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A Nonlinear Magneto-Mechanical Coupled Constitutive Model for the Magnetostrictive Material Galfenol

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Abstract: In order to predict the performance of magnetostrictive smart material and push its applications in engineering, it is necessary to build the constitutive relations for the magnetostrictive material Galfenol. For Galfenol rods under the action of the pre-stress and magnetic field along the axial direction, the one-dimensional nonlinear magneto-mechanical coupling constitutive model is proposed based on the elastic Gibbs free energy, where the Taylor expansion of the elastic Gibbs free energy is made to obtain the polynomial forms. And then the constitutive relations are derived by replacing the polynomial forms with the proper transcendental functions based on the microscopic magneto-mechanical coupling mechanism. From the perspective of microscopic mechanism, the nonlinear strain related to magnetic domain rotation results in magnetostrictive strain changing with the pre-stress among the elastic strains induced by the pre-stress. By comparison, the predicted stress-strain, magnetostrictive strain, magnetic induction and magnetization curves agreed well with experimental results under the different pre-stresses. The proposed model can describe not only the influences of pre-stress on magnetostrictive strain and magnetization curves, but also nonlinear magneto-mechanical coupling effect of magnetostrictive material systematically, such as the Young's modulus varying with stress and magnetic field. In the proposed constitutive model, the key material constants are not chosen to obtain a good fit with the experimental data, but are measured directly by experiments, such as the saturation magnetization, saturation magnetostrictive coefficient, saturation Young's modulus, linear magnetic susceptibility and so on. In addition, the forms of the new constitutive relations are simpler than the existing constitutive models. Therefore, this model could be applied conveniently in the engineering applications.

Keywords: Magnetostrictive materials, one-dimensional constitutive relations, galfenol rods, nonlinear magneto-mechanical coupling.

1 Introduction

Compared with giant magnetostrictive material Terfenol-D being brittle, lower magnetic permeability and high cost, the Galfenol alloys of iron substituted with non-magnetic gallium have obtained much attention due to the advantages of large magnetostriction under very low magnetic fields and high tension strength [Kellogg (2003); Atulasimha and

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Flatau (2011)], twisted extrusion and welding [Emdadi, Cifre, Dementeva et al. (2015); Weng, Li, Sun et al. (2017); Datta, Atulasimha, Mudivarthi et al. (2010); Liu, Wang, Jiang et al. (2010); Palacheva, Emdadi, Emeis et al. (2017); Scheidler and Dapino (2013)]. These unique advantages of Galfenol alloys make it drive flexibly by tension and compression in smart devices applications and be used to prepare various sensor and actuator applied in complex environment, such as sonar transducer elements, underwater acoustic sensors, hydrophone, robot medical sensor and tactile sensor and osteosynthesis plate bending [Palacheva, Emdadi, Emeis et al. (2017); Scheidler and Dapino (2013); Sauer, Marschner, Adolphi et al. (2012)]. However, it was found from lots of experimental results [Datta, Atulasimha, Mudivarthi et al. (2010); Estrine, Hein, Robbins et al. (2014); Wun-Fogle, Restorff and Clark (2006); Atulasimha (2006); Yoo, Na, Flatau et al. (2014); Evans and Dapino (2013); Weng, Walker, Deng et al. (2013)] that the Galfenol alloys exhibit complex enhanced nonlinear magneto-mechanical coupling effect under the interaction of pre-stress and magnetic field. Both magnetostriction and magnetization curves show saturation phenomenon. Moreover, the curves of magnetostriction and magnetization are influenced complicatedly by pre-stress. What's more, as a key mechanical parameter, the Young's modulus also varies with the pre-stress and bias magnetic field changing (i.e. ΔE effect [Datta, Atulasimha, Mudivarthi et al. (2010)]). Due to the nonlinear magneto-mechanical coupling behaviors, the related mechanical analysis and design in the engineering applications for smart structures and devices with Galfenol alloys become very complex. Thus, it is urgent to build the constitutive relations, which can describe accurately the magnetostrictive response of the Galfenol rods under the interaction of pre-stress and magnetic field.

Till now, much effort has been taken [Atulasimha and Flatau (2011)] related to magneto-mechanical constitutive model of magnetostrictive materials. The most typical Jiles-Atherton model [Jiles and Atherton (1984a, 1984b)] is to describe magnetostrictive behavior including domain rotation and domain wall pinning, but the model is an empirical model and its parameters were estimated from experimental data. Afterwards, Jiles introduced the concept of a stress equivalent field to model the effect of stress [Li and Jiles (2003); Jiles and Li (2004)], magnetocrystalline anisotropy [Ramesh, Bi and Jiles (1997); Raghunathan, Melikhov, Snyder et al. (2009)] and temperature [Raghunathan, Melikhov, Snyder et al. (2010)] on hysteresis, which was extended to simulate the response of magnetostriction and magnetization under multi-physical field loading. And Smith et al. [Smith, Dapino and Seelecke (2003); Smith, Seelecke, Dapino et al. (2006)] built a class of free-energy constitutive model based on a two-well Helmholtz potential as a function of magnetization for the uniaxial case. The approach incorporated the effect of material inhomogeneities and non-constant effective field through a stochastic distribution to evaluate magnetization and incorporated stress effect by introducing the appropriate term in the Gibbs energy, which has also been extended to model magneto-mechanical behavior in 3D [Oates (2007); Evans and Dapino (2008)] and then was developed by Armstrong [Armstrong (1997)]. In the Armstrong model, the magnetocrystalline, magnetoelastic and magnetic field energy terms corresponding to the magnetization vector being oriented along different directions are considered, which uses a brief description of mathematical formulation and numerical solution procedure by introducing the two constants of magnetocrystalline anisotropy energy and the direction cosines for orientation of the magnetic moment. Altogether, the energy model incorporated many parameters, which are not easy to measure directly by

experiment. But the modified free-energy constitutive model is well fit to simulate the actuation and sensing behavior of Galfenol alloys [Atulasimha, Flatau and Cullen (2008)], and its extension is further used to model the magneto-mechanical behavior of polycrystals [Atulasimha, Flatau and Summers (2007)] and modified for tetragonal symmetries to describe the magneto-mechanical behavior of highly textured stress annealed Galfenol alloys [Restorff and Wun-Fogle (2006)]. Later, Evans et al. [Evans and Dapino (2013)] have built a magneto-mechanical coupling constitutive model of magnetization and strain based on thermodynamics law of stochastic homogenization, but the computational process of magnetization and strain in the model touches upon some material physical parameters, such as magnetic orientation, domain orientation, smooth factor and internal variables. Recently, Weng et al. [Weng, Zhao, Sun et al. (2016)] measured hysteresis between the strain and magnetic field, which increases with frequency increasing. They proposed a dynamic hysteresis model of Galfenol based on the energy-weighted average hysteresis equation, eddy current loss and anomalous loss. Although the predicted results agree well with the experimental results when the frequency being below $200\,\mathrm{Hz}$, the free-energy computational formulation as well touches upon empirical parameters and parameters difficult to measure. And Shu et al. [Shu, Wu, Chen et al. (2016)] presented the modeling of galfenol bending actuator based on the Gibbs free energy by introducing cubic anisotropy constant and the internal energy of a magnetic domain with orientation. The calculation of the macroscopic magnetization and magnetostriction is very complex and difficult. So it is essential for engineering application of Galfenol alloys to build a phenomenological constitutive model, which can describe accurately complex magneto-mechanical coupling effect of Galfenol rods under the action of axial stress and magnetic field with only engineering material constants, which are measured directly by experiment. Also, the calculation is not difficult.

