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Numerical Study on Mechanical Properties of Steel Fiber Reinforced Concrete by Statistical Second-order Two-scale Method

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Abstract: The present study aims to evaluate the mechanical properties of steel fiber reinforced concrete (SFRC) by the statistical second-order two-scale (SSOTS) method. At first, the representation for microstructure of SFRC is described by a concept of statistical screen. According to the microstructure representation, the SSOTS method is displayed in a concise way. This method is on the basis of asymptotic expansion homogenization and Monte Carlo method, and can calculate the local strain and stress field through the two-order displacement solution. As the classical homogenization method, the expression of homogenized elastic modulus is derived analytically. Then combined with the appropriate strength criterion and correspondence principle, the homogenized strength and viscoelastic properties of SFRC are obtained respectively. The validity of the SSOTS method is confirmed by the comparison between numerical results and the available experiment data. Results show that the SSOTS method is effective to evaluate the elastic, strength and viscoelastic properties of SFRC. In addition, the influence of distribution of steel fibers on the macroscopic mechanical properties of SFRC is discussed.

Keywords: Steel fiber reinforced concrete, mechanical properties, SSOTS method, random distribution.

1 Introduction

With the rapid development of material science and technology, steel fiber reinforced concrete (SFRC) is widely used in constructions of high-rise buildings or critical protective structures due to its outstanding mechanical performance over conventional plain concrete. Since the steel fibers were first proposed as dispersed reinforcement for concrete, a certain number of experimental researches have been

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carried out to investigate the mechanical properties of fiber reinforced concrete. Song et al. [Song and Hwang (2004)] investigated the strength improving potentials of high-strength steel fiber reinforced concrete with steel fibers of volume fractions varying from 0.5% to 2.0% and concluded that the compressive strength increased with the fiber content. Nataraja et al. [Nataraja, Dhang and Gupta (1999)] carried out an experimental work to study the stress-strain behavior of steel fiber reinforced concrete with compressive strength ranging from 30 to 50 MPa. The toughness property of SFRC was also investigated in another experimental study carried out by Banthia et al. [Banthia and Sappakittipakorn (2007)] . The advantage of laboratory test is that it lies on the direct access to material information at the relevant scale for analysis of the structure. On the other hand, it has the complex experimental programs and costs a lot of time. The numerical simulation provides a powerful tool to handing this kind of problem.

Generally SFRC is considered as a typical example of randomly heterogeneous composite material in the view of material structure. So far, some numerical methods to evaluate the mechanical properties of randomly heterogeneous composite materials have been developed, such as the Hashin-Shtrikman bounds [Hashin and Shtrikman (1963)], the self-consistent method [Hill (1965); Budiansky (1965)], the Eshelby's equivalent inclusion method [Eshebly (1959)], and the Mori-Tanaka homogenization method [Mori and Tanaka (1973); Pasa-Dutra, Maghous, Campos-Filho and Pacheco (2010)]. Rencently, Dong et al. have developed a novel numerical tool " Computational Grains ", which enables a direct numerical simulation of a large number of heterogeneities of materials and structures, without using brutal force FEM, see [Dong, Gamal and Atluri (2013); Dong and Atluri (2012a); Dong and Atluri (2012b)]. Although these methods can be applied to predict the effective elastic properties of SFRC, the determination of strength properties of fiber reinforced concrete remains an important task. This is because that few of these methods can be employed to calculate the local strain and stress fields. So the strength properties can't be evaluated by these methods. Moreover, under constant mechanical load, the concrete material has a creep phenomenon which is usually described in the viscoelastic model. So evaluation of the viscoelastic properties of concrete material is a meaningful work. In addition, few of the above mentioned methods can take into account all the microstructure characteristics of SFRC, especially for the random distribution characteristic of steel fibers.

In this paper, a statistical second-order two-scale (SSOTS) analysis method is introduced to predict the elastic, strength and viscoelastic properties of SFRC. The SSOTS method [Li and Cui (2005); Han, Cui and Yu (2008); Yang, Cui, Nie and Ma (2012); Yang, Cui, Nie, Wu, Yang and Wu (2013)] is on the basis of asymptotic expansion homogenization and Monte Carlo method and it can take into account all the microstructure information of SFRC. Above all, compared with the traditional homogenization method, the SSOTS method can effectively catch the local stain and stress information in the structures through the two-order displacement solution. Naturally, the strength limit parameters can be evaluated by introducing the appropriate strength criterion. According to the correspondence principle, the viscoelastic properties can also be evaluated by this method.

