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Measurement Techniques of Torsional Vibration in Rotating Shafts

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Abstract: The measurement of torsional vibration is a common practice in certain fields, such as the automotive industry, power generation, or large alternative engines. Similarly, functional analysis and diagnostic of other equipment, which are not traditionally measured, can benefit greatly from this type of measurement. This review discusses some techniques used in industry to measure torsional vibration, briefly describing the types of sensors used and the transduction procedures. Choosing the most appropriate technique in each case not only responds to economic reasons, but also to other conditions of the given equipment, such as its design, coupled machines or devices, functional status and operating environment, and the possibilities to install the instrumentation.

Keywords: torsional vibrations, measurement techniques, analysis and diagnostic methods, laser interferometry.

1 Introduction

The measurement of vibration in the radial and axial directions has been performed consistently since the beginning of predictive maintenance, with great implementation and development related to instrumentation; yet the measurement of the torsional vibration has not seen a parallel development. Different defects or anomalous behaviour of the equipment, initially associated with a radial vibration, can sometimes be explained as coming from a source of torsional vibration [Al Bedoor (2001); Lee, Ha and Choi (2003); Gupta and Chawla(2004)]. Moreover, frequent defects in rotating machines, such as the misalignment of coupled machines, which are clearly characterised by radial and axial vibration behaviour, also carry cyclic torque variations [Meroño and Gómez de León (1998)], which cause irregularities in the angular velocity of the equipment. In fact, usually both phenomena, torsional

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and flexural deflections, happen simultaneously [Sapountzakis and Dourakopoulos (2010)].

Some machines and mechanisms, such as alternative engines [Brusa, Delprete and Genta (1997)] or crank-rocker mechanisms are, by their functional principle, a cause of torsional vibration. Adittionally, many mechanical components and load conditions or dynamic circumstances can also cause torsional vibration in rotating machines. Some notable cases are:

Simple Cardan joints (or double with an adjustment or incorrect assembly), whose cinematic introduces a driven shaft oscillation at double the frequency of the drive shaft's rotation [Dudita (1971)].

Gear drives, in which the elastic deformation of the material and the surface irregularities of the teeth caused by wear lead to angular frequency oscillations associated with the gear [Lee, Ha and Choi (2003); Li and Yu (2001); Bartelmus (2001)].

In electric motors, it is also easy to find variations of the angular velocity associated with the bar and groove steps, increased significantly due to an electrical imbalance or, in the squirrel-cage rotor, bars or short-circuit rings defects [Yen-Nien, Jyh-Cherng and Chih-Ming (2001)].

In pumps, compressors, turbines and fans, variations in rotating speed can often occur at a frequency equal to the blade pitch, on the average angular velocity, which are increased by uneven wear [Liu, Su, An and Huang (2011)]. Also periodic pulses of flow can induce this problem, either at the inlet or in the head [Singh and Rawtani (1979)].

Many theoretical studies, mainly based on the Finite Element Method [Genta (1988); Akbarov and Turan (2011)] or Timoshenko Beam Theory [Nelson (1980)], have been developed to model the effect of vibrations in the materials, what constitutes certainly a huge problem, especially in the shafts of rotating machines. The measurement of vibrations, to detect as soon as possible the damage in structures and the equipment industrial, is in practice very relevant. However, an accurate measurement of structural mode shapes and its severity is needed, which may not be easily satisfied in practice [Zhang and Xiang (2011)]. Torsional vibrations have even more difficulties to be measured than flexural modes. This paper shows a set of techniques to face the problem of measuring torsional vibration in shafts of rotating machines, analyzing its features and accuracy reachable.

2 Theory

2.1 Steady conditions and transient states

The circumstances described thus far relate to steady conditions, in which the variation of the rotational speed, which we will call angular vibration, occurs and remains periodic (or quasi-periodic) while the machine is operating. It should be clarified that when the angular vibration occurs in different ways, either in amplitude or phase, shaft torque occurs in two different sections of the machine and a torsional vibration appears. Thus, with a source of angular vibration, heavy rotors or low angular stiffness will cause more torsional vibration than light or very rigid rotors will. Therefore, the measurement of the angular vibration in one section of the shaft of the machine, is not itself a measurement of the torsional vibration. In steady conditions, some repetitive dynamic states are assumed, such as when there are faults in gears, misalignment between shafts or other circumstances that result in periodicity in the variations of the speed of the equipment. These alternating torsional stress conditions on rotating elements, especially the shaft and coupling, can significantly limit their working life as a result of material fatigue. The most common result is breakage or misalignment of the couplings and shaft cracking [Darpe, Gupta and Chawla (2004); Dimarogonas and Massouros (1981); Zou, Chen and Pu (2004)]. Besides the steady conditions discussed, some of which originate due to faulty equipment, in all rotating machines the start and stop are transient circumstances in which conditions for torsional vibrations exist. In addition, any variation in the load of the machine, that is, in the resisting torque, will cause transient changes in its rotating speed, compatible with the mechanical characteristics of the given machine, which are also a source of torsional vibration.

