

Normal Stresses in an Infinite Elastic Body with a Locally Curved and Hollow Nanofiber

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Abstract: In the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrically nonlinear exact equations of the theory of elasticity, the method developed for the determination of the stress distribution in the nanocomposites with unidirectional locally curved and hollow nanofibers is used to investigate the normal stresses acting along the nanofibers. Furthermore, it is assumed that the body is loaded at infinity by uniformly distributed normal forces which act along the nanofibers and the crosssection of the nanofibers and normal to its axial line, is a circle of constant radius along the entire nanofiber length. For the solution of considered boundary value problem, an approximate analytical method is developed by using the boundary form perturbation method. The numerical results related to stress distribution in considered body and the influence of geometrical nonlinearity to this distribution are presented and interpreted.

Keywords: Nanocomposites, nanofiber, stress distribution, geometrical nonlinearity, local curvature.

1 Introduction

As it had been noted in Tarnopolsky, Yu, Jigun and Polyakov (1987), Guz (1990), Kelly (1998), Akbarov and Guz (2000), Guz (2003), one of the basic reasons through which the strength of the unidirectional fibrous composites under loading along the fibers is determined principally, is the curvature of the fibers. According to the curving form, these curvatures can be classified as periodically or locally. The effective practical applications of the unidirectional fibrous composite materials, under service conditions, require intensive, systematic investigations to determine the stress deformed state in these materials, taking the curving of reinforcing fibers into account. For this purpose, in Akbarov and Guz (1985) within the framework of the piecewise homogeneous body model by the use of the three-

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dimensional exact equations of the theory of elasticity, the method for investigation of the mentioned stress state in the unidirectional composites was presented. This method is employed in a case where the curving of the fibers is periodic. The reviews of the results obtained by this method are detailed in Akbarov and Guz (2002).

Note that the method used in Akbarov and Guz (1985) is presented for the case where the concentration of the fibers in the composite is small enough to neglect the interactions between them. In Kosker and Akbarov (2003), the method used in Akbarov and Guz (1985) has been developed for two neighbouring periodically curved fibers and some corresponding numerical results have also been presented. In Akbarov and Kosker (2003), the method in Akbarov and Guz (1985), Akbarov and Guz (2002), Kosker and Akbarov (2003) has been developed for the geometrical nonlinear statement and numerical results for one and two neighbouring periodically curved fibers have been given.

That is all as far as the above mentioned periodical curving case is concerned. There are also few investigations, such as Djafarova (1992), Djafarova (1994), Djafarova (1995), which regard the local curving case. But these investigations have been carried out in the case where the concentration of the fibers is small enough to ignore any interaction between the fibers. Moreover, it is by the use of the linear theory of elasticity that these investigations have been carried out. According to the well-known mechanical considerations and to the results obtained in Akbarov and Kosker (2003) taking the geometrical nonlinearity into account influences significantly the values of the self-balanced stresses caused by fibers' curving.

In Kosker and Simsek (2006), Kosker and Simsek (2007), the normal stresses are analysed. The Method in Akbarov, Kosker and Simsek (2005) has been developed in Kosker and Simsek (2006) in such way as to obtain normal stress values up to the second approximation in the fiber matrix interface. In Kosker and Simsek (2007), the method has been developed for the determination of the stress distribution in the unidirectional fibrous composites with locally curved fibers is used to investigate normal stresses acting along the fibers for the case where there exists the bond covering cylinder with constant thickness between fiber and matrix materials are considered.

Alan and Akbarov (2010) developed the method for the determination of the stress distribution in the composites with unidirectional locally curved covered fibers is used for investigation of the shear stresses acting along the fibers.

The changeover from the microlevel to the nanolevel in the science of materials broke new ground in the mechanics of materials and structures. This was preceded by important events in the physics and chemistry of nanoformations, which were

partially described in Guz (2006) and in Guz, Rushchitsky, Guz (2007), a basic approach to study the mechanical properties of nanocomposite materials with polymer matrix was proposed and the deformation of nanocomposites and structural members made of them. In Alan and Akbarov (2011) the normal stresses acting along the nanofiber in the nanocomposites with unidirectional locally curved covered nanofibers are considered.

Nanofibers are defined as fibers with diameters on the order of 100 nanometers. In this study, within the framework of the piecewise homogeneous body model, with the use of the three dimensional geometrically nonlinear exact equations of the theory of elasticity, the method developed for the determination of the stress distribution in the nanocomposites with unidirectional locally curved and hollow nanofibers is used for investigation of the normal stresses acting along the nanofibers. The case is considered where a single locally curved and hollow nanofiber of infinite length is located in an infinite elastic body with a low concentration of nanofibers. The interaction between the nanofibers is neglected. Furthermore, it is assumed that the body is loaded at infinity by uniformly distributed normal forces which act along the nanofibers and the crosssection of the nanofibers and normal to its axial line, is a circle of constant radius along the entire fiber length. The numerous numerical results related to stress distribution in considered body and influence of geometrical nonlinearity to this distribution are obtained and interpreted.

2 Formulation of The problem

We consider an infinite body containing a single locally curved and hollow nanofiber with an initial local imperfection, which is illustrated schematically in Fig.1. The values related to the nanofiber are denoted by superscript (2) and those related to the matrix by superscript (1). With the middle line of the nanofiber, we associate Lagrangian rectilinear $Ox_1x_2x_3$ and cylindrical $Ox_1\theta z$ system of coordinates (Fig.1). We assume that the material of the nanofiber and the matrix are homogeneous, isotropic and linear elastic. The body is compressed or stretched at infinity by uniformly distributed normal forces with intensity p acting along the Ox_3 axis and the crosssection of the nanofiber normal to its axial line are formed by two circles of constant radius R_1 and R_2 along the entire length. Under this situation which has no motion, for the nanofiber and matrix, in the framework of the piecewise homogeneous body model with use of the three-dimensional geometrically nonlinear exact equations of the theory of elasticity of continuum mechanics, we write the equilibrium equations ;

$$\nabla_i \left[\sigma^{(k)in} \left(g_n^j + \nabla_n u^{(k)j} \right) \right] = 0, \quad k = 1, 2 \quad (1)$$

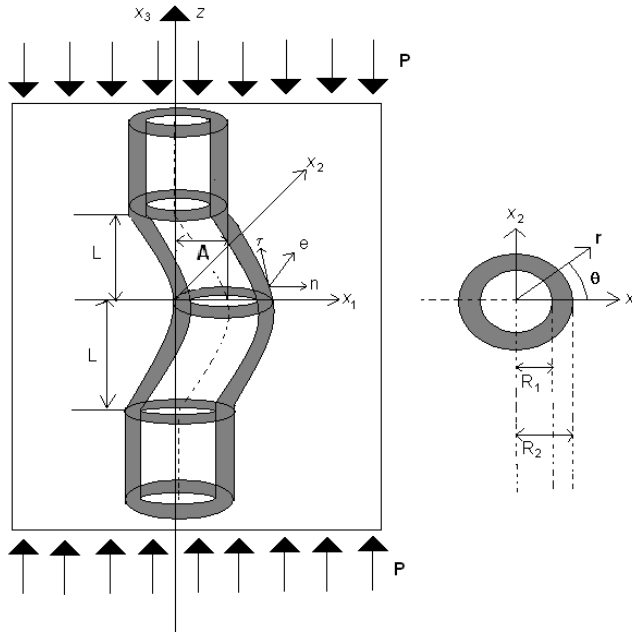


Figure 1: The geometry of material structure and chosen coordinates.

The strain-displacement relations as follows:

$$2\varepsilon_{jm}^{(k)} = \nabla_j u_m^{(k)} + \nabla_m u_j^{(k)} + \nabla_j u^{(k)n} \nabla_m u_n^{(k)}, \quad (2)$$

In Eqs.(1) and (2), the conventional tensor notation is used. $\sigma_{(in)}^{(k)}$ denote the k. constant of the physical components of the stress tensors and $\varepsilon_{jm}^{(k)}$ denote the k. constant of the physical components of the strain tensors. For the detailed explanation and formulation of the equations (1), (2), and any undefined terminology and notation ,we refer to Akbarov and Guz (2000).