In this paper, the one-dimensional nonlinear coupling phenomenological constitutive model for Galfenol alloys is proposed. Compared with the existing experimental results, the predicted results have shown that the model can predict effectively the strain-stress, magnetostriction and magnetization curves under magneto-mechanical coupling for Galfenol alloys of magnetostrictive materials, which demonstrates its validity. Moreover, the proposed model can also describe the ΔE effect of Galfenol rods. Due to higher order items in the Taylor series expansion of the elastic Gibbs free energy being reserved for the proposed model, both the square and fourth power terms of magnetization contribute to the strain. Moreover, the elastic strain induced by the pre-stress includes a nonlinear part related to the magnetic domain rotation, which produces ΔE effect as the magnetic field being 0. Because the Langevin function is from Boltzmann statistics and has better physical basis, it is used to describe the magnetization items independent of stress and simulate the magnetization curves well.

2 Constitutive model

Here considering the one-dimensional Galfenol rod, due to the pre-stress and the external magnetic field exerted along axial direction of Galfenol, the induced magnetization and strain are all along axial direction. Thus, the constitutive model can be simplified to the one-dimensional model to describe the relationship among the pre-stress σ, strain ε,

magnetic field H and magnetization M. Considering 18 at. % Ga content in Galfenol, the material anisotropy in magnetization is very low, which almost does not exist [Rafique, Cullen, Wuttig et al. (2004)]. Meanwhile, when the exerted compressive stress on Galfenol along axial direction is beyond jumping stress [Atulasimha (2006)], the stress anisotropy is dominant along the axial direction. Therefore, the considered Galfenol rods are isotropic material along axial direction in this paper.

Not considering the variation of temperature, the total differential of the internal energy density function $U(\varepsilon, M)$ of magnetostrictive material is written as $\mathrm{d}U = \sigma \mathrm{d}\varepsilon + \mu_0 H \mathrm{d}M$ [Parton and Kudryavtsev (1988)], where the vacuum permeability is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$. The elastic Gibbs free energy density function is defined as $G(\sigma, M) = U - \sigma\varepsilon$, so its total differential is $\mathrm{d}G = -\varepsilon \mathrm{d}\sigma + \mu_0 H \mathrm{d}M$. Based on the thermodynamic theory, the relations can be obtained as follows:

$$\varepsilon = -\frac{\partial G}{\partial \sigma}, \quad \mu_0 H = \frac{\partial G}{\partial M}. \tag{1}$$

This is the basis for deriving constitutive relations for the magnetostrictive material.

In order to obtain the polynomial constitutive relations, Taylor expansion is made at the reference point $(\sigma, M) = (0,0)$ for $G(\sigma, M)$. It is worth noting that the magnetic field H is always an odd function of the magnetization M, so all the odd-order terms of M are set to zero. Thus, the Taylor expansion of the elastic Gibbs free energy $G(\sigma, M)$ can be simplified as:

$$\begin{aligned} G(\sigma, M) = & \frac{1}{2}\frac{\partial^2 G}{\partial \sigma^2}\sigma^2 + \frac{1}{3!}\frac{\partial^3 G}{\partial \sigma^3}\sigma^3 + \frac{1}{4!}\frac{\partial^4 G}{\partial \sigma^4}\sigma^4 + \frac{1}{5!}\frac{\partial^5 G}{\partial \sigma^5}\sigma^5 + \frac{1}{6!}\frac{\partial^6 G}{\partial \sigma^6}\sigma^6 + \cdots \\ & + \frac{3}{3!}\frac{\partial^3 G}{\partial \sigma \partial M^2}\sigma M^2 + \frac{6}{4!}\frac{\partial^4 G}{\partial \sigma^2 \partial M^2}\sigma^2 M^2 + \frac{10}{5!}\frac{\partial^5 G}{\partial \sigma^3 \partial M^2}\sigma^3 M^2 \\ & + \frac{15}{6!}\frac{\partial^6 G}{\partial \sigma^4 \partial M^2}\sigma^4 M^2 + \frac{5}{5!}\frac{\partial^5 G}{\partial \sigma \partial M^4}\sigma M^4 + \frac{15}{6!}\frac{\partial^6 G}{\partial \sigma^2 \partial M^4}\sigma^2 M^4 + \cdots \\ & + \frac{1}{2}\frac{\partial^2 G}{\partial M^2}M^2 + \frac{1}{4!}\frac{\partial^4 G}{\partial M^4}M^4 + \frac{1}{6!}\frac{\partial^6 G}{\partial M^6}M^6 + \cdots \end{aligned} . \tag{2}$$

Where, the various order partial derivatives of $G(\sigma, M)$ related to σ and M are obtained at the reference point $(\sigma, M) = (0,0)$. In the above Taylor series expansion of function $G(\sigma, M)$, the constant term $G_0 = G(0,0)$, $\partial G / \partial M = 0$ and $\partial G / \partial \sigma = 0$ are neglected since they do not make any contribution to the strain ε, and the magnetic field H at the reference point $(\sigma, M) = (0,0)$, (that is $\varepsilon = 0$, $H = 0$ as $\sigma = 0$, $M = 0$.)

It can be seen from Eq. (2) that the all terms on the right can be classified into three categories: the first type is only dependent on the pre-stress σ, which describes the elastic property when $M=0$; the second type is related to both pre-stress σ and magnetization M, which describes the characteristic of magneto-elastic coupling (magnetostrictive effect and converse magnetostrictive effect); the third type is just relative to the magnetization M, which reflects the magnetic property when $\sigma=0$. For simplicity, according to the experimental result of Clark [Clark (1980)], Jiles [Jiles (1995)], Kuruzar et al. [Kuruzar and Cullity (1971)], only the terms containing M^2 and M^4 are kept for coupling terms in Eq. (2) (i.e. σM^2, $\sigma^2 M^2$, σM^4 and $\sigma^2 M^4$). Then substituting Gibbs free energy function Eq. (2) into the thermodynamic Eq. (1), the expressions of strain ε and magnetic field H can be obtained based on thermodynamic relations as follows:

$$\begin{aligned}\varepsilon=&-\frac{\partial^2 G}{\partial\sigma^2}\sigma-\frac{1}{2}\frac{\partial^3 G}{\partial\sigma^3}\sigma^2-\frac{1}{3!}\frac{\partial^4 G}{\partial\sigma^4}\sigma^3-\frac{1}{4!}\frac{\partial^5 G}{\partial\sigma^5}\sigma^4-\frac{1}{5!}\frac{\partial^6 G}{\partial\sigma^6}\sigma^5-\cdots\\&-\left(\frac{1}{2}\frac{\partial^3 G}{\partial\sigma\partial M^2}+\frac{1}{2}\frac{\partial^4 G}{\partial\sigma^2\partial M^2}\sigma+\frac{1}{4}\frac{\partial^5 G}{\partial\sigma^3\partial M^2}\sigma^2+\frac{1}{12}\frac{\partial^6 G}{\partial\sigma^4\partial M^2}\sigma^3+\cdots\right)M^2.\\&-\frac{1}{4!}\left(\frac{\partial^5 G}{\partial\sigma\partial M^4}+\frac{\partial^6 G}{\partial\sigma^2\partial M^4}\sigma+\cdots\right)M^4\end{aligned}\tag{3a}$$

$$\begin{aligned}\mu_0 H=&\frac{\partial^2 G}{\partial M^2}M+\frac{1}{3!}\frac{\partial^4 G}{\partial M^4}M^3+\frac{1}{5!}\frac{\partial^6 G}{\partial M^6}M^5+\cdots\\&+\left(\frac{\partial^3 G}{\partial\sigma\partial M^2}\sigma+\frac{1}{2}\frac{\partial^4 G}{\partial\sigma^2\partial M^2}\sigma^2+\frac{1}{6}\frac{\partial^5 G}{\partial\sigma^3\partial M^2}\sigma^3+\frac{1}{24}\frac{\partial^6 G}{\partial\sigma^4\partial M^2}\sigma^4+\cdots\right)M.\\&+\left(\frac{1}{6}\frac{\partial^5 G}{\partial\sigma\partial M^4}\sigma+\frac{1}{12}\frac{\partial^6 G}{\partial\sigma^2\partial M^4}\sigma^2+\cdots\right)M^3\end{aligned}\tag{3b}$$

Firstly, the terms independent on magnetization M in Eq. (3a) represent the elastic strain only induced by stress σ, while the terms dependent on magnetization M represent magnetostrictive strain $\lambda(M,\sigma)$, that is to say, it is the strain induced by magnetization M under the magnetic field and stress, or induced by applied stress after magnetization. For Galfenol before annealing, the relationship between magnetostrictive strain $\lambda(M,\sigma)$ and magnetization M is not only nonlinear, but also is affected by the pre-stress σ. From the microscopic physical mechanism (as shown in Fig. 1), when Galfenol rod is magnetized completely by the applied magnetic field after applying a certain pre-stress along axial direction, it can reach larger saturation magnetostrictive strain [Atulasimha (2006); Hale (2006)] than one in a free state ($H=0,\sigma=0$). The applied pre-stress makes the magnetic domain turn to an in-plane perpendicular to the axial direction, and then the magnetostrictive rod produces 'pre-compression' caused by the magnetic domain rotation before the magnetic field is applied. In this case, when the applied magnetic field is high enough to make magnetization get saturation, (that is to say, all of the magnetic domains turn to the axial direction), a greater relative elongation is obtained in the magnetostrictive

rod. This is why the maximum magnetostrictive strain under a given pre-stress is larger than that in a free state ($H=0,\sigma=0$). According to the analysis above, we might as well decompose the elastic strain of Galfenol rod produced by applied pre-stress into the part independent on magnetic domain rotation and the part dependent on magnetic domain rotation. The former is linear which is the inherent elastic property of magnetostrictive material and can be described with $\mathrm{E_s}$, that is inherent Young's modulus under saturation magnetization; the latter is nonlinear which can be described with non-linear function $\lambda_0(\sigma)$. So, the term of elastic strain in Eq. (3a) can be simplified as

$$-\frac{\partial^2 G}{\partial\sigma^2}\sigma-\frac{1}{2}\frac{\partial^3 G}{\partial\sigma^3}\sigma^2-\frac{1}{3!}\frac{\partial^4 G}{\partial\sigma^4}\sigma^3+\cdots=\frac{\sigma}{\mathrm{E_s}}+\lambda_0(\sigma). \tag{4}$$

For $\lambda_0(\sigma)$, it is usually difficult to give a precise expression, but it is easy to know the main characteristics from the stress-strain curve $\sigma\sim\varepsilon$ under demagnetization state ($M=0$). For example, the strain induced by the magnetic domain rotation is 0 without applying pre-stress, that is $\lambda_0(\sigma)|_{\sigma=0}=0$. Meanwhile, when the stress tends to negative infinity, $\lambda_0(\sigma)$ certainly tends to saturate, that is $\lambda_0(\sigma)|_{\sigma\to-\infty}=\frac{1}{2}\lambda_s$ as the magnetic domain rotation (here, λ_s is saturation magnetostrictive coefficient). The schematic of magnetic moments in Gafenol rods is shown in Fig. 1. Based on the physical property above, the hyperbolic tangent function is just fitful to describe the tendency of $\lambda_0(\sigma)$, so the hyperbolic tangent function is chosen to describe $\lambda_0(\sigma)$ as follow

$$\lambda_0(\sigma)=\frac{\lambda_s}{2}\tanh(2\sigma/\sigma_s). \tag{5}$$

where, σ_s is the saturation pre-stress.

The terms dependent on magnetization M in Eq. (3a) represent magnetostrictive strain $\lambda(M,\sigma)$. Usually, the magnetization of magnetostrictive materials mainly includes two stages, one is the domain wall moving, and the other is the domain rotation. For Galfenol, the magnetostrictive strain $\lambda(M,\sigma)$ is induced mainly by domain wall moving at smaller magnetic field under the combined action of magnetic field and pre-stress, which is usually determined by the square of magnetization. So the changing of magnetostriction strain can be described by nonlinear incremental function $\lambda_M(\sigma)$. That is to say, $\lambda(M,\sigma)$ is mainly determined by $\frac{\lambda_s+\lambda_M(\sigma)}{\mathrm{M}_s^2}M^2$ (M_s is the saturation magnetization). However, when the material is further magnetized to a certain extent, the magnetic domain rotation occurs with the external magnetic field increasing continually. At this time, the fourth power term of magnetization starts to work, and then the changing

of magnetostriction is induced by the magnetic domain rotation, which is described by nonlinear function $\lambda_0(\sigma)$ as mentioned in the previous article. That is to say, which is determined by $\frac{\lambda_s+\lambda_0(\sigma)}{M_s^4}M^4$. Therefore, the final magnetostrictive strain $\lambda(M,\sigma)$ is determined collectively by both $\frac{\lambda_s+\lambda_M(\sigma)}{M_s^2}M^2$ and $\frac{\lambda_s+\lambda_0(\sigma)}{M_s^4}M^4$.

