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Prediction of Fracture Parameters of High Strength and Ultra-High Strength Concrete Beams using Minimax Probability Machine Regression and Extreme Learning Machine

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Abstract: This paper deals with the development of models for prediction of facture parameters, namely, fracture energy and ultimate load of high strength and ultra high strength concrete based on Minimax Probability Machine Regression (MPMR) and Extreme Learning Machine (ELM). MPMR is developed based on Minimax Probability Machine Classification (MPMC). ELM is the modified version of Single Hidden Layer Feed Foreword Network (SLFN). MPMR and ELM has been used as regression techniques. Mathematical models have been developed in the form of relation between several input variables such as beam dimensions, water cement ratio, compressive strength, split tensile strength, notch depth, and modulus of elasticity and output is fracture energy and ultimate load A total of 87 data sets (input-output pairs) are used, 61 of which are used to train the model and 26 are used to test the models. The data-sets used in this study are derived from experimental results. A comparative study has been presented between the developed MPMR and ELM models. The results showed that the developed models give reasonable performance for prediction of fracture energy and ultimate load.

Keywords: High strength concrete, Ultra high strength concrete, Minimax Probability Machine Regression, Extreme Learning Machine, Fracture energy, Ultimate load.

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1 Introduction

A highly developed infrastructure is an important part of any country's growth and prosperity. It is important to study Reinforced Concrete Structures (RCS) to analyze and develop better methods and materials which are more resilient and durable. These Reinforced Concrete Structures have suffered many failures due to the combined effect of de-icing, alternate expansion and contraction, freeze-thaw cycles, creep and shrinkage failures, heavy live load impacts and harsh, aggressive environments. Consequently civil engineers today are facing immense problems when it comes to preserving, maintaining and retrofitting these structures. Traditionally normal strength concrete (NSC) was used for building structures. Thus to build complicated infrastructure edifices such as high rise buildings and long span cable stayed bridges, among other humungous development projects, it became necessary to develop High Strength Concrete (HSC) with compressive strength of 50 MPa or higher. The easiest way to achieve such high compressive strength was to decrease the water-cement ratio. The use of appropriate additives and admixtures was encouraged to develop such high strengths along with other characteristics. Thus HSC is considered as green high performance concrete (GHPC). Nowadays ultra-high strength concrete (UHSC) is also used, with axial compression of above 140 MPa. Ultra-High Performance Concrete (UHPC) is a high-strength, ductile material formulated by combining Portland Cement, silica fume, quartz flour, fine silica sand, high-range water reducer, water, and steel or organic fibers. The material provides compressive strengths up to 29,000 pounds per square inch (psi) and flexural strengths up to 7,000 psiThis was successfully developed by Richard and Cheyrezy (1994, 1995), Mingzhe et al. (2010).

Concrete being a quasi-brittle materials exhibit a nonlinear region before the peak of the stress–strain relationship and substantial post-peak strain softening. Linear elastic fracture mechanics cannot be applied directly to the quasi-brittle materials [Bazant, 2000]. Due to high heterogeneity nature in concrete, cracks follow the weakest matrix links in the material. They lead their way through the weak bonds, voids, mortar and get arrested on encountering a hard aggregate, forming crack face bridges. Micro cracking, crack bridging and aggregate interlocking are a few of many specific mechanisms that absorb energy during fracture process. These mechanisms contribute to the tendency of the main crack to follow a tortuous path [Bazant (2000); Barenblatt (1959); Dugdale (1960)]. This tortuous nature of the crack causes difficulty in computing the fracture energy. Therefore, modeling the exact nature of the fracture surface poses a new challenge to the researchers. In these days, most theoretical works in fracture mechanics are based on the fundamental assumption that cracks have smooth surfaces. This assumption is helpful to use analytical models in the field of fracture mechanics.

Over the past few years, researchers have used different statistical modelling methods such as Artificial Neural Network, Support Vector Regression, Multivariate Adaptive Regression Splines and Relevance Vector Machine for prediction of fracture characteristics of concrete. Yuvarajet al. (2013) used Support vector regression (2013), Artificial Neural Network (2012) and Multivariate Adaptive Regression Splines (2013) to predict the fracture characteristics of concrete beams. Though the performance of ANN is acceptable, its results are hard to interpret. Support vector machines do not directly provide probability estimates and in the case of MARS, parameter confidence intervals and other checks on the model cannot be calculated directly, unlike linear regression models.