This paper is organized as follows. In section 2, the microstructure of SFRC is represented by setting up a statistic screen point wise and the formulation of SSOTS method is derived. Section 3 deals with the evaluation of the mechanical properties of SFRC. Finally, some numerical results are shown to verify the feasibility and effectiveness of this numerical method for SFRC. The influence of distribution of steel fiber to mechanical properties for SFRC is also displayed by calculating the mechanical parameters in three typical cases.

2 Statistical second-order two-scale method

2.1 Microstructure Representation of SFRC

Steel fiber reinforced concrete is a typical randomly heterogeneous material. Suppose that it is made from the concrete matrix and steel fiber reinforcement with random distribution. Each steel fiber can be regarded as an ellipsoid specially. An ellipsoid in 3D space can be denoted by 9 random parameters, including the coordinates of the center (x_1, x_2, x_3), the sizes of the long, middle and short axis (a, b, c) and three Euler angles $(\theta_1, \theta_2, \theta_3)$ of the rotations. Then the microstructure of the steel fiber reinforced concrete can be represented as follow:

1) At an arbitrary point x in the macroscopic structure Ω, there exists a cell with size ε, satisfying $a \ll \varepsilon \ll L$, where L is the size of macroscopic structure Ω. And all the ellipsoids in the cell subject to a specific probability distribution model P.

2) Let the random vector $u = (x_1, x_2, x_3, a, b, c, \theta_1, \theta_2, \theta_3)$. Suppose that the cell εY contains I ellipsoids, where Y denotes the normalized cell. Then we define $\omega = (u_1, u_2, \cdots, u_{I-1}, u_I)$. Obviously, it is a sample of the probability distribution model P.

Then for a sample ω, the mechanical parameters of SFRC can be defined as follow:

$$A^{\varepsilon}(x,\omega) = \begin{cases} A^1, x \in \bigcup\limits_{i=1}^{I} e_i \\ A^2, x \in \varepsilon Y - \bigcup\limits_{i=1}^{I} e_i \end{cases} \tag{1}$$

where $\bigcup\limits_{i=1}^{I} e_i$ denotes the domain of fibers, $\varepsilon Y - \bigcup\limits_{i=1}^{I} e_i$ denotes the domain of concrete matrix. A^1 and A^2 are mechanical parameters of steel fiber and concrete matrix.

As the representation for the microstructure of SFRC is described, one of important issues is to repeatedly generate the samples subjected to prescribed probability distribution model in 3D domain. Yu et al. [Yu, Cui and Han (2008)] have developed an effective computer generation method for the composites with random distribution of large numbers of heterogeneous grains. In this paper, we will apply this computer method to generation the 3D model of SFRC. Taking the concept of scale separation, the SFRC is regarded as a homogenized medium in macroscopic scale. In the microscopic scale, it is heterogeneous. Fig. 1 shows the structure representation of SFRC in 3D domain.

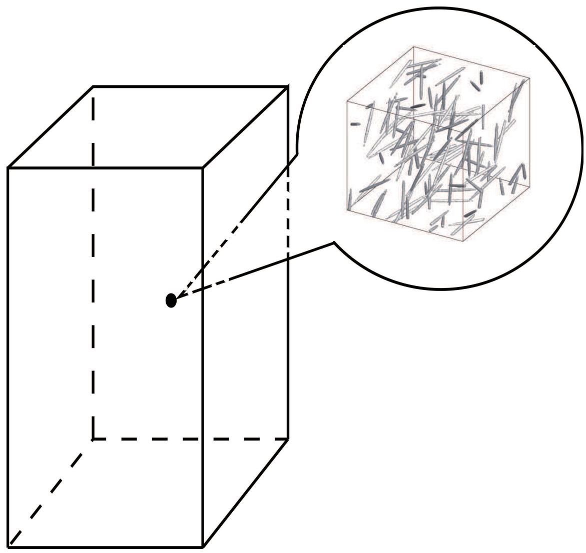

Figure 1: Structure representation of the steel fiber reinforced concrete

2.2 SSOTS formulation

According to the microstructure representation in Section 2.1, for the structure Ω, the boundary value problem of its mechanical behavior can be expressed as follows:

$$\begin{cases} -\frac{\partial}{\partial x_j}\left[a^{\varepsilon}_{ijhk}(x,\omega)\frac{1}{2}\left(\frac{\partial u^{\varepsilon}_h(x,\omega)}{\partial x_k}+\frac{\partial u^{\varepsilon}_k(x,\omega)}{\partial x_h}\right)\right]=f_i(x), x\in\Omega \\ \mathbf{u}^{\varepsilon}(x,\omega)=\bar{\mathbf{u}}(x), x\in\Gamma_1 \\ v_j a^{\varepsilon}_{ijhk}(x,\omega)\frac{1}{2}\left(\frac{\partial u^{\varepsilon}_h(x,\omega)}{\partial x_k}+\frac{\partial u^{\varepsilon}_k(x,\omega)}{\partial x_h}\right)=p_i(x), x\in\Gamma_2 \end{cases} \tag{2}$$

where $i,j,h,k=1,2,3$; $a^{\varepsilon}_{ijhk}(x,\omega)$ is the elastic coefficients; $f_i(x)$ is the body force assumed to be independent of ε; $\mathbf{u}^{\varepsilon}(x,\omega)$ is the displacement vector; Γ_1 and Γ_2 denote the boundary portions where displacement $\bar{\mathbf{u}}(x)$ and loads $p_i(x)$ are prescribed. Let $y = x/\varepsilon \in Y$, which denotes local coordinates on the normalized cell Y. Then $a^{\varepsilon}_{ijhk}(x,\omega) = a_{ijhk}(y,\omega)$. For a certain sample ω^s, the second-order two-scale approximate solution of displacement vector has the follow expansion:

$$\mathbf{u}^{\varepsilon}(x,\omega^s) \approx \mathbf{u}^0(x) + \sum_{l=1}^{2} \varepsilon^l \sum_{\alpha_1\cdots\alpha_l=1,2,3} \mathbf{N}_{\alpha_1\cdots\alpha_l}(y,\omega^s) \frac{\partial^l \mathbf{u}_0(x)}{\partial x_{\alpha_1}\cdots\partial x_{\alpha_l}} \tag{3}$$

where $\mathbf{u}^0(x)$ reflects the macroscopic behaviors of the structure, and is called as the homogenization solution; $N_{\alpha_1}(y,\omega^s)$ and $N_{\alpha_1\alpha_2}(y,\omega^s)$ involve the microstructural information, and are matrix-valued functions. And $\mathbf{u}^0(x)$, $N_{\alpha_1}(y,\omega^s)$ and $N_{\alpha_1\alpha_2}(y,\omega^s)$ are determined in the following ways:
1) For $l = 1$ and the sample ω^s, $N_{\alpha_1 m}(y,\omega^s)$ $(\alpha_1, m = 1,2,3)$ are solutions of the following problem:

$$\begin{cases} \frac{\partial}{\partial y_j}\left[a_{ijhk}(y,\omega^s)\frac{1}{2}\left(\frac{\partial N_{\alpha_1 hm}(y,\omega^s)}{\partial y_k} + \frac{\partial N_{\alpha_2 km}(y,\omega^s)}{\partial y_h}\right)\right] \\ = -\frac{\partial a_{ij\alpha_1 m}(y,\omega^s)}{\partial y_j}, y \in Y^s \\ N_{\alpha_1 m}(y,\omega^s) = 0, y \in \partial Y^s \end{cases} \tag{4}$$

2) The homogenized parameters $\hat{a}_{ijhk}(\omega^s)$ are calculated by the following formula

$$\hat{a}_{ijhk}(\omega^s) = \int_{Y^s}\left[a_{ijhk}(y,\omega^s) + a_{ijpq}(y,\omega^s)\frac{1}{2}\left(\frac{\partial N_{hpk}(y,\omega^s)}{\partial y_q} + \frac{\partial N_{hqk}(y,\omega^s)}{\partial y_p}\right)\right]dy \tag{5}$$

