CoF of the steady state with different displacement amplitudes under friction-increasing grease and dry friction conditions, *P*=24 N [Shen, Zhang, Duan et al. (2011)]

Experimental results presented above shows that fretting wear is a very complicated surface damage due to friction and that debris plays an important role. Main factors of fretting wear are materials of fretting couples (types, hardness), loading conditions (the normal load and the displacement amplitude) and environmental conditions (dry friction or with lubrication).

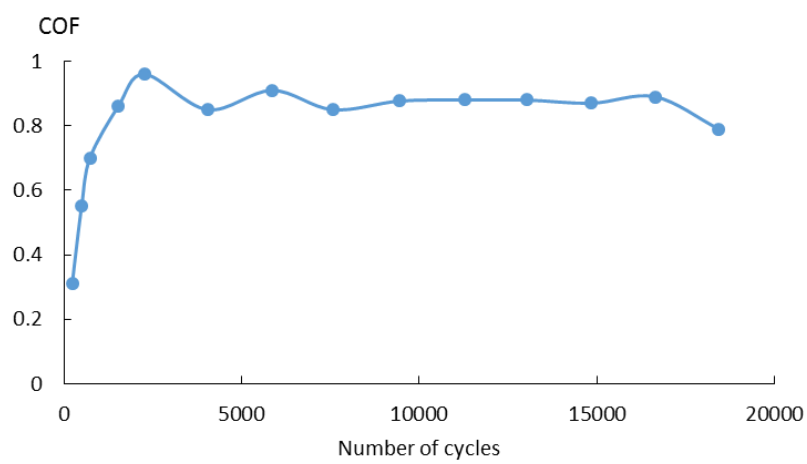

Figure 8: The evolution of CoF during fretting wear tests [McColl, Ding and Leen (2004)]. *R*=6 mm, *P*=185 N and *D*=25 μm

3 Fretting wear models

3.1 Impact factors

The property of fretting involves a large number of factors including material properties and the working environment of fretting couples. Fig. 9 lists main variables impacting fretting. Depending on combinations of these parameters, different fretting damages may occur in fretting couples.

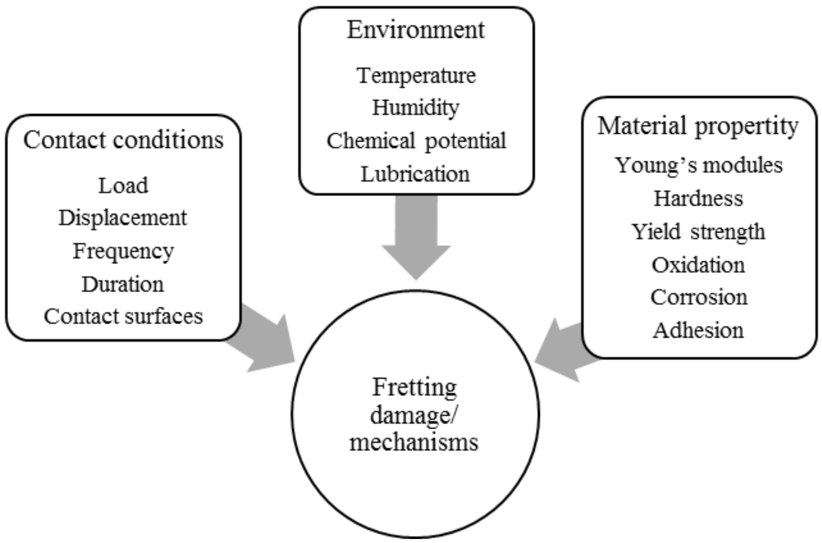

Figure 9: Impact factors in fretting [Braunovic (2009)]

Fretting wear problems cause the service lives reduction of practical applications considerably. Hence, it is important and necessary to simulate the fretting wear behaviour of different materials under various work environments. Under this condition, a reliable wear model connecting working parameters to the wear damage of a given fretting couple is really necessary.

Among the last ten years study by researchers, Archard model and energy model are the two main wear models to predict fretting wear process. Both of them are based on contact-mechanics [Meng and Ludema (1995)]. In the following sections, every model is introduced briefly.

