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Localization in Time of Solutions for Thermoelastic Micropolar Materials with Voids

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Abstract: In this study we want to decide whether the decay of the solutions of the mixed initial boundary value problem in the context of thermoelasticiy of micropolar bodies with voids is sufficiently fast to guarantee that they vanish after a finite time. In fact, we prove that the effect of the micropolar structure in combination with the thermal and porous dissipation can not determine the thermomechanical deformations vanish after a finite time.

Keywords: localization in time, thermoelasticity, micropolar, voids, backward in time.

1 Introduction

As is already known, the theory of materials with voids or vacuous pores is the simplest extension of the classical theory of elasticity and was first proposed by Nunziato and Cowin (1979). In this theory the authors introduce an additional degree of freedom in order to develope the mechanical behavior of a body in which the skeletal material is elastic and interstices are voids of material. It is worth recalling that porous materials have applications in many fields of engineering such as petroleum industry, material science, biology and so on. The intended applications of the theory are to geological materials like rocks and soil and to manufactured porous materials. The linear theory of elastic materials with voids was developed by Cowin and Nunziato (1983). Here the uniqueness and weak stability of solutions are also derived. Iesan (1986) has established the equations of thermoelasticity of materials with voids.

An extension of these results to cover other theories of materials with voids was been made in our studies [Marin (1999, 2010a, 2010b); Marin, Mahmoud and Al-Basyouni (2013); Munoz-Rivera and Quintanilla (2008)].

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In the case of systems where dissipation mechanism is sufficiently strong, is already proved the localization of solutions in the time variable, that is the decay of the solutions is quite accelerated to ensure that they vanish after a finite time. However, is several cases there is no proof of the impossibility of localization of the solutions. Even if there are cases of thermoelasticity for bodies with voids in which the decay of solutions can be controlled for exponential functions, and, also, for polynomials functions, still not proven the impossibility of localization for the solutions. In previous studies on this subject, the authors demonstrated different upper bounds for the decay of solutions.

In the case of the present study are important the lower bounds for the decay of solutions. It is worth to mention two studies [Casas and Quintanilla (2005); Nunziato (1979)], in which the authors have shown that after a certain period of time, the deformations of the thermoelastic bodies with voids become so small that they can be neglected. However, we are not sure that these deformations are null for every positive time. If we prove the uniqueness of solutions for the backward in time problem of the thermoelasticity of micropolar bodies with voids, then we are sure that the only solution for this problem which vanishes for every $t \geq 0$ is the null solution. In other words, we deduce the impossibility of localization of the solutions of the problem of the thermoelasticity of micropolar bodies with voids. Regarding the uniqueness of solutions for the backward in time problem, have noted the contribution of Ciarletta (2002), but he proved the uniqueness of solutions for the backward in time problem in the context of classical thermoelasticity.

Some results regarding back in time problems have been considered also by Ames and Payne (1991) in order to obtain stabilizing criteria for solutions of the boundary-final value problem.

Quintanilla (2007) improves the uniqueness result obtained by Ciarletta, using more concrete assumptions, in particular considering a strictly positive heat capacity. So, it is proved the impossibility of localization in time of the solutions of the forward in time problem for the linear thermoelasticity of Green and Naghdi's type.

Other results regarding the backward in time problem, but for classical porous elastic materials has been studied by Iovane and Passarella (2004).

As we can see, such backward in time problem has never been studied for the case of micropolar thermoelastic bodies with voids.

We still use the paper of Ciarletta as a guide in getting results of this study. The main result of our paper is based on Lagrange identities and energy arguments.

2 Organization of a paper

A paper for publication in CMC must contain a title, names and affiliations of the authors, a list of keywords, a brief abstract at the beginning of the main body, a conclusion section at the end of main body, and a list of references that follows the conclusions section.

In the main body of the paper, three different levels of headings (for sections, subsections, and subsubsections) may be used. The typesetting style for these headings is presented in the next section.

3 Basic equations

An anisotropic elastic material is considered. Assume a such body that occupies a properly regular region B of three-dimensional Euclidian space R^3 bounded by a piecewise smooth surface ∂B and we denote the closure of B by $\bar{B}$. The boundary ∂B is smooth enough to apply the divergence theorem.

We use a fixed system of rectangular Cartesian axes $Ox_i, (i = 1,2,3)$ and adopt Cartesian tensor notation. A superposed dot stands for the material time derivate while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates. Einstein summation convention on repeated indices is used. Also, the spatial argument and the time argument of a function will be omitted when there is no likehood of confusion.

We consider the mixed problem associated with the theory of thermoelasticity of micropolar bodies with voids on the time interval I. So, in the absence of supply terms, it is known that the basic equations on $B \times I$ are, (see, for instance [Marin, Mahmoud and Al-Basyouni (2013)])

$$t_{ij,j} = \rho \ddot{u}_i, \tag{1}$$

$$m_{ij,j} + \varepsilon_{ijk} t_{jk} = I_{ij} \ddot{\varphi}_j, \qquad h_{i,i} + g = J\ddot{\phi}, \tag{2}$$

$$\rho \dot{\eta} = q_{i,i}. \tag{3}$$

The equations (1) are the motion equations, (2) is the balance of the equilibrated forces and (3) is the energy equation.