The differences between the signals obtained from the torsional vibration in steady conditions and transient states impose some differences in the type of technique used for measurement and analysis. While the instrumentation to be used in both cases may be the same, it is worth analyzing the possibilities offered, mainly in terms of speed of acquisition, frequency response and resolution. Whereas during a steady state of the equipment the stationary condition (or quasi-stationary) of the acquired signal allows a classic Fourier frequency analysis; analyzing a transient state, by definition, requires the temporal characteristics of the signal to be considered, therefore processing techniques in which the temporal component “disappears” should not be used, such as in the case of FFT (Fast Fourier Transform). By the nature of performed abstractions on the signal, the functions become harmonic, which are not best suited to analysing transient events. However, the characteristics and duration of the transient events advise the type of analysis technique to use in each situation. In these cases, the most advisable alternative is resorting to an analysis by time-frequency transformations. Some options that are often successful are: the STFT (Short-Time Fourier Transform) which includes information about the frequency in a given time window, and is by contrast not adaptable; or WT (Wavelet Transform), whose multi-resolution analysis enables, as the window size variable, to obtain more accurate information on the full range of frequencies [Zou, Chen and Pu (2004)]. During transient states the stresses on all rotating elements instantaneously increase, particularly on the couplings and shafts, and can reach values above the design ones. Otherwise, these transients excite natural modes of vibration of the elements coupled to the rotor, such as blades or vanes [Singh and Rawtani (1979)].

2.2 Measuring methods

Until the arrival of laser technology in systems measuring vibration, all methods for the measurement of the torsional vibration were invasive: they needed to introduce or provide certain elements of the measuring system in the rotor, such as set strain gauges, gears, encoders and torque transducers. This circumstance represents, undoubtedly, an important drawback in cases where it is necessary to make changes to the original design of the equipment to insert these elements, Fig. 1. Nevertheless, there are several methods to measure the torsional vibration in rotating shafts of rotating equipment. According to the measured quantity, these methods can be classified into direct and indirect. Direct methods are those in which the torsional vibration is obtained from the intervention performed. Therefore, it is necessary to measure torque, that is to say, the angular deformations caused in the given element, typically the axis, over time. These methods are based on determining the relative angular positions between two different sections of the rotor. The difference between the angular positions at each instant of time gives the angular deformation of the rotor area between the two sections.

Depending on the technique used, the measured quantity may not correspond to the angular position but to its first derivative, i.e. the angular velocity, or secondly, to the angular acceleration. Although any of those three offer a valid representation of the torsional vibration, the measure of angular velocity is normally used in practice. In indirect methods torsional vibration is inferred or estimated from certain related magnitudes. The most significant magnitudes typically measured include the following:

- The torque, by a transducer intercalated in the rotor assembly.
- The deformations produced in two main torsion directions of the shaft along a given free area of it.

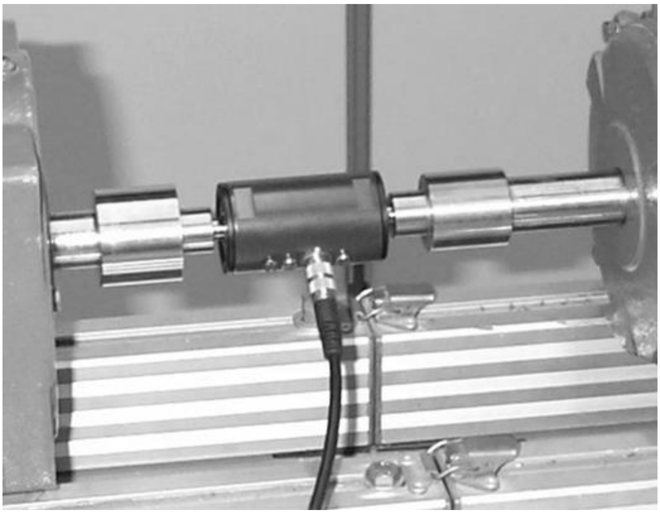

Figure 1: Torque transducer coupled between two shafts.

- Surface tensions arising from elastic elements arranged on the shaft or interspersed in the rotor assembly.
- The relative angular vibration of a seismic mass elastically coupled at a free extreme of the shaft.

Since, in general, the yield strength of the material is not exceeded, in most cases the obtained values are interpreted directly as a measurement of the torsional vibration, due to the proportionality between this and the measured quantity. The group of indirect techniques it could also include measuring the angular vibration, provided that it is understood that when only referring to a single section of the rotor assembly this magnitude is not strictly torsional vibration as discussed above. However, this method is considered within the direct techniques, since the measurement of the angular vibration in two sections of the rotor is the basis of torsional vibration measurement.