3.2 Archard model

Archard proposed Archard model in 1953 [Archard (1953)] which was validated later by Archard et al. [Archard and Hirst (1956)]. In this model, the wear volume per unit sliding distance $\frac{V}{s}$ is calculated by the normal load P and the flow pressure P_m as:

$$\frac{V}{s} = k\frac{P}{P_m}, \tag{1}$$

$\frac{V}{s}$ is named as the wear rate of a given sliding system. P_m is approximately equivalent to the hardness H of the soft material of this system, i.e., $P_m = H$. The physical meaning of $\frac{P}{P_m}$ is the real contact area for fully plastic asperities [Hutchings (1992)]

k is wear coefficient related to the probability of each contacting asperity led to the loosened particle leaving the system. From experimental observations, the stead-state wear rate is constant during the wear test. Thus, k of a specific sliding system is also constant and could be calculated based on experimental data the formula for the calculation of k is:

$$k = \frac{HV}{Ps}, \tag{2}$$

The ratio *k/H* named Archard wear coefficient or dimensional wear coefficient is more useful in engineering applications, for comparing wear rates of different classes of materials.

Hence, Archard equation could be rewritten as:

$$\frac{V}{s} = K_A P, \tag{3}$$

where K_A is the Archard wear coefficient, K_A= *k/H*.

3.3 Energy model

Introducing energy concept to simulate wear process is another popular method. This dissipated energy method can be dated back to 1960s, firstly proposed by Matveevsky [Matveevsky (1965)]. In this research, friction power intensity (frictional energy dissipated per unit area) was supposed to be related to wear when studying oil-lubricated Hertzian point contact and line contact. From 1990s, Fouvry and his co-workers do plenty of study on fretting wear and improves energy concept on fretting wear further. In their fundamental work [Fouvry, Kapsa and Vincent (1996)], they suggested that the frictional work was the global energy dissipated in initiation, stimulating and activating different processes of wear. Fig. 10 shows the concept ion of energy dissipation on fretting wear.

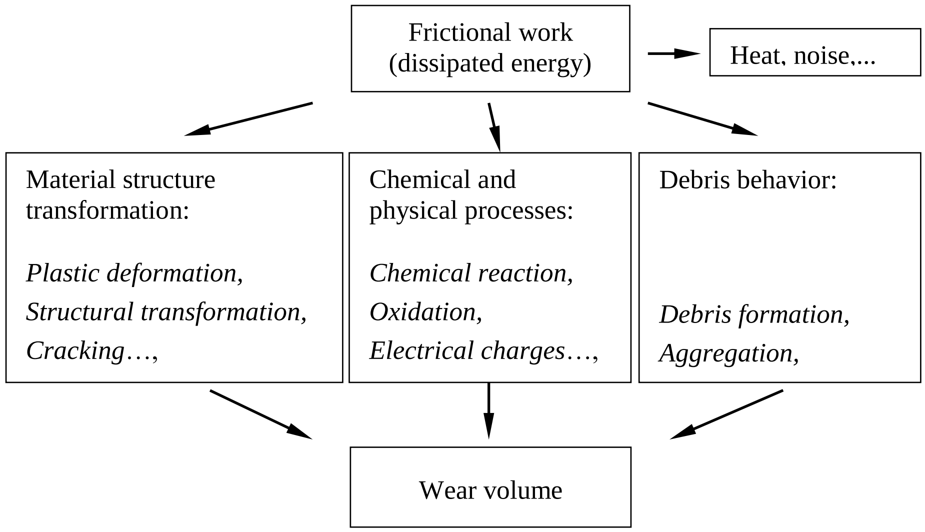

Figure 10: Dissipated energy concept on the wear process

In energy model, accumulated dissipated energy E_d of the whole wear duration is the summation of the frictional work of each wear cycle. This accumulated dissipated energy could be obtained from experiments by fretting loops. A fretting loop is determined by the friction force Q_i and the relative slip δ_i of one fretting wear cycle as shown in Fig. 11:

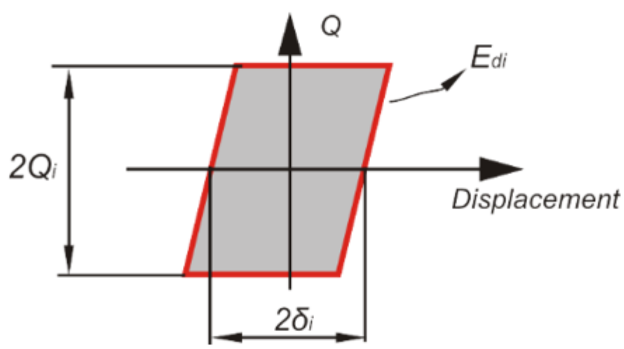

Figure 11: Calculation of the dissipated energy in one fretting wear cycle

Therefore, the accumulated dissipated energy E_d could be calculated as:

$$E_d = \sum_{i=1}^{N} E_{di} = \sum_{i=1}^{N} 4Q_i\delta_i, \tag{4}$$

Where E_{di} is the area of the fretting loop in the i^{th} fretting wear cycle and N is the total number of fretting wear cycles in an experiment. Q_i is the shear force and δ_i is the relative slip amplitude between contact surfaces in the i^{th} fretting wear cycle.