In the following, we restricte our considerations only to the case where the materials have a center of symmetry. Consequently, the constitutive tensors of odd order must vanish and the constitutive equations become

$$
\begin{aligned}
&t_{ij} = C_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + B_{ij}\phi + \beta_{ij}\theta,\\
&m_{ij} = B_{mnij}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + C_{ij}\phi + \alpha_{ij}\theta,\\
&h_i = A_{ij}\phi_{,j},\\
&g = -B_{ij}\varepsilon_{ij} - C_{ij}\gamma_{ij} - \xi\phi + m\theta - \tau\dot{\phi},\\
&\rho\eta = -\beta_{ij}\varepsilon_{ij} - \alpha_{ij}\gamma_{ij} + m\phi + c\theta,\\
&q_i = K_{ij}\theta_{,j};
\end{aligned}
\tag{4}
$$

where the strain tensors ε_{ij}, γ_{ij} and the temperature are defined by means of the kinetic relations

$$
\varepsilon_{ij} = u_{j,i} + \varepsilon_{jik}\varphi_k,\ \gamma_{ij} = \varphi_{j,\,i}. \tag{5}
$$

In the above equations we have used the following notations: ρ-the constant mass density; η-the specific entropy; T_0-the constant absolute temperature of the body in its reference state; I_{ij}-coefficients of microinertia; J- a positive function and it is the product of the mass density and the equilibrated inertia; u_i-the components of displacement vector; φ_i-the components of microrotation vector; ϕ-the volume distribution function which in the reference state is ϕ_0; θ-the temperature variation measured from the reference temperature T_0; $\varepsilon_{ij}, \gamma_{ij}$-kinematic characteristics of the strain; t_{ij}-the components of the stress tensor; m_{ij}-the components of the couple stress tensor; h_i-the components of the equlibrated stress vector; q_i-the components of the heat flux vector; g-the intrinsic equilibrated force; K_{ij}-the heat conductivity tensor; c=the heat capacity which is assumed positive; $A_{ijmn}, B_{ijmn}, ..., \alpha_{ij}$ from the constitutive equations are the characteristic functions of the material, and they obey the symmetry relations

$$
A_{ijmn} = A_{mnij},\ C_{ijmn} = C_{mnij},\ A_{ij} = A_{ji},\ K_{ij} = K_{ji}. \tag{6}
$$

As time interval I we take $I = (-\infty, 0]$ and consider the mixed boundary-final problem $\mathscr{P}$ defined by the system of equations (1)-(5), the final conditions in $\bar{B}$

$$
\begin{aligned}
&u_i(x,0) = u_i^0(x), && \dot{u}_i(x,0) = u_i^1(x),\\
&\varphi_i(x,0) = \varphi_i^0(x), && \dot{\varphi}_i(x,0) = \varphi_i^1(x),\\
&\phi(x,0) = \phi_i^0(x), && \dot{\phi}(x,0) = \phi_i^1(x),\\
&\theta(x,0) = \theta^0(x), && x \in \bar{B},
\end{aligned}
\tag{7}
$$

and the following null boundary conditions

$$
\begin{aligned}
&u_i(x,\ t) = 0 \ \text{ on } \ \partial B_1 \times (-\infty, 0],\ t_i = 0 \ \text{ on } \ \partial B_1^c \times (-\infty, 0]\\
&\varphi_i(x,\ t) = 0 \ \text{ on } \ \partial B_2 \times (-\infty, 0],\ m_i = 0 \ \text{ on } \ \partial B_2^c \times (-\infty, 0]\\
&\phi(x,\ t) = 0 \ \text{ on } \ \partial B_3 \times (-\infty, 0],\ h = 0 \ \text{ on } \ \partial B_3^c \times (-\infty, 0]\\
&\theta(x,\ t) = 0 \ \text{ on } \ \partial B_4 \times (-\infty, 0],\ q = 0 \ \text{ on } \ \partial B_4^c \times (-\infty, 0]
\end{aligned} \tag{8}
$$

with usual notations

$$
t_i = t_{ji} n_j, \quad m_i = m_{ji} n_j, \quad h = h_i n_i, \quad q = q_i n_i,
$$

where n_i are the components of the outward unit normal to the boundary surface. Also, ∂B_1, ∂B_2, ∂B_3 and ∂B_4 with respective complements ∂B_1^c, ∂B_2^c, ∂B_3^c and ∂B_4^c are the subsets of the surface ∂B such that

$$
\begin{aligned}
&\partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c = \partial B_3 \cap \partial B_3^c = \partial B_4 \cap \partial B_4^c = \emptyset\\
&\partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c = \partial B_3 \cup \partial B_3^c = \partial B_4 \cup \partial B_4^c = \partial B
\end{aligned}
$$

Introducing (5) and (4) into equations (1), (2) and (3), we obtine the following system of equations

$$
\begin{aligned}
\rho \ddot{u}_i &= \left(A_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + B_{ij}\phi + \beta_{ij}\theta\right)_{,j},\\
I_{ij}\ddot{\varphi}_j &= \left(B_{mnij}\varepsilon_{mn} + C_{ijmn}\gamma_{mn} + C_{ij}\phi + \alpha_{ij}\theta\right)_{,j}\\
&\quad + \varepsilon_{ijk}\left(A_{jkmn}\varepsilon_{mn} + B_{jkmn}\gamma_{mn} + B_{jk}\phi + \beta_{jk}\theta\right),\\
J\ddot{\phi} &= \left(A_{ij}\phi_{,i}\right)_{,j} - B_{ij}\varepsilon_{ij} - C_{ij}\gamma_{ij} - \xi\phi + m\theta - \tau\dot{\phi},\\
c\dot{\theta} &= \left(K_{ij}\theta_{,i}\right)_{,j} + \beta_{ij}\dot{\varepsilon}_{ij} + \alpha_{ij}\dot{\gamma}_{ij} - m\dot{\phi}.
\end{aligned} \tag{9}
$$

By a solution of the mixed boundary-final value problem of the theory of thermoelasticity of micropolar bodies with voids in the cylinder $\Omega_0 = B \times (-\infty, 0]$ we mean an ordered array $(u_i, \varphi_i, \theta, \sigma)$ which satisfies the system of equations (9) for all $(x,t) \in \Omega_0$, the boundary conditions (8) and the final conditions (7).