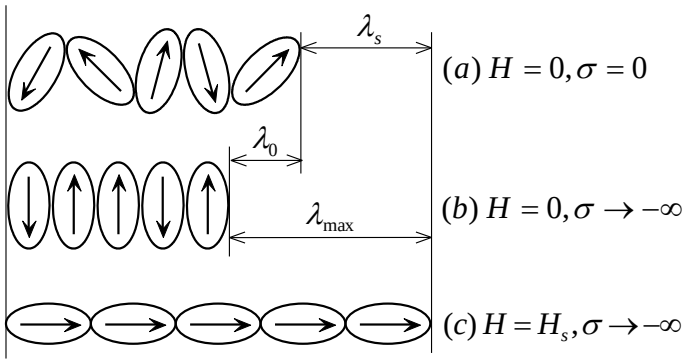


Figure 1: The schematic of the magnetic moments in Galfenol rod affected by the pre-stress and magnetic field (H_s is saturation magnetic field)

Based on the analysis, the applied magnetic field or pre-stress makes the magnetic domain wall of material move and then rotate because of the magnetic field or stress increasing continually. For Galfenol material, the continued wall movement of magnetic domain will be blocked under the strong magnetic field, and then the magnetic domain rotates, which will induce the magnetostriction reduction in certain extent. Here, the wall movement and rotation of magnetic domain occur at the same time. So, a domain rotation coefficient $\theta(0<\theta\le1)$ is introduced. When the term of pre-stress or magnetic field is zero, the coefficient is 1, which represents that the domain rotation is easy to occur. When the term of pre-stress or magnetic is larger and both of them are nonzero, the coefficient is less than 1, which represents that the domain rotation is hard to occur due to the wall movement and rotation of magnetic domain being blocked.

Also, Jiles mentioned an empirical model to describe the relation between bulk magnetostriction and bulk magnetization, which is $\lambda(M,\sigma)=\sum_{i=0}^{\infty}\gamma_i(\sigma)M^{2i}$ [Kuruzar and Cullity (1971)]. Here, a reasonable first approximation to the magnetrostriction of iron can be obtained by including the terms up to $i=2$. Therefore, based on the empirical model and above analysis, the stress dependent of the magnetostrictive strain $\lambda(M,\sigma)$ in Eq. (3a) can be denoted finally in terms of the pre-stress dependence of $\gamma_1(\sigma)>0$ and $\gamma_2(\sigma)<0$ as follow:

$$\lambda(M,\sigma)=-\left(\frac{1}{2}\frac{\partial^3 G}{\partial\sigma\partial M^2}+\frac{1}{2}\frac{\partial^4 G}{\partial\sigma^2\partial M^2}\sigma+\frac{1}{4}\frac{\partial^5 G}{\partial\sigma^3\partial M^2}\sigma^2+\frac{1}{12}\frac{\partial^6 G}{\partial\sigma^4\partial M^2}\sigma^3+\cdots\right)M^2.$$

$$-\frac{1}{4!}\left(\frac{\partial^5 G}{\partial\sigma\partial M^4}+\frac{\partial^6 G}{\partial\sigma^2\partial M^4}\sigma+\cdots\right)M^4=\gamma_1(\sigma)M^2+\gamma_2(\sigma)M^4 \tag{6}$$

$$=\frac{\lambda_s+\lambda_M(\sigma)}{\mathrm{M}_s^2}M^2-\frac{\theta\left[\lambda_s+\lambda_0(\sigma)\right]}{\mathrm{M}_s^4}M^4$$

Here, $\gamma_1(\sigma)$ and $\gamma_2(\sigma)$ are $\frac{\lambda_s+\lambda_M(\sigma)}{\mathrm{M}_s^2}$ and $-\frac{\theta\left[\lambda_s+\lambda_0(\sigma)\right]}{\mathrm{M}_s^4}$ respectively.

For $\lambda_M(\sigma)$ in a free state ($\sigma=0$), $\lambda_M(\sigma)$ can be increased near to λ_s when the material is magnetized. That is to say, $\sigma\to 0$, $\lambda_M(\sigma)\to\lambda_s$. For $\lambda_M(\sigma)$ under demagnetization state ($M=0$), $\lambda_M(\sigma)$ monotonically decreases from maximum to near zero with the exerted stress σ is near to $\pm\infty$. That is to say, $\sigma\to\pm\infty$, $\lambda_M(\sigma)\to 0$. Considering these properties and the tendency of $\lambda_M(\sigma)$, the nonlinear even function $\lambda_M(\sigma)$ can be determined as

$$\lambda_M(\sigma)=\lambda_s\mathrm{sech}(\sigma/\sigma_s). \tag{7}$$

The terms independent on σ in Eq. (3b) represent magnetization relationship $M=M(H)$ in a free state ($\sigma=0$), which is usually characterized by nonlinear function $f(x)$ (i.e., $M=\mathrm{M}_s f(kH)$, where k is the relaxation factor) due to its nonlinearity and saturation property. There are a variety of options for the function $f(x)$ to describe the magnetization curve. Because Langevin function is based on the Boltzmann statistics and has a clear physical background to simulate the magnetization curves better, the nonlinear function $f(x)$ is chosen as follows:

$$f(x)=\coth(x)-1/x. \tag{8}$$

In this case, the magnetization can be expressed as $M=\mathrm{M}_s\left[\coth(kH)-1/(kH)\right]$. Here, the relaxation factor is set as $k=3\chi_m/\mathrm{M}_s$ (χ_m is the linear magnetic susceptibility). Hence, the terms independent on the stress σ in Eq. (3b) can be expressed as:

$$\frac{\partial^2 G}{\partial M^2}M+\frac{1}{3!}\frac{\partial^4 G}{\partial M^4}M^3+\cdots=\frac{\mu_0}{k}f^{-1}\left(\frac{M}{\mathrm{M}_s}\right). \tag{9}$$

Meanwhile, the terms dependent on σ in Eq. (3b) represent the effect of stress on magnetization (i.e. converse magnetostrictive effect), according to the thermodynamic relations Eq. (1), which are same as the terms dependent on M in Eq. (3a) from the

coupling terms in Taylor expansion of $G(\sigma, M)$. Therefore, with the help of Eq. (6), the terms dependent on σ in Eq. (3b) can be expressed as

$$\left(\frac{\partial^3 G}{\partial\sigma\partial M^2}\sigma+\frac{1}{2}\frac{\partial^4 G}{\partial\sigma^2\partial M^2}\sigma^2+\cdots\right)M+\left(\frac{1}{6}\frac{\partial^5 G}{\partial\sigma\partial M^4}\sigma+\frac{1}{12}\frac{\partial^6 G}{\partial\sigma^2\partial M^4}\sigma^2+\cdots\right)M^3.$$

$$=-\frac{\partial}{\partial M}\int_0^\sigma\frac{\lambda_s+\lambda_M(\sigma)}{M_s^2}M^2d\sigma+\frac{\partial}{\partial M}\int_0^\sigma\frac{\lambda_s+\lambda_0(\sigma)}{M_s^4}\theta M^4d\sigma \tag{10}$$

$$=-\frac{2\left(\lambda_s\sigma+\Lambda_M(\sigma)\right)}{M_s^2}M+\frac{2\theta\left(\lambda_s\sigma+\Lambda_0(\sigma)\right)}{M_s^4}M^3$$