According to the Kolmogorov's strong law of large numbers, we defined the expected homogenized parameters $\widehat{a}_{ijhk}$ as follow

$$\widehat{a}_{ijhk} = \frac{\sum_{s=1}^{M} \hat{a}_{ijhk}(\omega^s)}{M} \tag{6}$$

where M is the number of samples.
3) $\mathbf{u}^0(x)$ is the homogenized displacement vector for the structure and is the solution of the following problem

$$\begin{cases} \frac{\partial}{\partial x_j}\left[\widehat{a}_{ijhk}\frac{1}{2}\left(\frac{\partial u_h^0(x)}{\partial x_k} + \frac{\partial u_k^0(x)}{\partial x_h}\right)\right] = f_i(x), x \in \Omega \\ \mathbf{u}^0(x) = \bar{\mathbf{u}}(x), x \in \Gamma_1 \\ v_j \widehat{a}_{ijhk}\frac{1}{2}\left(\frac{\partial u_h^0(x)}{\partial x_k} + \frac{\partial u_k^0(x)}{\partial x_h}\right) = p_i(x), x \in \Gamma_2 \end{cases} \tag{7}$$

4) For $l=2$, $N_{\alpha_1\alpha_2 m}(y,\omega^s)$ $(\alpha_1,\alpha_2,m=1,2,3)$ are solutions of the following problem

$$\begin{cases} -\frac{\partial}{\partial y_j}[a_{ijhk}(y,\omega^s)\frac{1}{2}(\frac{\partial N_{\alpha_1\alpha_2 hm}(y,\omega^s)}{\partial y_k}+\frac{\partial N_{\alpha_1\alpha_2 km}(y,\omega^s)}{\partial y_h})] \\ =-\widehat{a}_{ijhk}+a_{i\alpha_2 m\alpha_1}(y,\omega^s)+a_{i\alpha_2 hk}(y,\omega^s)\frac{\partial N_{\alpha_1 hm}(y,\omega^s)}{\partial y_k} \\ +\frac{\partial}{\partial y_j}(a_{ijh\alpha_2}(y,\omega^s)N_{\alpha_1 hm}(y,\omega^s)),y\in Y^s \\ N_{\alpha_1\alpha_2 m}(y,\omega^s)=0,y\in\partial Y^s \end{cases} \tag{8}$$

As the undetermined functions in (3) have been defined, the strain field in the whole structure can be approximately expressed as

$$\begin{aligned} &\varepsilon_{hk}^{\varepsilon}(x,\omega^s)=\frac{1}{2}(\frac{\partial u_h^{\varepsilon}(x,\omega^s)}{\partial x_k}+\frac{\partial u_k^{\varepsilon}(x,\omega^s)}{\partial x_h}) \\ &\approx\frac{1}{2}(\frac{\partial u_h^0(x)}{\partial x_k}+\frac{\partial u_k^0(x)}{\partial x_h}) \\ &+\frac{1}{2}(\frac{\partial N_{\alpha_1 hm}(y,\omega^s)}{\partial y_k}+\frac{\partial N_{\alpha_1 km}(y,\omega^s)}{\partial y_h})\frac{\partial u_m^0(x)}{\partial x_{\alpha_1}} \\ &+\varepsilon\frac{1}{2}(N_{\alpha_1 hm}(y,\omega^s)\frac{\partial^2 u_m^0(x)}{\partial x_{\alpha_1}\partial x_k}+N_{\alpha_1 km}(y,\omega^s)\frac{\partial^2 u_m^0(x)}{\partial x_{\alpha_1}\partial x_h}) \\ &+\varepsilon\frac{1}{2}(\frac{\partial N_{\alpha_1\alpha_2 hm}(y,\omega^s)}{\partial y_k}+\frac{\partial N_{\alpha_1\alpha_2 km}(y,\omega^s)}{\partial y_h})\frac{\partial^2 u_m^0(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}} \\ &+\varepsilon^2\frac{1}{2}N_{\alpha_1\alpha_2 hm}(y,\omega^s)\frac{\partial^3 u_m^0(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}\partial x_k}+\varepsilon^2\frac{1}{2}N_{\alpha_1\alpha_2 km}(y,\omega^s)\frac{\partial^3 u_m^0(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}\partial x_h} \end{aligned} \tag{9}$$

And by the generalized Hooke's law, the expression of the stress field can be obtained

$$\sigma_{ij}^{\varepsilon}(x,\omega^s)=a_{ijhk}^{\varepsilon}(x,\omega^s)\varepsilon_{hk}^{\varepsilon}(x,\omega^s) \tag{10}$$

Then the stresses at any point x are naturally used to evaluate the strength properties of SFRC.