4 Basic relations

We need to impose the positivity of several tensors. Also, in all what follows we shall use the following assumptions on the material properties

(i) $\rho(x) \geq \rho^0 > 0$, $I_{ij}(x) \geq I_{ij}^0 > 0$, $J(x) \geq J^0 > 0$, $c(x) \geq c^0 > 0$, $\tau(x) \geq \tau^0 > 0$,
(ii) $K_{ij}\eta_i\eta_j \geq K^0\eta_i\eta_i$, for every η_i, where $K^0 > 0$.

(iii) $A_{ijmn}x_{ij}x_{mn}+2B_{ijmn}x_{ij}y_{mn}+C_{ijmn}y_{ij}y_{mn}+2B_{ij}x_{ij}\omega+$
$+2C_{ij}y_{ij}\omega+\xi\omega^2\geq 0$, for every $x_{ij},\ y_{ji},\ \omega$;
(iv) $A_{ij}\xi_i\xi_j\geq 0$ for every ξ_i.

These assumptions are in agreement with the usual restrictions imposed in the mechanics of continua. While the interpretation of conditions (i) is obvious, the assumption (ii) represent a considerable strenghtening of the entropy production inequality.

Conditions (iii) and (iv) ensure that the internal energy is positive and the best of their interpretation finds its place in the theory of mechanical stability.

By using an appropiate change of variables and notations suitably chosen, we can transform the mixed boundary-final value problem $\mathscr{P}$ into the mixed initial boundary value problem $\mathscr{P}^*$. In other words, we set, generally speaking, $f^*(t^*)=f(t)$, with $t^*=-t$. For the sake of simplicity, we remove the sign * from the notations such that we obtain the problem $\mathscr{P}^*$, called the backward in time problem corresponding to the problem of thermoelasticity of micropolar bodies with voids. The problem $\mathscr{P}^*$ is defined by (see also Ciarletta (2002)):

- the system of equations

$$\begin{aligned}
&\rho\ddot{u}_i=\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)_{,j},\\
&I_{ij}\ddot{\varphi}_j=\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)_{,j}\\
&\qquad+\varepsilon_{ijk}(A_{jkmn}\varepsilon_{mn}+B_{jkmn}\gamma_{mn}+B_{jk}\phi+\beta_{jk}\theta),\\
&J\ddot{\phi}=\left(A_{ij}\phi_{,i}\right)_{,j}-B_{ij}\varepsilon_{ij}-C_{ij}\gamma_{ij}-\xi\phi+m\theta+\tau\dot{\phi},\\
&c\dot{\theta}=-\left(K_{ij}\theta_{,i}\right)_{,j}+\beta_{ij}\dot{\varepsilon}_{ij}+\alpha_{ij}\dot{\gamma}_{ij}-m\dot{\phi}.
\end{aligned}\tag{10}$$

that occur for any $(x,t)\in B\times[0,\infty)$;

- the constitutive equations (4) in $\bar{B}\times[0,\infty)$;

- the geometric equations (4) on $\bar{B}\times[0,\infty)$;

- the following homogeneous boundary conditions

$$\begin{aligned}
&u_i(x,\,t)=0\ \text{ on }\ \partial B_1\times[0,\infty),\quad t_i=0\ \text{ on }\ \partial B_1^c\times[0,\infty)\\
&\varphi_i(x,\,t)=0\ \text{ on }\ \partial B_2\times[0,\infty),\quad m_i=0\ \text{ on }\ \partial B_2^c\times[0,\infty)\\
&\phi(x,\,t)=0\ \text{ on }\ \partial B_3\times[0,\infty),\quad h=0\ \text{ on }\ \partial B_3^c\times[0,\infty)\\
&\theta(x,\,t)=0\ \text{ on }\ \partial B_4\times[0,\infty),\quad q=0\ \text{ on }\ \partial B_4^c\times[0,\infty)
\end{aligned}\tag{11}$$

where ∂B_i and ∂B_i^c are defined in Section 2;

- the initial conditions (7) where $u_i^0,\ u_i^1,\ \varphi_i^1,\ \varphi_i^0,\ \phi^0,\ \phi^1$ and θ^0 are prescribed continuous functions on B.

We now state and prove some basic relations which qbe used to prove the main result of our study. The first of these is an energy relation.