The constitutive equations (Hooke's Law) are as follows:

$$\sigma_{(in)}^{(k)} = \lambda^{(k)} e^{(k)} \delta_i^n + 2\mu^{(k)} \varepsilon_{(in)}^{(k)}, \quad e^{(k)} = \varepsilon_{(11)}^{(k)} + \varepsilon_{(22)}^{(k)} + \varepsilon_{(33)}^{(k)} \quad (3)$$

Where λ and μ the material constants.

Also, perfect contact conditions are assumed at the interfaces S_1 and S_2 :

$$\sigma^{(1)in} \left(g_n^j + \nabla_n u^{(1)j} \right) \Big|_{s_1} n_j = 0$$

$$\sigma^{(1)in} \left(g_n^j + \nabla_n u^{(1)j} \right) \Big|_{s_2} n_j = \sigma^{(2)in} \left(g_n^j + \nabla_n u^{(1)j} \right) \Big|_{s_2} n_j, \quad u^{(1)j} \Big|_{s_2} = u^{(2)j} \Big|_{s_2} \quad (4)$$

where n_j are the covariant components of the unit normal vector to the surfaces S_1 and S_2 . We assume that the materials of the matrix and nanofiber are comparatively rigid and therefore the nonlinear terms can be neglected in the equations obtained for the zeroth approximation and the term $(g_n^j + \nabla_n u^{(k)j,0})$ can be replaced by the Kronecker symbol δ_n^j in the first and subsequent approximations.

For the analyzed case, the conditions are also assumed.

$$\sigma_{zz}^{(1)} \xrightarrow{r \rightarrow \infty} p, \quad \sigma_{ij}^{(1)} \xrightarrow{r \rightarrow \infty} 0 \quad (ij) \neq zz, \quad (5)$$

It is known that

$$\sigma_{(ij)} = \sigma^{ij} H_i H_j = \sigma^{ij} \frac{1}{H_i H_j}, \quad \varepsilon_{(ij)} = \varepsilon_{ij} \frac{1}{H_i H_j} = \varepsilon^{ij} H_i H_j, \quad u_{(i)} = u^i H_i = u_i \frac{1}{H_i}, \quad (6)$$

where $(ij) = rr, \theta\theta, zz, r\theta, rz, z\theta$, $(i) = r, \theta, z$. H_i are the Lamé coefficients, and n_j are the covariant components of the unit normal to the surfaces S_1 and S_2 . σ and ε denote the covariant and contravariant components of the stress tensor and the strain tensor in the cylindrical system, respectively. u^i and u_i are the covariant and contravariant components of the displacement vector (\mathbf{u}) in the cylindrical system. In (6) formulas are given the relationships between the physical components of the tensor and vectors. The physical components of this stress and strain tensor in the cylindrical system are expressed according to the covariant and contravariant components of this stress and the strain tensors.

$$H_r = r, \quad H_\theta = 1; \quad H_z = 1$$

$$u_{(r)} = u^r = u_r, \quad u_{(\theta)} = r u^\theta = \frac{1}{r} u_\theta; \quad u_{(z)} = u^z = u_z;$$

$$\sigma_{(rr)} = \sigma_{rr} = \sigma^{rr}; \quad \sigma_{(r\theta)} = \frac{1}{r} \sigma_{r\theta} = r \sigma^{r\theta}; \quad \sigma_{(rz)} = \sigma_{rz} = \sigma^{rz}$$

$$\sigma_{(\theta r)} = \frac{1}{r} \sigma_{\theta r} = r \sigma^{r\theta}; \quad \sigma_{\theta\theta} = \frac{1}{r^2} \sigma_{\theta\theta} = r^2 \sigma^{\theta\theta}; \quad \sigma_{(\theta z)} = \frac{1}{r} \sigma_{\theta z} = r \sigma^{\theta z},$$

$$\sigma_{(zr)} = \sigma_{zr} = \sigma^{zr}; \quad \sigma_{(z\theta)} = \frac{1}{r} \sigma_{z\theta} = r \sigma^{z\theta}; \quad \sigma_{(zz)} = \sigma_{zz} = \sigma^{zz} \quad (7)$$

We define the initial imperfection form of the nanofiber by the equation of its axial line.

$$x_1 = A \exp\left(-\left(\frac{x_3}{L}\right)^2\right) \cos\left(m \frac{x_3}{L}\right) = \varepsilon L \exp\left(-\left(\frac{x_3}{L}\right)^2\right) \cos\left(m \frac{x_3}{L}\right) = \varepsilon \delta(x_3); \quad \varepsilon = \frac{A}{L} \quad (8)$$

Where A and L are geometrical parameter which are shown in Fig.1. ε is small parameter that specifies the curvature amplitude. Assuming that A is smaller than

L, we introduce a small parameter $\varepsilon = \frac{A}{L}$ ($0 \leq \varepsilon < 1$). The function δ_n^j is the local curving form of the nanofiber.

First, using Eq.(8) and the condition for the crosssection of the nanofiber, we derive the following equation of the interface S and the components of their normal vectors in the cylindrical system of coordinates $Or\theta z$.

$$r(\theta, t_3) = \frac{\varepsilon \delta(t_3) (1 + \varepsilon^2 (\delta'(t_3))^2) \cos \theta}{1 + \varepsilon^2 (\delta'(t_3))^2 \cos^2 \theta} + \left\{ \frac{\varepsilon^2 (\delta(t_3))^2 (1 + \varepsilon^2 (\delta'(t_3))^2)^2 \cos \theta}{(1 + \varepsilon^2 (\delta'(t_3))^2 \cos^2 \theta)^2} + R^2 - (\varepsilon^2 (\delta(t_3))^2 (1 + \varepsilon^2 (\delta'(t_3))^2))^2 \right\}^{\frac{1}{2}}$$

$$z(\theta, t_3) = t_3 - \varepsilon \delta'(t_3) (r(\theta, t_3) - \varepsilon \delta(t_3)), \quad \delta'(t_3) = \frac{d\delta(t_3)}{dt_3} \quad (9)$$

where t_3 is a parameter and $t_3 \in (-\infty, +\infty)$. Using the well-known operations of differential geometry, the components n_r , n_θ and n_z of the unit normal vector \mathbf{n} to the surfaces S_1 and S_2 are obtained as follows:

$$n_r = r(\theta, t_3) \frac{\partial z(\theta, t_3)}{\partial t_3} [A(\theta, z)]^{-1},$$

$$n_\theta = \left[\frac{\partial z(\theta, t_3)}{\partial \theta} \frac{\partial r(\theta, t_3)}{\partial t_3} - \frac{\partial r(\theta, t_3)}{\partial \theta} \frac{\partial z(\theta, t_3)}{\partial t_3} \right] [A(\theta, z)]^{-1},$$

$$n_z = -r(\theta, t_3) \frac{\partial r(\theta, t_3)}{\partial t_3} [A(\theta, t_3)]^{-1}$$

where

$$[A(\theta, t_3)] = \left[\left(r(\theta, t_3) \frac{\partial z(\theta, t_3)}{\partial t_3} \right)^2 + \left(\frac{\partial z(\theta, t_3)}{\partial \theta} \frac{\partial r(\theta, t_3)}{\partial t_3} - \frac{\partial z(\theta, t_3)}{\partial t_3} \frac{\partial r(\theta, t_3)}{\partial \theta} \right)^2 + \left(r(\theta, t_3) \frac{\partial z(\theta, t_3)}{\partial t_3} \right)^2 \right]^{1/2} \quad (10)$$