3 Macroscopic mechanical properties of steel fiber reinforced concrete

This section devotes to deal with the evaluation of the elastic, strength and viscoelastic properties of SFRC. As emphasized by Eq. (5) and (6), the homogenized elastic coefficients can be computed in the same way as the traditional homogenization method. And the evaluation of strength and viscoelastic properties is modeled as follow.

3.1 Strength limit of SFRC

To evaluate the strength limit properties, selecting a suitable strength criterion is very important. In this paper, the strength capacities of concrete matrix are assumed to be described by a Drucker-Prager [Pasa-Dutra, Maghous and Campos-

Filho (2013)]. The Drucker-Prager criterion can be expressed in the following form:

$$F^m(\sigma) = \sqrt{\frac{3}{2}}\,\|s\| + \alpha_m(tr\sigma - \sigma_m) - \sigma_m \le 0 \tag{11}$$

where σ represents the stress field which can be obtained by the Eq. (10), and $\|s\| = (s:s)^{1/2}$ is the norm of the deviatoric stress tensor; σ_m denotes the strength limit of the material; α_m is a scalar ranging between 0 and 1. When the scalar σ_m equals 0, the Drucker-Prager criterion will degenerate to the Von Misses criterion in particular. The later criterion is applied to the steel fibers, when evaluating the strength parameters of the SFRC. In this paper, we generate a set of samples ω^s to obtain their strength limit. Naturally, by using Kolmogorov's strong law of large numbers, the expected strength limit of the SFRC can be expressed as follow

$$\widehat{S} = \frac{\sum\limits_{s=1}^{M} S(\omega^s)}{M} \tag{12}$$

where M is the number of samples.

3.2 Viscoelastic behavior of SFRC

Under constant mechanical loading, the concrete specimen exhibits the creep phenomenon, which can be described as a non-ageing viscoelastic model. In non-ageing liner viscoelasticity, the stress tensor is related to the strain history, and has the following representation:

$$-\frac{\partial}{\partial x_j}\left[\frac{1}{2}\int_0^t G^{\varepsilon}_{ijhk}(x,\omega,t-\tau)\frac{\partial}{\partial \tau}\left(\frac{\partial u^{\varepsilon}_h(x,\omega,\tau)}{\partial x_k} + \frac{\partial u^{\varepsilon}_k(x,\omega,\tau)}{\partial x_h}\right)d\tau\right] = f_i(x) \tag{13}$$

where $G^{\varepsilon}_{ijhk}(x,\omega,t)$ refers to fourth-order relaxation modulus tensor. Recalling that the Laplace transformation is defined as

$$\hat{f}(s) = L(f) = \int_0^{\infty} f(t)e^{-st}dt \tag{14}$$

so that the viscoelastic relationship writes in terms of Laplace transformation

$$-\frac{\partial}{\partial x_j}\left[s\hat{G}_{ijhk}\frac{1}{2}\left(\frac{\partial \hat{u}^{\varepsilon}_h}{\partial x_k} + \frac{\partial \hat{u}^{\varepsilon}_k}{\partial x_h}\right)\right] = \hat{f} \tag{15}$$

which expresses an elastic type relationship formally in the Laplace domain space. The simple and commonly used model for linear viscoelastic solid consists in the association in series of a spring (stiffness E_1) and a Kelvin element with spring stiffness E_2 and dashpot viscosity η. The relaxation modulus $G(t)$ can be expressed as

$$G(t) = \frac{E_1}{E_1 + E_2}(E_2 + E_1 e^{-t/T}) \tag{16}$$

where $T = \frac{\eta}{E_1+E_2}$ is the relaxation characteristic time. It is observed that $G(0) = E_1$ and $G(\infty) = \frac{E_1 E_2}{E_1+E_2}$ respectively refers to the instantaneous and asymptotic relaxation modulus. The homogenized relaxation modulus in Laplace domain can be computed by the elastic homogenization procedure in Section 2. Then the homogenized relaxation and creep modulus can be obtained by the inverse Laplace transformation respectively.