2.3 Transducers for measuring torsional vibration

Transducers for measuring torsional vibration can also be classified according to the measurement method used. Thus, transducers which are employed as indirect methods usually include the following:

- Strain gauges

- Torque sensors
- Elastic coupled systems
    - Mechanical
    - Magnetic
- Electromagnetic Systems
- The direct methods include the following types of transducers:
- Encoders
    - Electrical
        - Contact
        - Capacitive
    - Electromagnetic
    - Optical
        - Opaque and transparent sectors
        - Interference fringe
        - Non-reflective and reflective sectors
- Laser Interferometers
    - Crossed laser beam
    - Parallel laser beam

The corresponding measurement techniques for each of the types of transducers indicated above are now briefly described.

2.3.1 Strain gauges

One of the most widely known systems is used by strain gauges firmly joined to the shaft surface in the direction of the principal torques, i.e. forming 45° with the axis, forming a full Wheatstone bridge, as shown in Fig. 2.

This particular arrangement eliminates the effects due to bending and temperature changes. The method is accurate if installation and calibration are correctly performed, but presents significant practical difficulties. The first is that the signal has to be transmitted from a rotation system (the shaft), to a stationary system feedback, either through a system of slip rings or via a radio signal. This means that factors such as the accumulation of dirt on the rings, or interference with other radio signals can significantly distort the measurement. A similar problem is that the system requires a power supply that is either incorporated into the rotor assembly, which is not always possible, or the necessary supply is given from outside via slip rings. Another drawback that may arise is that derived from an incorrect orientation of gauges. Even small deviations from the correct orientation can cause significant distortions in the measurement of torque by its interference with the effects produced by bending the shaft. Taking into account that the gauges can deteriorate or break away from the axis through use or due to environmental reasons, such as temperature or chemicals, it can be seen that the requirements for proper installation and maintenance of the measuring system are too complex and laborious from a practical standpoint. To avoid these drawbacks, sections of shafts with slip rings and gauges installed for this purpose, or even a radio telemetry system, are commercially available. Still, as can be imagined, this requires the replacement or adjustment of certain elements of the machine, causing interference in its original design which, in some cases can unnecessarily worsen safety and reliability.

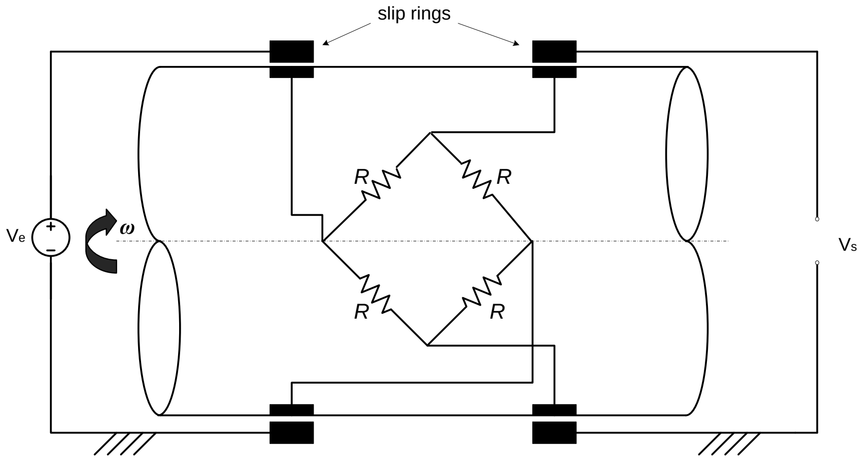

Figure 2: Strain gauges and slip rings on the shaft.

2.3.2 *Torque sensors*

Commercially, there are different types of sensors capable of determining the torque transmitted between the coupled machines, all of which are based on the deformation of an intermediate elastic element. Since the torsional stiffness of the shaft whose torque is to be measured is generally unknown, it is necessary to embed the sensor between machines. Torsion bars are the sensors traditionally used and they have an elastic element, a cylindrical rod, whose behaviour is well known. The transmitted torque is deduced from the surface deformation, as measured by extensiometry.

Another group of torque sensors, which are becoming more widely accepted, are those that use magneto-elastic technology. Essentially, they are based on measuring changes produced in a magnetic field as a result of the torsional deformation of an elastic element interposed between the shafts of the machines. This element is made of materials which change their magnetic permeability when they experience a crystal deformation, resulting in changes in a given magnetic field.

Several alternative technologies based on this operating principle have been proposed, a number of which are very original. One option consists of a ring or collar, of locally magnetized steel alloy joined to the shafts of the machines. The material has magneto-crystalline orthogonal anisotropy, and its main directions coincide with the circumferential and axial shaft. The sensing element is based on the Hall's effect. While the collar is not deformed, magnetic flux can flow easily in the same circumferential direction. When this deformation occurs, the path of the lines of magnetic flux also changes, and thus its size, with some of them circulating around the outside of the collar, which is detected by the electronic module of the sensor.

2.3.3 *Elastic coupled systems*

These transducers are made of a seismic mass elastically coupled to a free extreme of the rotor assembly whose torsional vibration needs to be determined, Fig. 3. While the shaft's rotation speed is strictly constant, the transducer experiences no angular oscillation. Conversely, when the rotation speed is variable, an angular vibrational response is excited in the elastic system. This response is caused by the inertia of the seismic mass which, if periodic, will have the same frequency as the excitation. The shaft and the seismic mass may be connected by mechanical elements, such as springs or torsion bars, or through magnetic fields.

