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Rotational Effects on Magneto-Thermoelastic Stoneley, Love and Rayleigh Waves in Fibre-Reinforced Anisotropic General Viscoelastic Media of Higher Order

A. M. Abd-Alla[1, 2] , S. M. Abo-Dahab[1, 3] and Aftab Khan[4]

Abstract: In this paper, we investigated the propagation of magneto-thermoelastic surface waves in fibre-reinforced anisotropic general viscoelastic media of higher order ofnth order, including time rate of strain under the influence of rotation and magnetic field.The general surface wave speed is derived to study the effects of rotation, magnetic field and thermal on surface waves. Particular cases for Stoneley, Love and Rayleigh waves are discussed. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. Our results for viscoelastic of order zero are well agreed to fibre-reinforced materials. Comparison was made with the results obtained in the presence and absence of rotation, magnetic field and parameters for fibre-reinforced of the material medium. It is also observed that, surface waves cannot propagate in a fast rotating medium. Numerical results for particular materials are given and illustrated graphically. The results indicate that the effect of rotation, magnetic field on fibre-reinforced anisotropic general viscoelastic media are very pronounced.

Keywords: Fibre-reinforced, viscoelastic, surface waves, rotation, anisotropic, thermoelastic, magnetic.

1 Introduction

These problems are based on the more realistic elastic model since thermoelastic waves are propagating on the surface of earth, moon and other planets which are rotating about an axis. Schoenberg and Censor (1973) were the first to study the propagation of plane harmonic waves in a rotating elastic medium where it is shown that the elastic medium becomes dispersive and anisotropic due to rotation. Later on, many researchers introduced rotation in different theories of thermoelasticity. Agarwal (1979) studied thermo-elastic plane wave propagation in an infinite non-rotating medium. The normal mode analysis was used to obtain the exact expression for the temperature distribution, the thermal stresses and the displacement components. The purpose of the present work is

[1]Math. Dept., Faculty of Science, Taif University 888, Saudi Arabia.
[2]Math. Dept., Faculty of Science, Sohag University, Egypt.
[3]Math. Dept., Faculty of Science, SVU, Qena 83523, Egypt
[4]Department of Mathematics, COMSATS, Institute of Information, Park Road, Chakshahzad, Islamabad, Pakistan.
Emails: mohmrr@yahoo.com, sdahb@yahoo.com, aftab.khan@comsats.edu.pk

to show the thermal and rotational effects on the surface waves.Surface waves have been well recognized in the study of earthquake, seismology, geophysics and Geodynamics. A good amount of literaturefor surface waves is available [Bullen (1965), Ewing and Jardetzky (1957), Rayleigh (1885), Stoneley (1924)]. Acharya and Singupta (1978), Pal and Sengupta (1987), Sengupta and Nath (2001) and his research collaborators have studied surface waves. These waves usually have greater amplitudes ascompared with body waves and travel more slowly than body waves. There are

many types of surface waves but we only discussed Stoneley, Love andRayleigh waves. Earthquakeradiate seismic energy as both body and surface waves. These are also used for detecting cracks and other defects in materials. The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. (1983). The superiority of fibre-reinforced composite materials over other structural materials attracted many authors to study different types of problems in this field. Fibre-reinforced composite structures are used due to their low weight and high strength. Two important components, namely concrete and steel of a reinforced medium are bound together as a single unit so that there can be no relative displacement between them i.e. they act together as a single anisotropic unit. The artificial structures on the surface of the earth are excited during an earthquake, which give rise to violent vibrations in some cases [Acharya (2009); Samaland and Chattaraj (3011)]. Engineers and architects are in search of such reinforced elastic materials for the structures that resist the oscillatory vibration. The propagation of waves depends upon the ground vibration and the physical properties of the material structure. Surface wave propagation in fiber reinforced media was discussed by various authors [Sing (2006); Kakar et al. (2013)]. Abd-Alla et al. (2012) investigated the transient coupled thermoelasticity of an annular fin.Reflection of quasi-P and quasi-SV waves at the free and rigid boundaries of a fibre-reinforced medium was also discussed by Chattopadhyay et al.(2012). Abd-Alla and Mahmoud (2011) investigated the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic, heat conduction model.The extensive literature on the topic is now available and we can only mention a few recent interesting investigations [Singh and Singh (2004); Abd-Alla (2013); Singh (2007); Abd-Alla (2011); Abo-Dahab et al. (2016), Alla et al. (2015); Kumar et al. (2016); Said and Othman (2016); Bakora and Tounsi (2015)]. The temperature-rate dependent theory of thermoelasticity, which takes into account two relaxation times, was developed by Green and Lindsay (1972); Kumar et al. (2016) investigated the thermomechanical interaction transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with the combined effects of rotation. Marin (1996) studied the Lagrange identity method in thermoelasticity of bodies with microstructure. Marin (1995) presented the existence and uniqueness in thermoelasticity of micropolar bodies. Marin and Marinescu (1998) investigated the thermoelasticity of initially stressed bodies. Asymptotic equipartition of energies.

The aim of this paper is to investigate the propagation of magneto-thermoelastic surface waves in a rotating fibre-reinforced viscoelastic anisotropic media of higher order. The general surface wave speed is derived to study the effect of rotation, magnetic field and thermal on surface waves.

The wave velocity equations have been obtained for Stoneley waves, Rayleigh waves and

Love waves, and are in well agreement with the corresponding classical result in the absence of viscosity, temperature, rotation as well as homogeneity of the material medium.The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. For order zero our results are well agreedto fibre-reinforced materials. It is also observed that the corresponding classical gcresults follow from this analysis, in viscoelastic media of order zero, by neglecting reinforced parameters, rotational and thermal effects. Numerical results are given and illustrated graphically. It is important to note that Love wave remains unaffected by thermal, magnetic field and rotational effects.

2 Formulation of the problem

The constitutive relation of an anisotropic and elastic solid is expressed by the generalized Hooke’s law, which can be written as

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl}, \qquad i, j, k, l=1, 2, 3. \tag{1}$$

Let To be the reference temperature at which the system is in equilibrium and let it be subjected to a temperature change $T - T_o$ where $|T - T_0| << T_0$. Thus the coupled thermoelastic equations for the material may be written as Kakar et al. (2013).

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} - \beta_{ij}\left(1 + \nu_o \frac{\partial}{\partial t}\right)(T - T_o) \tag{2}$$

$$\frac{\partial}{\partial x_i}\left(\kappa_{ij}\frac{\partial T}{\partial x_j}\right) = \rho c_v\left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)T + T_o\beta_{ij}\left(\frac{\partial}{\partial t}\right)\varepsilon_{ij} \tag{3}$$

The thermal constant ν_o and τ_o appearing in the above equations satisfy the inequalities $\nu_o \geq \tau_o \geq 0$. It is evident that if $\tau_o > 0$, consequently $\nu_o > 0$, the Eq. (3) predicts a finite speed of propagation of thermal signals and that if $\nu_o = \tau_o = 0$, the Eq. (2) and (3) reduce to the coupled theory. The assumption $\tau_o = 0$ and $\nu_o > 0$ is also a valid one; in this case the equation of motion continues to be affected by the temperature rate, while Eq. (3) predicts an infinite speed for the propagation of heat.

In Eq. (3) we have made use of the condition $|T - T_o| << T_o$ to replace T by T_o in the last term of Eq. (3). The κ_{ij} is the conductivity tensor, c_v is the specific heat at constant deformation, β_{ij} are the thermal moduli, σ_{ij} are the Cartesian components of the stress and ε_{kl} is the strain tensor which is related with the displacement vector, u_i, C_{ijkl} are the components of a fourth-order tensor called the elasticities of the medium. The Einstein convention for repeated indices is used.

For a homogeneous elastic body equation of motion may be taken as follows

$$\tau_{ij,j} = C_{ijkl}\varepsilon_{kl,j} - \beta_{ij}(1+\nu_o \frac{\partial}{\partial t})T_{,j} \tag{3a}$$

where the comma denotes differentiation with respect to the appropriate component of **x** . If a body is rotating about an axis with a constant angular velocity Ω in the presence of externally applied force $\bar{F}$, then equation of motion can be written as follows [(Abd-Alla et al. (2013)].

$$\tau_{ij,j} + F_i = \rho\left\{\ddot{u}_i + \Omega_j u_j \Omega_i - \Omega^2 u_i + 2\varepsilon_{ijk}\Omega_j \dot{u}_k\right\} \tag{4}$$

For a slowly moving electrically conducting homogeneous elastic medium in which the variation of magnetic and electric fields is given by Maxwell's equations as follows:

$$\begin{aligned} &\mathrm{curl}\bar{\mathrm{H}} = \bar{\mathrm{J}} + \varepsilon_0 \dot{\bar{\mathrm{E}}}, \quad \mathrm{curl}\bar{\mathrm{E}} = -\mu_0 \dot{\bar{\mathrm{H}}}, \quad \mathrm{div}\bar{\mathrm{H}} = 0, \ \ \mathrm{div}\bar{\mathrm{E}} = 0, \\ &\bar{\mathrm{E}} = -\mu_0\left(\dot{\bar{\mathrm{u}}} \times \bar{\mathrm{H}}\right), \quad \bar{\mathrm{F}} = \mu_0\left(\bar{\mathrm{J}} \times \bar{\mathrm{H}}\right), \ \bar{\mathrm{b}} = \mathrm{curl}(\bar{\mathrm{u}} \times \bar{\mathrm{H}}_0) \end{aligned} \tag{4a}$$

where $\bar{\mathrm{H}} = \bar{\mathrm{H}}_0 + \bar{\mathrm{b}}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}), \qquad \bar{\mathrm{H}}_0 = (0, 0, \mathrm{H}_0)$

$\bar{\mathrm{E}}$ is electric intensity, $\bar{\mathrm{F}}$ is Lorentz's body forces, $\dot{\bar{\mathrm{u}}}$ is the velocity vector, $\bar{\mathrm{b}}$ is perturbed magnetic field, $\bar{\mathrm{H}}$ is magnetic field vector, $\bar{\mathrm{H}}_0$ is primary constant magnetic field vector, H_0 is the absolute magnetic field, $\bar{\mathrm{J}}$ is an electric current density vector, and μ_0 is magnetic permeability, ε_0 is the electric permeability.