4 Numerical examples

In this section, some numerical examples are given to show that the present algorithm is feasible and effective in predicting the mechanical properties of SFRC.

Ashour et al. and Thomas et al. have tested the elastic and strength properties of the steel fiber reinforced concrete by mechanics experiment respectively [Ashour, Wafa and Kamal (2000); Thomas and Ramaswamy (2007)]. The steel fibers used in their experiments have the same mechanical properties: Young's modulus 210000Mpa, Poisson's ratio 0.30. The values of mechanical properties of the concrete as provided in their experiments are list in Tab. 1. The numerical results obtained by the SSOTS method are shown in follows, and the results are based on the statistics of 50 samples. At first, the results of homogenized Young's modulus estimated by

Table 1: Mechanical properties of concrete in experiments [Thomas and Ramaswamy (2007); Ashour, Wafa and Kamal (2000)]

Materials	Young's modulus (MPa)	Poisson's ratio	Compressive strength (MPa)	Split tensile strength (MPa)
C35	28700	0.182	29.80	3.93
C65	37500	0.201	56.00	5.19
C85	41700	0.210	72.40	5.76
NSC	24612	0.200	48.61	3.69
MSC	35443	0.200	78.50	5.05
HSC	38423	0.200	102.40	5.59

SSOTS are obtained. Fig. 2 and Fig. 3 respectively display the estimates of Young's modulus of SFRC, together with the experimental data. As it appears from the follow figures, the numerical results of SSOTS method agree with the experimental data and the Young's modulus increases as the volume fraction of steel fiber increasing.

Next, the numerical results of the strength properties of SFRC predicted by the SSOTS method are shown. In engineering, the compressive strength and tensile strength of concrete are commonly concerned. Unlike other materials, the compressive strength of concrete is far larger than its tensile strength. The two strength limit parameters of SFRC are evaluated by the above SSOTS method and the comparison of numerical results and experimental results is shown in Tab. 2. The numerical results have the same tendency with the experimental data and the discrepancy of them remains lower than 10%.

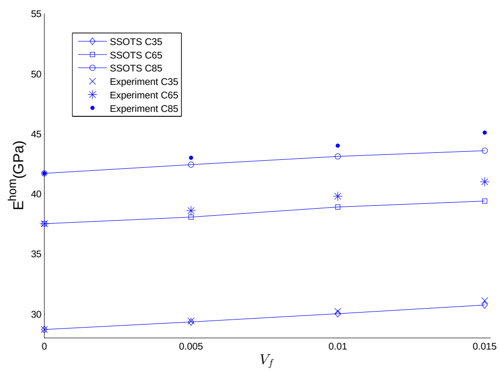

Figure 2: Young's modulus predicted by SSOTS method and experiment data [Thomas and Ramaswamy (2007)]]

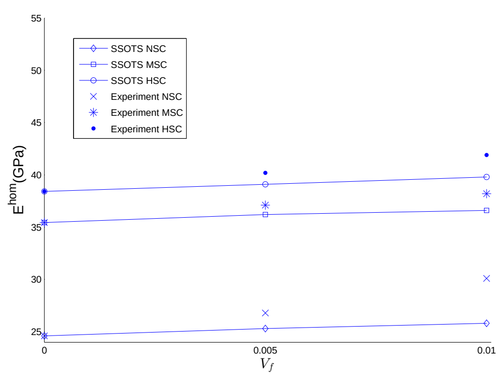

Figure 3: Young's modulus predicted by SSOTS method and experiment data [Ashour, Wafa and Kamal (2000)]

Table 2: Numerical and experimental results for strength properties of SFRC [Ashour, Wafa and Kamal (2000); Thomas and Ramaswamy (2007)]