In the first case, the angular oscillation of the shaft produces a relative movement between this and the mass which causes the elastic deformation of the mechanical element. Such deformation is picked up by strain gauges or piezoelectric elements giving an output proportional to the angular acceleration. In the second case, the oscillations occur between a permanent magnet, which acts as a seismic mass, and a coil of ferromagnetic core integral with the shaft. In the absence of torsional vibration, both rotate together since the magnetic field of the magnet holds it stitched to the coil. However, due to the mass of the magnet, an angular shaft vibration will cause fluctuations in the magnetic circuit, that is, a relative movement between the magnet and the coil, which generates an electromotive force in the coil proportional to the speed of oscillation. Therefore, measurement systems, based on this type of

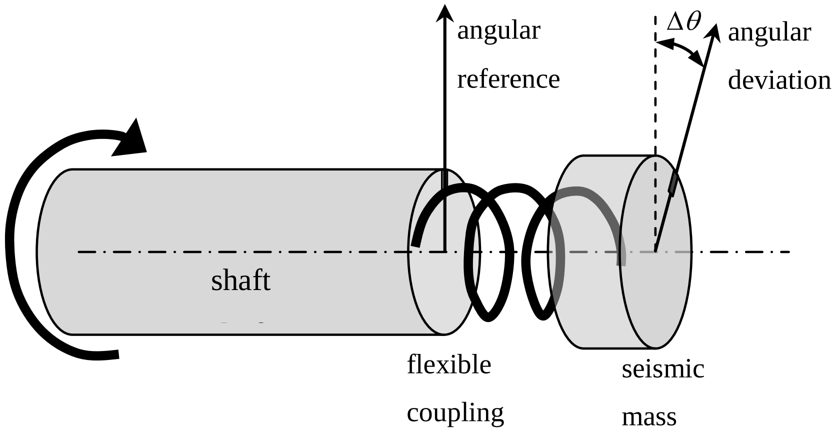

Figure 3: Elastic coupled system.

vibration transducers, measure the angular shaft from the angular vibration which is promoted by the seismic mass in the transducer, just as occurs with an accelerometer for measuring a linear vibration. Within the range of linearity of the transducer, it can be assumed that the vibrational response of the transducer is directly proportional to the angular shaft vibration. The non-linearity becomes more significant the closer the oscillation frequency is to the first resonant frequency of the transducer. The upper limit of the valid frequency range of the transducer, which is set by the manufacturer, is usually below 1/3 of its resonance frequency.

2.3.4 Electromagnetic systems

One type of transducer similar to the latter, in a slightly different operation, employs a solidary metal disc to the shaft, a stationary permanent magnet and a stationary coil as a sensing element. The disc, rigidly connected to the shaft to be measured, rotates close to a permanent magnet, Fig. 4, which generates Eddy currents in the disc. On the opposite side, a coil, which acts as a secondary circuit, maintains a constant value output current while the disc speed is constant, whereas it captures changes in these Eddy currents when there is torsional oscillation. The output current of this sensor is proportional to the angular acceleration of the shaft.

2.3.5 Encoders

2.3.5.1 Signal processing

Encoders used for torsional vibration measurement are based on the same principle as the non-contact method for measuring the speed of rotation of the rotor. In

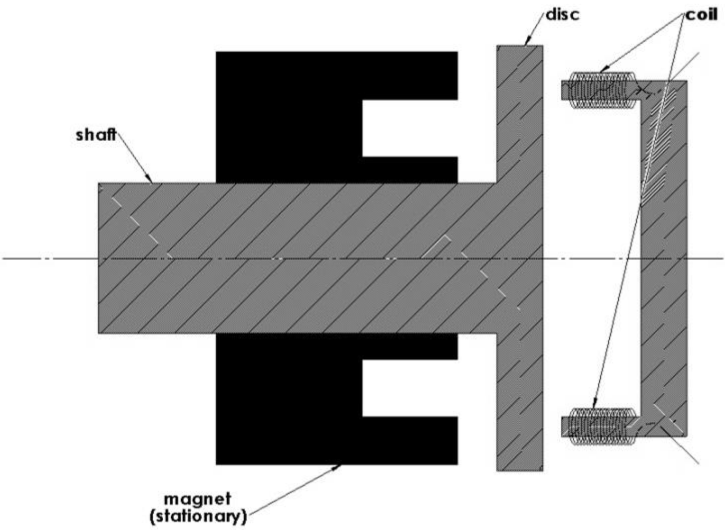

Figure 4: Electromagnetic transducer of angular vibration.

this case, a mark provided on the shaft is detected at every turn by a given sensor, located in a fixed position of the stator. This mark may be: an optical-type reflective tape or of an unidentifiable colour; mechanical, -a mechanical cleavage or tooth-; or an electrical discontinuity alternating insulated sectors with conducting sectors. The average speed is obtained as the inverse of the time between two consecutive steps of the mark time, named the period of a revolution.