Then magnetic force is defined as follows[13]

$$\vec{F} = \mu_0 H_0^2 \left(\frac{\partial e}{\partial x} - \varepsilon_0 \mu_0 \ddot{u}_1, \frac{\partial e}{\partial y} - \varepsilon_0 \mu_0 \ddot{u}_2, 0\right), \ \bar{b}(x,y,z,t) = (0, 0, -e)$$

where $e = \dfrac{\partial u_1}{\partial x} + \dfrac{\partial u_2}{\partial y}$ (5a)

In an incompressible material $e = 0$, here ε_{ijk} is the Levi-Civita tensor, by using (4), the equation of motin in a thermoelastic medium becomes

$$C_{ijkl}u_{k,jl} + F_i = \rho\left\{\ddot{u}_i + \Omega_j u_j \Omega_i - \Omega^2 u_i + 2\varepsilon_{ijk}\Omega_j \dot{u}_k\right\} + \beta_{ij}(1+\nu_o \frac{\partial}{\partial t})T_{,j} \tag{5b}$$

In isotropic medium $\kappa_{ij} = \kappa\delta_{ij}$ and $\beta_{ij} = \beta\delta_{ij}$, β is the coefficient of linear thermal expansion and κ is the thermal conductivity of the medium. Thus Above equation becomes

$$C_{ijkl}u_{k,jl} + F_i - \beta(1+\nu_o \frac{\partial}{\partial t})T_{,i} = \rho\left\{\ddot{u}_i + \Omega_j u_j \Omega_i - \Omega^2 u_i + 2\varepsilon_{ijk}\Omega_j \dot{u}_k\right\} \quad (5c)$$

Medium is consisting of two homogeneous anisotropic fibre-reinforced semi-infinite elastic solid media M and M_1 with different elastic and reinforcement parameters. The two media are perfectly welded in contact at a plane interface. Let us take orthogonal Cartesian axes $Ox_1x_2x_3$ with the origin at O. Ox_2 is pointing vertically upwards into the medium M(x_2 >0). Each of the media M(x_2 >0) and M_1 (x_2 <0) separated at x_2 =0. Both media are rotating about an axis.

It is assumed that the waves travel in the positive direction of the x_1-axis and at any instant, all particles have equal displacements in any direction parallel to Ox_3. In view of those assumptions, the propagation of waves will be independent of x_3.

The general equation for a fibre-reinforced linearly elastic anisotropic media w. r. t. a direction $\bar{a} = (a_1, a_2, a_3)$.

$$C_{ijkl}\varepsilon_{kl} = D_\lambda \varepsilon_{kk}\delta_{ij} + 2D_{\mu_T}\varepsilon_{ij} + D_\alpha(a_k a_m \varepsilon_{km}\delta_{ij} + \varepsilon_{kk}a_i a_j)_+ 2(D_{\mu_L} - D_{\mu_T})(a_i a_k \varepsilon_{kj} + a_j a_k \varepsilon_{ki}) + D_\beta(a_k a_m \varepsilon_{km} a_i a_j), \quad (6)$$

Strain tensor is $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ and D_λ, D_{μ_T} are elastic parameters. D_α, D_β and $(D_{\mu_L} - D_{\mu_T})$ are reinforced anisotropic viscoelastic parameters of higher order, s , defined as

$$D_\lambda = \lambda_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_\mu = \mu_k \left(\frac{\partial}{\partial t}\right)^k$$

$$D_\alpha = \alpha_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_{\mu_L} = \mu_{L_k} \left(\frac{\partial}{\partial t}\right)^k$$

$$D_\beta = \beta_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_{\mu_T} = \mu_{T_k} \left(\frac{\partial}{\partial t}\right)^k$$

$k = 0,1,2...s.$

An Einstein summation convention for repeated indices upon "*k*" is used and comma followed by an index denotes the derivative with respect to coordinate.

u_i are the displacement vectors components. By choosing the fibre direction as $\bar{a} = (1,0,0)$, the components of stress becomes as follows

$$\tau_{11} = (D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)\varepsilon_{11} + (D_\lambda + D_\alpha)\varepsilon_{22} + (D_\lambda + D_\alpha)\varepsilon_{33} ?\ \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o\ ,$$

$$\tau_{22} = (D_\lambda + D_\alpha)\varepsilon_{11} + (D_\lambda + 2D_{\mu_T})\varepsilon_{22} + D_\lambda\varepsilon_{33} - \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o),$$

$$\tau_{33} = (D_\lambda + D_\alpha)\varepsilon_{11} + D_\lambda\varepsilon_{22} + (D_\lambda + 2D_{\mu_T})\varepsilon_{33} ?\ \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o\ ,$$

$$\tau_{13} = 2D_{\mu_L}\varepsilon_{13},$$

$$\tau_{12} = 2D_{\mu_L}\varepsilon_{12},$$

$$\tau_{23} = 2D_{\mu_T}\varepsilon_{23}.$$

By using strain tensor, we get

$$\tau_{11} = (D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)u_{1,1} + (D_\lambda + D_\alpha)u_{2,2} + (D_\lambda + D_\alpha)u_{3,3} - \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o)$$

$$\tau_{11,1} = (D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)u_{1,11} + (D_\lambda + D_\alpha)u_{2,11} - \beta(1+\nu_o \frac{\partial}{\partial t})T_{,1}$$

$$\tau_{12,2} = D_{\mu_T}(u_{1,22} + u_{2,21}),$$

$$\tau_{13,3} = 0. \tag{7}$$

It is assumed that body is rotating about z-axis with an angular frequency Ω i.e. $\mathbf{\Omega} = \Omega(0,0,1)$ and by choosing the fibre direction as $\bar{a} = (1,0,0)$, Also by taking all derivatives w.r.t. x_3 zero. The equations (6) of motion takes the following form

$$(D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta + \mu_0 H_0^2)u_{1,11} + (D_\alpha + D_\lambda + D_{\mu_L} + \mu_0 H_0^2)u_{2,21} + D_{\mu_L}u_{1,22} =$$
$$(\rho + \varepsilon_0 \mu_0^2 H_0^2)\ddot{u}_1 - \{\Omega^2 u_1 + 2\Omega\dot{u}_2\} + \beta(1+\nu_o \frac{\partial}{\partial t})T_{,1} \tag{8}$$

$$(D_\alpha + D_{\lambda_k} + D_{\mu_L} + \mu_0 H_0^2)u_{1,12} + D_{\mu_L}u_{2,11} + (D_{\lambda_k} + 2D_{\mu_T} + \mu_0 H_0^2)u_{2,22} =$$
$$(\rho + \varepsilon_0 \mu_0^2 H_0^2)\ddot{u}_2 - \{\Omega^2 u_2 - 2\Omega\dot{u}_1\} + \beta(1+\nu_o \frac{\partial}{\partial t})T_{,2} \tag{9}$$

$$(D_{\mu_L} u_{3,11} + D_{\mu_T} u_{3,22}) = \rho\ddot{u}_3, \tag{10}$$

From Eq. (2), we have

$$\kappa T_{,ii} = \rho c_v \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)T + T_o\beta\left(\frac{\partial}{\partial t}\right)u_{i,i} \tag{11}$$

Similarly, we can get similar relations in M_1 with $\rho, \alpha, \beta, \kappa, c_v\ D_\alpha, D_\lambda,\ D_{\mu_L},\ D_{\mu_T}$ and D_β are replaced by $\rho', \beta', \kappa', c'_v\ D_{\alpha'}, D_{\lambda'}, D_{\mu'_L}\ D_{\mu'_T}$ and $D_{\beta'}$ i.e. all the parameters in medium M_1 are denoted by super script " dash "