Materials	Volume fractions (%)	Compressive strength (MPa)		Split tensile strength(MPa)	
		Experiment	SSOTS results	Experiment	SSOTS results
C35	0.5	30.5	30.3	4.37	4.03
	1.0	31.0	30.8	4.87	4.40
	1.5	32.3	31.5	5.43	5.00
C65	0.5	57	56.5	5.81	5.34
	1.0	57.8	56.5	5.81	5.34
	1.5	59.4	58.8	7.33	6.65
C85	0.5	73.6	73.1	6.48	6.14
	1.0	74.8	74.0	7.20	6.80
	1.5	77.0	76.1	7.98	7.40
NSC	0.5	55.82	52.60	4.67	4.30
	1.0	65.16	59.50	6.72	6.20
MSC	0.5	81.99	81.60	6.01	5.50
	1.0	87.37	83.48	7.69	6.95
HSC	0.5	106.91	105.0	6.53	6.08
	1.0	111.44	109.80	8.13	7.32

An experimental study on creep of steel fiber reinforced concrete has been performed by Mangat and Azari [Mangat and Azari (1985)]. The viscoelastic model parameters are identified as follows. The instantaneous relaxation modulus is determined by condition $G(0) = E_1 = \sigma_c / \varepsilon(0)$ and the value of E_2 is fixed by the asymptotic condition $G(\infty) = \frac{E_1 E_2}{E_1+E_2} = \sigma_c / \varepsilon(\infty)$. The dashpot viscosity η is estimated by curve fitting from the experimental creep curve of plain concrete. Then $E_1 = 20.4GPa$, $E_2 = 7.7GPa$ and $\eta = 184.1GPa \times days$. Through the present method in Section 2, the homogenized viscoelastic properties of SFRC can be evaluated. Fig. 4 shows the relationship between the creep strain of SFRC and time under load at 0.3 stress/strength ratio. Chern and Young also investigated the creep performance of steel fiber reinforced concrete experimentally [Chern and Young (1989)]. The viscoelastic model parameters is identified as follow, $E_1 = 23.5GPa$, $E_2 = 4.4GPa$ and $\eta = 184.7GPa \times days$. Fig. 5 shows the creep function of SFRC under load at 0.25 stress/strength ratio. The results show that the steel fiber reduces creep deformation clearly and the effect is more obvious as the volume fraction of steel fiber increasing. The above numerical results demonstrate that the present model is applicable and the SSOTS method is valid to predict the mechanical properties of steel fiber reinforced concrete.

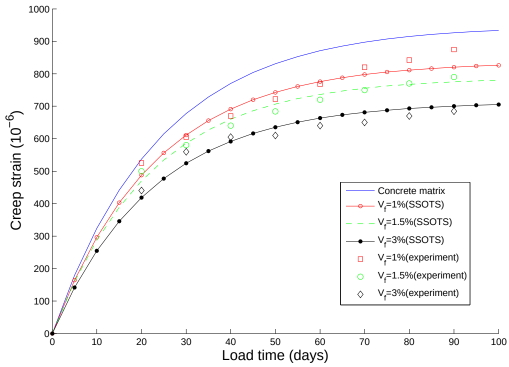

Figure 4: Creep strain predicted by SSOTS method and experimentally measured at a stress/strength ratio 0.3 [Mangat and Azari (1985)]

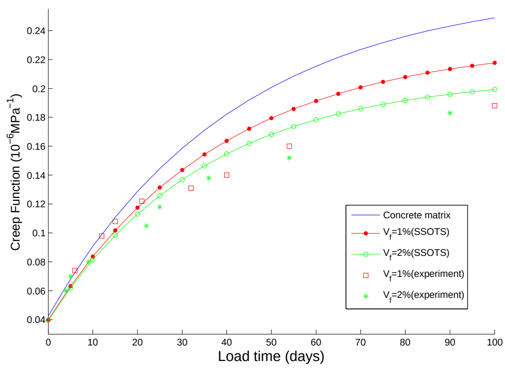

Figure 5: Creep function predicted by SSOTS method and experimentally measured at a stress/strength ratio 0.25 [Chern and Young (1989)]

4.1 Discussion on the distribution of steel fiber

In order to investigate the influence of distribution of steel fibers on the mechanical properties of SFRC, three typical distribution models of fibers are considered, as shown in Fig. 6. We calculate the mechanical properties of steel fiber reinforced concrete with these three different distribution models respectively. Fig. 7, Fig. 8 and Fig. 9 show the numerical results predicted by SSOTS method. The results show that the distribution of fibers has a certain impact on the mechanical properties of steel fiber reinforced concrete, especially when the steel fibers hold in a high volume fraction relatively. Due to the heterogeneous distribution of steel fiber, the anisotropy of SFRC is emerged and the enhancement effect is more obvious in the specific direction when the steel fiber has an orientation distribution. These results may provide a reference for manufacturing the steel fiber reinforced concrete.