Proposition 1. *If* $(u_i,\ \varphi_i,\ \psi,\ \theta)$ *is a solution of the problem consists of system of equations (10), initial conditions (7) and the boundary conditions (11), then we have the following equality*

$$\begin{aligned}&\frac{1}{2}\int_B\big(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\varphi}_i\dot{\varphi}_j+J\dot{\phi}^2+c\theta^2+A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}\\&+C_{ijmn}\gamma_{mn}\gamma_{ij}+2B_{ij}\varepsilon_{ij}\phi+2C_{ij}\gamma_{ij}\phi+A_{ij}\phi_{,i}\phi_{,j}+\xi\phi^2\big)\,dV\\&=\int_0^t\int_B\big(K_{ij}\theta_{,i}\theta_{,j}+\tau\dot{\phi}^2\big)\end{aligned}\tag{12}$$

Proof. First, we multiply $(10)_1$ by $\dot{u}_i$ and after simple calculations we obtain

$$\begin{aligned}\rho\dot{u}_i\ddot{u}_i=&\left[\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)\dot{u}_i\right]_{,j}\\&-\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)\dot{u}_{i,j}\end{aligned}\tag{13}$$

If we multiply $(10)_2$ by $\dot{\varphi}_i$ then using the product rule derivation we deduce

$$\begin{aligned}I_{ij}\ddot{\varphi}_j\dot{\varphi}_i=&\left[\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)\dot{\varphi}_i\right]_{,j}\\&-\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)\dot{\varphi}_{i,j}\\&+\varepsilon_{ijk}\left(A_{jkmn}\varepsilon_{mn}+B_{jkmn}\gamma_{mn}+B_{jk}\phi+\beta_{jk}\theta\right)\dot{\varphi}_i\end{aligned}\tag{14}$$

Now, we multiply $(10)_3$ by $\dot{\phi}$ and after simple calculations we lead to the result

$$J\dot{\phi}\ddot{\phi}=\left(A_{ij}\phi_{,i}\dot{\phi}\right)_{,j}-A_{ij}\phi_{,i}\dot{\phi}_{,j}-B_{ij}\varepsilon_{ij}\dot{\phi}-C_{ij}\gamma_{ij}\dot{\phi}-\xi\phi\dot{\phi}+m\theta\dot{\phi}+\tau\dot{\phi}^2\tag{15}$$

Finally, we multiply $(10)_4$ by θ and obtain

$$c\theta\dot{\theta}=-\left(K_{ij}\theta_{,i}\theta\right)_{,j}+K_{ij}\theta_{,i}\theta_{,j}+\beta_{ij}\dot{\varepsilon}_{ij}\theta+\alpha_{ij}\dot{\gamma}_{ij}\theta-m\dot{\phi}\theta\tag{16}$$

If we add the equalities (13) to (16), term by term, and use geometric equations (5), we find the relation

$$\begin{aligned}&\rho\dot{u}_i\ddot{u}_i+I_{ij}\ddot{\varphi}_j\dot{\varphi}_i+J\dot{\phi}\ddot{\phi}+c\theta\dot{\theta}=\left[\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)\dot{u}_i\right]_{,j}\\&+\left[\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)\dot{\varphi}_i\right]_{,j}\\&-\left(A_{ijmn}\varepsilon_{mn}\dot{\varepsilon}_{ij}+B_{ijmn}\gamma_{mn}\dot{\varepsilon}_{ij}+B_{ij}\phi\dot{\varepsilon}_{ij}+\beta_{ij}\theta\dot{\varepsilon}_{ij}\right)\\&-\left(B_{mnij}\varepsilon_{mn}\dot{\gamma}_{ij}+C_{ijmn}\gamma_{mn}\dot{\gamma}_{ij}+C_{ij}\phi\dot{\gamma}_{ij}+\alpha_{ij}\theta\dot{\gamma}_{ij}\right)\\&+\left(A_{ij}\phi_{,i}\dot{\phi}\right)_{,j}-A_{ij}\phi_{,i}\dot{\phi}_{,j}-B_{ij}\varepsilon_{ij}\dot{\phi}-C_{ij}\gamma_{ij}\dot{\phi}-\xi\phi\dot{\phi}+\tau\dot{\phi}^2-\left(K_{ij}\theta_{,i}\theta\right)_{,j}\\&+K_{ij}\theta_{,i}\theta_{,j}+\beta_{ij}\dot{\varepsilon}_{ij}\theta+\alpha_{ij}\dot{\gamma}_{ij}\theta\end{aligned}\tag{17}$$

Equality (17) can be rewritten in the form

$$\begin{aligned}
&\rho\dot{u}_i\ddot{u}_i+I_{ij}\dot{\varphi}_i\ddot{\varphi}_j+J\dot{\phi}\ddot{\phi}+c\theta\dot{\theta}\\
&+A_{ijmn}\varepsilon_{mn}\dot{\varepsilon}_{ij}+B_{ijmn}\left(\gamma_{mn}\dot{\varepsilon}_{ij}+\dot{\gamma}_{mn}\varepsilon_{ij}\right)+C_{ijmn}\gamma_{mn}\dot{\gamma}_{ij}\\
&+B_{ij}\left(\phi\dot{\varepsilon}_{ij}+\dot{\phi}\varepsilon_{ij}\right)+C_{ij}\left(\phi\dot{\gamma}_{ij}+\dot{\phi}\gamma_{ij}\right)+A_{ij}\phi_{,i}\dot{\phi}_{,j}+\xi\phi\dot{\phi}\\
&=\left[\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)\dot{u}_i\right]_{,j}\\
&+\left[\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)\dot{\varphi}_i\right]_{,j}\\
&+\left(A_{ij}\phi_{,i}\dot{\phi}\right)_{,j}-\left(K_{ij}\theta_{,i}\theta\right)_{,j}+K_{ij}\theta_{,i}\theta_{,j}+\tau\dot{\phi}^2
\end{aligned}\tag{18}$$

Now we integrate in (18) over $[0,t]\times B$ and apply the divergence theorem. Given the boundary and the initial conditions, we are led to the desired result (12). □

In next proposition we prove a second useful relation which is also an energy relation.