Thus above set of equation becomes (For convenious dashes are omitted)

$$h_3u_{1,11}+h_2u_{2,21}+h_1u_{1,22}=\rho(1+\varepsilon_0\ \mu_o c_A^2)\ddot{u}_1-\rho(\Omega^2u_1+2\Omega\dot{u}_2)+\beta(1+\nu_o\frac{\partial}{\partial t})T_{,1} \quad (12)$$

$$h_4u_{2,22}+h_2u_{1,12}+h_1u_{2,11}=\rho(1+\varepsilon_0\ \mu_0 c_A^2)\ddot{u}_2-\rho\Omega^2u_2+2\rho\Omega\dot{u}_1+\beta(1+\nu_o\frac{\partial}{\partial t})T_{,2} \quad (13)$$

$$h_1u_{3,11}+h_5u_{3,22})=\rho\ddot{u}_3, \quad (14)$$

$$\kappa T_{,ii}=\rho\,c_v\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T+T_o\beta\left(\frac{\partial}{\partial t}\right)u_{i,i} \quad (15)$$

where

$$h_1=D_{\mu_L},$$

$$h_2=\mathrm{D}_\alpha+D_\lambda+D_{\mu_L}+\rho c_A^2,$$

$$h_3=\mathrm{D}_\lambda+2D_\alpha+4D_{\mu_L}-2D_{\mu_T}+D_\beta+\rho c_A^2,$$

$$h_4=D_\lambda+2D_{\mu_T}+\rho c_A^2,$$

$$h_5=D_{\mu_T}$$

and $c_A^2=\dfrac{\mu_0 H_0^2}{\rho}$, $c_v$is the Alfven wave speed

3 Solution of the problem

To solve the coupled thermoelastic equations, we make the assumptions :

$$\mathrm{u}_1,\mathrm{u}_2,\mathrm{u}_3=\hat{\mathrm{u}}_1(\mathrm{x}_2),\hat{\mathrm{u}}_2(\mathrm{x}_2),\hat{\mathrm{u}}_3(\mathrm{x}_2)\exp\{\mathrm{i}\omega(\mathrm{x}_1\text{-ct})\}$$

$$\theta=\hat{\theta}(\mathrm{x}_2)\ \exp\{\mathrm{i}\omega(\mathrm{x}_1\text{-ct})\} \quad (16)$$

where $\theta=T-T_0$

Hence the initially uniform magnetic field $\overline{\mathrm{H}}_0$ is transverse to the direction of wave propagation.

Thus coupled Eq. (8a, b, c) becomes

$$(\mathrm{h}_1\mathrm{D}^2\text{-}\omega^2\mathrm{h}_3+\omega^2(1+\varepsilon_0\,\mu_o\mathrm{c}_A^2)\rho\mathrm{c}^2+\rho\Omega^2)\hat{\mathrm{u}}_1+\mathrm{i}\omega(\mathrm{h}_2\mathrm{D}\text{-}2\mathrm{c}\rho\,\Omega)\hat{\mathrm{u}}_2\text{-}\mathrm{i}\omega\beta(1\text{-}\mathrm{i}\omega\mathrm{c}\nu_o)\hat{\theta}=0 \quad , \quad (17)$$

$$(\mathrm{h}_4\mathrm{D}^2\text{-}\omega^2\mathrm{h}_1+\omega^2(1+\varepsilon_0\,\mu_o\mathrm{c}_A^2)\rho\mathrm{c}^2+\rho\Omega^2)\hat{\mathrm{u}}_2+\mathrm{i}\omega(\mathrm{h}_2\mathrm{D}+2\mathrm{c}\rho\,\Omega)\hat{\mathrm{u}}_1\text{-}\beta(1\text{-}\mathrm{i}\omega\mathrm{c}\nu_o)\mathrm{D}\hat{\theta}=0 \quad , \quad (18)$$

$$\{h_5D^2-\omega^2(h_1-\rho c^2)\})\hat{u}_3=0\ , \quad (19)$$

and

$$\beta T_o\left(i\omega c_o\right)(D\hat{u}_1\text{-}i\omega\hat{u}_2)+\left\{\kappa(D^2\text{-}\omega^2)+i\omega c+\omega^2c^2\tau_o\right\}\hat{\theta}=0 \tag{19a}$$

where

$$\hbar_1=\mu_{Lk}(-i\omega c)^k,$$

$$\hbar_2=(\alpha_k+\lambda_k+\mu_{Lk})(-i\omega c)^k+\rho c_A^2,$$

$$\hbar_3=(\lambda_k+2\alpha_k+4\mu_{Lk}-2\mu_{Tk}+\beta_k)(-i\omega c)^k+\rho c_A^2,$$

$$\hbar_4=(\lambda_k+2\mu_{Tk})(-i\omega c)^k+\rho c_A^2,$$

$$\hbar_5=\mu_{Tk}(-i\omega c)^k$$

Above 3rd equation has the following solution,

$$u_3=Ee^{-\eta\omega x_2}e^{i\omega(x_1-ct)}, \tag{20}$$

where $\eta^2=\dfrac{\hbar_1-\rho c^2}{\hbar_5}$.

for positive real root η, it is necessary that $0<\rho c^2<\hbar_1$.

Remaining above set of equation can be written as

$$\left.\begin{array}{l}(h_1D^2\text{-}A_1)\hat{u}_1+i\omega(h_2D\text{-}2c\rho\Omega)\hat{u}_2\text{-}i\omega Q\hat{\theta}=0\\(h_4D^2\text{-}A_2)\hat{u}_2+i\omega(h_2D+2c\rho\Omega)\hat{u}_1\text{-}QD\hat{\theta}=0\\A_4(i\omega\hat{u}_1+D\hat{u}_2)+(D^2\text{-}A_3)\hat{\theta}=0\end{array}\right\} \tag{21}$$

where

$$A_1=\omega^2\hbar_3-\omega^2(1+\varepsilon_0\,\mu_o c_A^2)\rho c^2-\rho\Omega^2$$

$$A_2=\omega^2\hbar_1-\omega^2(1+\varepsilon_0\,\mu_o c_A^2)\rho c^2-\rho\Omega^2$$

$$A_3=\omega^2\left(1-c^2\tau_o\right)-i\omega c$$

$$A_4=i\omega c\beta T_o$$

$$Q=\beta(1-i\omega c\nu_o)$$

From avove set of equations, we have

$$\begin{vmatrix}(\hbar_1D^2-A_1) & i\omega(\hbar_2D-2c\rho\Omega) & -i\omega Q\\ i\omega(\hbar_2D+2c\rho\Omega) & (\hbar_4D^2-A_2) & -DQ\\ i\omega A_4 & DA_4 & (D^2-A_3)\end{vmatrix}(u_1,u_2,\theta)=0 \tag{22}$$

This implies

$$(D^6 - AD^4 + BD^2 - C)(u_1, u_2, \theta) = 0 \tag{23}$$

where

$$A = \frac{1}{\hbar_1 \hbar_4}\left(A_1 + \hbar_1(A_2 + \hbar_4 A_3 - A_4 Q) - \omega^2 \hbar_2^2\right)$$

$$B = \frac{1}{\hbar_1 \hbar_4}\left\{\left(A_1 A_2 + \hbar_4 A_1 A_3 + \hbar_1 A_2 A_3 - \omega^2 \hbar_2^2 A_3\right) - QA_4(A_1 - 2\omega^2 \hbar_2 + \hbar_4 \omega^2) - 4c^2 \omega^2 \rho^2 \Omega^2)\right\}$$

$$C = \frac{1}{\hbar_1 \hbar_4}\left(A_1 A_2 A_3 - \omega^2 A_2 A_4 Q - 4c^2 \omega^2 \rho^2 \Omega^2\right).$$

$$Let \quad D^2 = m$$

Auxiliary equation becomes

$$m^3 - Am^2 + Bm - C = 0 \tag{24}$$

A, *B* and *C* must be positive for real positive roots (*m*). If there is no thermal effect then the above equation is quadratic in *m* and it is easy to solve. But in the case of thermoelastic, it is cubic. *A*, *B* and *C* must be positive impose a necessary and sufficient condition upon the frequency of rotation of the medium. Through which a surface wave cannot propagate in a fast rotating medium. If there is no thermal effect then

$$0 < c^2 < \min\left\{\frac{\omega^2 \hbar_3 - \rho\Omega^2}{(1 + \varepsilon_0\, \mu_o c_A^2)\rho},\ \frac{\hbar_1}{(1 + \varepsilon_0\, \mu_o c_A^2)\rho}\right\}$$

From 1st term $\rho\Omega^2 < \omega^2 \hbar_3$, and from second term speed of the wave approaches to zero as c_A *i.e.* H_o approached to infinite. Thus in a fast rotating medium or in the presence of highly initially applied magnetic field, the surface wave cannot propagate. Thus earth quakes can be stoped by increasing the frequency of rotation of the earth or by increasing the gravity of the earth. But human cannot do that.