Meanwhile, linear relationships between the wear volume V and the accumulated dissipated energy were observed from experiments by Fouvry et al. [Fouvry and Kapsa

(2001)], presented in Fig. 12. These results were from experiments of different fretting couples, i.e., different steels/Alumina and different hard TiN, TiC coatings with substrate of a high-speed steel/Alumina under fretting and reciprocating condition. With Coulomb's friction law, the energy model could be written as:

$$V = K_E \sum_{i=1}^{N} E_{di} = K_E \sum_{i=1}^{N} 4Q_i\delta_i = K_E \sum_{i=1}^{N} 4\mu_i P_i \delta_i \tag{5}$$

where K_E is the energy wear volume coefficient of the studied interface under a given displacement amplitude. This factor relates the evolution of wear volume to the additional energy dissipated during fretting wear process. μ_i and P_i are the CoF and the normal load of the i^{th} fretting wear cycle, respectively. The research group of Fouvry also explained the formation of Tribologically Transformed Structure (TTS) in fretting wear process and found a specific threshold of dissipated energy E_{dth} exist before starting wear (Fig. 12) [Sauger, Fouvry, Ponsonnet et al. (2000); Fouvry, Kapsa and Vincent (2001)].

Both of Archard model and energy are classic wear models for fretting wear prediction. Archard model links the wear volume to the normal load and the sliding distance of a given tribology system. However, the main weakness of this model is without explicitly including the CoF of the fretting wear process. While, energy model is from the concept of the conservation of energy, i.e. part of the frictional work is dissipated by the wear process. The evolution of CoF of fretting wear is explicitly imported to calculate wear volume, and this makes it possible to investigate influences of CoF on fretting wear by FEM. Furthermore, the structure transformation of material, chemical and physical processes, and debris behaviour may change during the wear process. In this case, energy model is also convenient to explain different wear mechanisms.

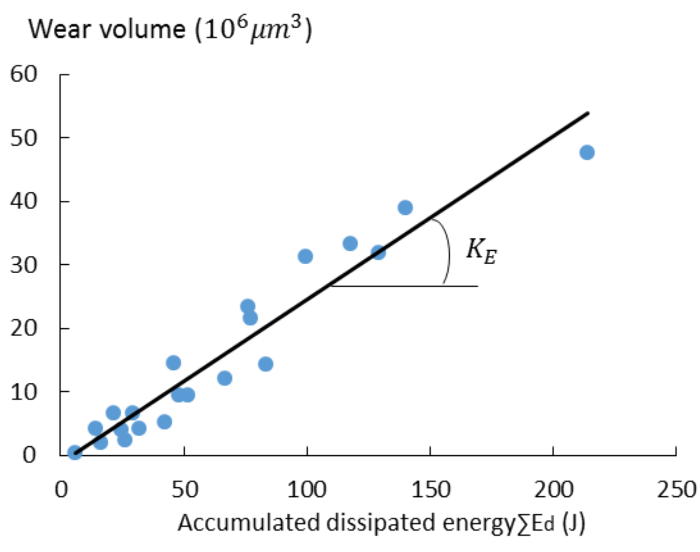

Figure 12: The linear relations between the accumulated energy and the wear volume. (*P*=50-200 N, *D*=25-200 μm, room humidity=50%) [Fouvry and Kapsa (2001)]

4 Fretting wear simulation of line contact

Using wear models introduced in Section 3 and FEM makes it possible to predict the fretting wear process. To date a number of studies have been examined this problem in various aspects by FEM. In this section, main research and findings are reviewed by two parts: FE models with and without debris effects.

4.1 Neglecting debris effects

Loading conditions: As presented in Section 3, loading conditions, such as the imposed normal load and the tangential displacement amplitude, affect fretting behaviour significantly. Therefore, most of early studies on fretting wear by FEM focused on the effects of loading conditions. In 2003, McColl et al. [McColl, Ding and Leen (2004)] firstly developed a FE model of fretting wear, using Archard model for wear calculation. It predicted the evolution of contact profile, contact variables and sub-surface stresses during wear simulation under various normal load conditions. Predicted results showed an underestimation of wear volume in higher normal load cases comparing to experimental results. Authors argued that using global wear coefficient and ignoring debris effects may induce this disparity. Influences of tangential displacement amplitudes were also studied, under both partial slip [Ding (2004)] and gross sliding condition [Paulin, Fouvry and Meunier (2008)]. Research in Ding [Ding (2004)] confirms that fretting wear is the primary damage in the gross sliding condition, however, fretting fatigue plays a more important role in the partial slip regime [Bhatti and Abdel Wahab (2018); Bhatti, Pereira and Abdel Wahab (2018)].