3 Solution Method

The considered problem is the boundary-value problem for the nonlinear partial differential equations. For the analysis of the this problem, the boundary form perturbation method given in Akbarov and Guz (2000) are used. According to this method, the unknown values are presented as series in the small parameter ε :

$$\sigma_{rr}^{(k)} = \sum_{q=0}^{\infty} \varepsilon^q \sigma_{rr}^{(k),q}, \dots, \varepsilon_{rr}^{(k)} = \sum_{q=0}^{\infty} \varepsilon^q \varepsilon_{rr}^{(k),q}, \dots, u_r^{(k)} = \sum_{q=0}^{\infty} \varepsilon^q u_r^{(k),q} \quad (11)$$

We expand the expression (9)-(10) as series in ε ; the quantities r , z , n_r , n_θ and n_z are also presented in series forms:

$$r = R + \sum_{k=1}^{\infty} \varepsilon^k a_{rk}(\theta, t_3), \quad z = t_3 + \sum_{k=1}^{\infty} \varepsilon^k a_{zk}(\theta, t_3),$$

$$n_r = 1 + \sum_{k=1}^{\infty} \varepsilon^k b_{rk}(\theta, t_3), \quad n_\theta = \sum_{k=1}^{\infty} \varepsilon^k b_{\theta k}(\theta, t_3), \quad n_z = \sum_{k=1}^{\infty} \varepsilon^k b_{zk}(\theta, t_3) \quad (12)$$

The expressions for the coefficients of the ε^k in (12) can be determined by employing some routine operations, whose details are left out here.

Substituting the expression (11) in Eq.(2) and Eq.(3) and grouping terms with identical powers, we obtain a complete system of equations for each approximation. In this case, Eq.(1) and (2) hold for the zeroth approximation, and the equations derived for the first and subsequent approximations contain the values of the previous approximations.

Now we consider the contact conditions for each approximation, which are derived from Eqs. (4). For this purpose, we substitute the expressions (11) and (12) into Eq. (4) and expand the quantities $\sigma_{rr}^{(k),q}, \dots, u_r^{(k),q}$ involved in eqs.(11) into Taylor series in vicinity of a point (R, θ, t_3) .

Then, grouping the terms with identical powers of the parameter ε and taking into account the foregoing assumptions, we derive the contact conditions for each approximation:

for the zeroth approximation

$$\sigma_{(ij)}^{(2),0} \Big|_{r_q=R_1} = \sigma_{(ij)}^{(1),0} \Big|_{r_q=R_1}, \quad u_{(i)}^{(2),0} \Big|_{r_q=R_1} = u_{(i)}^{(1),0} \Big|_{r_q=R_1},$$

$$\sigma_{(ij)}^{(2),0} \Big|_{r_q=R_2} = \sigma_{(ij)}^{(1),0} \Big|_{r_q=R_2}, \quad u_{(i)}^{(2),0} \Big|_{r_q=R_2} = u_{(i)}^{(1),0} \Big|_{r_q=R_2}; \quad (ij) = rr, r\theta, rz, (i) = r, \theta, z \quad (13)$$

for the first approximation

$$[\sigma_{(i)r}]_{1,1}^{2,1} + f_1 \left[\frac{\partial \sigma_{(i)r}}{\partial r} \right]_{1,0}^{2,0} + \varphi_1 \left[\frac{\partial \sigma_{(i)r}}{\partial z} \right]_{1,0}^{2,0} + \gamma_r [\sigma_{(i)r}]_{1,0}^{2,0} + \gamma_\theta [\sigma_{(i)\theta}]_{1,0}^{2,0} + \gamma_z [\sigma_{(i)z}]_{1,0}^{2,0} = 0$$

$$[u_{(i)}]_{1,1}^{2,1} + f_1 \left[\frac{\partial u_{(i)}}{\partial r} \right]_{1,0}^{2,0} + \varphi_1 \left[\frac{\partial u_{(i)}}{\partial z} \right]_{1,0}^{2,0} = 0 \quad (14)$$

In Eqs. (13) and (14) the following notation is used:

$$[\mathbf{X}]_{1,q}^{2,q} = \mathbf{X}^{(1),q}(R, \theta, t_3) - \mathbf{X}^{(1),q}(R, \theta, t_3), \quad q = 0, 1,$$

$$f_1 = \delta(t_3) \cos \theta, \quad \varphi_1 = -R \frac{d\delta(t_3)}{dt_3} \cos \theta, \quad \gamma_r = \left(\frac{\delta(t_3)}{R} - \frac{d^2\delta(t_3)}{dt_3^2} R \right) \cos \theta,$$

$$\gamma_\theta = \frac{\delta(t_3)}{R} \sin \theta, \quad \gamma_z = -\frac{d\delta(t_3)}{dt_3} \cos \theta, \quad \delta(t_3) = \exp\left(\frac{-x_3}{L}\right)^2 \cos\left(m \frac{-x_3}{L}\right) \quad (15)$$

The zeroth approximation. Taking into account $n_r=1$, $n_\theta=0$ and $n_z=0$ in the equations (1), (2) and (3), the contact condition (4) will be provided. Therefore, the equations and the contact conditions obtained for the zeroth approximation are non-linear equations. The zeroth approximation is the boundary-value problem for the determination of stresses and strain in the curvature-free nanofiber in our model. To the determine the zeroth approximation, we obtain the equations (16) and the contact condition (17).

$$\nabla_i \sigma^{(k)ij,0} = 0, \quad 2\varepsilon_{jm}^{(k),0} = \nabla_j u_i^{(k),0} + \nabla_m u_k^{(k),0},$$

$$\sigma_{(ij)}^{(k)} = (\lambda^{(k)} e^{(k)}) \delta_i^n + 2(\mu^{(k)} \varepsilon_{(ij)}^{(k)}), \quad e^{(k),0} = \varepsilon_{(rr)}^{(k),0} + \varepsilon_{(\theta\theta)}^{(k),0} + \varepsilon_{(zz)}^{(k),0} \quad (16)$$

$$\sigma_{ij}^{(1),0} \Big|_{R_q=R_1} = \sigma_{ij}^{(1),0} \Big|_{R_q=R_1}, \quad u_i^{(1),0} \Big|_{R_q=R_1} = u_i^{(1),0} \Big|_{R_q=R_1},$$

$$\sigma_{ij}^{(1),0} \Big|_{R_q=R_2} = \sigma_{ij}^{(1),0} \Big|_{R_q=R_2}, \quad u_i^{(1),0} \Big|_{R_q=R_2} = u_i^{(1),0} \Big|_{R_q=R_2}, \quad (ij) = rr, r\theta, rz, (i) = r, \theta, z \quad (17)$$

In this case, an exact analytical solution is given in Akbarov and Guz (2000), Guz (2003), Akbarov and Guz (1985). According to which, at $v^{(1)} = v^{(1)}$, we have the following relations:

$$\varepsilon_{zz}^{(1),0} = \varepsilon_{zz}^{(1),0} = \frac{p}{E^{(1)}}, \quad \sigma_{zz}^{(1),0} = p, \quad u_z^{(1),0} = u_z^{(1),0} = \frac{p}{E^{(1)}} z, \quad u_r^{(1),0} = -v^{(1)} \varepsilon_{zz}^{(1),0} r,$$

$$u_r^{(1),0} = -v^{(1)} \varepsilon_{zz}^{(1),0} r, \quad u_\theta^{(1),0} = u_\theta^{(1),0} = 0, \quad \sigma_{rr}^{(1),0} = \sigma_{rr}^{(1),0} = \sigma_{\theta\theta}^{(1),0} = \sigma_{\theta\theta}^{(1),0} = 0,$$

$$\sigma_{zz}^{(1),0} = p \frac{E^{(1)}}{E^{(1)}}, \quad \sigma_{\theta z}^{(1),0} = \sigma_{\theta z}^{(1),0} = \sigma_{rz}^{(1),0} = \sigma_{rz}^{(1),0} = \sigma_{r\theta}^{(1),0} = \sigma_{r\theta}^{(1),0} = 0 \quad (18)$$

$E^{(1)}$, $E^{(2)}$ in (18) represent the elasticity modulus of the matrix, nanofiber material, respectively.