Llet m_1, m_2 *and* m_3 be three positive real roots, then solution by normal mode method has the following form

$$\hat{u}_1 = \sum_{n=1}^{3} M_n\ e^{-m_n x_2}, \tag{25}$$

$$\hat{u}_2 = \sum_{n=1}^{3} M_{1n}\ e^{-m_n x_2}, \tag{26}$$

$$\hat{\theta} = \sum_{n=1}^{3} M_{2n} \; e^{-m_n x_2}, \tag{27}$$

where M_n, M_{1n} and M_{2n}, are some parameters depending on c and ω. By using Eqs. (10a,b,c) into Eqs. (9), we get the following relations,

$$M_{1n} = H_{1n} M_n$$
$$M_{2n} = H_{2n} M_n \tag{28}$$

Where

$$H_{1n} = \frac{i\omega\left(A_2 + (\hbar_2 - \hbar_4) m_n^2 + 2\rho c \Omega m_n\right)}{\hbar_1 m_n^3 + (\hbar_2 \omega^2 - A_1) m_n + 2\rho c \omega^2 \Omega},$$

$$H_{2n} = \frac{m_n^2 - A_3}{A_4 (m_n H_{1n} - i\omega)} \qquad n = 1, 2, 3.$$

Hence we obtain the expressions of the displacement components, temperature distribution function and stresses as follows

$$u_1 = \sum_{n=1}^{3} M_n \; e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\}, \tag{29}$$

$$u_2 = \sum_{n=1}^{3} H_{1n} M_n \; e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\}, \tag{30}$$

$$u_3 = E e^{-\eta \omega x_2} \; exp\left\{ i\omega (x_1 - ct) \right\}, \tag{31}$$

$$\theta = \sum_{n=1}^{3} H_{2n} M_n \; e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\}, \tag{32}$$

and

$$\tau_{11} = \sum_{n=1}^{3} \left\{ i\omega h_3 - (\hbar_2 - \hbar_1) m_n H_{1n} - \beta (1 - i\omega c \nu_o) H_{2n} \right\} M_n e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\} \tag{33}$$

$$\tau_{22} = \sum_{n=1}^{3} \left\{ i\omega (\hbar_2 - \hbar_1) - (\hbar_4) m_n H_{1n} - \beta (1 - i\omega c \nu_o) H_{2n} \right\} M_n e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\} \tag{34}$$

$$\tau_{12} = \sum_{n=1}^{3} \hbar_1 \left(-m_n + i\omega H_{1n} \right) M_n e^{-m_n x_2} \; exp\left\{ i\omega (x_1 - ct) \right\} \tag{35}$$

Similar expressions can be obtained for second mediun and present them with dashes as follows

$$u_1' = \sum_{n=1}^{3} M_n' \; e^{-m_n' x_2} \; exp\left\{ i\omega (x_1 - ct) \right\}, \tag{36}$$

$$u_2' = \sum_{n=1}^{3} H_{1n}' M_n' \ e^{-m_n' x_2} \ exp\left\{i\omega(x_1 - ct)\right\}, \tag{37}$$

$$u_3' = Fe^{-\eta'\omega x_2} \ exp\left\{i\omega(x_1 - ct)\right\}, \tag{38}$$

$$\theta' = \sum_{n=1}^{3} H_{2n}' M_n' \ e^{-m_n' x_2} \ exp\left\{i\omega(x_1 - ct)\right\}, \tag{39}$$

Also it is found that

$$\tau_{11}' = \sum_{n=1}^{3} \left\{i\omega h_3' - (\hbar_2' - \hbar_1')m_n' H_{1n}' - \beta'(1 - i\omega c v_o')H_{2n}'\right\} M_n' e^{-m_n' x_2} \ exp\left\{i\omega(x_1 - ct)\right\} \tag{40}$$

$$\tau_{22}' = \sum_{n=1}^{3} \left\{i\omega(\hbar_2' - \hbar_1') - (\hbar_4')m_n' H_{1n}' - \beta'(1 - i\omega c v_o')H_{2n}'\right\} M_n' e^{-m_n' x_2} \ exp\left\{i\omega(x_1 - ct)\right\} \tag{41}$$

$$\tau_{12}' = \sum_{n=1}^{3} \hbar_1' \left(-m_n' + i\omega H_{1n}'\right) M_n' e^{-m_n' x_2} \ exp\left\{i\omega(x_1 - ct)\right\} \tag{42}$$

In order to determine the secular equations, we have the following boundary conditions.

4 Boundary conditions

1) The displacement components between the mediums are continuous, i.e.
$u_1 = u_1'$, $u_2 = u_2'$, $u_3 = u_3'$ and $\theta = \theta'$ on $x_2 = 0$, for all x_1 and *t*.

2) Stress continuity exists, i.e. $\tau_{12} + \overline{\tau_{12}} = \tau_{12}' + \overline{\tau_{12}'}$, $\tau_{22} + \overline{\tau_{22}} = \tau_{22}' + +\overline{\tau_{22}'}$, $\tau_{23} + \overline{\tau_{23}} = \tau_{23}' + \overline{\tau_{23}'}$ on $x_2 = 0$, for all x_1 and *t*.

where, Maxwell's stress equation

$\overline{\tau_{ij}} = \mu_0 \left[\mathrm{H_i b_j + H_j b_i - H_k b_k} \delta_{ij} \right]$, this implies

$$\overline{\tau_{ij}} = \mu_0 \mathrm{H}_0 \begin{bmatrix} -\mathrm{b}_3 & 0 & \mathrm{b}_1 \\ 0 & -\mathrm{b}_3 & \mathrm{b}_2 \\ \mathrm{b}_1 & \mathrm{b}_2 & \mathrm{b}_3 \end{bmatrix}, \qquad \overline{\tau_{ij}'} = \mu_0' H_0 \begin{bmatrix} -b_3' & 0 & b_1' \\ 0 & -b_3' & b_2' \\ b_1' & b_2' & b_3' \end{bmatrix}$$

Thermal boundary conditions [Abd-Alla and Mahmoud (2010)], gives

$$\left(\frac{\partial \theta}{\partial x_2} + h\theta\right)_{medium\,M} = \left(\frac{\partial \theta'}{\partial x_2} + h'\theta'\right)_{medium\ M_1}, \ \text{on the plane } x_2 = 0, \forall\, x_1 \ and \ t,$$

where h and h' are non nagative thermal constant.

Boundary conditions implies the following equatios.

$$\left.\begin{aligned}
&M_1+M_2+M_3=M_1'+M_2'+M_3'\\
&H_{11}M_1+H_{12}M_2+H_{13}M_3=H_{11}'M_1'+H_{12}'M_2'+H_{13}'M_3'\\
&H_{21}M_1+H_{22}M_2+H_{23}M_3=H_{21}'M_1'+H_{22}'M_2'+H_{23}'M_3'\\
&E=F
\end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned}
&\sum_{n=1}^{3}\hbar_1\left(-m_n+i\omega H_{1n}\right)M_n=\sum_{n=1}^{3}\hbar_1'\left(-m_n'+i\omega H_{1n}'\right)M_n',\\
&\sum_{n=1}^{3}\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_n H_{1n}-\beta(1-i\omega c\nu_o)H_{2n}\right\}M_n-\mu_0H_0b_3=\\
&\qquad\sum_{n=1}^{3}\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_n' H_{1n}'-\beta'(1-i\omega c\nu_o')H_{2n}'\right\}M_n'-\mu_0'H_0b_3',\\
&\hbar_5\eta_3E+\mu_0H_0b_2=\hbar_5'\eta_3'F+\mu_0'H_0b_2',\\
&(h-m_n)H_{2n}M_n=(h'-m_n')H_{2n}'M_n'
\end{aligned}\right\} \tag{44}$$

From the above equations containing E and F, we have

$$E=F=\frac{H_0\left(\mu_0'b_2'-\mu_0b_2\right)}{\left(\hbar_5\eta_3+\hbar_5'\eta_3'\right)}$$

But from Eq.(5) we have $b_2=0$ and similarly for second medium $b_2'=0$, Thus $E=F=0$.

This implies that there is no propagation in the transverse component of displacement. From others equations one can find M_n M_n' very easily. Also if $\mu_0b_3=\mu_0'b_3'$, then elimination of constants M_n *and* M_n', $(n=1,2,3)$ from above set of relation , gives the following secular equation for thermoelastic surface wave in a rotating fibre reinforced viscoelastic material of order n.

$$\det(a_{pq})=0;\qquad p=q=1,2,3,4,5,6. \tag{45}$$

where

$$a_{11}=1,\ a_{12}=1,\ a_{13}=1,\ a_{14}=-1,\ a_{15}=-1,\ a_{16}=-1,$$
$$a_{21}=H_{11},\ a_{22}=H_{12},\ a_{23}=H_{13},\ a_{24}=-H_{11}',\ a_{25}=-H_{12}',\ a_{26}=-H_{13}',$$
$$a_{31}=H_{21},\ a_{32}=H_{22},\ a_{33}=H_{23},\ a_{34}=-H_{21}',\ a_{35}=-H_{22}',\ a_{36}=-H_{23}',$$
$$a_{41}=\hbar_1\left(-m_1+i\omega H_{11}\right),\ a_{42}=\hbar_1\left(-m_2+i\omega H_{12}\right),\ a_{43}=\hbar_1\left(-m_3+i\omega H_{13}\right),$$
$$a_{44}=-\hbar_1'\left(-m_1'+i\omega H_{11}'\right),\ a_{45}=-\hbar_1'\left(-m_2'+i\omega H_{12}'\right),\ a_{46}=-\hbar_1'\left(-m_3'+i\omega H_{13}'\right)$$

$$a_{51}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_1 H_{11}-\beta(1-i\omega c v_o)H_{21}\right\}$$
$$a_{52}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_2 H_{12}-\beta(1-i\omega c v_o)H_{22}\right\}$$
$$a_{53}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_3 H_{13}-\beta(1-i\omega c v_o)H_{23}\right\}$$
$$a_{54}=-\left\{i\omega(\hbar'_2-\hbar'_1)-\hbar'_4 m_1 H'_{11}-\beta'(1-i\omega c v_o)H'_{21}\right\}$$
$$a_{55}=-\left\{i\omega(\hbar'_2-\hbar'_1)-\hbar'_4 m_2 H'_{12}-\beta'(1-i\omega c v_o)H'_{22}\right\}$$
$$a_{56}=-\left\{i\omega(\hbar'_2-\hbar'_1)-\hbar'_4 m_3 H'_{13}-\beta'(1-i\omega c v_o)H'_{23}\right\}$$
$$a_{61}=(h-m_1)H_{22},\qquad a_{62}=(h-m_2)H_{22},\qquad a_{63}=(h-m_3)H_{23},$$
$$a_{64}=-(h'-m'_1)H'_{21},\qquad a_{65}=-(h'-m'_2)H'_{22},\qquad a_{66}=-(h'-m'_3)H'_{23},$$