Proposition 2. *If $(u_i,\ \varphi_i,\ \psi,\ \theta)$ is a solution of the problem consists of system of equations (10), initial conditions (7) and the boundary conditions (11), then we have the following equality*

$$\begin{aligned}
&\frac{1}{2}\int_B\left(\rho\dot{u}_i\dot{u}_i+I_{ij}\dot{\varphi}_i\dot{\varphi}_j+J\dot{\phi}^2-c\theta^2+A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}\right.\\
&\left.+C_{ijmn}\gamma_{mn}\gamma_{ij}+2B_{ij}\phi\varepsilon_{ij}+2C_{ij}\phi\gamma_{ij}+\xi\phi^2+A_{ij}\phi_{,i}\phi_{,j}\right)dV\\
&=-\int_0^t\int_B\left(K_{ij}\theta_{,i}\theta_{,j}-2\beta_{ij}\dot{\varepsilon}_{ij}\theta-2\alpha_{ij}\dot{\gamma}_{ij}\theta-2m\theta\dot{\phi}-\tau\dot{\phi}^2\right)dV
\end{aligned}\tag{19}$$

Proof. First, we multiply $(10)_1$ by $\dot{u}_i$ and obtain the equality (13). If we multiply $(10)_2$ by $\dot{\varphi}_i$ we deduce the equality (14). Now, we multiply $(10)_3$ by $\dot{\phi}$ and we lead to the equality (15). Finally, we multiply $(10)_4$ by $-\theta$ and obtain

$$-c\theta\dot{\theta}=\left(K_{ij}\theta_{,i}\theta\right)_{,j}-K_{ij}\theta_{,i}\theta_{,j}+-\beta_{ij}\dot{\varepsilon}_{ij}\theta-\alpha_{ij}\dot{\gamma}_{ij}\theta+m\dot{\phi}\theta \tag{20}$$

If we add the equalities (13) to (15) and (20), term by term, and use geometric equations (5), we find the relation

$$\begin{aligned}
&\rho\dot{u}_i\ddot{u}_i+I_{ij}\dot{\varphi}_i\ddot{\varphi}_j+J\dot{\phi}\ddot{\phi}-c\theta\dot{\theta}\\
&+A_{ijmn}\varepsilon_{mn}\dot{\varepsilon}_{ij}+B_{ijmn}\left(\gamma_{mn}\dot{\varepsilon}_{ij}+\dot{\gamma}_{mn}\varepsilon_{ij}\right)+C_{ijmn}\gamma_{mn}\dot{\gamma}_{ij}\\
&+B_{ij}\left(\phi\dot{\varepsilon}_{ij}+\dot{\phi}\varepsilon_{ij}\right)+C_{ij}\left(\phi\dot{\gamma}_{ij}+\dot{\phi}\gamma_{ij}\right)+A_{ij}\phi_{,i}\dot{\phi}_{,j}+\xi\phi\dot{\phi}\\
&=\left[\left(A_{ijmn}\varepsilon_{mn}+B_{ijmn}\gamma_{mn}+B_{ij}\phi+\beta_{ij}\theta\right)\dot{u}_i\right]_{,j}\\
&+\left[\left(B_{mnij}\varepsilon_{mn}+C_{ijmn}\gamma_{mn}+C_{ij}\phi+\alpha_{ij}\theta\right)\dot{\varphi}_i\right]_{,j}\\
&+\left(A_{ij}\phi_{,i}\dot{\phi}\right)_{,j}+\left(K_{ij}\theta_{,i}\theta\right)_{,j}-K_{ij}\theta_{,i}\theta_{,j}+\tau\dot{\phi}^2\\
&+2\beta_{ij}\dot{\varepsilon}_{ij}\theta+2\alpha_{ij}\dot{\gamma}_{ij}\theta+2m\theta\dot{\phi}
\end{aligned}\tag{21}$$

Now we integrate in (21) over $[0,t]\times B$ and apply the divergence theorem. Given the boundary and the initial conditions, we are led to the desired result (19).

Third relation is obtained using the Lagrange identity method.

Proposition 3. *If $(u_i,\ \varphi_i,\ \psi,\ \theta)$ is a solution of the problem consists of system of equations (10), initial conditions (7) and the boundary conditions (11), then we have the following equality*

$$\begin{aligned}
&\int_B \left(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + J\dot{\phi}^2 - c\theta^2\right)dV\\
&= \int_B \Big(A_{ijmn}\varepsilon_{mn}\varepsilon_{ij} + 2B_{ijmn}\gamma_{mn}\varepsilon_{ij} + C_{ijmn}\gamma_{mn}\gamma_{ij}\\
&+ 2B_{ij}\varepsilon_{ij}\phi + 2C_{ij}\gamma_{ij}\phi + \xi\phi^2 + A_{ij}\phi_{,i}\phi_{,j}\Big)\,dV
\end{aligned} \tag{22}$$

Proof. Using the equation $(10)_1$ we have

$$\begin{aligned}
&\frac{\partial}{\partial s}\Big[\rho\dot{u}_i(s)\dot{u}_i(2t-s)\Big] = \rho\ddot{u}_i(s)\dot{u}_i(2t-s) - \rho\dot{u}_i(s)\ddot{u}_i(2t-s)\\
&= \left[\left(A_{ijmn}\varepsilon_{mn}(s) + B_{ijmn}\gamma_{mn}(s) + B_{ij}\phi(s) + \beta_{ij}\theta(s)\right)\dot{u}_i(2t-s)\right]_{,j}\\
&- \left(A_{ijmn}\varepsilon_{mn}(s) + B_{ijmn}\gamma_{mn}(s) + B_{ij}\phi(s) + \beta_{ij}\theta(s)\right)\dot{u}_{i,j}(2t-s)\\
&- \left[\left(A_{ijmn}\varepsilon_{mn}(2t-s) + B_{ijmn}\gamma_{mn}(2t-s) + B_{ij}\phi(2t-s) + \beta_{ij}\theta(2t-s)\right)\dot{u}_i(s)\right]_{,j}\\
&+ \left(A_{ijmn}\varepsilon_{mn}(2t-s) + B_{ijmn}\gamma_{mn}(2t-s) + B_{ij}\phi(2t-s) + \beta_{ij}\theta(2t-s)\right)\dot{u}_{i,j}(s)
\end{aligned} \tag{23}$$