The First approximation. Taking into consideration the fact that the solution obtained for the zeroth approximations, we assume that the materials of the matrix and nanofiber are comparatively rigid, therefore, the nonlinear terms can be neglected in the equations obtained for the first approximation and the term $(\delta_n^j + \nabla_n u^{(k)j,0})$ can be replaced with δ_n^j in the first and subsequent approximations. For the first approximation, we obtain the equations (19), (20) and (21).

$$\nabla_i \left[\sigma^{(k)ij,n} + \sigma^{(k)in,0} \nabla_n u^{(k)j,1} \right] = 0, \quad k = 1, 2 \quad (19)$$

$$2\varepsilon_{ij}^{(k)} = \nabla_j u_i^{(k),1} + \nabla_i u_j^{(k)}, \quad (20)$$

$$\sigma_{(in)}^{(k)} = \lambda^{(k)} e^{(k)} \delta_i^n + 2(\mu^{(k)} \varepsilon_{(in)}^{(k)}), \quad e^{(k)} = \varepsilon_{rr}^{(k)} + \varepsilon_{\theta\theta}^{(k)} + \varepsilon_{zz}^{(k)} \quad (21)$$

The expressions of the physical components of the equations (19) and (20) are given in Kosker (2002).

Taking into consideration the fact that the solution obtained for the zeroth approximation, other assumptions and the right-side function to be obtained for the second approximation would have values small enough to be neglected, governing equations for the first and second approximation can be defined as follows in terms of the physical components of the tensors and vectors:

$$\begin{aligned} \frac{\partial \sigma_{rr}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{rz}^{(k),1}}{\partial z} + \frac{1}{r} \left(\sigma_{rr}^{(k),1} - \sigma_{\theta\theta}^{(k),1} \right) + \sigma_{zz}^{(k),0} \frac{\partial^2 u_r^{(k),1}}{\partial z^2} &= 0, \\ \frac{\partial \sigma_{r\theta}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{\theta z}^{(k),1}}{\partial z} + \frac{2}{r} \sigma_{r\theta}^{(k),1} + \sigma_{zz}^{(k),0} \frac{\partial^2 u_\theta^{(k),1}}{\partial z^2} &= 0, \\ \frac{\partial \sigma_{rz}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta z}^{(k),1}}{\partial \theta} + \frac{\partial \sigma_{zz}^{(k),1}}{\partial z} + \frac{1}{r} \sigma_{rz}^{(k),1} + \sigma_{zz}^{(k),0} \frac{\partial^2 u_z^{(k),1}}{\partial z^2} &= 0 \end{aligned} \quad (22)$$

These equations coincided with the 3-dimensional linearized elasticity equations.

The mechanical and geometrical relations for the foregoing approximation are

$$\sigma_{(in)}^{(k),1} = \lambda^{(k)} e^{(k),1} \delta_i^n + 2\mu^{(k)} \varepsilon_{(in)}^{(k),1},$$

$$e^{(k),1} = \varepsilon_{(rr)}^{(k),1} + \varepsilon_{(\theta\theta)}^{(k),1} + \varepsilon_{(zz)}^{(k),1},$$

$$\varepsilon_{rr}^{(k),1} = \frac{\partial u_r^{(k),1}}{\partial r}, \quad \varepsilon_{\theta\theta}^{(k),1} = \frac{\partial u_\theta^{(k),1}}{r \partial \theta} + \frac{u_r^{(k),1}}{r},$$

$$\varepsilon_{zz}^{(k),1} = \frac{\partial u_z^{(k),1}}{\partial z}, \quad \varepsilon_{r\theta}^{(k),1} = \frac{1}{2} \left(\frac{\partial u_r^{(k),1}}{r \partial \theta} + \frac{\partial u_\theta^{(k),1}}{\partial r} - \frac{u_\theta^{(k),1}}{r} \right),$$

$$\varepsilon_{\theta z}^{(k),1} = \frac{1}{2} \left(\frac{\partial u_{\theta}^{(k),1}}{\partial z} + \frac{\partial u_z^{(k),1}}{r \partial \theta} \right), \quad \varepsilon_{zr}^{(k),1} = \frac{1}{2} \left(\frac{\partial u_z^{(k),1}}{\partial r} + \frac{\partial u_r^{(k),1}}{\partial z} \right) \quad (23)$$

The contact conditions for the first approximation become

$$\begin{aligned} (\sigma_{rr}^{(1),1} - \sigma_{rr}^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \quad (\sigma_{r\theta}^{(1),1} - \sigma_{r\theta}^{(2),1}) \Big|_{(R_2, \theta, t_3)} = 0, \\ (\sigma_{rz}^{(1),1} - \sigma_{rz}^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= (\sigma_{zz}^{(1),0} - \sigma_{zz}^{(2),0}) \frac{d\delta(t_3)}{dt_3} \cos \theta, \\ (u_r^{(1),1} - u_r^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \quad (u_{\theta}^{(1),1} - u_{\theta}^{(2),1}) \Big|_{(R_2, \theta, t_3)} = 0, \\ (u_z^{(1),1} - u_z^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \quad \sigma_{rr}^{(1),1} \Big|_{(R_1, \theta, t_3)} = 0, \\ \sigma_{r\theta}^{(1),1} \Big|_{(R_1, \theta, t_3)} &= 0, \quad \sigma_{rz}^{(1),1} \Big|_{(R_1, \theta, t_3)} = \sigma_{zz}^{(1),0} \frac{d\delta(t_3)}{dt_3} \cos \theta, \end{aligned} \quad (24)$$

To solve the problem (22), (23), and (24), we employ the representations in Guz (1999)

$$\begin{aligned} u_r^{(k)} &= \frac{1}{r} \frac{\partial}{\partial \theta} \psi^{(k)} - \frac{\partial^2}{\partial r \partial z} \chi^{(k)}; \quad u_{\theta}^{(k)} = -\frac{\partial}{\partial r} \psi^{(k)} - \frac{1}{r} \frac{\partial^2}{\partial \theta \partial z} \chi^{(k)}; \\ \Delta_1 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \\ u_z^{(k)} &= (\lambda^{(k)} + \mu^{(k)})^{-1} \left((\lambda^{(k)} + 2\mu^{(k)}) \Delta_1^{(k)} + (\mu^{(k)} + \sigma_{zz}^{(k),0}) \frac{\partial^2}{\partial z^2} \right) \chi^{(k)} \end{aligned} \quad (25)$$

The functions of $\psi^{(k)}$, $\chi^{(k)}$ are determined from the equations

$$\begin{aligned} \left(\Delta_1^{(k)} + \left(\xi_1^{(k)} \right)^2 \frac{\partial^2}{\partial z^2} \right) \psi^{(k)} &= 0; \\ \left(\Delta_1^{(k)} + \left(\xi_2^{(k)} \right)^2 \frac{\partial^2}{\partial z^2} \right) \left(\Delta_1^{(k)} + \left(\xi_3^{(k)} \right)^2 \frac{\partial^2}{\partial z^2} \right) \chi^{(k)} &= 0 \end{aligned} \quad (26)$$

Where, $\xi_i^{(k)}$ ($k=1,2; i=1,2,3$) given in the following equations:

$$\xi_1^{(k)} = \sqrt{\frac{\mu^{(k)} + \sigma_{zz}^{(k),0}}{\mu^{(k)}}}, \quad \xi_2^{(k)} = \sqrt{\frac{\mu^{(k)} + \sigma_{zz}^{(k),0}}{\mu^{(k)}}}, \quad \xi_3^{(k)} = \sqrt{\frac{\lambda^{(k)} + 2\mu^{(k)} + \sigma_{zz}^{(k),0}}{\lambda^{(k)} + 2\mu^{(k)}}}$$

We apply the exponential Fourier transform with respect to z , i.e.