5 Particular cases

5.1 Stoneley waves

Eq. (14) is the secular equation for Stonely waves in a fibre reinforced viscoelastic media of order s if $\mu_0 b_3=\mu'_0 b'_3$, For k=0, results are similar to Abd-Alla (2013) and Lotfy (2012). If rotational, thermal and fiber-reinforced parameters are ignored, then for k=0, the results are same as Stoneley (1924).

Then equation (45) reduces to,

$$\left|b_{ij}\right|=0,\qquad i,j=1,2,3,4,5,6$$

where

$$b_{11}=1,\quad b_{12}=1,\quad b_{13}=1,\quad b_{14}=-1,\quad b_{15}=-1,\quad b_{16}=-1, \tag{46}$$

$$b_{21}=H_{11},\quad b_{22}=H_{12},\quad b_{23}=H_{13},\quad b_{24}=-H'_{11},\quad b_{25}=-H'_{12},\quad b_{26}=-H'_{13},$$
$$b_{31}=H_{21},\quad b_{32}=H_{22},\quad b_{33}=H_{33},\quad b_{34}=-H'_{21},\quad b_{35}=-H'_{22},\quad b_{36}=-H'_{23},$$
$$b_{41}=D_{\mu L}(-m'_1+i\omega H_{11}),\quad b_{42}=D_{\mu L}(-m_2+i\omega H_{12}),\quad b_{43}=D_{\mu L}(-m_3+i\omega H_{13}),$$
$$b_{44}=-D'_{\mu L}(-m'_1+i\omega H'_{11}),\quad b_{45}=-D'_{\mu L}(-m'_2+i\omega H'_{12}),\quad b_{46}=-D'_{\mu L}(-m'_3+i\omega H'_{13}),$$
$$b_{51}=\{i\omega(h_2-h_1)-h_4 m_1 H_{11}-\beta(1-i\omega c v_0)H_{21}\},$$
$$b_{52}=\{i\omega(h_2-h_1)-h_4 m_2 H_{12}-\beta(1-iwc v_0)H_{22}\},$$
$$b_{53}=\{i\omega(h_2-h_1)-h_4 m_3 H_{13}-\beta(1-i\omega c v_0)H_{23}\},$$
$$b_{54}=-\{i\omega(h'_2-h'_1)-h'_4 m'_1 H'_{11}-\beta'(1-i\omega c v_0)H'_{21}\},$$
$$b_{55}=-\{i\omega(h'_2-h'_1)-h'_4 m'_2 H'_{12}-\beta'(1-i\omega c v_0)H'_{22}\},$$
$$b_{56}=-\{i\omega(h'_2-h'_1)-h'_4 m'_3 H'_{13}-\beta'(1-i\omega c v_0)H'_{33}\},$$
$$b_{61}=(h-m_1)H_{22},\quad b_{62}=(h-m_2)H_{22},\quad b_{63}=(h-m_3)H_{23},$$
$$b_{64}=-(h'-m'_1)H'_{21},\quad b_{65}=-(h'-m'_2)H'_{22},\quad b_{66}=-(h'-m'_3)H'_{23}$$

Eq. (46) gives the wave velocity equation of Stoneley waves in a viscoelastic medium of Voigt type where the viscosity is of Ist order involving time rate of change of strain which is completely in agreement with classical results given by Sengupta and Nath (2001). Further equation (46), of course, is in complete agreement with the corresponding classical result, when the effect of rotation, viscosity and parameters of fibre-reinforcement are ignored.

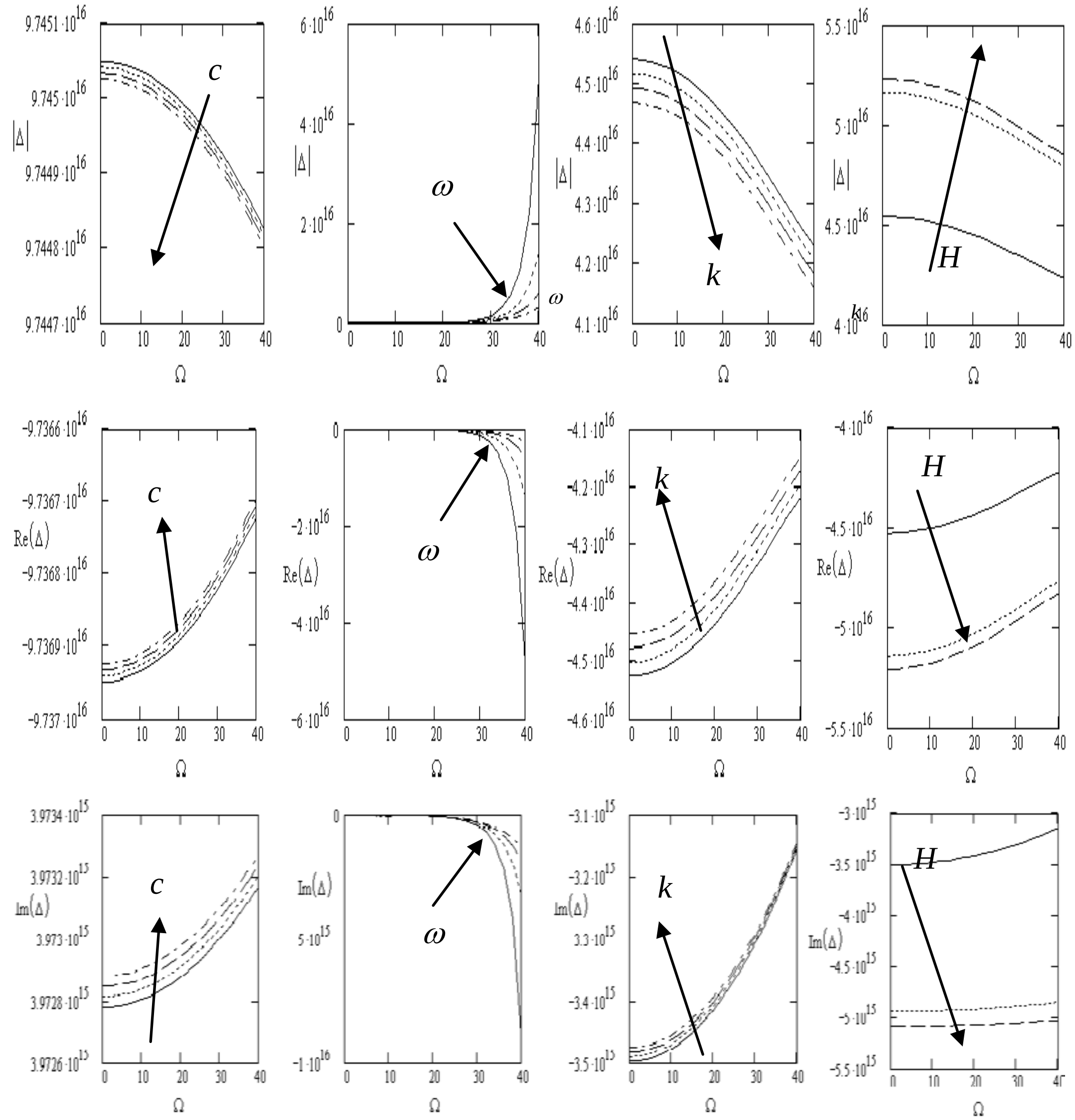

Figure 1: Variation of $|\Delta|$, velocity ($\mathrm{Re}(|\Delta|)$) and attenuation coefficient ($\mathrm{Im}(|\Delta|)$) for stoneley waves with respect to Ω with variation of c, ω and k

5.2 Love waves

To investigate the rotational effects on Love waves in a fibre reinforced viscoelastic media of higher order, we replace medium M_1 by an infinitely extended horizontal plate of finite thickness d and bounded by two horizontal plane surfaces $x_2=0$ and $x_2=d$. Medium M is semi infinite as in the general case.

The boundary conditions of Love wave are as follows

The displacement component u_3 and τ_{12} between the mediums are continuous, i.e.