This article examines the applicability of Minimax Probability Machine Regression (MPMR) and Extreme Learning Machine (ELM) for prediction of fracture energy and ultimate load. MPMR is a new model based on Minimax Probability Machine Classification (MPMC). There are several applications of MPMR in different domains [Utkin et al. (2012); Takeda et al. (2013); Yang and Ju (2014)]. ELM is developed based on the concept of single hidden layer forward network [Huang et al. (2006)]. Researchers have successfully applied ELM for solving different problems in engineering [Jiang et al. (2012); Li et al. (2013); Du et al. (2014)].

2 Minimax Probability Machine Regression

This section will serve the details of MPMR. It is developed by constructing dichotomy classifier [Strohmann and Grudic (2002)]. The relation between input(x) and output(y) is given below.

$$y = \sum_{i=1}^{N} \beta_i K(x_i, x) + b \tag{1}$$

Where N is the number of datasets, K(x_i,x) is kernel function, β_i and b is the output of MPMR. In this, beam dimensions, fck, split tensile strength, notch depth and Young's modulus have been used as inputs of the MPMR. The output of MPMR is fracture energy (G_F) and ultimate load ($P_{\max}$).

The total datasets will be divided into the following two classes.

$$u_i = (y_i + \varepsilon, x_{i1}, x_{i2}, ..., x_{in}) \tag{2}$$

$$v_i = (y_i - \varepsilon, x_{i1}, x_{i2}, ..., x_{in}) \tag{3}$$

The classification boundary between u_i and v_i is regression surface.

For developing MPMR, the dataset have been divided into the following two groups:

Training Dataset: This is adopted to develop the MPMR model. This article uses 61 dataset as training dataset.

Testing Dataset: This is used to verify the developed MPMR. The remaining 26 dataset has been used as testing dataset.

The dataset is normalized between 0 and 1. Radial basis function has been adopted as kernel function. The program of MPMR has been constructed by using MATLAB.

3 Extreme Learning Machine

The basic concept of ELM has been taken from single hidden layer forward network (SLFN) [Huang et al. (2012)]. In SLFN, the relation between input(x) and output(y) is given below:

$$\sum_{i=1}^{L} \beta_i g_i(x_j) = \sum_{i=1}^{L} \beta_i G(a_i, b_i, x_j) = y_j \tag{4}$$

Where L is the number of hidden layers, g denotes the non-linear activation function and β_i is weight.

The above equation (4) can be written in the following way.

$$H\beta = T \tag{5}$$

Where

$$H = \begin{bmatrix} G(a_1, b_1, x_1) & \cdots & G(a_L, b_{L,} x_1) \\ \vdots & \ddots & \vdots \\ G(a_1, b_1, x_N) & \cdots & G(a_L, b_L, x_N) \end{bmatrix}_{N\times L},$$

$$\beta = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{bmatrix} \text{and} T = \begin{bmatrix} y_1^T \\ \vdots \\ y_N^T \end{bmatrix}_{N\times m}.$$

The value of β is determined from the following equation

$$\beta = H^{-1} T \tag{6}$$

where H^{-1} is the Moore–Penrose generalized inverse of hidden layer output matrix.

ELM employs the same training dataset, testing dataset and normalization technique as used by the MPMR model. Radial basis function has been used as activation function for developing the ELM model. The program of ELM has been constructed by using MATLAB.