Similarly, if m markings are arranged regularly distributed on the periphery of the shaft, or other rotor element of circular section, as many times as there are partial marks can be obtained. The inverse of the time between the step of any two marks will be the average angular velocity in the covered angular range between those marks.

$$\omega = \frac{\Delta\varphi}{\Delta t} = \frac{n}{\Delta t}\frac{2\pi}{m} \tag{1}$$

where ‘Δt’ is the time measured, ‘n’ is the number of marks that have gone against the sensor for the measurement of time and ‘$\Delta\varphi$’ is the angle between two consecutive marks.

The method described is called TIMS (Time Interval Measurement System). The time is usually measured by an electronic high frequency oscillator clock. The method consists on the one hand of counting the pulses, produced by the sensor to the step of a mark, and on the other of counting the pulses from the oscillator. Both counters start when a pulse arrives and stop after n-pulses. Since all the marks are identical and regularly arranged, they are incremental encoders. The angular position of the shaft can only be known, from a given starting position, by counting the number of marks passed. By contrast, in the absolute position encoders each mark is actually a complete code indicating a specific angular position of the rotor. As is clear from the above methodology, measuring the torsional vibration using incremental encoders is enough since they provide all the information necessary for the measurement. The TIMS method described is not the only possible method of measurement from the signal provided by the encoder. Since the angular vibration is a variation of the instantaneous velocity of the shaft about its mean value, the period between marks will change causing a frequency modulated signal. The average speed acts as a carrier signal while the angular vibration does the modulating signal. This is the fundamental principle of the Frequency Modulation Systems (FMS) [Meroño and Gómez de León (2005)].

Instruments used in this procedure perform frequency demodulation to obtain an analogical value of voltage proportional to the instantaneous rotating speed of the shaft, resulting in an alternating signal representing the angular vibration around the average rotating speed of the shaft. This signal can be composed of the same instrument to obtain the angular displacement, or be derived for the rotor angular acceleration. The output signal is modulated in both amplitude and frequency, if the electromagnetic encoder is used, so amplitude modulation can also be employed to measure torsional vibration. They are the Amplitude Modulation Systems (AMS). In these systems, an electronic procedure known as ‘envelope detection’ is employed in which, basically, the signal is rectified and then subjected to a low pass filter to, after various stages of electronic processing, produce a voltage proportional to the angular oscillation speed.

The accuracy of measurements made of instantaneous angular velocity by this system can be improved substantially if, instead of considering the set of pulses as a sequence of zeros and ones, it is regarded as an analogical signal that, from the transducer, an encoder, is taken to an ADC system which is embedded in a computer. Then, the sampled signal is processed to obtain a sequence of values of the angular velocity along the temporary segment acquired [Gómez de León, Meroño and Aguirre (2002)]. The precision and high resolution that can be achieved with this system are possible due to the high sampling frequency of existing A/D boards along with the potential for mass storage of information, which means that the number of samples of the signal acquired can easily reach hundreds of thousands.

The process to obtain the measure, which is denominated generically DTIMS (Discrete Time Interval Measurement System), is described below.

The measurement principle is based on capturing and processing the signal provided by the transducer, as if it were an analogical signal, in order to detect and measure the duration of each pulse. Since the geometry of the encoder is fixed, the angle that every pulse covers on the shaft is known beforehand. Consequently, once the time it takes for each pulse to pass before the sensor is obtained, the mean angular velocity on the shaft, during the angular interval that it occupies, can be determined. In this way, the method provides as many instantaneous values of the angular velocity as there are pulses available.

It should be noted that a full pulse consists of two distinct states of voltage. Since this technique for acquiring the signal can discern both states, an encoder with p pulses per revolution (ppr) can deliver 2p values of velocity in a revolution, that is, it provides twice as many values of angular velocity as pulses it has. Therefore, by using this technique we can double the resolution of the encoder, Fig. 5.

The analogical signal must be sampled at a frequency, f_s, sufficiently high to provide several samples, s, for each pulse of the encoder. The greater the number of samples per pulse is, the greater the accuracy that will be achieved in the measurement. The number, n, of samples needed to register a complete revolution of the shaft, using 2p states per revolution, will be:

$$n = 2\mathrm{p} \cdot \sum_{i=1}^{2\mathrm{p}} s_i \qquad (2)$$

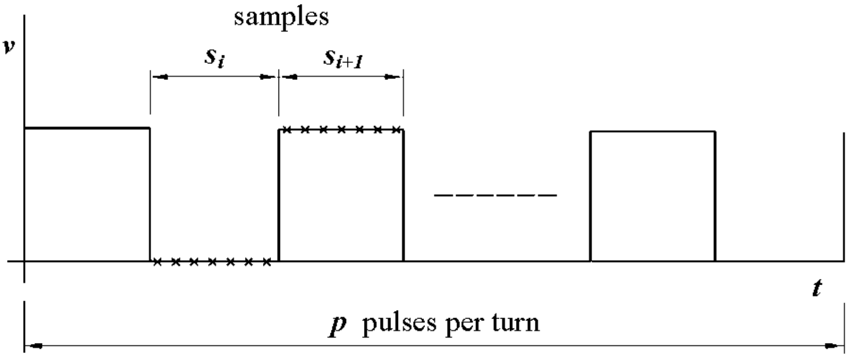

Figure 5: Signal sampling.