With the help of the equation $(10)_2$ we deduce

$$\begin{aligned}
&\frac{\partial}{\partial s}\Big[I_{ij}\dot{\varphi}_j(s)\dot{\varphi}_i(2t-s)\Big] = I_{ij}\ddot{\varphi}_j(s)\dot{\varphi}_i(2t-s) - I_{ij}\dot{\varphi}_j(s)\ddot{\varphi}_i(2t-s)\\
&= \left[\left(B_{mnij}\varepsilon_{mn}(s) + C_{ijmn}\gamma_{mn}(s) + C_{ij}\phi(s) + \alpha_{ij}\theta(s)\right)\dot{\varphi}_i(2t-s)\right]_{,j}\\
&- \left(B_{mnij}\varepsilon_{mn}(s) + C_{ijmn}\gamma_{mn}(s) + C_{ij}\phi(s) + \alpha_{ij}\theta(s)\right)\dot{\varphi}_{i,j}(2t-s)\\
&+ \varepsilon_{ijk}\left(A_{jkmn}\varepsilon_{mn}(s) + B_{jkmn}\gamma_{mn}(s) + B_{jk}\phi(s) + \beta_{jk}\theta(s)\right)\dot{\varphi}_i(2t-s)\\
&- \left[\left(B_{mnij}\varepsilon_{mn}(2t-s) + C_{ijmn}\gamma_{mn}(2t-s) + C_{ij}\phi(2t-s) + \alpha_{ij}\theta(2t-s)\right)\dot{\varphi}_i(s)\right]_{,j}\\
&+ \left(B_{mnij}\varepsilon_{mn}(2t-s) + C_{ijmn}\gamma_{mn}(2t-s) + C_{ij}\phi(2t-s) + \alpha_{ij}\theta(2t-s)\right)\dot{\varphi}_{i,j}(s)\\
&- \varepsilon_{ijk}\left(A_{jkmn}\varepsilon_{mn}(2t-s) + B_{jkmn}\gamma_{mn}(2t-s) + B_{jk}\phi(2t-s) + \beta_{jk}\theta(2t-s)\right)\dot{\varphi}_i(s)
\end{aligned} \tag{24}$$

Using the equation $(10)_3$ we have

$$\begin{aligned}
&\frac{\partial}{\partial s}\left[J\dot{\phi}(s)\dot{\phi}(2t-s)\right]=J\ddot{\phi}(s)\dot{\phi}(2t-s)-\phi\dot{\phi}(s)\ddot{\phi}(2t-s)\\
&=\left[A_{ij}\phi_{,i}(s)\dot{\phi}(2t-s)\right]_{,j}-A_{ij}\phi_{,i}(s)\dot{\phi}_{,j}(2t-s)-B_{ij}\varepsilon_{ij}(s)\dot{\phi}(2t-s)\\
&-C_{ij}\gamma_{ij}(s)\dot{\phi}(2t-s)-\xi\phi(s)\dot{\phi}(2t-s)+m\theta(s)\dot{\phi}(2t-s)\\
&-\left[A_{ij}\phi_{,i}(2t-s)\dot{\phi}(s)\right]_{,j}+A_{ij}\phi_{,i}(2t-s)\dot{\phi}_{,j}(s)+B_{ij}\varepsilon_{ij}(s)\dot{\phi}(2t-s)\\
&+C_{ij}\gamma_{ij}(2t-s)\dot{\phi}(s)-\xi\phi(2t-s)\dot{\phi}(s)-m\theta(2t-s)\dot{\phi}(s)
\end{aligned}\tag{25}$$

Similarly, with the help of the equation $(10)_4$ we obtain

$$\begin{aligned}
&\frac{\partial}{\partial s}\left[c\theta(s)\theta(2t-s)\right]=c\dot{\theta}(s)\theta(2t-s)-c\theta(s)\dot{\theta}(2t-s)\\
&=-[K_{ij}\theta_{,i}(s)\theta(2t-s)]_{,j}+\beta_{ij}\dot{\varepsilon}_{ij}(s)\theta(2t-s)+\alpha_{ij}\dot{\gamma}_{ij}(s)\theta(2t-s)-m\dot{\theta}(s)\theta(2t-s)\\
&+[K_{ij}\theta_{,i}(2t-s)\theta(s)]_{,j}-\beta_{ij}\dot{\varepsilon}_{ij}(2t-s)\theta(s)-\alpha_{ij}\dot{\gamma}_{ij}(2t-s)\theta(s)+m\dot{\theta}(2t-s)\theta(s)
\end{aligned}\tag{26}$$