Where, $\Lambda_M(\sigma)=\int_0^\sigma\lambda_M(\sigma)d\sigma$ is the primitive function of $\lambda_M(\sigma)$, and $\Lambda_0(\sigma)=\int_0^\sigma\lambda_0(\sigma)d\sigma$ is the primitive function of $\lambda_0(\sigma)$. Substituting Eqs. (5) and (7), the expression of $\Lambda_M(\sigma)$ and $\Lambda_0(\sigma)$ can be obtained as follows:

$$\Lambda_M(\sigma)=\sigma_s\arctan\left[\sinh\left(\frac{\sigma}{\sigma_s}\right)\right]. \tag{11}$$

$$\Lambda_0(\sigma)=\frac{1}{4}\sigma_s\ln\left[\cosh\left(\frac{2\sigma}{\sigma_s}\right)\right]. \tag{12}$$

Here, based on Eqs. (4), (5), (6), (7), (9), (10), (11) and (12), Eqs. (3a) and (3b) could be re-written finally as:

$$\varepsilon=\frac{\sigma}{E_s}+\frac{\lambda_s}{2}\tanh\left(\frac{2\sigma}{\sigma_s}\right)+\left[1+\operatorname{sech}\left(\frac{\sigma}{\sigma_s}\right)\right]\frac{\lambda_s}{M_s^2}M^2-\left[1+\frac{1}{2}\tanh\left(\frac{2\sigma}{\sigma_s}\right)\right]\frac{\theta\lambda_s}{M_s^4}M^4 \tag{13a}$$

$$H=\frac{1}{k}f^{-1}\left(\frac{M}{M_s}\right)-2\left\{\sigma+\sigma_s\arctan\left[\sinh\left(\frac{\sigma}{\sigma_s}\right)\right]\right\}\frac{\lambda_s M}{\mu_0 M_s^2}+\left\{\sigma+\frac{1}{4}\sigma_s\ln\left[\cosh\left(\frac{2\sigma}{\sigma_s}\right)\right]\right\}\frac{4\theta\lambda_s M^3}{\mu_0 M_s^4} \tag{13b}$$

This is the final form of the proposed one-dimensional constitutive model for Galfenol material. The new model just requires the parameters of k, E_s, M_s, λ_s and σ_s, which can be measured by experiment directly. What's more, the proposed model is simpler in form and in calculation compared with the existing model of Galfenol [Atulasimha and Flatau (2011, 2008)], which will be convenient for practical engineering applications.

In order to facilitate the engineering application, the three important material constants expressions, piezoelectric coefficient, the Young's modulus and permeability can be derived easily by the constitutive relations of Eqs. (13a) and (13b). Based on the definition of piezoelectric coefficient $d_m=\dfrac{\partial\varepsilon(\sigma,H)}{\partial H}$, the expression of $d_m=\dfrac{\partial\varepsilon(\sigma,H)}{\partial H}$ can be obtained:

$$d_m = \frac{\left[1+\sec h(\frac{\sigma}{\sigma_s})\right]\frac{2M\lambda_s}{M_s^2} - \left[1+\frac{1}{2}\tanh(\frac{2\sigma}{\sigma_s})\right]\frac{4\lambda_s\theta M^3}{M_s^4}}{\frac{1}{kM_s(-\csc h^2 M_x + 1/M_x^2)} - \left\{\sigma+\sigma_s \arctan\left[\sinh(\frac{\sigma}{\sigma_s})\right]\right\}\frac{2\lambda_s}{\mu_0 M_s^2} + \left\{\sigma+\frac{1}{4}\sigma_s \ln\left[\cosh(\frac{2\sigma}{\sigma_s})\right]\right\}\frac{12\theta\lambda_s M^2}{\mu_0 M_s^4}}. \quad (14)$$

Where,

$$M_x = kH + 2k\left\{\sigma+\sigma_s \arctan\left[\sinh(\frac{\sigma}{\sigma_s})\right]\right\}\frac{\lambda_s M}{\mu_0 M_s^2} - 4k\left\{\sigma+\frac{1}{4}\sigma_s \ln\left[\cosh(\frac{2\sigma}{\sigma_s})\right]\right\}\frac{\lambda_s\theta M^3}{\mu_0 M_s^4}$$

Based on the definition $E_m = \frac{\partial\sigma}{\partial\varepsilon}$, the expression of the Young's modulus can be obtained as follow:

$$\frac{1}{E_m} = \frac{\partial\varepsilon(\sigma,H)}{\partial\sigma} = \frac{1}{E_s} - \frac{\lambda_s M^2 \sec h(\frac{\sigma}{\sigma_s})\tanh(\frac{\sigma}{\sigma_s})}{\sigma_s M_s^2} + \frac{\lambda_s}{\sigma_s}\sec h^2(\frac{2\sigma}{\sigma_s})(1-\frac{\theta M^4}{M_s^4}). \quad (15)$$
$$+\left\{\left[1+\sec h(\frac{\sigma}{\sigma_s})\right]\frac{2\lambda_s M}{M_s^2} - [1+\frac{1}{2}\tanh(\frac{2\sigma}{\sigma_s})]\frac{4\theta\lambda_s M^3}{M_s^4}\right\}\frac{\partial M}{\partial\sigma}$$

Where,

$$\frac{\partial M}{\partial\sigma} = \frac{2k\lambda_s M M_s^2[1+\sec h(\frac{\sigma}{\sigma_s})] - 4k\lambda_s\theta M^3[1+\frac{1}{2}\tanh(\frac{2\sigma}{\sigma_s})]}{\frac{\mu_0 M_s^3}{-\csc h^2 M_x + 1/M_x^2} - 2k\lambda_s M_s^2[\sigma+\sigma_s \arctan(\sinh(\frac{\sigma}{\sigma_s}))] + 12k\theta\lambda_s M^2[\sigma+\frac{1}{4}\sigma_s \ln(\cosh(\frac{2\sigma}{\sigma_s}))]}$$

Based on the definition of $\mu_m = \frac{\partial B}{\partial H} = \mu_0(1+\frac{\partial M}{\partial H})$, the expression of permeability can be obtained as follow:

$$\mu_m = \mu_0(1+\frac{\partial M}{\partial H})$$
$$= \mu_0 + \frac{\mu_0}{\frac{1}{kM_s(-\csc h^2 M_x + 1/M_x^2)} - \left\{\sigma+\sigma_s \arctan\left[\sinh(\frac{\sigma}{\sigma_s})\right]\right\}\frac{2\lambda_s}{\mu_0 M_s^2} + \left\{\sigma+\frac{1}{4}\sigma_s \ln\left[\cosh(\frac{2\sigma}{\sigma_s})\right]\right\}\frac{12\lambda_s\theta M^2}{\mu_0 M_s^4}}. \quad (16)$$