In this form, the angular velocity provided by measuring the time interval of the whole pulse is calculated by the following expression:

$$\omega = \frac{2\pi \cdot f_s}{p \cdot s} \qquad (3)$$

2.3.5.2 Types of encoders

According to their operating principles, the following types of incremental position encoders can be highlighted:

2.3.5.2.1 Electromagnetic

They consist of a ferromagnetic material gear joined to the shaft and a coil placed near the extreme of the teeth, Fig. 6. There are three basic constructions:

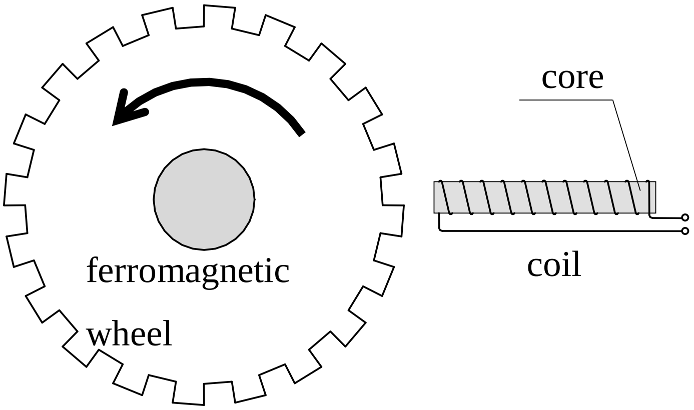

Figure 6: Electromagnetic encoder in a toothed wheel.

(1) The core of the coil is a permanent magnet. Each time a tooth passes against the magnet the reluctance of the medium is changed, which involves variations in the magnetic flux. These pulsating variations generate an electromotive force in the coil with the same pulsating character. These variations in the voltage output of the sensor, which can be measured directly with a voltmeter, are the trigger signal of the pulse counter.

(2) The core is ferromagnetic and the coil is fed by a sinusoidal radio frequency current. As a result of this current, a magnetic field is induced in the surrounding area. As the tooth approaches the sensor, Eddy currents are generated within the material, and therefore, energy is absorbed leading to an increase in the current absorbed by the coil. A signal conditioner measures the radiofrequency voltage envelope and provides a DC output signal proportional to the current envelope. Alternating between teeth and gaps produces a pulsed output signal.

(3) A third type, which can be considered as a combination of the above, is the toroidal core. In this case, the coil, fed by a high-frequency current, is wound on a split toroidal core, with the opening near the toothed wheel. A second coil acts as the secondary of a transformer. Each time a tooth is facing the core opening it

channels the magnetic flux, decreasing the reluctance of the medium and causing an increase in the current of the secondary coil.

The measuring method used is not that stated above, TIMS, but in this case the angular oscillations are measured by a frequency-voltage converter, according to any of the methods discussed below.

2.3.5.2.2 Electrical

In this case the marks are conductive metal areas, separated by insulating ones, generally photo-printed or metal traces discs. The main exponents are:

(1) Contact. The simplest construction is formed by an electric circuit with two brushes. One is in constant contact with a disc of conductive material, the other with the encoder disc. This second brush closes the electrical circuit whenever it makes contact with a track of conductive material and opens in the area of insulating material. Inaccuracies of the contact and gradual wear occur, so this type has become virtually obsolete.

(2) Capacitive. These sensors are based on the change in capacitance of an electrical circuit by alternating conductive and insulating strips of the encoder in its movement. Such capacity variations are transformed into voltage pulses acting on the pulse counter.

2.3.5.2.3 Optical

Despite being less robust than those described above, they are currently the most widely used, due to their accuracy and wide range of commercial uses. Three types can be distinguished:

(1) Opaque and transparent sectors. Their constitution is essentially as shown in Fig. 7. They consist of a disc fixed to the shaft having a number of windows through which a beam of light can pass. A second fixed disc with a single window, which avoids interference and channels the light beam, is positioned so that the light beam impinges on a photosensitive transducer which generally responds varying its electrical conductivity. An electronic circuit converts these current pulses variations. Since the size of a photodetector, based on semiconductor technology, can be extremely small, the number of pulses to be derived from these encoders can be one or two orders of magnitude higher than those of the electric or electromagnetic type.

(2) Interference fringes. When high frequency resolution is required in the measurement of the angular velocity, the number of marks that the transducers discussed thus far can offer is insufficient. It should be noted that the highest frequency –strictly speaking, the harmonic order of the average rotating speed- that could be analysed in the torsional vibration spectrum is limited by the number of encoder marks. In such circumstances, interference fringe encoders are used. They consist of two superposed discs: one is integral to the shaft while the other is fixed.