Material properties: Wear mechanisms under fretting are also based on material properties of fretting couples. The role of plasticity playing in fretting wear was studied by several groups. In an A357 aluminium alloy/AISI 52100 steel contact, it is found that elasto-plasticity of contact surfaces could not explain the sliding dependence of the wear rate, by simulating five wear cycles of fretting wear with a Prager linear kinematics hardening model [Elleuch and Fouvry (2005)]. For Ti-6Al-4V fretting couples, studies of Dick et al. indicted that ratcheting could induce stress redistribution, geometry evolution and the formation of residual stress, by predicting fretting wear of first 100 cycles with a cyclic plasticity model [Dick and Cailletaud (2006); Dick, Paulin, Cailletaud et al. (2006)]. In the study of Mohd et al. [Mohd Tobi, Ding, Bandak et al. (2009)], the evolution of plastic variables and effects of plasticity during fretting wear were analyzed using a kinematic hardening plasticity model for the cyclic plasticity behaviour description. Mohd et al. also investigated the accumulation of plastic strain in fretting wear decoupling wear effects [Mohd Tobi, Sun and Shipway (2017)]. Predicted results showed that significant plasticity accumulation existed due to plastic shakedown in the partial slip condition, while a saturation of plastic deformation occurs under gross sliding condition. In addition, the lifetime of coatings was predicted in Fouvry et al. [Fouvry, Paulin and Liskiewicz (2007)] and Mohd Tobi et al. [Mohd Tobi, Shipway and Leen (2011)] by FEM.

Fretting wear and fretting fatigue: Numerical modelling of fretting fatigue has been also intensively reported recently in the literature, including stress analysis [Pereira, Bordas, Tomar et al. (2016)] [Ferjaoui, Yue, Abdel Wahab et al. (2015); Kumar, Biswas, Poh et al. (2017)], damage initiation [Bhatti and Abdel Wahab (2017); Pereira, Bhatti and Abdel Wahab (2018)]and crack propagation [Martínez, Vanegas Useche and Wahab (2017); Pereira and Abdel Wahab (2017)]. The interaction between fretting wear and fretting fatigue has attracted some researchers. Madge et al. [Madge, Leen, McColl et al. (2007); Madge, Leen and Shipway (2007)] created a FEA tool integrating wear calculation with fretting fatigue analysis to predict influences of fretting wear on fretting fatigue life. This method not only predicted the evolution of contact profile and contact stressescaused by fretting wear but also calculated a multi-axial fatigue damage parameter with cumulative damage effects, which is a function of slip amplitude. Zhang et al. [Zhang, McHugh and Leen (2011)] presented a FEA model with energy method to compare fretting behaviour of different contact geometries, i.e., cylinder/flat contact and rounded punch /flat contact. This model was able to predict plasticity and fatigue damage parameters. It was found that the fatigue crack initiation was more sensitive to effects of slip regime and wear in the case of the cylinder/flat contact configuration, comparing to the case of rounded punch /flat contact configuration.

CoF: as explained in Section 2.2, CoF changes during fretting wear process. However, all the FE fretting wear models introduced above assume CoF is a constant. In 2007, Cheikh et al. [Cheikh, Quilici and Cailletaud (2007)] proposed a friction model, named KI-COF (Kinematic Isotropic Coefficient of Friction), to describe the evolution of COF. In this model, evolution of CoF is governed by the local history of the contact and the amount of slip at the interface. The difference of fretting loops between experiments and simulations decreased with this model. In 2014, KI-COF model was implemented to simulate deformation behaviour during torsional fretting in Liu et al. [Liu, Shen and Yang (2014)], which was focused on fretting fatigue and only ten cycles of fretting wear were simulated. Recently, a FE model considering effects of variable CoFs measured in experiments was generated in both gross sliding and partial slip conditions of fretting wear [Yue and Abdel Wahab (2017)]. Results indicated that, when considering partial slip or running-in stage of gross sliding conditions, FE models with variable CoF achieve predictions that are closer to experimental results. To date, fretting wear by FEM have been investigated intensively including loading conditions, material properties and fretting fatigue.