3 Results and discussions

Datta et al. [Datta, Atulasimha, Mudivarthi, et al. (2010), Atulasimha [Atulasimha (2006)], Atulasimha et al. [Atulasimha, Flatau and Cullen (2008)], Atulasimha et al. [Atulasimha, Flatau and Summers (2007)] have given the curves of magnetostrictive strain and magnetic induction of Galfenol rods varying with the bias magnetic field under the different compressive stresses. Because these classical experimental results have been widely used to testify all kinds of applicability range and prediction precision of magnetostrictive

constitutive model, these representative experimental datums are chosen to testify the validity of the proposed constitutive model. Since Atulasima et al. [Atulasimha, Flatau and Cullen (2008)] chose $Fe_{81}Ga_{19}$ alloys as the experimental sample, the basic parameters of materials in proposed model are same as ones in reference [Atulasimha, Flatau and Cullen (2008)]. The detailed values are $\lambda_s = 212\text{ppm}$, $\mu_0 M_s = 1.66\text{T}$, $\sigma_s = 200\text{MPa}$, $E_s = 59\text{GPa}$, θ=0.6 . The strain-stress curves of prediction and experiment are shown as Figs. 2(a) and 2(b) while the bias magnetic field being 0, 22.3, 44.6, 66.9, 89.1, 111, 223, 446 Oe respectively. By comparison, the proposed model can describe the experimental results well both in qualitatively and quantitatively. It can be seen from Fig. 2 (b) that the strain-stress curve is close to a straight line as the bias magnetic field being 0 Oe, and the curves only move up slightly in the range of compressive stress less than 20 MPa. It also can be seen from Eq. (13a) that the magnetization is 0 when the bias magnetic field is 0 Oe. Thus only the first two items are left on the right of Eq. (13a), i.e. $\varepsilon = \sigma/\mathrm{E_s} + \lambda_s \tanh\left(2\sigma/\sigma_s\right)/2$. Here, the nonlinear strain is mainly induced by $\lambda_s \tanh\left(2\sigma/\sigma_s\right)/2$. Even though the influence of $\lambda_s \tanh\left(2\sigma/\sigma_s\right)/2$ is not very obvious due to smaller saturation magnetostrictive coefficient, the predicted values of strain agree well with the experimental results as the compressive stress being 115 MPa, which was usually neglected in previous model [Atulasimha, Flatau (2011); Atulasimha, Flatau and Cullen (2008)**].** When smaller bias magnetic fields are exerted (i.e. the middle curves as the magnetic field being from 0 Oe-446 Oe in Fig. 2 (b)), the items related to magnetization on the right of Eq.(13a) start to work, and then a positive magnetostrictive strain is produced in advance by bias magnetic field, which results in nonzero strain while the stress being 0 MPa. The magnetostrictive strain tends to decrease gradually from positive to negative strain with the compressive stress increasing. When the wall moving and rotation of magnetic domain induced by bias magnetic field are counteracted by the compressive stress, the strain curves tend to an agreement with the ones as the magnetic field being 0 Oe. If the magnetic field is very large enough to reach saturation, the magnetic domain cannot be rotated to the status before magnetization under the smaller compressive stress. Consequently, the strain-stress curve approaches to a straight line (i.e. the curve as magnetic field being 446 Oe).

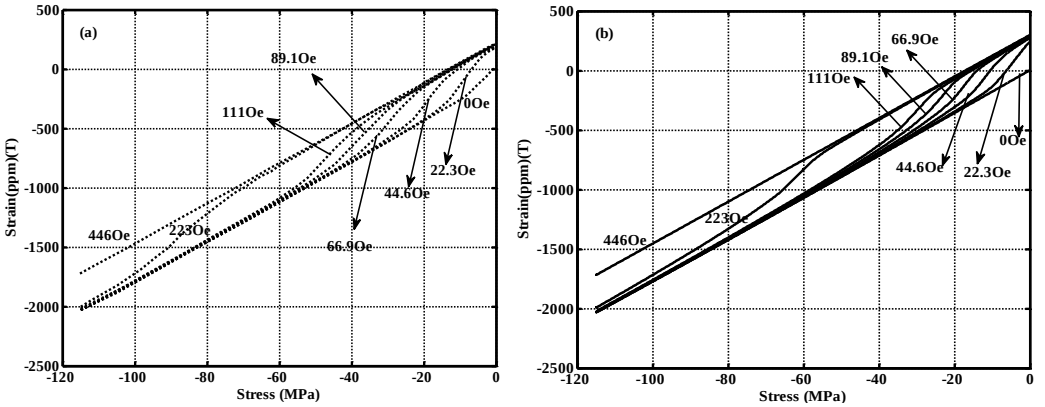


Figure 2: The experimental (a) [Atulasimha, Flatau and Cullen (2008)] and prediction (b) strain *vs.* compressive stress at bias magnetic field of 0, 22.3, 44.6, 66.9, 89.1, 111, 223, 446 Oe

Next the magnetization curves determined by Eq. (13b) would be discussed. The experimental and prediction results are shown as Figs. 3(a) and (b) when the compressive stresses are 0 MPa, 15 MPa, 30 MPa, 45 MPa, 60 MPa, 80 MPa respectively. By comparison with Figs. 3(a) and (b), it can be seen that the model can describe effectively the magnetization curves changing from steep to flat and describe effectively the tendency of saturation magnetic induction tending to a stable value with compressive stresses changing. In fact, the first item of Eq. (13b) on the right has nothing to do with stress, which describes mainly the magnetization curve as pre-stress being 0 MPa, so the curve as pre-stress being 0 MPa in Fig. 3 is just determined by the first item of Eq. (13b) on the right. If compressive stress is applied, it would do work to rotate magnetization direction of magnetic domain of Galfenol to a hard axis in advance and thus make magnetizing difficult. With the applied static magnetic field increasing, the influence of pre-stress must be overcome firstly, so a larger exerting magnetic field is necessary in order to reach to a same magnetic induction under the applied compressive stress. Yet, in the case of nonzero compressive stress, the second and third items of Eq. (13b) on the right start to work, which makes the constitutive model be used to describe the magnetization effectively.