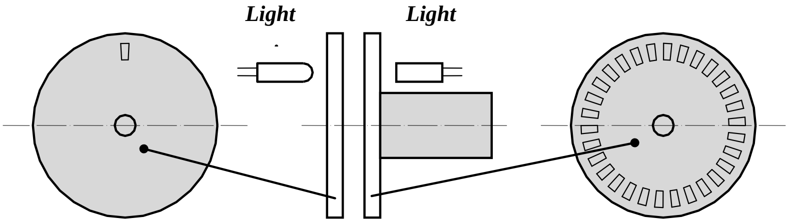

Figure 7: Optical incremental position encoder.

The number of windows of both discs can be the same, in which case the windows of one of the discs are inclined, or have a window apart. The relative movement of one window with respect to another causes the signal obtained in the photodetector to be a substantially sinusoidal function. By arranging some of these sinusoidal outputs out of phase from each other, it can be interpolated between them for much higher resolution [Pallas Areny (1998)], easily in the order of 104 pulses per revolution.

(3) Reflective and non-reflective areas. In such transducers, the light emitter and receiver are located on the same side of the marks, whose purpose in this case is to reflect light now. Depending on the characteristics of the light source (usually infrared LED or laser), but mainly the photodetector (LDR, phototransistor, etc.), the encoder will consist of reflective and non-reflective areas or simply alternating light and dark areas (usually black and white).

2.3.6 Laser interferometers

The use of laser technology for measuring speed has been applied since its discovery in the 1960s. LDV (Laser Doppler Velocimetry) was initially used for non-invasive measurements in the field of Fluid Mechanics. However, it was not until 1983, at the Institute of Sound and Vibration Research, University of Southampton, where this technique was applied to measure the torsional vibration, due to the development of the so-called cross-beam laser torsional vibrometer.

2.3.6.1 Crossed-beam laser vibrometer

This consists of a laser blaster and a system of lenses and mirrors through which the light beam is split into two beams incident on a point on the surface of the shaft, whose angular velocity is to be measured, forming between them a specific angle, ‘θ’, which can be seen in Fig. 8. The two reflected light beams are directed by mirrors to a dual photodetector, as shown in Fig. 8, where the signals are processed. Since the angle between the direction of incidence of each of the beams and the tangential velocity vector is different, the Doppler frequencies of the reflected beams will be different, allowing to obtain a measurement of the tangential velocity of the surface; even without knowing the angles of incidence, it is sufficient to know the angle between them, which is determined by the instrument optics.

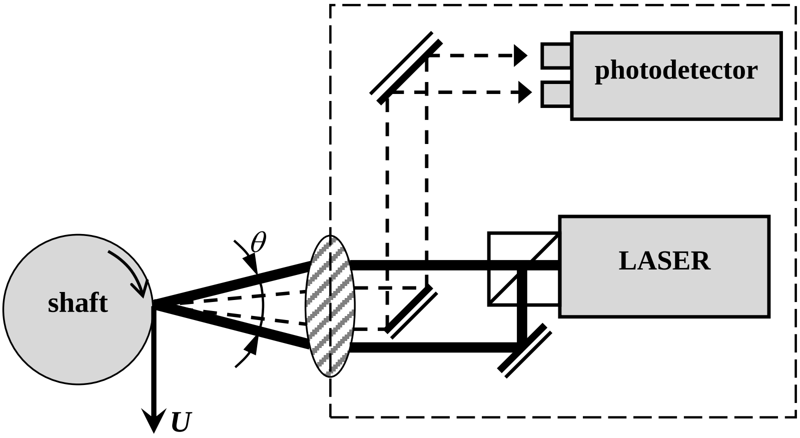

Figure 8: Crossed-beam laser vibrometer.

It can be shown that for the arrangement of the figure, the absolute difference between the frequencies of the incident light and that reflected called Doppler frequency is given by the expression:

$$f_D = \left(2 \cdot {}^{U}\!/_{\lambda}\right) \cdot \sin\left({}^{\theta}\!/_{2}\right) \tag{4}$$

where ‘U’ is the tangential velocity of the surface at the point of intersection of the beams; λ, the wavelength of the incident laser beam, and ‘θ’, the angle between the incident beams. Demodulating the Doppler signal, a voltage equivalent to the speed ‘U’ occurs; the alternating part is proportional to the speed of torsional oscillation. The instantaneous angular velocity of the rotor is given by the ratio of the tangential velocity, ‘U’, and the radius of the shaft in the measuring point. This system has some practical drawbacks. First, the intersection of the incident beams must occur at a point on the shaft surface, or very close to it, so said surface must have a circular cross section and the laser blaster should be arranged as closely as possible at a fixed distance from it. Secondly, any movement of the shaft in the tangential direction, not due to rotation, that is to say the radial vibration, introduces an error into the measurement because it is interpreted as angular oscillation.