$u_3 = u_3'$ and $\tau_{23} = \tau_{23}'$ on $x_2 = 0$

$\tau_{23}' = 0$ on $x_2 = d$, for all x_1 and t,

where

$$u_3 = E e^{-\eta\omega x_2} e^{i\omega(x_1 - ct)},$$

$$u_3' = E' e^{\eta'\omega x_2} e^{i\omega(x_1 - ct)} + F' e^{-\eta'\omega x_2} e^{i\omega(x_1 - ct)},$$

Boundry conditions implies

$$E - E' - F' = 0,$$

$$\hbar_5 \eta E + \hbar_5' \eta' E' - \hbar_5' \eta' F' = H_0(\mu_0 b_2 - \mu_0' b_2'),$$

$$\hbar_5' \eta' e^{\omega\eta' d} E' - \hbar_5' \eta' e^{-\omega\eta' d} F' = -\mu_0' H_0 b_2'.$$

This implies

$$E = \frac{H_0 \left\{ \left(\hbar_5' \eta' (\mu_0 b_2 - \mu_0' b_2') Cosh\omega\eta' d \right) - \mu_0 H_0 b_2 \hbar_5' \right\}}{\eta_3 \hbar_5 Cosh\omega\eta' d - \eta_3' \hbar_5' Sin\omega h\eta' d}$$

$$E' = \frac{H_0 \left\{ \hbar_5' \eta' e^{-\omega\eta' d} (\mu_0 b_2 - \mu_0' b_2') - \mu_0 b_2 (\hbar_5 \eta - \hbar_5' \eta') \right\}}{2\eta' \left\{ \eta_3 \hbar_5 Cosh\omega\eta d - \eta_3' \hbar_5' Sin\omega h\eta d \right\}}$$

$$F' = \frac{H_0 \left\{ \hbar_5' \eta' e^{\omega\eta' d} (\mu_0 b_2 - \mu_0' b_2') + \mu_0 H_0 b_2 (\hbar_5 \eta + \hbar_5' \eta') \right\}}{2\eta' \left\{ \eta_3 \hbar_5 Cosh\omega\eta d - \eta_3' \hbar_5' Sin\omega h\eta d \right\}}$$

Since $b_2 = 0$ and $b_2' = 0$

This implies $E = E' = F' = 0$

Thus non trivial solution gives

$$\begin{vmatrix} 1 & -1 & -1 \\ \hbar_5 \eta_3 & \hbar_5' \eta_3' & -\hbar_5' \eta_3' \\ 0 & e^{\omega\eta_3' d} & -e^{-\omega\eta_3' d} \end{vmatrix} = 0,$$

On simplification yields $\hbar_5' \eta_3' \tan(\omega d \eta_3') + \hbar_5 \eta_3 = 0$ or

$$\hbar_5 \left(\frac{\hbar_5 - \rho c^2}{\hbar_5} \right)^{\frac{1}{2}} + \hbar_5' \left(\frac{\hbar_5' - \rho c^2}{\hbar_5'} \right)^{\frac{1}{2}} \tan\left[\omega d \left(\frac{\hbar_5' - \rho' c^2}{\hbar_5'} \right)^{\frac{1}{2}} \right] = 0 .$$

This gives the wave velocity of Love waves propagating in a fiber-reinforced viscoelastic medium of order *s*. For k= 0, the results are exactly same as in literature. It is interesting to note that the magnetic field, thermal and rotation did not interrupt the propagation of Love waves.

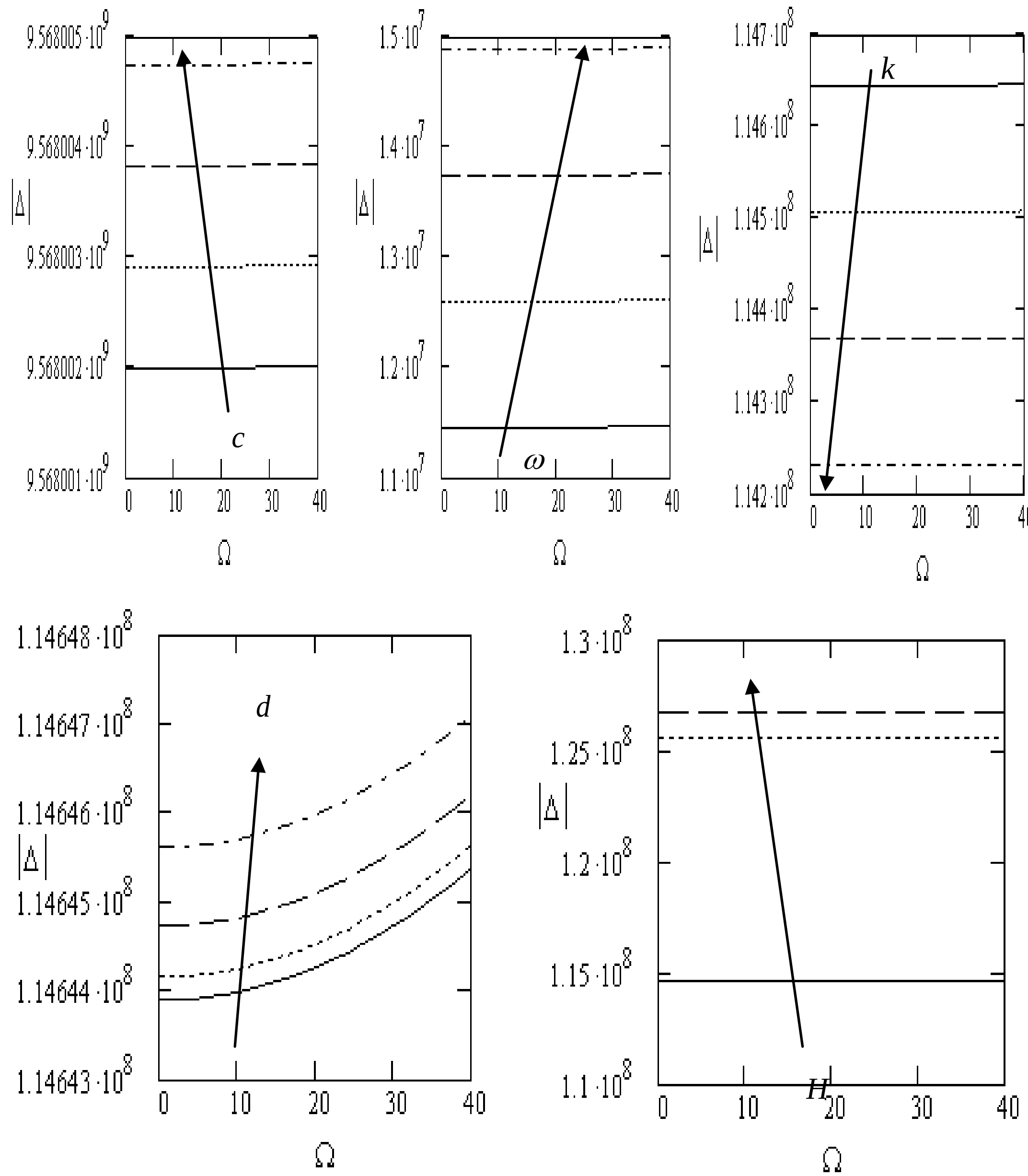

Figure 2: Variation of $|\Delta|$, for Love waves with respect to Ω with variation of c, ω, k , d and H