4 Development of MPMR & ELM Models

Out of the 87 data sets which are available, 61 datasets (Table 1) are used to train the models and 26 datasets (Table 2) are used to test the accuracy of the models. Tables1 and 2show the training and testing data-sets respectively. The data was normalized between 0 and 1 before being used in the model as following:

$$\mathbf{D}_{\text{norm}} = \frac{\mathbf{D} - \mathbf{D}_{\text{min}}}{\mathbf{D}_{\text{max}} - \mathbf{D}_{\text{min}}} \tag{7}$$

The assessment of the model is done on the basis of coefficient of regression value R which is calculated using the formula:

$$\mathbf{R} = \frac{\sum_{i=1}^{n} (E_{ai} - \bar{E}_a)(E_{pi} - E_p)}{\sqrt{\sum_{i=1}^{n} (E_{ai} - \bar{E}_a)}\sqrt{\sum_{i=1}^{n} (E_{pi} - \bar{E}_p)}} \tag{8}$$

where E_{ai} and E_{pi} are the actual and predicted values, respectively, $\bar{E}_a$ and $\bar{E}_p$ are mean of actual and predicted E values. For an effective and good model the R value should be close to one. Also while comparing the models the values of R is compared and the model with R value closer to one and higher than the other is considered better and used.

5 Results and discussion

The present study uses Coefficient of Correlation(R) to asses the performance of the developed MPMR & ELM. For a good model, the value of R should be close to one. For developing MPMR, the design values of σ and ε have been determined by trial and error approach. The developed MPMR gives best performance at P1 = 0.15 and ε= 0.07 for prediction of P_{max}. Figure 1 depicts the performance of MPMR for prediction of P_{max}. It is observed from figure 1 and Table 3 that the value of R is close to 1 for training as well as testing datasets. For prediction of G_F, the design values of P1 and ε are 0.07& 0.05 respectively. Figure 2 illustrates the performance of MPMR for prediction of G_F. As shown in figure 2 and Table 3, the value of R is close to one for training as well as testing datasets. So, the developed proves his ability for prediction of P_{max} and G_F.

The performance of ELM depends on the number of hidden nodes. The design number of hidden nodes is determined by trail and error approach. For prediction of P_{max}, the developed ELM gives best performance for 10 hidden nodes. Figure 3 depicts the performance of ELM for prediction of P_{max}. The developed ELM gives best performance at 9 hidden nodes For prediction of G_F. Figure 4 illustrates the performance of ELM for prediction of G_F. As shown in figures 3 and 4 and Table 3, the value of R is close to one.

Table 1: Training data-sets.

S. No	L (mm)	A (cm^2)	a_0 (mm)	w/c	f_{ck} (MPa)	σ_t (MPa)	E (GPa)	P_{max} (KN)	G_F (N/m)
1	250	25	5	0.45	57.14	3.96	35.78	2.71	115.84
2	250	25	4	0.45	57.14	3.96	35.78	2.62	123.31
3	250	25	10	0.45	57.14	3.96	35.78	1.98	91.12
4	250	25	9	0.45	57.14	3.96	35.78	1.98	86.65
5	250	25	10	0.45	57.14	3.96	35.78	1.84	74.32
6	250	25	16	0.45	57.14	3.96	35.78	1.14	55.18
7	250	25	15	0.45	57.14	3.96	35.78	1.42	68.61
8	500	50	9	0.45	57.14	3.96	35.78	4.53	144.02
9	500	50	10	0.45	57.14	3.96	35.78	4.10	130.26
10	500	50	18	0.45	57.14	3.96	35.78	3.79	92.72
11	500	50	19	0.45	57.14	3.96	35.78	3.63	115.42
12	500	50	28	0.45	57.14	3.96	35.78	2.58	89.12
13	1000	100	19	0.45	57.14	3.96	35.78	7.27	165.25
14	1000	100	19	0.45	57.14	3.96	35.78	7.32	146.28
15	1000	100	19	0.45	57.14	3.96	35.78	6.99	148.25
16	1000	100	39	0.45	57.14	3.96	35.78	6.01	135.85
17	1000	100	39	0.45	57.14	3.96	35.78	6.32	140.56
18	1000	100	58	0.45	57.14	3.96	35.78	4.54	115.12
19	1000	100	60	0.45	57.14	3.96	35.78	4.70	104.22
20	250	25	5	0.33	87.71	15.38	37.89	4.20	4157.28
21	250	25	5	0.33	87.71	15.38	37.89	4.15	4102.2
22	250	25	10	0.33	87.71	15.38	37.89	3.37	3464.6
23	250	25	10	0.33	87.71	15.38	37.89	3.26	3880.1
24	250	25	15	0.33	87.71	15.38	37.89	2.79	3301.2
25	250	25	15	0.33	87.71	15.38	37.89	2.88	3410
26	250	25	20	0.33	87.71	15.38	37.89	1.98	2892.06
27	250	25	20	0.33	87.71	15.38	37.89	2.05	2988.52
28	500	50	10	0.33	87.71	15.38	37.89	8.35	4811
29	500	50	10	0.33	87.71	15.38	37.89	8.20	4200.1
30	500	50	20	0.33	87.71	15.38	37.89	5.10	4516.1