2.3.6.2 Parallel-beam laser vibrometer

This technique appeared almost a decade after the above mentioned technique and it was possible to overcome the disadvantages that had arisen. Firstly, because the cross-sectional shape may be any and secondly, and more importantly from the practical standpoint, because the interference produced in the measurement by the radial vibration is prevented. As in the previous case, the laser beam is split into two beams of equal intensity and separated by a distance d, but, unlike what happened before, now the two light beams are incident in parallel on the surface of the shaft, Fig. 9.

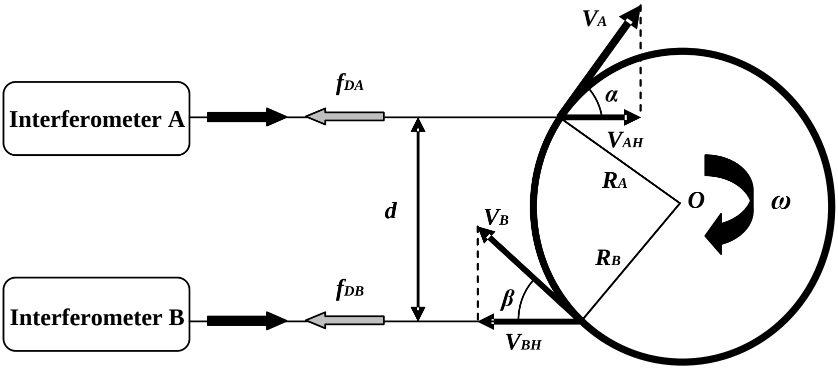

Figure 9: Parallel-beam laser interferometry.

When the plane determined by the two laser beams is perpendicular to the axial direction, it is possible to determine the angular velocity of solid –shaft- by measuring the velocity components of two points of the same in the direction of incidence of the laser.

Applying vector expressions:

$$\begin{aligned} V_{AH} &= V_A \cdot \cos\alpha = \omega \cdot R_A \cdot \cos\alpha \\ V_{BH} &= V_B \cdot \cos\beta = \omega \cdot R_B \cdot \cos\beta \end{aligned} \tag{5}$$

where:

- ‘V_{AH}’ and ‘V_{BH}’ are the velocity components in the directions of the rays impinging.
- ‘R_A’ and ‘R_B’ are the turning radius of the rays impinging points A and B, respectively.
- **‘ω’** is the angular velocity in rad/s.

As it can be known, the velocity components in the ray direction give the Doppler frequencies, 'f_{DA}' and 'f_{DB}', in the reflected rays:

$$\begin{aligned} f_{DA} &= 2 \cdot V_A/\lambda = 1 \cdot \omega \cdot R_A \cdot \cos\alpha/\lambda \\ f_{DB} &= 2 \cdot V_B/\lambda = 1 \cdot \omega \cdot R_B \cdot \cos\alpha/\lambda \end{aligned} \tag{6}$$

And the distance between rays is:

$$d = R_A \cdot \cos\alpha + R_B \cdot \cos\beta \tag{7}$$

And then:

$$f_D = f_{DA} + f_{Db} = 2 \cdot d \cdot \omega/\lambda \tag{8}$$

The Doppler frequency depends on 'λ', 'd' and 'ω', where the value of the first two parameters are known, and then the value of 'ω' is obtained without relation neither the shape nor the transversal displacements of the shaft.

In this case, it can be shown that it is possible to determine the angular velocity of the solid –the shaft- by measuring the velocity components of two points of that shaft in the direction of incidence of the laser. The expression obtained for the Doppler frequency is, in revolutions/second:

$$f_D = \left({}^{4\pi d}/_{\lambda} \right) \cdot n \tag{9}$$

Proceeding as set out above, i.e. demodulating this frequency, a voltage output proportional to the shaft rotating speed is obtained; the fluctuating part will be the angular oscillation. The particular geometry of the system used makes it insensitive to movement of the shaft to be measured, as well as to a possible movement of the instrument, as to be parallel to the incident beams; any movement of the shaft produces an identical change in the Doppler frequency of both. In addition, the change in frequency does not depend on the distance to the rotation shaft and therefore is independent of the shape of the cross section. This allows the system to be used successfully also on gears or any section which is not strictly circular. This technique has definitively replaced the cross beam laser for measuring torsional vibration.

3 Conclusions

The fact that torsional vibration causes a great deal of problems and failures in certain rotating equipment is well known. While just over a decade ago considering the torsional vibration measurement supposed facing cumbersome equipment modifications to couple the appropriate sensors, today there are enough alternatives on the market to address the problem much more easily, thus obtaining more accurate measurements and at a lower cost. In practice, non-contact transducers have completely displaced contact transducers. The best technique that is easily portable and can be quickly installed, without the need for any intervention on the machine, laser interferometry, still supposes a high cost. It also requires a portion of the shaft to be visible so a light laser blaster can be projected. The development of electronics has also led to a considerable increase in the use of incremental encoders, especially of the optical type. The high clock frequencies and computing speeds, along with increasing market supply, mean that torsional vibration can be measured with such a range or even higher resolution than with laser interferometry. Electromagnetic encoders, but generally with less resolution, also have their place due to their great robustness, which also applies to the torque sensors.

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