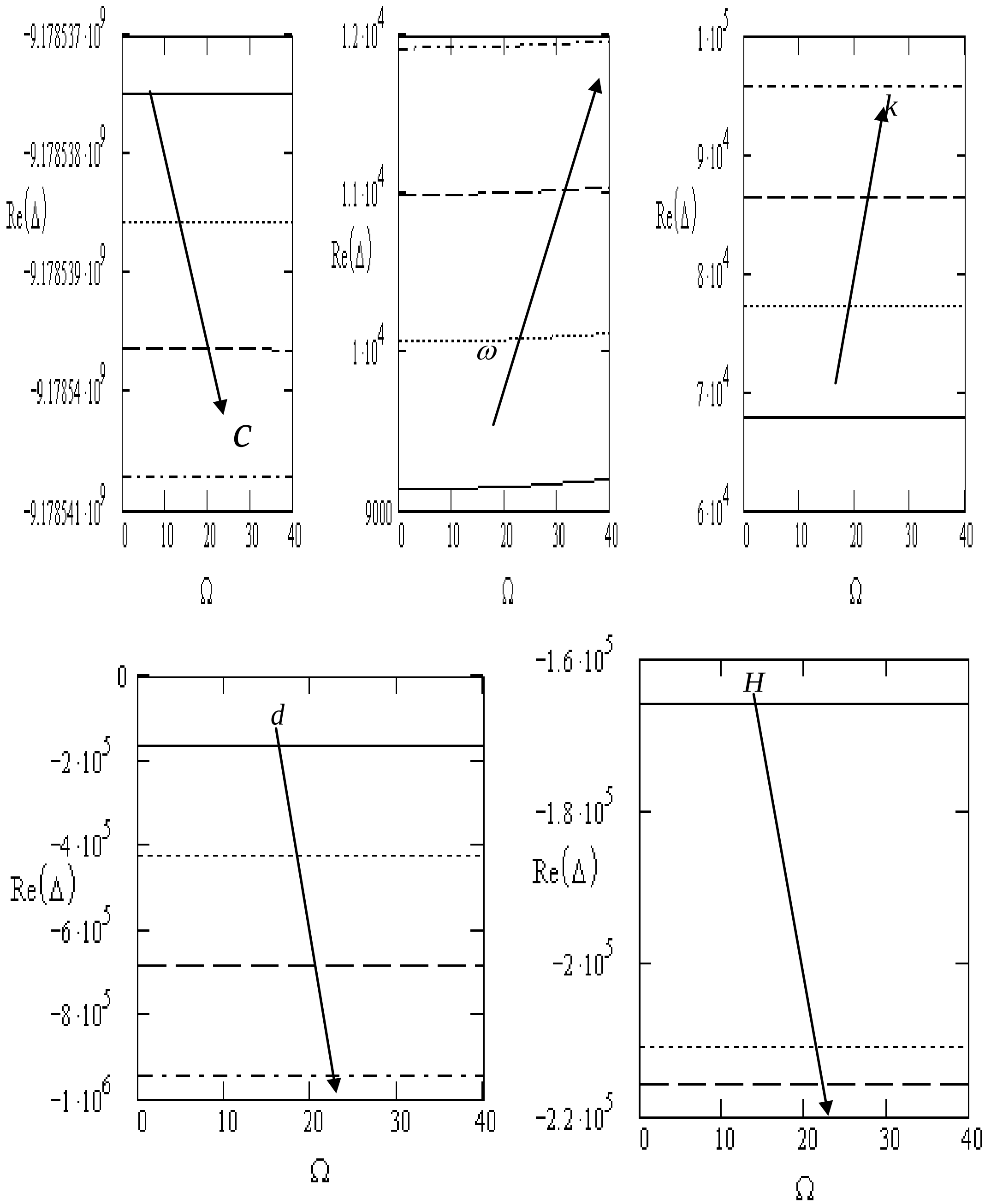

Figure 3: Variation of velocity ($\mathrm{Re}(\Delta)$) for Love waves with respect to Ω with variation of c, ω , k , d and H

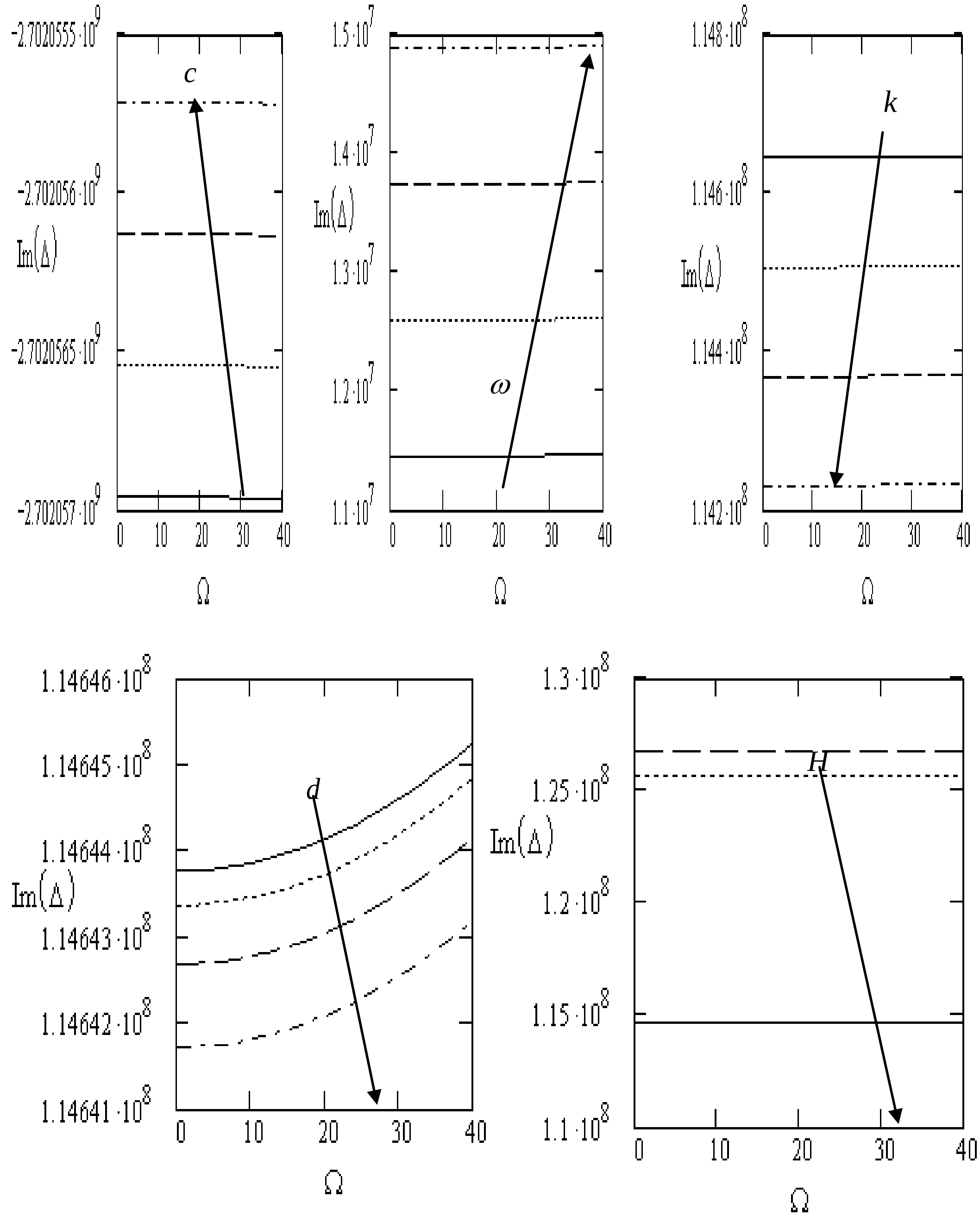

Figure 4: Variation of attenuation co-efficient ($\mathrm{Im}(\Delta)$) for Love waves with respect to Ω with variation of c, ω , k , d and H

5.3 Rayleigh waves

Rayleigh wave is a special case of the above general surface wave. In this case we consider a model where the medium M_1 is replaced by vacuum. Since the boundary $x_2 = 0$ is adjacent to vacuum. It is free from surface traction. So the stress boundary condition in this case may be expressed as

$\tau_{12} + \overline{\tau}_{12} = 0$, $\tau_{22} + \overline{\tau}_{22} = 0$ on $x_2 = 0$, for all x_1 and *t*.

$$\frac{\partial \theta}{\partial x_2} + h\theta = 0, \text{ on the plane } x_2 = 0, \forall x_1 \text{ and } t,$$

Thus above set of equations reduces to

$$\sum_{n=1}^{3} \left(-m_n + i\omega H_{1n}\right) M_n = 0,$$

$$\sum_{n=1}^{3} \left\{ i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_n H_{1n} - \beta(1 - i\omega c \nu_o) H_{2n} \right\} M_n - \mu_0 H_0 b_3 = 0$$

$$\sum_{n=1}^{3} (h - m_n) H_{2n} M_n = 0$$

From the above set one can easly find out the values of M_1, M_2 and M_3 as follows

$$M_1 = \frac{\mu_0 H_0 b_3 \left(d_{13} d_{23} - d_{12} d_{33}\right)}{\det(d_{ij})}$$

$$M_2 = \frac{\mu_0 H_0 b_3 \left(d_{11} d_{33} - d_{13} d_{31}\right)}{\det(d_{ij})}$$

$$M_3 = \frac{\mu_0 H_0 b_3 \left(d_{12} d_{13} - d_{11} d_{32}\right)}{\det(d_{ij})}$$

If $b_3 = 0$, this mean that induced magnetic field is not present then for non trival solution we have

$$\det(d_{lm}) = 0; \quad l = m = 1, 2, 3. \tag{47}$$

where

$$d_{11} = \left(-m_1 + i\omega H_{11}\right),\ d_{12} = \left(-m_2 + i\omega H_{12}\right),\ d_{13} = \left(-m_3 + i\omega H_{13}\right),$$

$$d_{21} = \left\{ i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_1 H_{11} - \beta(1 - i\omega c \nu_o) H_{21} \right\}$$

$$d_{22} = \left\{ i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_2 H_{12} - \beta(1 - i\omega c \nu_o) H_{22} \right\}$$

$$d_{23} = \left\{ i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_3 H_{13} - \beta(1 - i\omega c \nu_o) H_{23} \right\}$$

$$d_{31} = (h - m_1) H_{21}, \quad d_{32} = (h - m_2) H_{22}, \quad d_{33} = (h - m_3) H_{23},$$

So as a special case i.e. in the absence of induced magnetic field, the Eq. (15) is the secular equation for Rayleigh wave for the medium M. For k=0, that is, our results are similar to AbAlla et al. (2013). For a non-rotating media we have to put $\Omega = 0$, then for k = 0 our results are similar to Singh (2006) . In the absence of rotational, themal and magnetic field results are same as Sengupta and Nath (2001). If one also ignor the fibre-reinforced parameters then results are same as Rayleigh (1885).