S. No	L (mm)	A (cm^2)	a_0 (mm)	w/c	f_{ck} (MPa)	σ_t (MPa)	E (GPa)	P_{max} (KN)	G_F (N/m)
31	500	50	20	0.33	87.71	15.38	37.89	4.99	4266.5
32	500	50	20	0.33	87.71	15.38	37.89	5.07	3828.57
33	500	50	30	0.33	87.71	15.38	37.89	3.80	3579.89
34	500	50	30	0.33	87.71	15.38	37.89	3.79	3865.2
35	500	50	40	0.33	87.71	15.38	37.89	2.99	3970.95
36	500	50	40	0.33	87.71	15.38	37.89	3.08	3406.67
37	250	25	4	0.23	122.52	20.65	42.987	9.99	10349.24
38	250	25	5	0.23	122.52	20.65	42.987	10.01	10376.22
39	250	25	10	0.23	122.52	20.65	42.987	7.81	8308.49
40	250	25	9	0.23	122.52	20.65	42.987	7.43	7900
41	250	25	15	0.23	122.52	20.65	42.987	6.20	6925.54
42	250	25	15	0.23	122.52	20.65	42.987	5.99	6694.51
43	250	25	20	0.23	122.52	20.65	42.987	4.07	4386.6
44	250	25	19	0.23	122.52	20.65	42.987	3.99	4306.29
45	250	25	20	0.23	122.52	20.65	42.987	4.18	4511.36
46	400	40	9	0.23	122.52	20.65	42.987	14.23	11557.07
47	400	40	8	0.23	122.52	20.65	42.987	13.98	11354.02
48	400	40	16	0.23	122.52	20.65	42.987	10.85	8888.75
49	400	40	15	0.23	122.52	20.65	42.987	10.62	8700.84
50	400	40	25	0.23	122.52	20.65	42.987	7.58	7145.19
51	400	40	24	0.23	122.52	20.65	42.987	7.61	7171.63
52	400	40	32	0.23	122.52	20.65	42.987	5.56	5021.25
53	400	40	31	0.23	122.52	20.65	42.987	5.60	5058.14
54	650	65	13	0.23	122.52	20.65	42.987	19.49	12052.38
55	650	65	12	0.23	122.52	20.65	42.987	19.31	11944.13
56	650	65	25	0.23	122.52	20.65	42.987	13.37	8076
57	650	65	25	0.23	122.52	20.65	42.987	13.51	8892.69
58	650	65	39	0.23	122.52	20.65	42.987	10.12	6965.9
59	650	65	39	0.23	122.52	20.65	42.987	10.30	7085.13
60	650	65	52	0.23	122.52	20.65	42.987	7.46	5919.23
61	650	65	52	0.23	122.52	20.65	42.987	7.69	6109.05

Table 2: Testing data-sets.