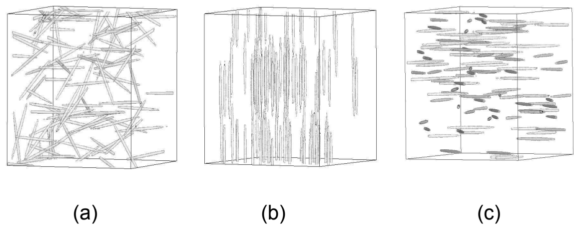

Figure 6: Microstructure representation of fiber distribution: (a) random distribution, (b) orientation distribution, (c) layered distribution

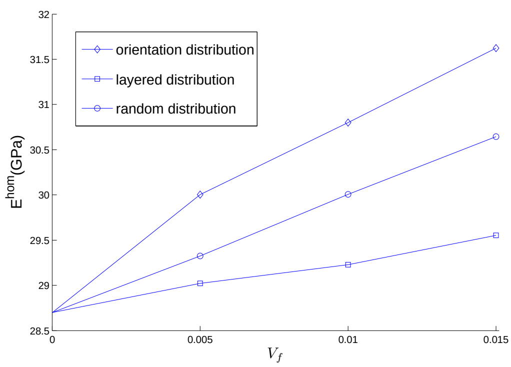

Figure 7: Influence of fiber distribution on the homogenized Young's modulus

5 Conclusions

In this paper, a SSOTS method is introduced to predict the macroscopic mechanical properties of steel fiber reinforced concrete. The microstructure of SFRC is first described by the statistical random model. Then the SSOTS formulation is derived to estimate the elastic properties analytically. This model successfully predicts the values of the Young's modulus of SFRC, as emphasized by comparison of the numerical results in the present work and the experiment data. Through the two-order displacement solution, the expression of the stress field is obtained. Then the strength limit parameters are evaluated by introducing the Drucker-Prager strength

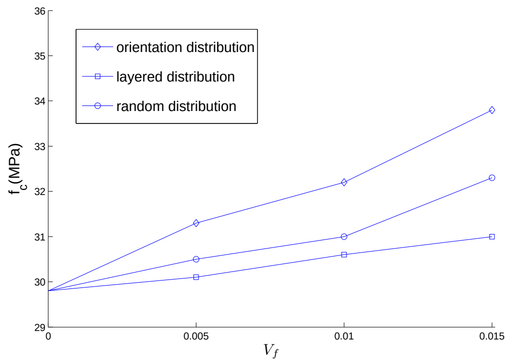

Figure 8: Influence of fiber distribution on the compressive strength

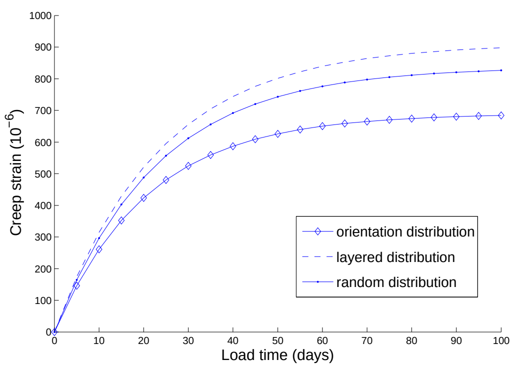

Figure 9: Influence of fiber distribution on the creep strain of SFRC

criterion. In addition, the viscoelastic behavior of SFRC is also investigated by the presented method and the correspondence principle. The numerical results of strength and viscoelastic properties are also compared with the available experiment data. Moreover, the influence of distribution of steel fibers on the mechanical properties is investigated. The results show that the SSOTS method is feasible and effective for predicting the mechanical properties of SFRC.

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