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(z)e^{-isz} dz \quad (27)$$

to the Eqs. (22)-(26). In this case, we have the governing equations as follows :

$$\begin{aligned} \frac{\partial \bar{\sigma}_{rr}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \bar{\sigma}_{r\theta}^{(k),1}}{\partial \theta} + \frac{is}{L} \bar{\sigma}_{rz}^{(k),1} + \frac{1}{r} \left(\bar{\sigma}_{rr}^{(k),1} - \bar{\sigma}_{\theta\theta}^{(k),1} \right) - \frac{s^2}{L^2} \bar{\sigma}_{zz}^{(k),0} u_r^{(k),1} &= 0, \\ \frac{\partial \bar{\sigma}_{r\theta}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \bar{\sigma}_{\theta\theta}^{(k),1}}{\partial \theta} + \frac{is}{L} \bar{\sigma}_{\theta z}^{(k),1} + \frac{2}{r} \bar{\sigma}_{r\theta}^{(k),1} - \frac{s^2}{L^2} \bar{\sigma}_{zz}^{(k),0} u_{\theta}^{(k),1} &= 0, \\ \frac{\partial \bar{\sigma}_{rz}^{(k),1}}{\partial r} + \frac{1}{r} \frac{\partial \bar{\sigma}_{\theta z}^{(k),1}}{\partial \theta} + \frac{is}{L} \bar{\sigma}_{zz}^{(k),1} + \frac{1}{r} \bar{\sigma}_{rz}^{(k),1} - \frac{s^2}{L^2} \bar{\sigma}_{zz}^{(k),0} u_z^{(k),1} &= 0 \end{aligned} \quad (28)$$

The strain are obtained as follows :

$$\begin{aligned} \bar{\varepsilon}_{rr}^{(k),1} &= \frac{\partial \bar{u}_r^{(k),1}}{\partial r}, \quad \bar{\varepsilon}_{\theta\theta}^{(k),1} = \frac{\partial \bar{u}_{\theta}^{(k),1}}{r \partial \theta} + \frac{\bar{u}_r^{(k),1}}{r}, \\ \bar{\varepsilon}_{zz}^{(k),1} &= \frac{is}{L} \frac{\partial u_z^{(k),1}}{\partial z}, \quad \varepsilon_{r\theta}^{(k),1} = \frac{1}{2} \left(\frac{\partial \bar{u}_r^{(k),1}}{r \partial \theta} + \frac{\partial \bar{u}_{\theta}^{(k),1}}{\partial r} - \frac{\bar{u}_{\theta}^{(k),1}}{r} \right), \\ \bar{\varepsilon}_{\theta z}^{(k),1} &= \frac{1}{2} \left(\frac{\partial \bar{u}_z^{(k),1}}{\partial \theta} + \frac{is}{L} u_{\theta}^{(k),1} \right), \quad \varepsilon_{zr}^{(k),1} = \frac{1}{2} \left(\frac{\partial \bar{u}_z^{(k),1}}{\partial r} + \frac{is}{L} \frac{\partial u_r^{(k),1}}{\partial z} \right) \end{aligned} \quad (29)$$

The Constitutive equations are given as follows :

$$\sigma_{(in)}^{(k),1} = (\lambda^{(k)} e^{(k),1}) \delta_i^n + 2(\mu^{(k)} \varepsilon_{(in)}^{(k),1}), \quad e^{(k)} = \varepsilon_{(11)}^{(k)} + \varepsilon_{(22)}^{(k)} + \varepsilon_{(33)}^{(k)} \quad (30)$$

The Contact condition are as follows ;

$$\begin{aligned} (\bar{\sigma}_{rr}^{(1),1} - \bar{\sigma}_{rr}^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \quad (\bar{\sigma}_{r\theta}^{(1),1} - \bar{\sigma}_{r\theta}^{(2),1}) \Big|_{(R_2, \theta, t_3)} = 0, \\ (\bar{\sigma}_{rz}^{(1),1} - \bar{\sigma}_{rz}^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= (\sigma_{zz}^{(1),0} - \sigma_{zz}^{(2),0}) \frac{is}{L} \bar{\delta}(t_3) \cos \theta, \\ (\bar{u}_r^{(1),1} - \bar{u}_r^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \quad (\bar{u}_{\theta}^{(1),1} - \bar{u}_{\theta}^{(2),1}) \Big|_{(R_2, \theta, t_3)} = 0, \\ (\bar{u}_z^{(1),1} - \bar{u}_z^{(2),1}) \Big|_{(R_2, \theta, t_3)} &= 0, \end{aligned}$$

$$\begin{aligned} \bar{\sigma}_{rr}^{(2),1} \Big|_{(R_1, \theta, t_3)} &= 0, & \bar{\sigma}_{r\theta}^{(2),1} \Big|_{(R_1, \theta, t_3)} &= 0, \\ \bar{\sigma}_{rz}^{(2),1} \Big|_{(R_1, \theta, t_3)} &= \sigma_{zz}^{(2),0} \frac{is}{L} \bar{\delta}(t_3) \cos \theta, \end{aligned} \tag{31}$$

We have the expressions (25) as follows ;

$$\begin{aligned} \bar{u}_r &= \frac{1}{r} \frac{\partial}{\partial \theta} \bar{\psi} - \frac{is}{L} \frac{\partial}{\partial r} \bar{\chi}; & \bar{u}_\theta &= -\frac{\partial}{\partial r} \bar{\psi} - \frac{is}{rL} \frac{\partial}{\partial \theta} \bar{\chi}; & \Delta_1 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \\ \bar{u}_z &= (\lambda + \mu)^{-1} \left((\lambda + 2\mu) \Delta_1 - \frac{s^2}{L^2} (\mu + \sigma_{zz}^0) \right) \bar{\chi} \end{aligned} \tag{32}$$

The equations (26) are as follows ;

$$\left(\Delta_1 - \frac{s^2}{L^2} (\xi_1)^2 \right) \bar{\psi} = 0; \quad \left(\Delta_1 - \frac{s^2}{L^2} (\xi_2^{(k)})^2 \frac{\partial^2}{\partial z^2} \right) \left(\Delta_1 - \frac{s^2}{L^2} (\xi_3)^2 \right) \bar{\chi} = 0 \tag{33}$$

Considering the equations (28) and the contact condition (31) , the equations (33) turns into the following equations

$$\begin{aligned} \bar{\psi}^{(1),1} &= \bar{A}_1^{(1)}(s) K_1 \left(\xi_1^{(1)} s \frac{r}{L} \right) \sin \theta, \\ \bar{\chi}^{(1),1} &= i \left[\bar{A}_2^{(1)}(s) K_1 \left(\xi_2^{(1)} s \frac{r}{L} \right) + \bar{A}_3^{(1)}(s) K_1 \left(\xi_3^{(1)} s \frac{r}{L} \right) \right] \cos \theta \\ \bar{\psi}^{(2),1} &= \left[\bar{A}_{11}^{(2)}(s) I_1 \left(\xi_1^{(2)} s \frac{r}{L} \right) + \bar{A}_{12}^{(2)}(s) K_1 \left(\xi_1^{(2)} s \frac{r}{L} \right) \right] \sin \theta, \\ \bar{\chi}^{(2),1} &= i \left[\bar{A}_{21}^{(2)}(s) I_1 \left(\xi_2^{(2)} s \frac{r}{L} \right) + \bar{A}_{22}^{(2)}(s) K_1 \left(\xi_2^{(2)} s \frac{r}{L} \right) \right. \\ &\quad \left. + \bar{A}_{31}^{(2)}(s) I_1 \left(\xi_3^{(2)} s \frac{r}{L} \right) + \bar{A}_{32}^{(2)}(s) K_1 \left(\xi_3^{(2)} s \frac{r}{L} \right) \right] \cos \theta \end{aligned} \tag{34}$$

where $I_n(x)$ are Bessel functions of a purely imaginary argument and $K_n(x)$ are the Macdonald functions. The derivatives of this function are given as follows ;

$$I'_n = \frac{1}{2} (I_{n-1} + I_{n+1}), \quad I'_0 = I_1, \quad K'_n = \frac{-1}{2} (K_{n-1} + K_{n+1}), \quad K'_0 = K_1 \tag{35}$$

The function (34) is employed in (32) and in (29) and substituted the contact condition (30) ; an 9x9 linear equation system is obtained which depends on the unknowns $\bar{A}_1^{(1)} = \frac{\bar{A}_1^{(1)}}{L}$, $\bar{A}_2^{(1)} = \frac{\bar{A}_2^{(1)}}{L^2}$, $\bar{A}_3^{(1)} = \frac{\bar{A}_3^{(1)}}{L^2}$, $\bar{A}_{ik}^{(2)} = \frac{\bar{A}_{ik}^{(2)k}}{L^2}$, $k=1,2, i=1,2,3$.