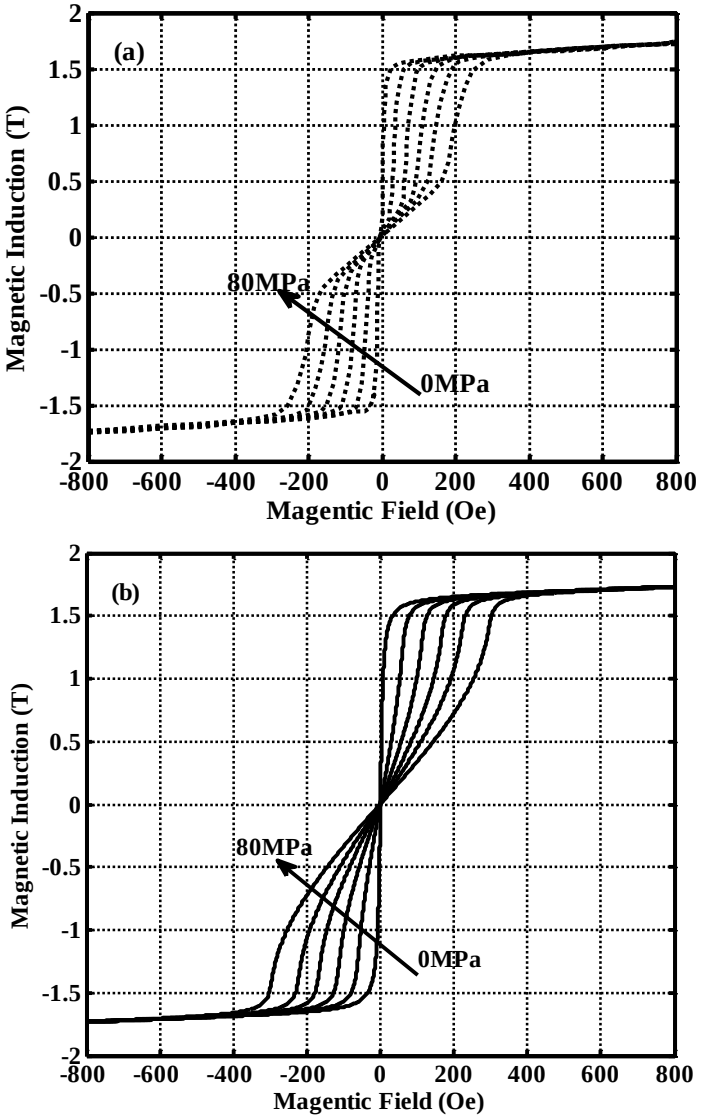


Figure 3: The experimental (a) [Atulasimha, Flatau and Cullen (2008)] and prediction (b) magnetic induction *vs.* magnetic field at compressive stress of 0 MPa, 15 MPa, 30 MPa, 45 MPa, 60 MPa, 80 MPa

Since the constitutive model can predict the magnetization curve effectively, as such, it can describe the magnetic induction curves. The magnetic induction curves of experimental measurement and prediction varying with pre-compressive stress at bias magnetic fields from 0 to 891 Oe are shown in Figs. 4 (a) and 4(b). It can be found from Fig. 4 that at bias magnetic field from 0 to 111 Oe, the magnetic induction reduces sharply with the compressive stress increasing as pre-compressive stress being smaller ($\sigma < 60\text{MPa}$), and then it tends to be stable gradually as pre-compressive stress being larger. However, as the magnetic field increasing, the magnetic induction keeps almost constant under smaller

compressive stress, and then it decreases and finally tends to be stable when the compressive stress increases to a certain degree, that is to say, the magnetic induction curve becomes smoothly. When the magnetic field reaches to 446 Oe, the magnetic induction is almost unchanged and linear tendency with compressive stress changing. It is because at smaller bias magnetic field Galfenol would be magnetized first, and then be demagnetized and finally tend to saturate by exerting compressive stress; yet at larger bias magnetic field, Galfenol would be magnetized and get to saturate directly, and then it is difficult to be demagnetized by exerting smaller compressive stress.

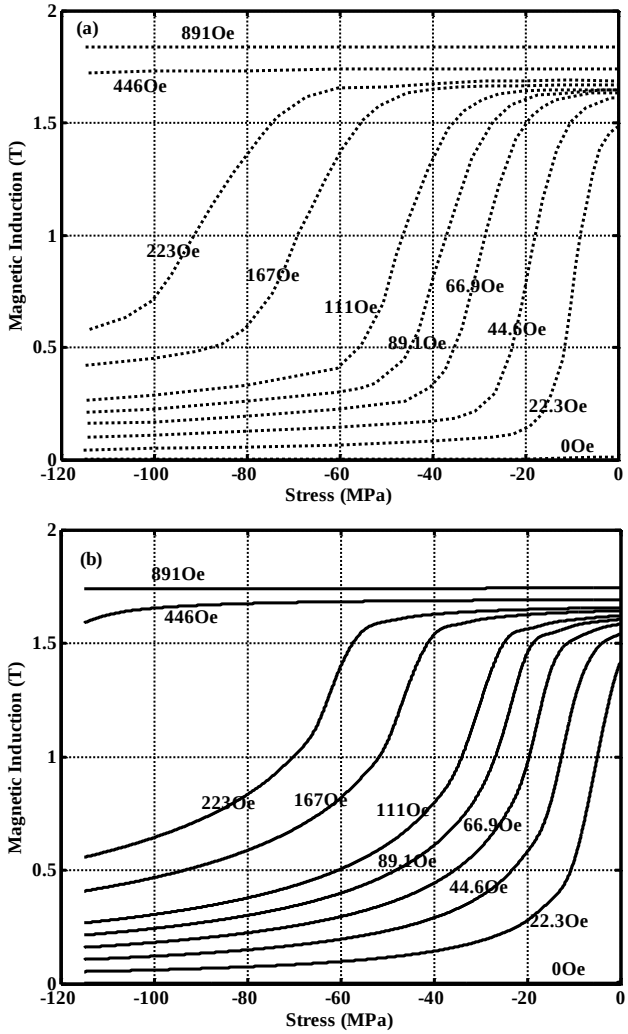
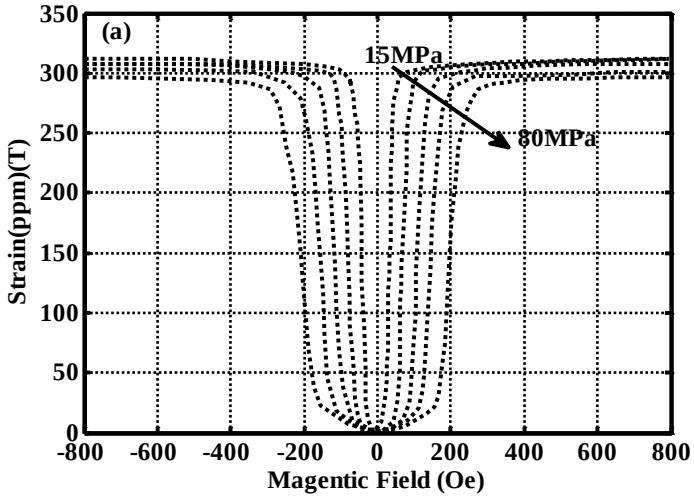


Figure 4: The experimental (a) [Atulasimha, Flatau and Cullen (2008)] and prediction (b) magnetic induction *vs.* compressive stress at bias magnetic field of 0, 22.3, 44.6, 66.9, 89.1, 111, 167, 223, 446, 891 Oe