Using the equalities (22)-(25) and the geometric equations (5) we are lead to

$$\begin{aligned}
&\frac{\partial}{\partial s}\left[\rho\dot{u}_i(s)\dot{u}_i(2t-s)+I_{ij}\dot{\varphi}_j(s)\dot{\varphi}_i(2t-s)+J\dot{\phi}(s)\dot{\phi}(2t-s)-c\theta(s)\theta(2t-s)\right]\\
&=\left[\left(A_{ijmn}\varepsilon_{mn}(s)+B_{ijmn}\gamma_{mn}(s)+B_{ij}\phi(s)+\beta_{ij}\theta(s)\right)\dot{u}_i(2t-s)\right]_{,j}\\
&-\left[\left(A_{ijmn}\varepsilon_{mn}(2t-s)+B_{ijmn}\gamma_{mn}(2t-s)+B_{ij}\phi(2t-s)+\beta_{ij}\theta(2t-s)\right)\dot{u}_i(s)\right]_{,j}\\
&+\left[\left(B_{mnij}\varepsilon_{mn}(s)+C_{ijmn}\gamma_{mn}(s)+C_{ij}\phi(s)+\alpha_{ij}\theta(s)\right)\dot{\varphi}_i(2t-s)\right]_{,j}\\
&-\left[\left(B_{mnij}\varepsilon_{mn}(2t-s)+C_{ijmn}\gamma_{mn}(2t-s)+C_{ij}\phi(2t-s)+\alpha_{ij}\theta(2t-s)\right)\dot{\varphi}_i(s)\right]_{,j}\\
&+\left[A_{ij}\phi_{,i}(s)\dot{\phi}(2t-s)\right]_{,j}-\left[A_{ij}\phi_{,i}(2t-s)\dot{\phi}(s)\right]_{,j}\\
&+\left[K_{ij}\theta_{,i}(s)\theta(2t-s)\right]_{,j}-\left[K_{ij}\theta_{,i}(2t-s)\theta(s)\right]_{,j}\\
&+\frac{\partial}{\partial s}\left[A_{ijmn}\varepsilon_{ij}(s)\varepsilon_{mn}(2t-s)+B_{ijmn}\left(\varepsilon_{ij}(s)\gamma_{mn}(2t-s)+\varepsilon_{ij}(2t-s)\gamma_{mn}(s)\right)\right.\\
&\left.+C_{ijmn}\gamma_{ij}(s)\gamma_{mn}(2t-s)\right]+\frac{\partial}{\partial s}\left[B_{ij}\left(\varepsilon_{ij}(s)\phi(2t-s)+\varepsilon_{ij}(2t-s)\phi(s)\right)\right]\\
&+\frac{\partial}{\partial s}\left[C_{ij}\left(\gamma_{ij}(s)\phi(2t-s)+\gamma_{ij}(2t-s)\phi(s)\right)\right]+\frac{\partial}{\partial s}\left(\xi\phi(s)\phi(2t-s)\right)
\end{aligned}\tag{27}$$

Now we integrate in (27) over $[0,t]\times B$ and apply the divergence theorem. Given the boundary and the initial conditions, we are led to the desired result (22) such that Proposition 3 is now concluded.

5 Main result

In this section we will prove the main result of our paper, that is, it is not possible the localization of the solutions of the mixed initial boundary value problem in thermoelasticity of micropolar osies with voids. For this we use relations (12), (19) and (22) and, also, the result of the following proposition.

Proposition 4. *Let $(u_i,\ \varphi_i,\ \psi,\ \theta)$ be a solution of the mixed problem consists of system of equations (10), initial conditions (7) and the boundary conditions (11). If the standard assumptions (i)-(iv) are satisfied, then we have*

$u_i(x,t)=\varphi_i(x,t)=\phi(x,t)=\theta(x,t)=0,\ \forall(x,t)\in B\times[0,\infty).$

Proof. First of all, from (18) and (21) we deduce

$$\begin{aligned}&\int_B\Big(A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}+C_{ijmn}\gamma_{mn}\gamma_{ij}\\&+2B_{ij}\varepsilon_{ij}\phi+2C_{ij}\gamma_{ij}\phi+\xi\phi^2+A_{ij}\phi_{,i}\phi_{,j}\big)\,dV\\&=-\int_0^t\int_B\left(K_{ij}\theta_{,i}\theta_{,j}-2\beta_{ij}\dot\varepsilon_{ij}\theta-2\alpha_{ij}\dot\gamma_{ij}\theta-2m\theta\dot\phi-\tau\dot\phi^2\right)dV\end{aligned}\tag{28}$$

Closely related to relations (12) and (28) we define the functions

$$\begin{aligned}E_1(t)=&\frac12\int_B\left(\rho\dot u_i\dot u_i+I_{ij}\dot\varphi_i\dot\varphi_j+J\dot\phi^2+A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}\right.\\&\left.+C_{ijmn}\gamma_{mn}\gamma_{ij}+2B_{ij}\varepsilon_{ij}\phi+2C_{ij}\gamma_{ij}\phi+\xi\phi^2+A_{ij}\phi_{,i}\phi_{,j}+c\theta^2\right)dV\\E_2(t)=&\int_B\Big(A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}+C_{ijmn}\gamma_{mn}\gamma_{ij}\\&+2B_{ij}\varepsilon_{ij}\phi+2C_{ij}\gamma_{ij}\phi+\xi\phi^2\big)\,dV+\int_B A_{ij}\phi_{,i}\phi_{,j}dV\\E(t)=&E_2(t)+\varepsilon E_1(t)\end{aligned}\tag{29}$$

where ε is a small positive constant.
It is easy to deduce that

$$\begin{aligned}E(t)=&\frac12\int_B\varepsilon\left(\rho\dot u_i\dot u_i+I_{ij}\dot\varphi_i\dot\varphi_j+J\dot\phi^2+c\theta^2\right)\\&+(2+\varepsilon)\Big(A_{ijmn}\varepsilon_{mn}\varepsilon_{ij}+2B_{ijmn}\gamma_{mn}\varepsilon_{ij}+C_{ijmn}\gamma_{mn}\gamma_{ij}\\&+2B_{ij}\varepsilon_{ij}\phi+2C_{ij}\gamma_{ij}\phi+\xi\phi^2+A_{ij}\phi_{,i}\phi_{,j}\big)\,dV\end{aligned}\tag{30}$$