To solving this equation system, the unknowns

$\bar{A}_1^{(1)}(s), \bar{A}_2^{(1)}(s), \bar{A}_3^{(1)}(s), \bar{A}_{11}^{(2)}(s), \bar{A}_{12}^{(2)}(s), \bar{A}_{21}^{(2)}(s), \bar{A}_{22}^{(2)}(s), \bar{A}_{31}^{(2)}(s), \bar{A}_{32}^{(2)}(s)$ are determined. By using the values $\bar{A}_1^{(1)}(s), \bar{A}_2^{(1)}(s), \dots, \bar{A}_{32}^{(2)}(s)$, the expressions $\bar{\sigma}_{rr}^{(1),1}, \dots, \bar{\sigma}_{zz}^{(2),1}$ are obtained. The inverse transform for the stresses, for example, for the stress $\sigma_{rr}^{(1),1}$ is determined by

$$\sigma_{rr}^{(1),1} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{\sigma}_{rr}^{(1),1} e^{isz} ds \quad (36)$$

Equation (4) shows that the function $\delta(t_3)$ is even, and therefore the expression (36) can be replaced with

$$\sigma_{rr}^{(1),1} = \frac{1}{\pi} \int_0^{+\infty} \bar{\sigma}_{rr}^{(1),1} \cos z ds \quad (37)$$

By similar manner we obtain the expressions to calculate the other sought values. Thus, by the above-described method we determine completely the values in the first approximation. Note that the values of the second and subsequent approximations can also be determined by this method. According to the investigations analyzed in Akbarov and Guz (2004) the main effect of the fiber curving on the distribution of stresses is manifested within the framework of the first approximation. The second and subsequent approximations give only some insignificant quantitative corrections to these results. However, to determine of the values of these approximations requires some very complicated and cumbersome mathematical procedures. Taking the above stated into account the investigations in the present paper are made only within the framework of the zeroth and first approximations.

4 Numerical Results and Discussion

First, we consider some remarks on the calculation of the improved integral (37). Note that under the calculation procedure these improved integrals are replaced by the corresponding definite ones, i.e. we use the relation $\int_0^{+\infty} (\cdot) ds \cong \int_0^{S_*} (\cdot) ds = \sum_{i=0}^N \int_{S_i}^{S_{i+1}} (\cdot) ds$, $S_0 = 0$, $S_N = S_*$. The values of N and S_* are determined from the convergence criterion of the improved integrals. Further, for the calculation of the integrals $\int_{S_i}^{S_{i+1}} (\cdot) ds$ the Gauss integration algorithm is employed. All these procedures are made automatically in PC by using programmes written in FTN77. Thus, we consider numerical results regarding the normal stress $\sigma_{\tau\tau}$, which operate along the tangent vectors τ to surface S , which constitute the matrix and nanofiber

intersection surfaces. In view of corresponding symmetry, we consider the distribution of these stresses only for $x_3 \geq 0$ and $0 \leq \theta \leq \pi$ (Fig. 1). If $\varepsilon = 0$ (i.e. if the curving is absent), the stresses $\sigma_{\tau\tau}$ coincide with σ_{zz} .

We introduce the parameter $\kappa = R_2/L$, $bk=h/L$ and assume that $v^{(1)} = v^{(2)} = 0.3$, $\varepsilon = 0.07$, $\theta = 0$, $\kappa = 0.3$, $E^{(2)}/E^{(1)}=300$. To illustrate the influence of the geometrical nonlinearity on the distribution of the considered stresses we will use the parameter $\alpha = p/E^{(1)}$.

The graphs given in Figs. 2 show the relationships between $\sigma_{\tau\tau}/p$ and h/L at point of the S_1 surface in tension (a) and compression (b) for various values of α at $\chi_3/L = 1.0$ and $m=1$. The intermittently lines in these figures represent the lines obtained in the problem of the single locally curved nanofiber in an infinite body given in Kosker and Simsek (2006). Here, it can be concluded that when the thickness of nanofiber material is increased, it is possible to obtain the same result as those obtained in the case of the single locally curved nanofiber in an infinite body in the same parameter values.

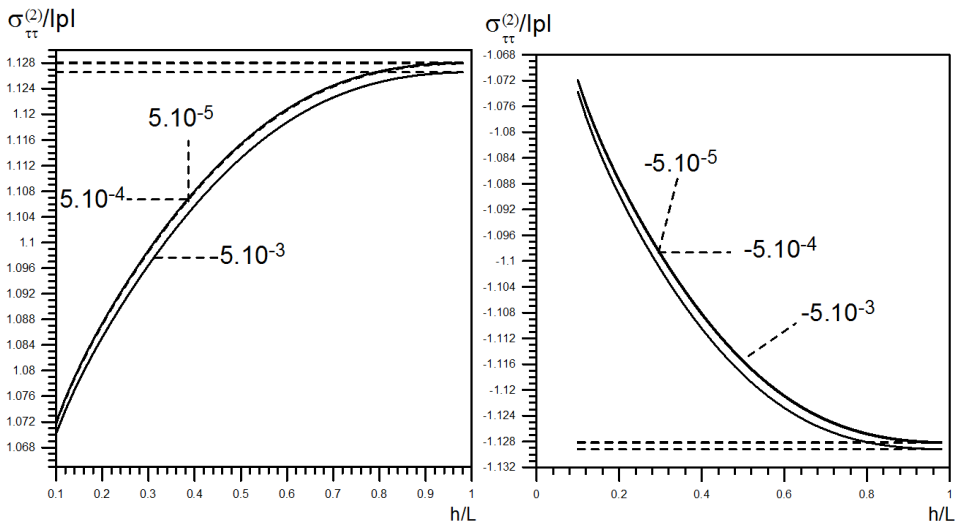


Figure 2: The relationships between $\sigma_{\tau\tau}/|p|$ and h/L for values α at $\chi_3/L=1.0$, and $m=1$ (---) for a single locally curved nanofiber in an infinite body.

Let us consider the graphs given in Figure 3, which illustrate the distribution of $\sigma_{\tau\tau}/|p|$ with respect to h/L on the surfaces S_2 in tension (a) and compression (b) for various values of α and m at $\chi_3/L = 1.0$. It follows from the graphs that as a result of the geometrical nonlinearity, the absolute values of the stresses $\sigma_{\tau\tau}/|p|$ in compression (tension) increase (decrease) monotonically with absolute values of

α . Note that under compression of the considered infinite body we assume that the selected values of α are smaller than those of critical values α_{cr} , which correspond to the micro-buckling of the fiber in the matrix in Guz (1990).

The same figures show also the influence of the parameter m on the distribution of the normal stresses. According to the graphs in Figure 3, absolute maximal values of the normal stresses $\sigma_{\tau\tau}/|p|$ increase monotonically with m .