4.2 Debris models

Since debris generated from fretting wear plays critical role in the process, several fretting wear numerical models integrating debris impacts have been developed. Based on experimental observations, Elleuch et al. [Elleuch and Fouvry (2005)] proposed a modified Archard model considering the ejection process based on the idea of the debris ejection controlling fretting wear as shown in Fig. 13. This modified model predicts a parabolic evolution between the applied displacement amplitude and the wear volume. This model introduced PSD parameter to describe the debris ejection, however, the debris itself was not explicitly combined to the fretting wear FE model. Therefore, this model could not specifically obtain the evolution of stresses or wear profile due to debris during the wear process.

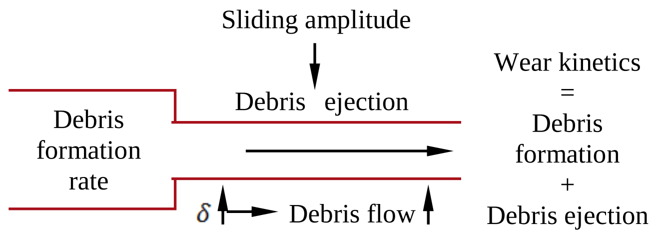

Figure 13: The debris flow of wear kinetics under gross sliding condition [Elleuch and Fouvry (2005)]

In 2007, Ding et al. [Ding, McColl, Leen et al. (2007)] firstly developed a FE model integrated a debris layer explicitly to a FE tool of fretting wear created by McColl et al. [McColl, Ding and Leen (2004)]. Fig. 14 shows this model including two contact interfaces generated by introducing a debris layer. The contact constraint of interface between the bottom debris and the top surface of the flat specimen Γ_1 was assumed rigid connection. While for the other interface, which is between the bottom surface of cylinder Γ_2 and the top surface of the debris Γ_3, the basic Coulomb's friction law was assumed as the contact constraint. Be means of this model, geometry evolutions of the debris such as the thickness and the width, and the normal movement of the debris layer, were analyzed. Based on Archard model and Hill's yield model, this simulation tool predicted debris effects on wear damage by redistributing the contact pressure and the relative slip between contact surfaces

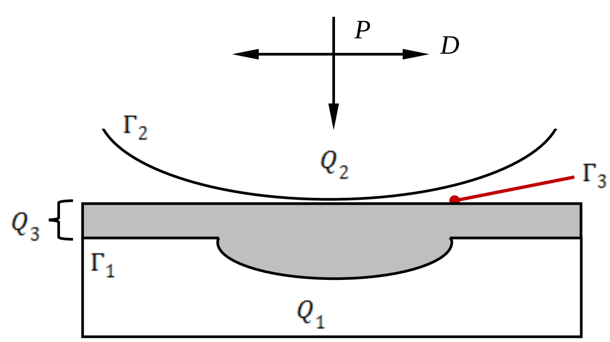

Figure 14: The simplified fretting wear contact model with a debris layer, Q_1 and Q_2: the contacting bodies, Q_3: debris. Γ_1: top surface of Q_1, Γ_2: bottom surface of Q_2, Γ_3: top surface of debris [Ding, McColl, Leen et al. (2007)]

Two years later, the same group presented a novel fretting wear FE tool based on multi-scale modeling [Shipway, Williams, Leen et al. (2009)]. The macro model simulates global wear process calculated with Archard model, and the micro model describes the roughness characteristics of contact surfaces presented by an asperity contact model.. Fig. 15 depicts the schematic diagram of the micro model, λ is the wavelength of the asperity spacing estimated by the roughness information of contact surfaces. d_{sub} is the instantaneous thickness of the debris layer. Both the normal load p^{sub} and the displacement with amplitude $\lambda/2$ were applied to the micro model. This micro model could determine the local plastic deformation under the debris layer, by which the insightful understanding of fretting wear mechanics could be possibly obtained. Although some assumptions were made, i.e., a) asperities were distributed uniformly, b) asperities were spherical with uniform radius and c) asperities were rigid, this multi-scale model successfully predicted the fretting wear process with evolution of interface between the debris and the substrate, which was closer to the realistic situation.