Since we can write $E(t)$ in the form

$$\begin{aligned}E(t)=&-(1-\varepsilon)\int_0^t\int_B K_{ij}\theta_{,i}\theta_{,j}\,dVds+2\int_0^t\int_B\Big(\beta_{ij}\dot\varepsilon_{ij}\theta+\alpha_{ij}\dot\gamma_{ij}\theta\Big)\,dVds\\&+2\int_0^t\int_B m\theta\dot\phi\,dVds+(1+\varepsilon)\int_0^t\int_B\tau\dot\phi^2dVds\end{aligned}$$

we obtain

$$\begin{aligned}\frac{dE(t)}{dt} = &-(1-\varepsilon)\int_B K_{ij}\theta_{,i}\theta_{,j}\,dV - 2\int_B\Big(\beta_{ij}\dot{\varepsilon}_{ij}\theta + \alpha_{ij}\dot{\gamma}_{ij}\theta\Big)\,dV \\ &+2\int_B m\theta\dot{\phi}\,dV + (1+\varepsilon)\int_B \tau\dot{\phi}^2 dV\end{aligned} \tag{31}$$

If we choose ε_1 sufficiently small, we can deduce

$$\begin{aligned}&\int_B\Big(\beta_{ij}\dot{\varepsilon}_{ij}\theta + \alpha_{ij}\dot{\gamma}_{ij}\theta\Big)\,dV \\ &\leq \varepsilon_1\int_B K_{ij}\theta_{,i}\theta_{,j}\,dV + K_1\int_B\left(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + c\theta^2\right)dV\end{aligned} \tag{32}$$

where the positive constant K_1 can be calculated in terms of constitutive coefficients and ε_1.
Also, we can compute a positive constant K_2 such that

$$\int_B m\theta\dot{\phi}\,dV \leq K_2\int_B\left(J\dot{\phi}^2 + c\theta^2\right)dV \tag{33}$$

With the help of the relations (31) to (33) we deduce that if we choose $\varepsilon_1 \leq 1-\varepsilon$, there exists a positive constant C such that

$$\frac{dE(t)}{dt} \leq C\int_B\left(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + J\dot{\phi}^2 + c\theta^2\right)dV \tag{34}$$

If we take into account the inequalities (32) to (34) we obtain the estimate

$$\frac{dE(t)}{dt} \leq C^*E(t) \tag{35}$$

which is satisfied for every $t \geq 0$.
In the inequality (35) C^* is a positive constant which can be calculated taking into account the above inequalities.
Of course, from (35) we deduce

$$E(t) \leq E(0)e^{C^*t}, \quad \forall\, t \geq 0$$

But the initial conditions, (7), were presumed null, therefore we have

$$E(t) \equiv 0, \quad \forall\, t \geq 0$$

Taking into account the definition (29) of function $E(t)$, it follows that

$$\dot{u}_i \equiv 0, \quad \dot{\varphi}_i \equiv 0, \quad \dot{\phi} \equiv 0, \quad \theta \equiv 0, \quad \forall\, t \geq 0$$

Considering again the null initial conditions, we deduce that our mixed problem has only null solution and the proof of Proposition 4 is concluded.

Let us consider the mixed initial boundary value problem consists of the equations (1) to (5), the homogeneous boundary conditions (11) and the initial conditions (7). We want to show the impossibility of localization in time of solutions of this problem. This is equivalent to show that the only solution for this problem that vanishes after a finite time is the null solution.

Theorem 1. *Suppose the standard assumptions (i)-(iv) and the symmetry relations (6) are satisfied. Consider* $(u_i,\ \varphi_i,\ \phi,\ \theta)$ *a solution of the mixed problem, previous defined, that vanishes after a finite time* $t_1 \geq 0$*, that is*

$$u_i = \varphi_i = \phi = \theta \equiv 0, \quad \forall\, t \geq t_1.$$

then

$$u_i = \varphi_i = \phi = \theta \equiv 0, \quad \forall\, t \geq 0.$$

Proof. To this aim we consider the corresponding backward in time problem in the time interval $(-\infty, t_1]$, consists of the equations (1) to (5), the homogeneous boundary conditions (11) and the following null final conditions

$$u_i(x,t_1) = 0,\ \dot{u}_i(x,t_1) = 0,\ \varphi_i(x,t_1) = 0,\ \dot{\varphi}_i(x,t_1) = 0,$$

$$\phi(x,t_1) = 0,\ \dot{\phi}(x,t_1) = 0,\ \theta(x,t_1) = 0,\ x \in B$$

Taking into account Proposition 4, this problem has only the null solution.

6 Conclusion

The main result of our study prove the uniqueness of the solution for the backward in time problem in the context of thermoelasticity of micropolar bodies with voids. This means that the only solution to the backward in time problem that vanishes for every $t \geq t_1 > 0$ is the null solution. In other words, we have shown the impossibility of localization in time of the solutions of the mixed intial boundary value problem for thermoelastic micropolar bodies.

Thus, the combination of the micropolar structure with the thermal and porous dissipation is not so strong to ensure that the mechanical deformations vanish a finite time.

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