In Fig.4 and 5, relationships between $\sigma_{\tau\tau}/|p|$ and x_3/L on the surfaces S_1 and surfaces S_2 in tension (a) and compression (b) are shown for various values of α at $h/L=0.5$ value. It follows from the graphs that as a result of the geometrical nonlinearity the absolute values of the stresses $\sigma_{\tau\tau}/|p|$ increase (decrease) monotonically with absolute values of α under compression (tension). Note that under compression of the considered infinite body we assume that the selected values of α are smaller than those of α_{cr} . which correspond to the micro-buckling of the fiber in the matrix in Guz (1990).

Table 1 shows the values of $\sigma_{\tau\tau}/|p|$ for various $E^{(2)}/E^{(1)}$, m and α . In this case the values of $\sigma_{\tau\tau}/|p|$ are calculated under $\kappa = 0.25$, $\chi_3/L = 1.0$ for $m = 0, 1$ and 3 respectively. The table shows that $\sigma_{\tau\tau}/|p|$ normal stresses increase with $E^{(2)}/E^{(1)}$ and m . We assume that α is smaller than its critical values α_{cr} corresponding to microbuckling of the fiber in the matrix in Guz (1990).

m	$\frac{E^{(2)}}{E^{(1)}}$	$\alpha = \frac{p}{E^{(1)}}$							
		Tension				Compression			
		$5 \cdot 10^{-5}$	$5 \cdot 10^{-4}$	$3 \cdot 10^{-3}$	$5 \cdot 10^{-3}$	$-5 \cdot 10^{-5}$	$-5 \cdot 10^{-4}$	$-3 \cdot 10^{-3}$	$-5 \cdot 10^{-3}$
0	300	1.0526	1.05241	1.05134	1.05051	-1.05265	-1.05284	-1.05395	-1.05485
	400	1.06444	1.06408	1.06213	1.06062	-1.06452	-1.06489	-1.06697	-1.06871
	500	1.07551	1.07498	1.0721	1.06991	-1.07563	-1.07618	-1.0793	-1.08194
1	300	1.06213	1.0622	1.06258	1.06286	-1.06211	1.06204	-1.06159	-1.0612
	400	1.0723	1.07225	1.07195	1.07171	-1.07231	-1.07236	-1.07266	-1.07291
	500	1.08172	1.08152	1.08049	1.07972	-1.08176	-1.08195	-1.08308	-1.08403
3	300	1.0913	1.09223	1.09728	1.10113	-1.09109	-1.09014	-1.0847	-1.08013
	400	1.09483	1.09578	1.10093	1.10485	-1.09461	-1.09365	-1.08809	-1.08342
	500	1.0977	1.09865	1.10379	1.10771	-1.09749	-1.09652	-1.09097	-1.0863

5 Conclusion

In the present paper in the framework of the piecewise homogeneous body model with the use of the three-dimensional geometrically nonlinear exact equations of the theory of elasticity, the method is developed for the determination of the

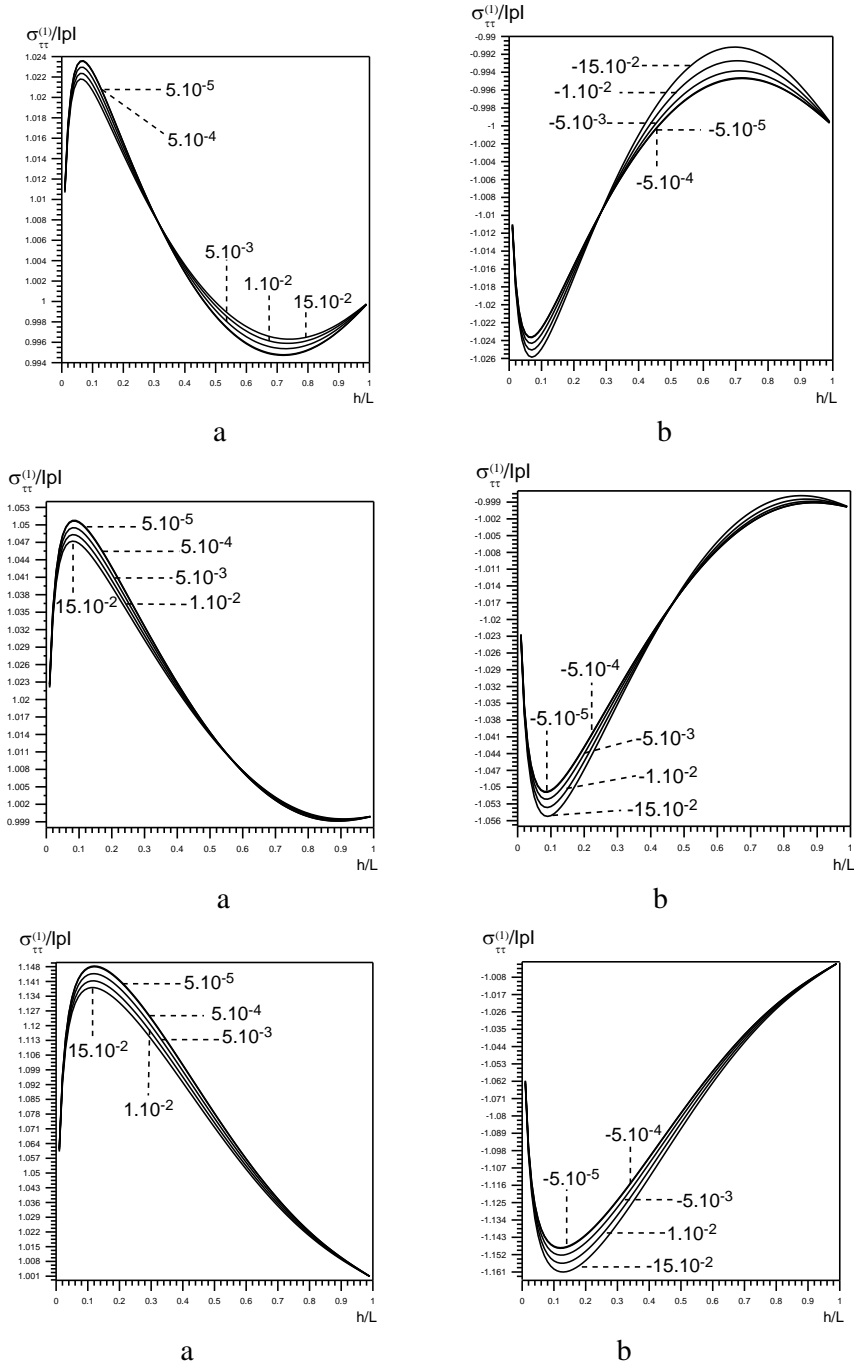


Figure 3: Distribution of the $\frac{\sigma_{\tau\tau}^{(1)}}{|p|}$ with respect to h/L for various $\alpha = p/E^{(1)}$ where $E^{(2)}/E^{(1)}=300$, $\varepsilon = 0.07$ and $\kappa = 0.3$, under $m = 0; m = 1; m = 3$.