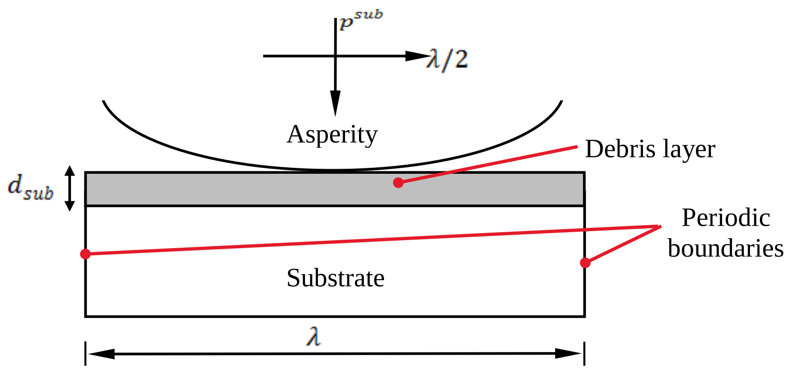

Figure 15: The micro model-asperity model used in multiscale modelling of fretting wear presented in Shipway et al. [Shipway, Williams, Leen et al. (2009)]

A plane strain fretting wear model with a debris layer was developed to investigate effects of debris on fretting wear damage [Yue and Abdel Wahab (2016)]. In this study, both the Young's modulus and the thickness of the debris layer were variable in this study. Meanwhile, the influence of the importing time of the layer was also investigated. Recently, a new FE model of fretting wear with the third body layer was developed [Arnaud, Fouvry and Garcin (2017)]. A third body conversion parameter was introduced to this model to quantify the debris layer in fretting wear. The results showed that the friction energy wear model without considering wear debris underestimates maximum wear depth and promotes dangerous non-conservative crack nucleation predictions.

Besides modelling the debris as a layer, the particle-shape debris is also introduced to FE models of fretting wear. In 2011, Basseville et al. [Basseville, Héripré and Cailletaud (2011)] developed a fretting wear model explicitly including rectangular particles with fixed number as the third body, as shown in Fig. 16. The wear calculation were conducted for both contact surfaces of substrate and particles based on dissipated energy method.The conservation of matter linked the substrate and particles, i.e., the amount of matter lost induced by wear was added to the debris. Although authors also simplified this model from real fretting wear process by neglecting the oxidation, choosing the fixed number of particles and only simulating 50 wear cycles, the predicted results provided some information of debris movement from physical aspect during wear process. It is found that debris might be trapped in the contact interface under partial slip condition, and they ejected from the interface if gross sliding occurred.

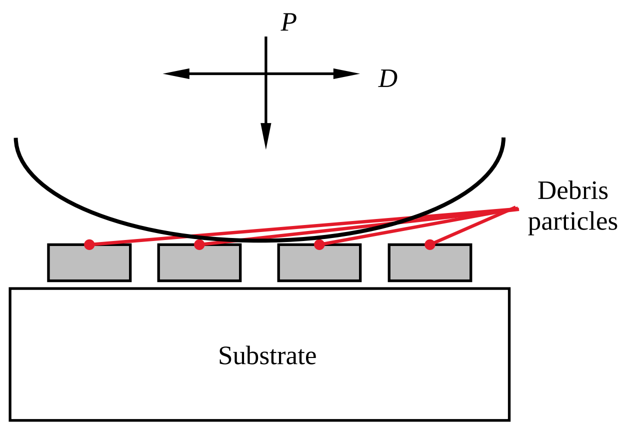

Figure 16: Schematic of the fretting wear model applied in Basseville et al. [Basseville, Héripré and Cailletaud (2011)]

A similar fretting wear FE model as shown in Fig. 16 but with sphere particles was developed in the research by Ghosh et al. [Ghosh, Wang and Sadeghi (2016)]. Comparing to the study of Basseville et al. [Basseville, Héripré and Cailletaud (2011)], they focused on the effects of material properties under partial slip condition, i.e., elastic plastic deformation, and the number of debris particles between interfaces to fretting behaviors. In this model, some assumptions were made for simplification, such as without considering the evolution of wear profile and keeping the stick zone as constant size. Predicted results reveal that the debris particles were underwent significant part of the load applied on the first bodies and plastically deformed, in addition, fretting wear had no direct relation with the number of debris particles.

As it reviewed from the above researches, the available studies have some limitations which could be improved in future work. For the debris model assumed as a layer, the contact interactions of interfaces between debris and first bodies are usually simplified as one is rigid connection and the other is followed Coulomb's friction law, which both should be actually controlled by Coulomb's friction law. For the particle-shape debris model, the difficulty is predicting full cycles of fretting wear, e.g., 10,000 cycles.