Now the magnetostrictive strain under magneto-mechanical coupling would be discussed. Obviously, the magnetostrictive strain is the sum of the third term and fourth term related

to magnetization on the right of Eq. (13a), which is the rest part without considering the strain induced directly by stress in formula of constitutive model. The magnetostrictive strain curves of experimental results and prediction under the different compressive stresses appear obvious nonlinear as shown in Figs. 5(a) and 5(b). By comparison, from Fig. 5, in quantitative, the predicted results are in agreement with the experimental results basically; in qualitative, the model can predict effectively the experimental phenomenon of magnetostrictive strain decreasing at initial segment with compressive stress increasing. However, the predicted results of saturated magnetostrictive strain appear reversal at very large magnetic field. That is to say, at very smaller compressive stress, the initial segment of magnetostrictive strain is very large and the final saturated magnetostrictive strain is very small; Whereas, at very larger compressive stress, the initial segment of magnetostrictive strain is very small and the saturated magnetostrictive strain is much greater. But the experimental results of magnetostrictive strain in Fig. 5(a) at high magnetic field did not show reversal at all. As we know, the experimental Galfenol sample measured by Atulasima [Atulasimha, Flatau and Cullen (2008)] was annealed under stress, so there is no reversal for the saturated magnetostrictive strain. In fact, if there is no annealing treatment, magnetostrictive strain will exhibit reversal. For example, the magnetostrictive strain curves of Galfenol material without annealing treatment measured by Evans et al. [Evans and Dapino (2010)] and Clark et al. [Clark, Wun-Fogle, Restorff et al. (2002)] appeared reversal phenomenon, which is certainly consistent with predictions by the proposed model. Therefore, the Galfenol alloys exhibit complex nonlinear magneto-mechanical coupling effect under the interaction of pre-stress and magnetic field.



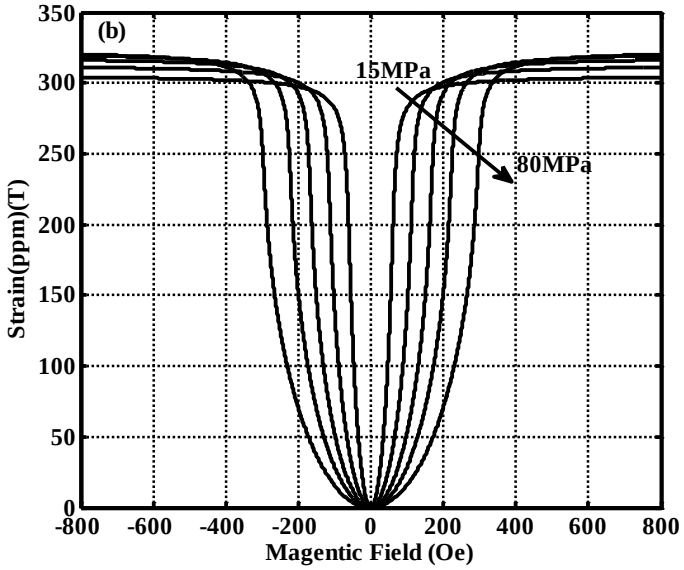


Figure 5: The experimental (a) [Atulasimha, Flatau and Cullen (2008)] and prediction (b) strain *vs.* magnetic field at compressive stress of 15 MPa, 30 MPa, 45 MPa, 60 MPa, 80 MPa

From the analysis above, the proposed model can describe accurately the nonlinear magneto-mechanical coupling effect of magnetization and magnetostrictive strain under the action of stress and magnetic field. Besides, the model can also describe the coupling characteristics in aspect of magnetism, magnetostriction properties and elasticity of magnetostrictive materials. Fig. 6 shows the predicted ΔE effect in this paper, which is the Young's modulus varying with the different magnetic fields and stresses. It can be seen from Fig. 6 that the predicted Young's modulus is almost not affected by pre-stress as the magnetic field being zero or saturation (i.e. $H = 0$ or $H = 891\text{Oe}$). Whereas, the predicted Young's modulus is with non-monotonicity varying with compressive stress as the magnetic fields being other values. For given nonzero magnetic fields in Fig. 6, all the values of Young's modulus descend first from the initial values and then go up and last tend to a common saturation value E_s. It also can be seen that the Young's modulus of Galfenol rod approaches to intrinsic Young's modulus E_s only when the compressive is enough large or the magnetic field is enough high. For these two cases, the direction of magnetic domain will be confine in plane perpendicular to axial direction or axial line direction, then any tiny stress variation cannot make magnetic domain rotate. Thus, the corresponding strain variation is just provided by the intrinsic elastic property of material, and of course its Young's modulus is equal to intrinsic Young's modulus E_s. And yet, at the moderate magnetic field and compressive stress, the Young's modulus of Galfenol rods is usually much lower than the saturated value of $59\,\text{GPa}$, and the lowest values are in the range of $28 \sim 38\,\text{GPa}$, which is consistent well with the lowest Young's modulus of

29 GPa done by experiment mentioned in literature [Atulasimha, Flatau and Cullen (2008)] and appears highly significant ΔE effect. In this case, the influence of the compressive stress and magnetic field on magnetic domain rotation is mutual to some extent. Because the magnetic domain is easy to rotate for the strain variation produced by each unit stress change, the contribution of magnetic domain rotation is very large. Therefore, the proposed model in this paper can give the variation law of Young's modulus closest to the truth, which shows the validity of the model again.

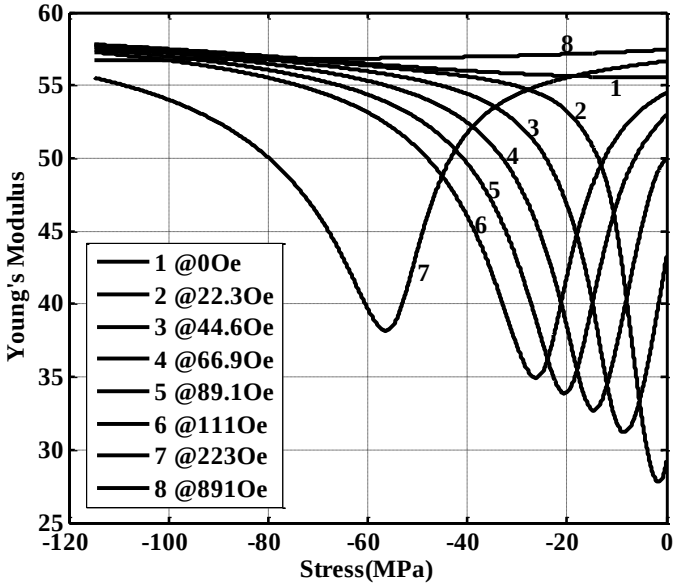


Figure 6: The prediction Young's Modulus *vs.* compressive stress at bias magnetic field of 0, 22.3, 44.6, 66.9, 89.1, 111, 223, 891 Oe

4 Conclusions

This paper proposed a magneto-mechanical coupling constitutive model of the magnetostrictive smart material Galfenol alloys, which can predict accurately the nonlinear magneto-mechanical coupling effect under the different compressive stresses and magnetic fields. Furthermore, the model can fully describe the nonlinear magneto-mechanical coupling characteristics in aspect of magnetism, magnetostriction properties and elasticity of Galfenol rods, such as the influence of the interaction of pre-stress and magnetic field on magnetization and magnetostrictive strain curves, on strain-stress curves and the Young's modulus (that is ΔE effect). Compared with the existing model, the proposed model is not only a macroscopical and phenomenological constitutive model based on the micromechanism, but also simple in form and in calculation. In addition, the key material constants can easily be measured by experiments. Therefore, this model would be used very conveniently in the practice engineering applications.

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