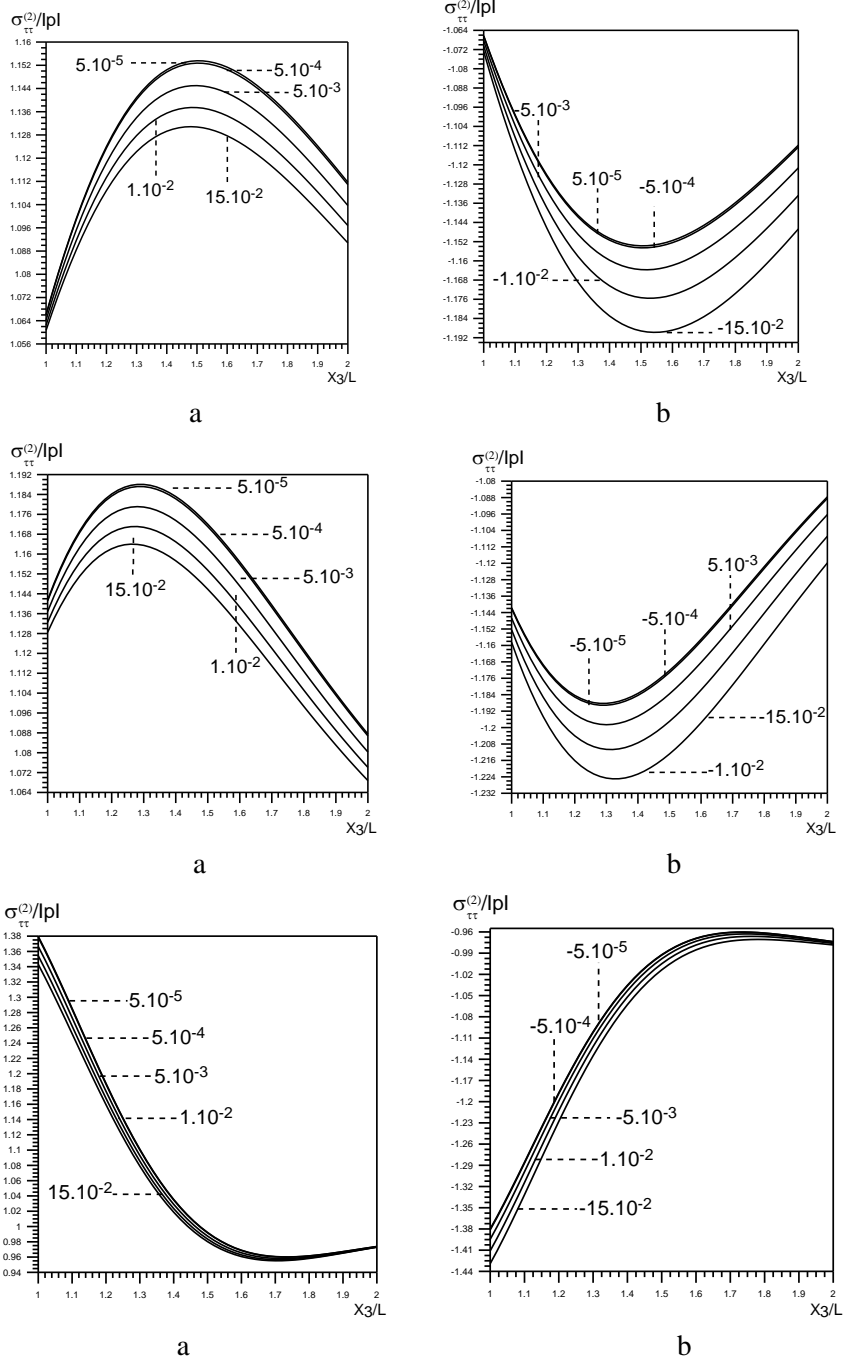
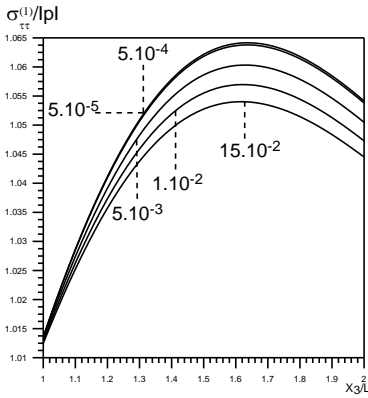
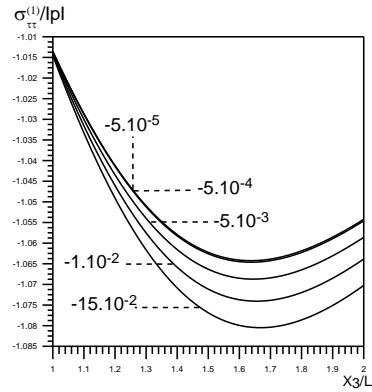


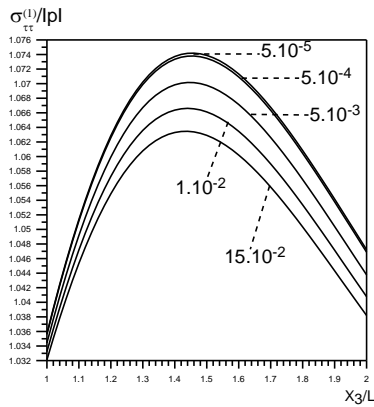
Figure 4: Distribution of the $\sigma_{\tau\tau}^{(2)}/p$ with respect to x_3/L for various $\alpha = p/E^{(1)}$ where $E^{(2)}/E^{(1)}=300$, $\varepsilon = 0.07$ and $\kappa = 0.3$, under $m = 0; m = 1; m = 3$.



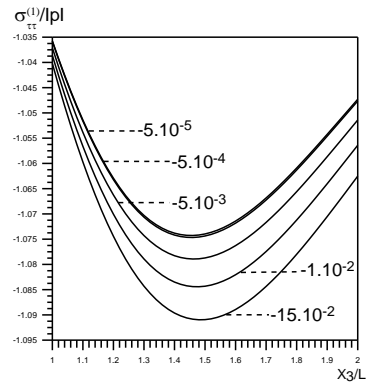
a



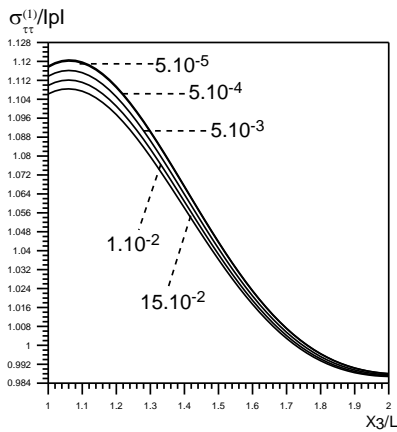
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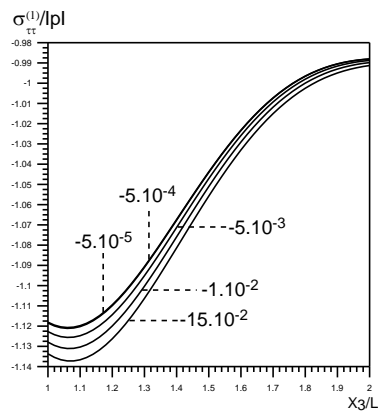
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a



b

Figure 5: Distribution of the $\sigma_{\tau\tau}^{(1)}/p$ with respect to x_3/L for various $\alpha = p/E^{(1)}$ where $E^{(2)}/E^{(1)}=300$, $\varepsilon = 0.07$ and $\kappa = 0.3$, under $m = 0$; $m = 1$; $m = 3$.

stress-strain state in the nanocomposites with unidirectional locally curved and hollow nanofibers is used for investigation of the normal stresses acting along the nanofibers. The case is considered where a locally curved and hollow nanofiber of infinite length is located in an infinite elastic body with a low concentration of nanofibers. The interaction between the nanofibers is neglected.

The numerical results related normal stresses, which act on the interface, are given. From the analyses of the numerical results are derived the following conclusions:

1. when the thickness between the hollow and nanofiber goes to 0 as a limit, it was obtained the same normal stresses values in the locally curved nanofiber in a nanocomposite material in same parameter values.
2. As a result of the geometrical nonlinearity the absolute values of normal stresses $\sigma_{\tau\tau}/|p|$, increase (decrease) in compression (tension) with $|\alpha|$.
3. The absolute maximum values of the considered stresses increase with monotonically with m .
4. An increase in $E^{(2)}/E^{(1)}$ results in an increase in the absolute values of the normal stresses on the Hollow and nanofiber interface.

The locally curvature of nanofiber is one of characteristics of nanocomposites as noted in Guz, Rushchitsky and Guz (2007), Guz and Chekhov (2007), Xiao, Zhang, Zarudi (2007). These results are also important for estimating the adhesion stress between the nanofibers and a polymer matrix. In this case, it is necessary to take into account the restrictions noted in Guz, Rushchitsky and Guz (2007).

The numerical results obtained agree with well-known mechanical consideration and, in some particular cases, coincide with known results.

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