4.3 Stress singularity in contact simulation

Investigating the optimization of computational cost on fretting wear FE simulations is necessary, since it contains great number of fretting wear cycles and considerable fine meshes in the contact zone, which induces time-consuming tasks. Given to fretting wear cycles, an accelerating technique called jump cycle is developed to keep reasonable computational time without reducing much accuracy. In this technique, a jump cycle factor ΔN is introduced assuming that contact variables are kept constant during the next ΔN fretting wear cycles to cut down computational time. This method was firstly proposed by McColl et al. [McColl, Ding and Leen (2004)] in 2004 and have been widely used for FE fretting wear simulation, such as research of Ding et al. [Ding (2004); Elleuch and Fouvry (2005); Zhang, McHugh and Leen (2011); Tang, Ding, Xie et al.

(2013)]. Besides this cycle reducing technique, mesh size influencing simulation time and FE results should also be considered to investigate if the mesh size is fine enough to make stresses converged or divergence occurs induced by singularity. However, little research has been done for the mesh sensitivity of FE fretting wear problems.

Stress singularity, showing stress infinite, is due to concentrated loads or discontinuity, such as geometric discontinuity, boundary condition discontinuity or discontinuities of the material properties [Carpinteri and Paggi (2009)]. In fact, stress singularity does not exist in the real world, as no stress could reach to infinity. In FEM simulations, however, it could be pretended as a specific value in computational results. Hence, it is worthy studying if there is stress singularity in a specific FEM task. Singularity problems in elasticity are grouped into two type: power singularity and logarithmic singularity, which are presented in Sinclair's review paper [Sinclair (2004)].

Power singularity: The relation between local stress σ and the dimensionless radial distance r from a singular point can be written as:

$$\sigma = O(\sigma_0 r^{-\gamma}), as \quad r \to 0, \tag{6}$$

where γ is the singularity exponent.

For logarithmic singularity, this relation behaves like:

$$\sigma = O(\sigma_0 \ln r), \quad as\ r \to 0. \tag{7}$$

where in both Eqs. (6) and (7), σ_0 is the applied stress.

Two methodology are applied to identify stress singularity in research, namely asymptotic method and numerical method. Asymptotic method is an analytical method focusing on special locations, such as the tip of a crack, the apex of a sharp notch or the corner of some slipping in complete contact [Hills, Dini, Magadu et al. (2004)]. Stress singularity signature is a numerical method proposed by Sinclair [Sinclair (2004)], identifying whether or not there is a stress singularity in FEM simulations. By means of stress singularity signature, the divergence of peak stresses is calculated based on numerical results with different mesh sizes to check singularity. This method has been employed in the field of computational fluid dynamics [Sinclair, Chi and Shih (2009)] to confirm the local asymptotic identification of singularities induced by flow.

In order to study whether or not fretting provides additional damage in fretting fatigue problems, asymptotic method was employed to identify whether the threshold of stress intensity exists [Hills, Thaitirarot, Barber et al. (2012)]. However, in the case of fretting wear, it is difficult to apply asymptotic method to distinguish stress singularity, since contact surfaces are continuously changing during wear process. In addition, as FEA is widely used in fretting wear simulation, it will be very useful and efficient if stress singularity could be detected by results of numerical simulations. A singularity study of FE model under fretting or fretting wear condition was conducted to distinguish stress singularity at cylinder/flat contact, computing based on different variables, such as applied displacement and fretting wear cycles [Yue and Abdel Wahab (2014)]. This stress singularity signature study implied that the existence of stress singularity was related to fretting regime, and the mesh size of contact surface under partial slip condition should be chosen carefully.

5 Other numerical models for fretting wear

Besides using FEA, few other numerical modeling techniques were applied to model fretting wear process. Gallege et al. [Gallego, Nélias and Jacq (2006); Gallego and Nélias (2007)] proposed a semi-analytical method for fretting wear simulation of cylinder/flat and sphere/flat contact. Both gross sliding and partial slip conditions are simulated. This contact solver based on conjugate gradient method and optimized fast Fourier transfer techniques reduce computation time. Kasarekar et al. [Kasarekar, Bolander, Sadeghi et al. (2007)] proposed a numerical method for simulating fretting wear with rough surfaces. Results from sinusoidal rough and random rough surfaces indicate roughness plays a significant role in wear process. Dhia et al. [Dhia and Torkhani (2011)] presented a numerical modelling method by Arlequin framework to solve wear problem under sharp contact with flexibility and low cost of simulation. Lehtovaara et al. [Lehtovaara and Lonnqvist (2011)] developed a numerical model evaluating fretting wear in rough point contact under partial slip condition. The results indicate that the amplitude and wavelength in wavy surfaces affect the original size of the stick zone. Rodriguez-Tembleque et al. [Rodríguez-Tembleque, Abascal and Aliabadi (2011)] proposed a 3D boundary element method formulation with Archard wear model for simulating 3D fretting wear problems, without remeshing the surface of solids. The comparisons between BEM method and FE method and analytical models presented in literature have a good agreement. Leonard et al. [Leonard, Ghosh, Sadeghi et al. (2014)] presented a combined finite-discrete element method to modeling fretting wear based on Archerd wear model. The FEA is used to calculate the internal stresses of the contacting bodies, and discrete element method is employed to determine the interaction between the bodies. In addition, this method is applied to fretting wear model taking into account the effect of the third body [Leonard, Ghosh, Sadeghi et al. (2014)]. In this model, FEM was employed for the calculation of substrate bodies, while the debris and contact interactions between debris and substrates were simulated by the discrete element method. Also this method is extended to analyse fretting wear of rough contacts [Leonard, Sadeghi, Shinde et al. (2013)].