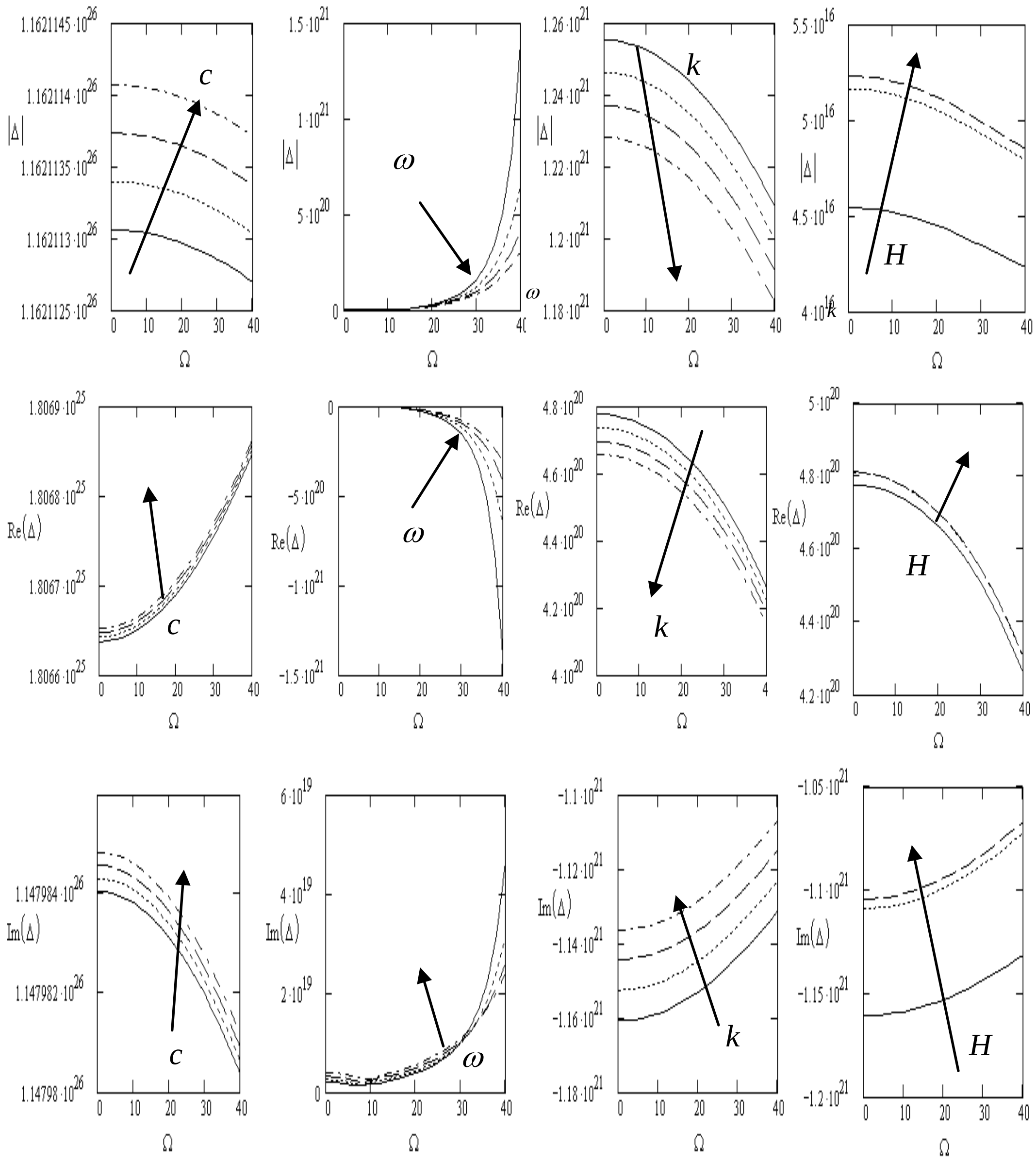

Figure 5: Variation of $|\Delta|$, velocity ($\mathrm{Re}(|\Delta|)$) and attenuation co-efficient ($\mathrm{Im}(|\Delta|)$) for Rayleigh waves with respect to Ω with variation of c, ω and k

6 Numerical results and discussion

The following values of elastic constants are considered Chattopadhyay et al. (2002) and Singh (2006), for mediums M and M_1 respectively.

$$\rho = 2660kG/m^3,\quad \lambda = 5.65x10^{10}\,Nm^{-2},\quad \mu_T = 2.46x10^9\,Nm^{-2},\quad \mu_L = 5.66x10^9\,Nm^{-2}$$
$$\alpha = -1.28x10^9\,Nm^{-2},\quad \beta = 220.90x10^9\,Nm^{-2},$$
$$\rho = 7800kG/m^3,\quad \lambda = 5.65x10^{10}\,Nm^{-2},\quad \mu_T = 2.46x10^{10}\,Nm^{-2},\quad \mu_L = 5.66x10^{10}\,Nm^{-2}$$
$$\alpha = -1.28x10^{10}\,Nm^{-2},\quad \beta = 220.90x10^{10}\,Nm^{-2}$$
$$T_0 = 293K,\quad \tau_0 = 0.1,\quad \nu_0 = 0.2$$

Taking into consideration Green-Linsay theory, the numerical technique outlined above was used to obtain secular equation, surface wave velocity and attenuation coefficients under the effects of rotation in two models. For the sake of brevity some computational results are being presented here. The variations are shown in Figure 1-5 respectively.

Figure1a-1l Show that the variation of the secular equation Stoneley wave, Stoneley wave velocity and attenuation coefficient of Stoneley wave with respect to rotation Ω for different values of phace velocity c, frequency ω, wave number k and magnetic field H. The secular equation decreases with increasing of rotation except when effect of frequency it increases with increasing of rotation, while it decreases with increasing of phase velocity, frequency and wave number, as well it increases with increasing of magnetic field, the Stoneley wave velocity increases with increasing of rotation except when effect of frequency it decreases with increasing of rotation, while it increases with increasing of phase velocity, frequency and wave number, as well it decreases with increasing of magnetic field, the attenuation coefficient increases with increasing of rotation except when effect of frequency it decreases with increasing of rotation, while it increases with increasing of phase velocity, frequency and wave number, as well it decreases with increasing of magnetic field.

Figure 2a-2e Show that the variation of the secular equation of Love wave with respect to rotation Ω for different values of phace velocity c, frequency ω, wave number k, thickness d and magnetic field H. There is no effect of rotation on the sculer equation except when effect of thickness it increases with increasing of thickness and rotation, while it increases with increasing of phase velocity, frequency and magnetic field, as well it decreases with increasing of wave number.

Figure 3a-3e Show that the variation of Love wave velocity with respect to rotation Ω for different values of phace velocity c, frequency ω, wave number k, thickness d and magnetic field H. There is no effect of rotation on the sculer equation except when effect of frequency it increases with increasing of frequency and rotation, while it decreases with increasing of phase velocity, thickness and magnetic field, as well it increases with increasing of wave number.

Figure 4a-4e) Show that the variation of attenuation coefficient of Love wave with respect to rotation Ω for different values of phace velocity c, frequency ω, wave number

k, thickness d and magnetic field H. There is no effect of rotation on the sculer equation except when effect of thickness it increases with increasing of thickness and rotation, while it increases with increasing of phase velocity and frequency, as well it decreases with increasing of wave number and magnetic field.

Figure 5a-5lShow that the variation of the secular equation of Rayleigh wave, Rayleigh wave velocity and attenuation coefficient of Rayleigh wave with respect to rotation Ω for different values of phace velocity c, frequency ω, wave number k and magnetic field H.The secular equation decreases with increasing of rotation except when effect of frequency of frequency it decreases with increasing of rotation, while it increases with increasing of phase velocity and magnetic field, as well it decreases with incrasing of frequency and wave number, Rayleigh wave velocity increases with increasing of rotation and phase velocity, while it decreases with increasing of rotation, as well it increases with increasing of frequency and magnetic field, while it decreases with increasing of wave number, the attenuation coefficient increases with increasing of rotation except when effect of phase velocity it decreases with increasing of phase velocity, while it increases with increasing of phase velocity, frequency, wave number and magnetic field.

7 Conclusion

The analysis of graphs permits us some concluding remarks.

1. The surface waves in a homogeneous, anisotropic, fibre-reinforced viscoelastic solid media under the rotation and higher order of nth order including time rate of strain are investigated. k

2. Love waves do not depend on temperature; these are only affected by viscosity, rotation, magnetic field, frequency, higher order of net order, including time rate of strain, phase velocity and thicknessof the medium. In the absence of all fields, the dispersion equation is incomplete agreement with the corresponding classical result. k

3. Rayleigh waves in a homogeneous, general magneto-thermo viscoelastic solid medium of higher order, including time rate of change of strain we find that the wave velocity equation, proves that there is a dispersion ofwaves due to the presence of rotation, magnetic field, temperature, frequency, phase velocity and viscosity. The results are incomplete agreement with the corresponding classical results in the absence of all fields.

4. The wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theoryof elasticity. The dispersion of waves is due to the presence of rotation, phase velocity, frequency, temperatureand viscosity of the solid. Also, wave velocity equation of this generalized type of surface waves is incomplete agreement with the corresponding classical result in the absence of all fields.

5. The results presented in this paper will be very helpful for researchers in geophysics, designers of new materials and the study of the phenomenon of rotation is also used to improve the conditions of oil extractions.

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