S. No	L (mm)	A (cm^2)	a_0 (mm)	w/c	f_{ck} (MPa)	σ_t (MPa)	E (GPa)	P_{max} (kN)	G_F (N/m)
1	250	25	4	0.45	57.14	3.96	35.78	2.412	114.9
2	250	25	17	0.45	57.14	3.96	35.78	1.321	47.4
3	500	50	29	0.45	57.14	3.96	35.78	2.575	96.2
4	500	50	28	0.45	57.14	3.96	35.78	2.321	100.3
5	500	50	10	0.33	87.71	15.38	37.89	8.102	4142.2
6	1000	100	40	0.45	57.14	3.96	35.78	6.278	110.2
7	500	50	10	0.45	57.14	3.96	35.78	4.312	137.0
8	250	25	10	0.33	87.71	15.38	37.89	3.121	3763.1
9	650	65	51	0.23	122.52	20.65	42.987	7.312	5806.5
10	500	50	30	0.33	87.71	15.38	37.89	3.991	4623.5
11	250	25	9	0.23	122.52	20.65	42.987	7.667	8155.0
12	250	25	14	0.23	122.52	20.65	42.987	6.128	6844.0
13	400	40	8	0.23	122.52	20.65	42.987	14.08	11435.2
14	400	40	16	0.23	122.52	20.65	42.987	10.514	8613.2
15	650	65	24	0.23	122.52	20.65	42.987	13.498	8155.1
16	650	65	13	0.23	122.52	20.65	42.987	19.126	11829.1
17	650	65	39	0.23	122.52	20.65	42.987	10.013	6889.1
18	250	25	5	0.23	122.52	20.65	42.987	10.136	10504.7
19	250	25	20	0.33	87.71	15.38	37.89	2.102	2894.0
20	400	40	31	0.23	122.52	20.65	42.987	5.312	4797.2
21	250	25	5	0.33	87.71	15.38	37.89	4.101	4056.4
22	500	50	40	0.33	87.71	15.38	37.89	3.194	2897.9
23	1000	100	58	0.45	57.14	3.96	35.78	4.412	111.9
24	400	40	25	0.23	122.52	20.65	42.987	7.31	6887.1
25	500	50	18	0.45	57.14	3.96	35.78	3.87	105.3
26	250	25	15	0.33	87.71	15.38	37.89	2.841	3685.1

Note: L-length, A-c/s area, a_0-Notch depth, w/c-Water-cementations material ratio, f_{ck}-compressive strength, σ_t-Split tensile strength, E-modulus of elasticity, P_{max}-Ultimate load, G_F-Fracture energy.

Table 3: Values of R for training and testing.

	MPMR		ELM	
	P_{max}	G_F	P_{max}	G_F
Rtrain	0.9993	0.99929	0.938	0.983
Rtest	0.9988	0.9953	0.932	0.983

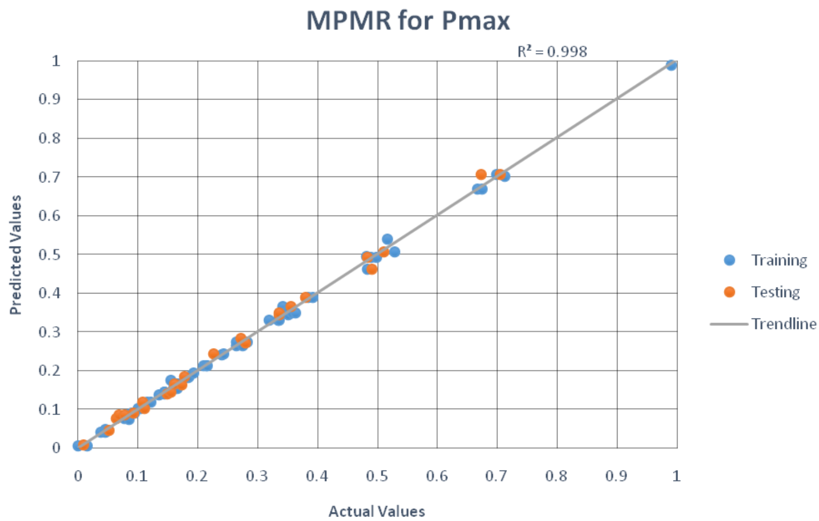

Figure 1: Comparison of predicted ultimate load - MPMR.