6 Conclusion

The currently available literature on fretting wear is reviewed based on various aspects including: impact factors, wear mechanisms, and the research by numerical modelling. A summary of literature on fretting wear regimes, models and numerical methods is presented in Tab. 1. Regarding numerical modelling, many aspects should be considered for the future works and are listed in the following points:

1) Wear coefficient models could be improved based on experimental data.

2) Coulomb's friction law is not always observed during fretting wear experiments. Other friction laws describing evolution of CoF may also be considered in fretting wear FE model in future.

3) Multiscale analysis could be applied for simulation of fretting wear with debris.

4) Singularity signature could be extended to 3D analysis of fretting wear. Applying the proposed singularity analysis to 3D FE wear models will help in balancing efficiency and

accuracy of it by identifying the location of stress singularity and hence adjusting mesh sizes.

Table 1: Summary of literature on fretting wear regimes, models and numerical methods

References	Method	Material	Fretting regime	Wear model	Objective
[McColl, Ding and Leen (2004)]	FEM	High strength steel	Gross sliding	Archard model	Develop a fretting wear simulation tool
[Ding (2004)]	FEM	High strength steel	Gross sliding/partial slip	Archard model	Study the effects of slip amplitude on fretting wear and fretting fatigue
[Dick, Paulin, Cailletaud et al. (2006)]	FEM	Ti6Al4V	Gross sliding	Not mentioned	Refined material model with strain ratcheting
[Paulin, Fouvry and Meunier (2008)]	FEM	Ti6Al4V	Gross sliding	Energy model	Develop a fretting wear simulation tool
[Mohd Tobi, Ding, Bandak et al. (2009)]	FEM	Ti6Al4V	Gross sliding/partial slip	Archard model	Refined material model with cyclic plasticity
[Mohd Tobi, Shipway and Leen (2011)]	FEM	High strength steel/Ti6Al4V with W-DLC coating	Gross sliding	Archard model	Prediction of the coating life based on the normal load, total slip distance and the worn coating thickness
[Zhang, McHugh and Leen (2011)]	FEM	Ti6Al4V	Gross sliding/partial slip	Energy model	Prediction of fretting crack nucleation
[Mohd Tobi, Sun and Shipway (2017)]	FEM	Ti6Al4V	Gross sliding/partial slip	Archard model	Refined material model decoupling with fretting wear
[Yue and Abdel Wahab (2017)]	FEM	High strength steel	Gross sliding/partial slip	Energy model	Study the effects of variable CoF on fretting wear
[Pereira, Yue and Abdel Wahab (2017)]	FEM	High strength steel	Gross sliding	Energy model	Influence of roughness on FE simulation of fretting wear
[Gallego, Nélias and Jacq (2006)],[Gallego and Nélias (2007)]	Semi-analytical method	Ti6Al4V	Gross sliding/partial slip	Energy model	Develop a fretting wear simulation tool
[Kasarekar,	Numerical	Bearing steel	Partial slip	Archard	Study the effects

Bolander, Sadeghi et al. (2007)]	modelling			model	of rough surfaces and plasticity material model on fretting wear
[Dhia and Torkhani (2011)]	Multimodel Arlequin	Not mentioned	Not mentioned	Archard model	Study wear under sharp contact
[Lehtovaara and Lonnqvist (2011)]	Numerical modelling	Steel	Partial slip	Archard model	Evaluate the fretting wear in rough point contact
[Rodríguez-Tembleque, Abascal and Aliabadi (2011)]	BEM	Not mentioned	Gross sliding/partial slip	Archard model	Develop a 3D BEM for modelling fretting wear
[Leonard, Sadeghi, Shinde et al. (2013)]	Combined finite-discrete elements	Steel	Gross sliding/partial slip	Archard model	Investigation the fretting wear of rough and smooth Hertzian contacts

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