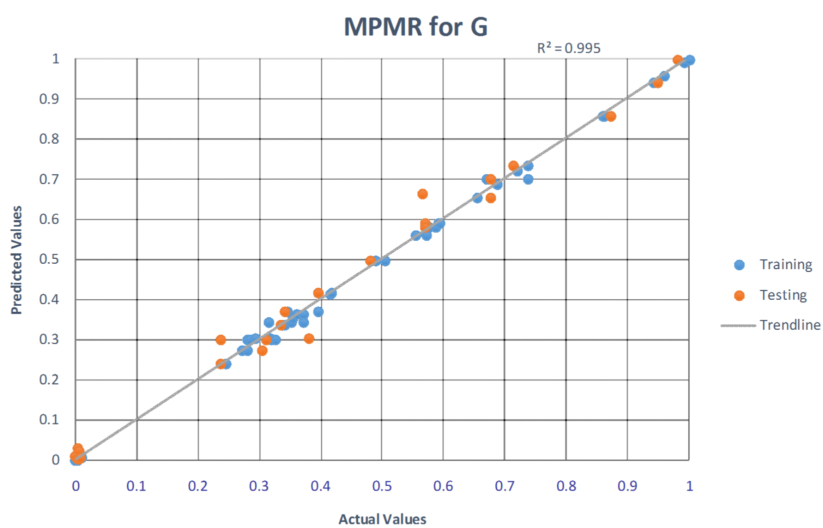

Figure 2: Comparison of predicted fracture energy – MPMR.

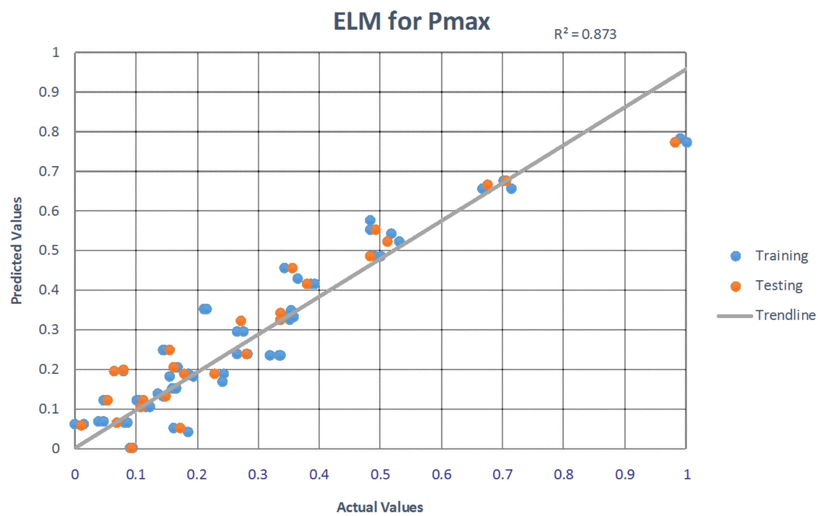

Figure 3: Comparison of predicted ultimate load - EML.

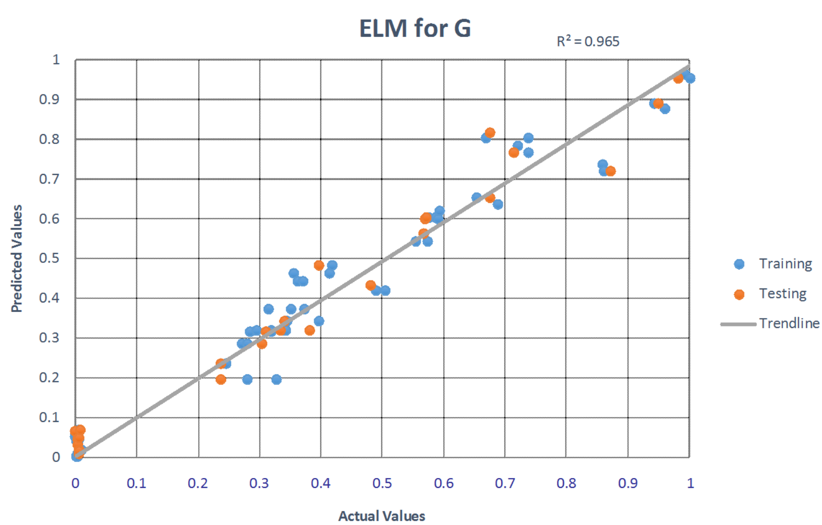

Figure 4: Comparison of predicted fracture energy - EML.

6 Conclusions

This study describes two alternative methods based of MPMR and ELM for prediction of facture energy and ultimate load. The methodology of MPMR and ELM has been described. Two types of dataset have been utilized to construct the MPMR and ELM models. The performance of MPMR and ELM is encouraging. Researchers can use the developed models as quick tools for prediction of facture energy and ultimate load. The developed models can be employed to solve different problems in structural engineering.

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